**Optical resolution of phase measurements of laser Fizeau interferometer**

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## ABSTRACT

Accurate interferometric measurement of large laser slabs requires spatial frequencies of 1mm/cycle to 33mm/cycle over a 100mm field of view to be passed by the system with no more than 25% loss in modulation. To eliminate noise and artifacts due to strictly coherent imaging, many commercial interferometers employ a rotating diffuser on an intermediate image plane and relay this image incoherently onto a detector. Unfortunately, this process may adversely affect the resolution of the instrument. Through measurement of a sinusoidal phase grating and fused silica step, the transfer function a laser Fizeau interferometer was measured for both a system with and without the incoherent relay system. Results are compared to those predicted by diffraction theory. Studies of the effects of defocus and propagation on the measurement were also made. Using strictly coherent imaging dramatically increases the system's ability to measure features of high spatial frequency and allows the measurement requirements for laser slabs to be met.

**Keywords:** Transfer Function, Interferometry, Resolution, Fresnel Propagation, Phase Grating, Power Spectral Density

## 1. INTRODUCTION

The optics in an interferometer have a dual function: to produce a collimated beam and to image the sample to the detector. Typically, commercial interferometers employ a rotating diffuser in an intermediate image plane and relay this image to the detector, often with a zoom lens to allow parts of various sizes to be easily measured. The system is therefore a combination of a coherent and incoherent system. The system transfer function is the product of the coherent transfer function of the interferometer, the incoherent transfer function associated with the ground glass and zoom system, and the detector.

High power laser systems require that the system optics be free from any periodic structure in order to avoid focusing effects within the system.[1] Thus, while typical laser Fizeau interferometric measurements are primarily concerned with low frequency figure errors, measurement of laser slabs requires accurate measurement of much higher spatial frequencies. Any increase in the attenuation of higher spatial frequencies caused by the incoherent relay system becomes critical. The first part of this paper will analyze the transfer function of a WYKO 6000 laser Fizeau interferometer both with and without the rotating diffuser and zoom system. After this, the effect of defocus and propagation on the measurements will be discussed.

## 2. SYSTEM CHARACTERIZATION

The first method used to determine the system transfer function uses three beam interference to create an effective sinusoidal phase grating. Figure 1 illustrates the setup both with and without the incoherent relay system. The AR coated side of the third transmission flat is aligned such that there is a very small wedge between it and the second transmission flat. Considering the last two flats, standard two beam interference equations give the phase of the combination as[2]:

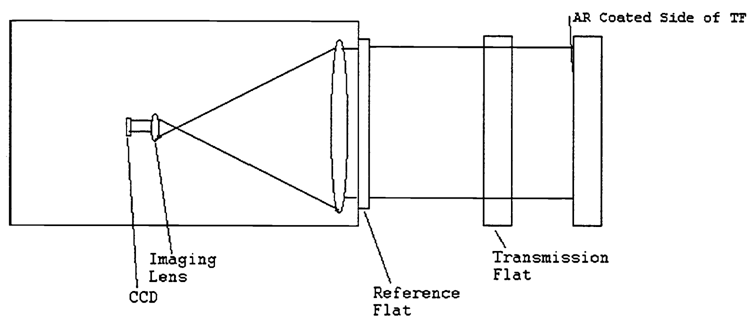
$$\Phi = \arctan\left[\frac{A_1 * \sin(\frac{2\pi}{\lambda}\sin(\theta_1) * x) + A_2 * \sin(\frac{2\pi}{\lambda}\sin(\theta_2) * x)}{A_1 * \cos(\frac{2\pi}{\lambda}\sin(\theta_1) * x) + A_2 * \cos(\frac{2\pi}{\lambda}\sin(\theta_2) * x)}\right] \tag{1}$$

where $A_1$ and $A_2$ are the amplitude reflectances of the two beams, $\lambda$ is the wavelength, and $\theta_1$ and $\theta_2$ are the angles the two beams make with respect to the coordinate system. If the coordinates are defined such that $\theta_1 = 0$ and $\theta_2 = \theta$ is the difference in angles between the two beams, one gets:

$$\Phi = \arctan\left[\frac{A_2 * \sin(\frac{2\pi}{\lambda}\sin(\theta) * x)}{A_1 + A_2 * \cos(\frac{2\pi}{\lambda}\sin(\theta) * x)}\right] \tag{2}$$

Provided that the difference between $A_1$ and $A_2$ is at least a factor of two, $\Phi$ is approximately sinusoidal. For the setup in Figure 1, $A_1$ and $A_2$ were .20 and .05 so this condition is met. Thus an effective sinusoidal phase grating is presented to the interferometer for measurement. Typical phase shifting techniques are used to measure this effective grating. By varying the tilt of the AR coated flat, the period of the phase grating may be modified over a broad range while the amplitude of the phase grating remains constant.

