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Image blur for rainbow holograms

James C. Wyant

Optical Sciences Center, University of Arizona, Tucson, Arizona 85721
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An analysis is presented for selecting the important rainbow-hologram formation-setup parameters for minimization of the image blur.

Introduction

The rainbow hologram invented by Benton has the property of producing excellent images even when illuminated with an extended white-light source.[1-3] Although the image produced by a rainbow hologram is rather sharp, image blur does exist, and the purpose of this Letter is to compare the image blur resulting from the spectral-wavelength spread present in the image with the image blur resulting from finite source size and diffraction. The results of the analysis can be used to determine the optimum experimental setup parameters for minimization of image blur.

The basic rainbow hologram is made by using a two-step process. First, a conventional hologram is recorded. In the reconstruction step this primary hologram is illuminated with the conjugate of the reference wavefront used in recording the hologram to produce a pseudoscopic real image of the original object. A narrow horizontal slit is placed over the primary hologram, as shown in Fig. 1, so only a small range of angles of light rays in the vertical direction is used to form the image. A second hologram is recorded in a plane near the position of the image produced by the primary hologram. The reference wave used in recording the second hologram should have the largest component of its propagation vector parallel to the y-z plane shown in Fig. 1. In the reconstruction stage for the second hologram, the hologram is illuminated with the conjugate of the reference wave used in recording the hologram. The image produced is thus pseudoscopic; however, since the object used in recording the hologram was pseudoscopic, the final image is orthoscopic. In addition to an image of the original object, a real image is formed of the slit placed on the primary hologram. If the viewer's eyes are placed in the position where the image of the slit is formed, using the exposing wavelength, the image will appear to be the same color as the wavelength used to expose the hologram, even if a white-light source is used in the reconstruction step. If the viewer moves his head up or down, the color of the image will change if a white-light source is used in the reconstruction step. If the viewer places his eyes in a plane other than the plane in which the image of the slit is formed, the color of the image varies across the image.

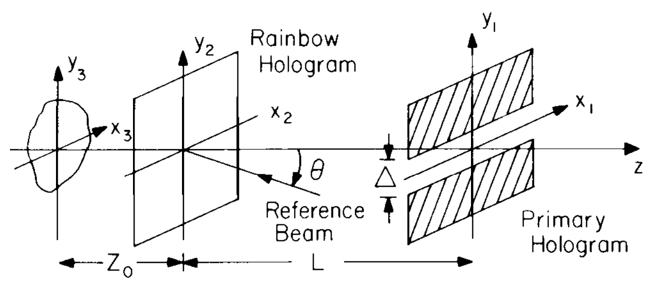

Fig. 1. Rainbow-hologram formation setup. The primary hologram is illuminated with the conjugate of the reference wavefront used in exposing the hologram. A slit of width A is placed over the primary hologram. The image produced by the primary hologram is formed a distance 20 to the left of the rainbow hologram.

Wavelength Spread in Image

Although the image appears sharp if the hologram is made properly, there is some image blur present, especially if a white-light extended source is used in the reconstruction process. As an example of the image blur resulting from a finite spectral-wavelength spread, consider a hologram made by using the setup shown in Fig. 1, where a slit of width A is placed over the primary hologram. If the second hologram is recorded using wavelength λ_0, then

$$\hat{\imath}\sin\alpha + \hat{\jmath}(\sin\beta + \sin\theta) = \lambda_0(\hat{\imath}\nu_x + \hat{\jmath}\nu_y), \qquad (1)$$

where ν_x and ν_y are the x and y components, respectively, of the spatial frequencies of the fringes making up the rainbow hologram. α and β are angles used to determine the direction cosines of the rays from the primary hologram to the rainbow hologram. θ is the reference-beam angle.

If, in the reconstruction step, the rainbow hologram is illuminated with the conjugate of the original reference wave at wavelength λ, then

$$\hat{\imath}\sin\alpha' + \hat{\jmath}(\sin\beta' + \sin\theta) = \lambda(\hat{\imath}\nu_x + \hat{\imath}\nu_y), \qquad (2)$$

where α' and β' are the angles used to determine the direction cosines of the diffracted rays.

Combining Eqs. (1) and (2) and writing the $\hat{\imath}$ and j components separately yields

$$\sin \alpha' = \frac{\lambda}{\lambda_0} \sin \alpha, \tag{3a}$$

$$\sin \beta' + \sin \theta = \frac{\lambda}{\lambda_0} (\sin \beta + \sin \theta). \tag{3b}$$

Taking derivatives to determine the range of the angles of the diffracted rays yields

$$\partial \alpha' = \frac{\partial \lambda}{\lambda_0} \frac{\sin \alpha}{\cos \alpha'} + \frac{\lambda \cos \alpha \partial \alpha}{\lambda_0 \cos \alpha'}, \tag{4a}$$

$$\partial \beta' = \frac{\partial \lambda}{\lambda_0} \frac{\sin \beta + \sin \theta}{\cos \beta'} + \frac{\lambda}{\lambda_0} \frac{\cos \beta \partial \beta}{\cos \beta'}. \tag{4b}$$

In taking the derivatives it is necessary to remember that α and β consisted of a range of values in recording the hologram.