**Schematic Configuration for Coherent Imaging of Phase Grating**




**Schematic Configuration for Incoherent Imaging of Phase Grating**

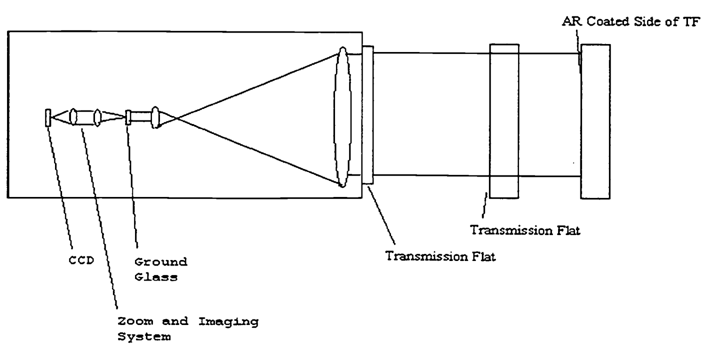



Figure 1 - Strictly Coherent Imaging System Configuration for Effective Phase Grating and Configuration with Ground Glass and Zoom System.

In order to determine the transfer function of the system, measurements were taken over a range of approximately .1 lines/mm to 2 lines/mm over a 100 mm field of view. A digital CCD camera with 740 pixels in the direction of interest was used for data acquisition for this and all measurements described in this paper. The Nyquist Frequency was therefore 3.7 lines/mm. After each measurement, the power spectral density of the data was calculated using the program Phanaly, written by Janice Lawson. While theoretically the PSD should show a single spike located at the spatial frequency of the sine wave[3], in practice this spike was somewhat broadened due to system noise and was superimposed on a background caused by noise, imperfections in the surface, and finite Fourier transform effects. A typical PSD plot is shown in Figure 2. The area under the spike due to the sinusoid was calculated for each spatial frequency measured. Results were normalized to the maximum value obtained, which occurs at the lowest spatial frequency, as expected. Data was taken with both the incoherent relay system and strictly coherent configuration.

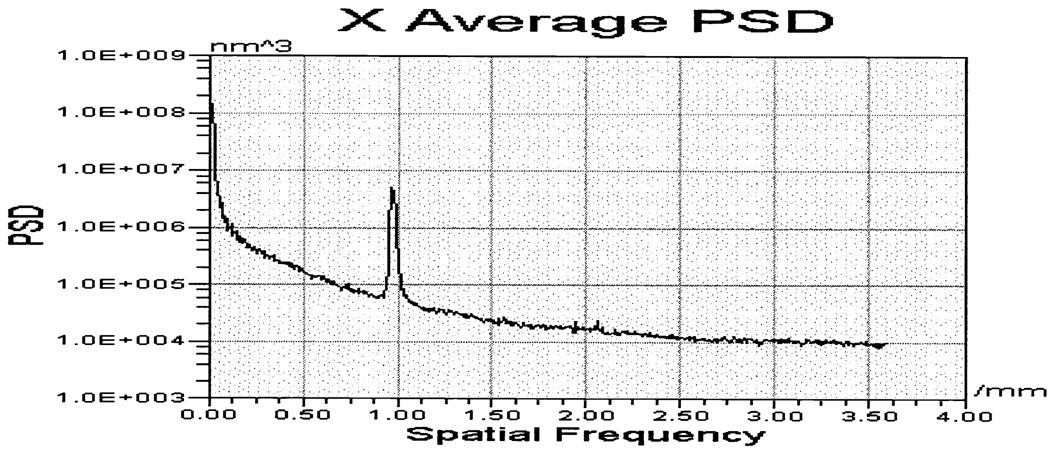



Figure 2: Typical PSD Output from Phase Grating Measurement. Spike is Superimposed on a Background Caused by Noise, Imperfect Flats, and FFT effects.

Results of the measurements are illustrated in Figure 3. It can clearly be seen that the incoherent imaging system drastically reduces the transfer function of the system. If one defines an acceptable transfer function as passing a given spatial frequency with better than 75% modulation, then the coherent system has a cutoff frequency of 1.5 lines/mm while the incoherent system has a cutoff frequency of .35 lines/mm. It should be noted that the coherent transfer function is not a rectangle function as predicted by theory due primarily to the camera employed by the system; when fringes are projected directly onto the camera and the modulation measured, the falloff with spatial frequency is nearly identical to the coherent configuration results here presented.

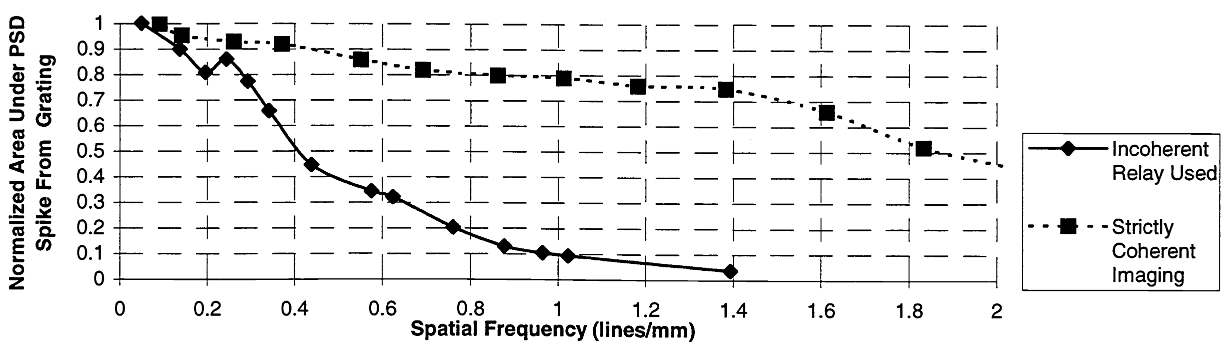



Figure 3: System transfer function determined by comparing the area under the PSD spike for sine gratings of varying frequency. The strictly coherent system has far better performance than when the incoherent relay system is used.

To further confirm the results achieved above, measurements were taken on a fused silica step measured in reflection with the system well-focused on the step. The PSD of a step of height H, measured with N data points over distance L is given by[4]:

$$S_1(f_m) = \frac{2}{L} * W * H^2 * \frac{1}{f_m^2} \tag{3}$$

where W is a weighting factor which depends on the type of FFT window function used on the data.

A single step thus contains all spatial frequencies. Comparison of the PSD of a measured step to the theoretical PSD will allow the system transfer function to be calculated at all spatial frequencies with one measurement. This procedure was followed for both the incoherent and coherent imaging configurations. Measurements of the actual step were taken and analyzed and compared with those obtained for a mathematically generated step of the same height run through the same analysis procedure.

Results are presented in Figure 4. The plot was normalized to a spatial frequency of .3 lines/mm for the strictly coherent imaging case and to .2 lines/mm for the case with the incoherent relay system. This normalization is somewhat arbitrary, but noise in the system causes ripples in the calculated transfer function. For the coherent imaging case, normalization at this frequency gives a transfer function close to one at lower spatial frequencies, as expected, and only one area, most likely due to coherent noise, where the transfer function is above its allowed maximum of one. For the case with the incoherent relay system, normalization at that value gives a steadily decreasing transfer function as one would expect.