In the reconstruction step, let the person viewing the image place his eyes in the position where the image of the slit is formed using wavelength λ_0. Let D be the eye pupil diameter and L be the distance between the primary and rainbow holograms. As long as θ is much larger than α, the predominant effect of changing the wavelength is to move the image of the slit in the plus or minus y direction with only a small motion in the x direction. Thus, in calculating the wavelength spread observed by the eye, only dispersion in the β' direction need by considered. Since $-\Delta/2L \leq \beta \leq \Delta/2L$, $-D/2L \leq \beta' \leq D/2L$, $\sin \beta \cong 0$, $\cos \beta \cong 1$, $\cos \beta' \cong 1$, and $\lambda \cong \lambda_0$, it follows from Eq. (4b) that the maximum wavelength spread, $\delta\lambda_{max}$, observed by the eye is given by

$$\partial \lambda|_{\max} \sim \frac{\lambda}{\sin \theta} \left(\frac{D}{L} + \frac{\Delta}{L} \right). \tag{5}$$

Equation (5) shows that it is the ratio of the angular subtense of the slit and the eye pupil, as measured from the hologram, to the sine of the reference-beam angle that determines the observed-wavelength spread present in the image. As an example, if $\lambda = 500$ nm, $\theta = 45°$, L = 30 cm, and D = Δ = 3 mm, then $\delta\lambda_{max} = 14$ nm.

Image Blur Due to Wavelength Spread

Equation (4b) can also be used to calculate the image blur resulting from the wavelength spread present in the image. Since we are now calculating the blur as a function of wavelength spread passing through the slit, $\delta\beta$ can be set equal to zero. Using the same approximations as given above, the blur $\delta_{\delta\lambda}$ is given by

$$\delta_{\partial \lambda} = Z_0 \partial \beta' = Z_0 \frac{\partial \lambda}{\lambda} \sin \theta \tag{6}$$

or

$$\delta_{\partial \lambda} = Z_0 \left(\frac{D + \Delta}{L} \right), \tag{7}$$

where Z_0 is the hologram-to-image distance.

As with all holograms, as the hologram-to-image distance goes to zero, the image blur due to wavelength spread also goes to zero. For the rainbow hologram, the image blur is equal to the image-to-rainbow hologram distance times the sum of the angular subtense of the slit used in making the rainbow hologram and the pupil used in viewing the rainbow-hologram image. For the parameters given above, and an image-to-rainbow hologram distance of 5 cm, the image blur is approximately 1 mm, which corresponds to an acceptable angular blur of about 3 mrad as measured from the eye pupil.

Image Blur Due to Source Size

Additional image blur results if an extended source, such as the sun, is used to illuminate the rainbow hologram. The use of an extended light source causes an angular blur of the image, as viewed from the hologram, of an amount approximately equal to the angular spread of the source, as viewed from the hologram. Thus, the approximate blur, δ_s, resulting from using a source of angular subtense Ω_s, is given by

$$\delta_s = \Omega_s Z_0. \tag{8}$$

It follows from Eqs. (7) and (8) that, if the blur due to wavelength spread is approximately equal to the blur due to source size,

$$\Omega_s = \frac{D}{L} + \frac{\Delta}{L}. \tag{9}$$

That is, the finite source size introduces less image blur than the slit-width and viewing-pupil size if the angular subtense of the source is smaller than the sum of the angular subtense of the slit and the viewing pupil, as measured from the hologram plane.

Image Blur Due to Diffraction

As a final source of image blur, consider the blur due to diffraction, which results from both the finite slit size and the eye pupil diameter. Since only an approximate answer is required, let us assume that the slit width and pupil diameter are equal and furthermore that the slit is imaged onto the eye pupil, so only the diffraction by a single aperture need be considered. The image blur, δ_D, due to diffraction can be approximated as

$$\delta_D = \frac{2\lambda(Z_0 + L)}{\Delta}. \tag{10}$$

The slit width Δ, for which the blur due to diffraction is equal to the blur due to wavelength spread, is found from Eqs. (7) and (10) to be given by

$$\Delta = \left(\frac{\lambda(Z_0 + L)L}{Z_0} \right)^{1/2}. \tag{11}$$

As an example, let $\lambda = 500$ nm, $L = 30$ cm, and $Z_0 = 5$ cm; then $\Delta = 1$ mm. For this example, which should be typical, as long as the slit width is greater than 1 mm, image blur due to the wavelength spread is larger than the image blur due to diffraction.

Conclusions

In the making of rainbow holograms as well as conventional holograms, the image blur due to spectral bandwidth can be reduced by reducing the image-to-hologram distance. If the hologram-to-image distance must be reasonably large, the image blur for a rainbow hologram can be reduced to acceptable values by reducing the angular subtense, as measured from the hologram, of the slit and pupil used in recording and viewing the hologram, respectively. As long as the angular subtense of the illuminating source is smaller than the sum of the angular subtense of the slit and the viewing pupil, again as measured from the hologram plane, the finite source size introduces less image blur than the wavelength spread. Although image blur due to wavelength spread can always be reduced by decreasing the slit width and the viewing pupil diameter, if they are reduced too much, image blur due to diffraction is no longer negligible.

References

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