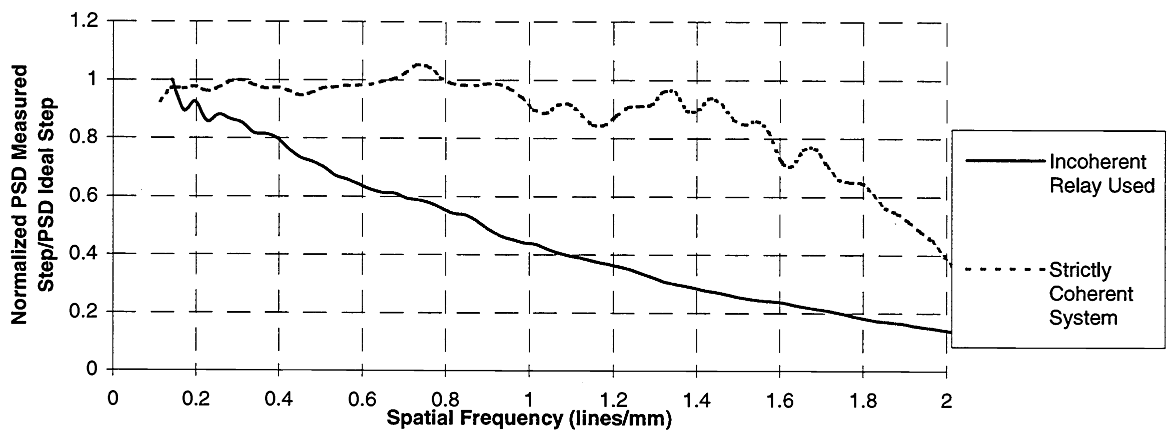



Figure 4: System transfer function determined from dividing the measured PSD of a fused silica step by the PSD of an ideal step.

The elimination of the incoherent relay system again improves the ability to measure higher spatial frequencies. The coherent system results are, however, noisier than the incoherent results due to dust, spurious reflections, and other problems typically associated with purely coherent imaging; If the three flat measurements were taken over a continuum of spatial frequencies results would most likely also be noisy for the strictly coherent configuration. Though both the sinusoidal grating and step measurements show a great difference between the coherent and incoherent transfer functions, the transfer functions for the step measurement are better than for the three flat measurements. Using the same criteria as above we now have cutoff frequencies of 1.75 lines/mm for the coherent imaging case and .4 lines/mm for with the incoherent relay system.

The difference in the transfer functions between the two measurement methods could have several causes. Only a small area near the edge is critical for PSD calculations involving the step, while the sinusoidal grating measurement uses the entire field. Thus aberrations could play a greater role in measurements involving the grating. Furthermore, sampling problems could cause the step to appear steeper in the measurement than might be the case with infinite sampling near the edge, effectively increasing the calculated transfer function. Finally, the fact that there is some structure and noise associated with the step makes normalization to the ideal step value somewhat arbitrary, as indicated earlier. Further study must be made to determine exactly why differences in the two techniques arise.

## 3. DEFOCUS AND PROPAGATION EFFECTS

After the above measurements were taken to determine system characteristics, the effects of defocus and propagation were studied. Some of the optics in large laser systems are used at Brewster's angle and it is thus important to also test them at this angle. For a test area of 100 square mm, mounting considerations, tilt of the part, and the large size of the test piece require that parts of the wavefront will propagate at least 20cm from the part to the return flat. Other parts of the beam will travel only a short distance to the return flat and back.. This means that any variations in the measurement due both to the defocus of the test part and Fresnel propagation of the wavefront must be characterized.

Since all functions may be broken down into sums of appropriately scaled sine waves, a consideration of the propagation of a sinusoidal phase disturbance is in order. A uniform, normalized beam passing through a sinusoidal phase grating may be represented by the following formula[5]:

$$u_i(x) = \exp\left[ j\frac{m}{2}\sin(2\pi f_o x)\right] \tag{4}$$

In the above m represents the peak to valley extent of the phase delay and $f_o$ is the spatial frequency of the grating. Provided that m is sufficiently small (less than 0.1), we may approximate the above as

$$u_i(x) \approx J_o(m/2) + J_1(m/2)\exp(j2\pi f_o x) + J_{-1}(m/2)\exp(-j2\pi f_o x) \tag{5}$$

The Fourier transform of this is:

$$U_i(f) = J_o(m/2)\delta(f) + J_1(m/2)\delta(f - f_o) + J_{-1}(m/2)\delta(f + f_o) \tag{6}$$

Finally, under propagation in the Fresnel regime we get in our final observation plane:

$$\begin{aligned} U_f(f) &= \exp(jkz)\exp(-j\pi\lambda z f^2) * T_i(f) \\ U_f(f) &= \exp(jkz)\begin{bmatrix} J_o(m/2)\delta(f) + J_1(m/2)\exp(-j\pi\lambda z f_o^2)\delta(f - f_o) \\ + J_{-1}(m/2)\exp(-j\pi\lambda z f_o^2)\delta(f + f_o) \end{bmatrix} \end{aligned} \tag{7}$$

which gives:

$$u_f(x) = \exp(jkz) * \begin{bmatrix} J_o(m/2) + \exp(-j\pi\lambda z f_o^2)[J_1(m/2)\exp(j2\pi f_o x) + \\ J_{-1}(m/2)\exp(-j2\pi f_o x)] \end{bmatrix} \tag{8}$$

This distribution is the same as $u_i(x)$ except for the frequency dependent phase terms multiplying the last two terms. This phase term causes the expression for $u_f(x)$ to represent a sinusoidal phase grating only when that term is real. When the frequency dependent phase term is strictly imaginary the distribution is that of a sinusoidal amplitude grating, and a combination of the two at other points.

When measuring such a grating, the distance the wavefront travels is critical. In reflection, if the grating is defocused, certain frequencies will be poorly measured or not measured at all due to this effect. In transmission, where the beam passes through the test piece twice, different positions of the return flat will cause different effects. The final beam entering the interferometer may have twice the peak to valley phase extent as expected, no phase modulation (if the reflected beam is exactly out of phase with the part), or any value in between. Figure 5 illustrates the theoretical peak to valley phase variation of a sinusoidal grating under different amounts of propagation. For a round trip propagation distance of 80cm, the 1 line/mm grating would have almost no phase modulation upon returning to the test object. The interferometer would see phase variation only of the second pass through the grating. Thus the measurement would be in error by a factor of two if this were not taken into account.

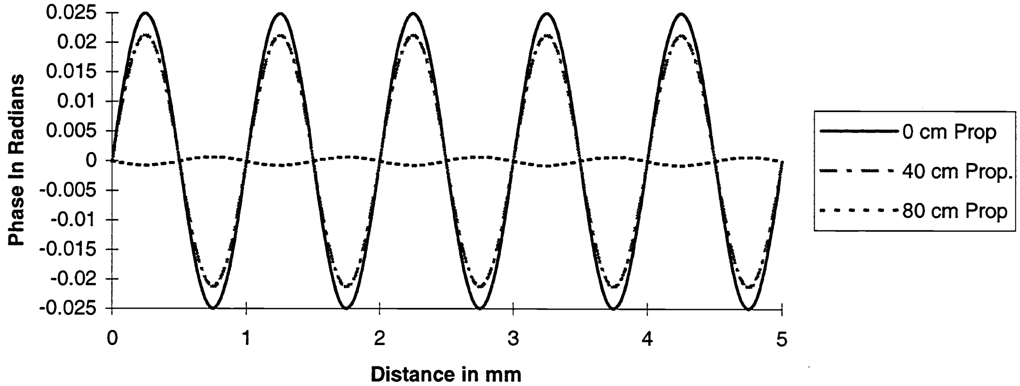



Figure 5: Theoretical phase variation of a sinusoidal phase disturbance under different amounts of propagation. Wavelength was taken to be 632.8 nm.

More complex phase objects will suffer similar problems upon propagation of the beam over large distances, since they may be constructed by summing many sinusoidal phase disturbances of varying frequencies and amplitudes. To study propagation and defocus effects, the fused silica step was used. A similar procedure was followed as for the initial transfer function characterization. The step was oriented in the vertical direction. The measurements were again evaluated by taking the PSD of the measurement, dividing by the PSD of an ideal step and then normalizing the plots.

Figure 6 shows how the profile of a step measured in reflection changes with defocus of the step for the strictly coherent system. The in-focus measurement shows a steep step with no noticeable ringing near the edge. The out-of-focus measurement shows a less steep step, with ringing near the edge due to the loss of some of the higher spatial frequencies.

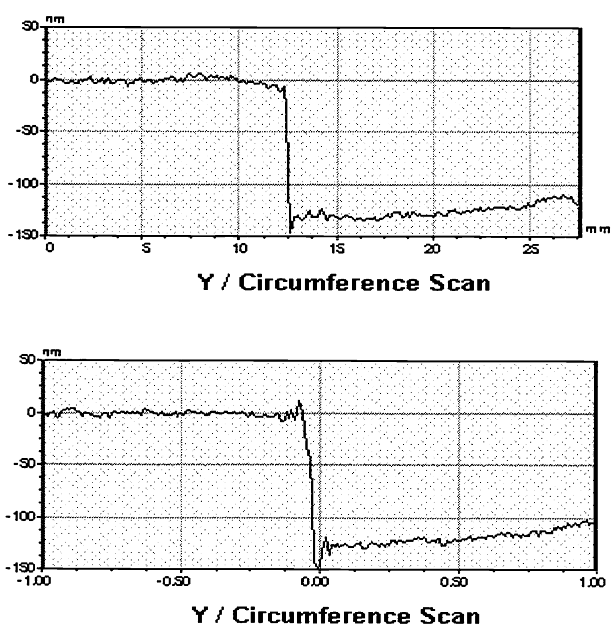



Figure 6: Profile of silica step measured in focus (top) and with step 30cm from interferometer focus (bottom). Step does not appear as sharp and exhibits ringing near the edge when not in focus.

Figure 7 shows the effect of moving the silica step from the focal plane of the interferometer on the calculated transfer function of the strictly coherent imaging system. As expected, as the step is moved farther from focus, the transfer function is reduced for higher spatial frequencies. The plot shown is from measurements of the step in reflection. Similar plots may be obtained by measuring the step in transmission and moving the step away from the return flat. The plot demonstrates that in order to maintain a transfer function greater than 75% at 1 line/mm, the sample may not be displaced more than about 20cm from the focal plane of the interferometer. Thus, to achieve the desired resolution on measurements of large laser slabs, the return flat must be as close to the test part as possible.

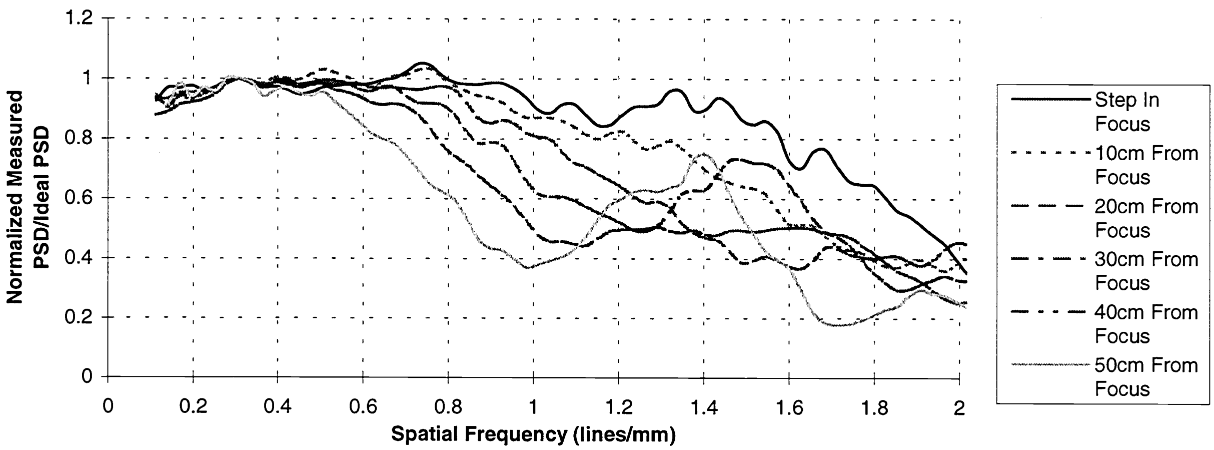



Figure 7: Transfer function of strictly coherent imaging configuration as a function of step location. Measurements were taken in reflection. Greater defocus causes the transfer function to fall off more severely at higher spatial frequencies.

## 4. CONCLUSIONS

Accurate measurement of laser slabs requires that the system be able to measure features of 1 line/mm with 75% of their true modulation. To achieve this goal, the ground glass and zoom system typically employed in phase shifting interferometers cannot be used. While strictly coherent imaging creates added difficulty due to spurious fringes from dust and stray reflections, the increase in the system transfer function is significant at higher frequencies. Although the two techniques used to calculate the transfer functions give different results at high spatial frequencies, both techniques show unacceptable transfer of high spatial frequencies with the incoherent relay system. Further study must still be done to determine why the two methods of transfer function calculation give different results.

Since some of the large laser slabs are to be tested at Brewster's angle, defocus and propagation effects on the measurement were studied. It was seen that in order to pass higher spatial frequencies with the required fidelity, the distance from the test piece to the focal plane of the interferometer had to be kept as small as possible. The system transfer function was found to be acceptable up to about 20cm distance between part and focus position. While the decrease in transfer function with added defocus is expected, further study must be done to determine whether diffraction theory alone predicts the observed results or if other factors influence the measurement as well. More phase objects will also hopefully be studied to determine whether the decrease in transfer function is object dependent.

## 5. Acknowledgments

The authors wish to thank Lawrence Livermore National Laboratories for providing the fused silica step and the computer program Phanaly which greatly assisted this research. The authors also wish to thank Bob Knowlden and the other employees of WYKO who assisted with the research and writing of this paper.

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