Optics 513 Exam #1

October 16, 2012
Closed Book Exam
Calculate the quantities asked for, don't just give equations.

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1) (10 Pts)

The reciprocal magnification technique is used to measure the focal length of a thick lens. The magnitude of the magnification for the first lens position is 2. The lens is moved a distance of 5 cm between the two positions used in the reciprocal magnification measurement. What is the focal length of the lens?

▪ Solution

Let p be distance from object to first principal plane for position 1. q is the distance from the second principal plane to the image for position 2.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

From the magnification we know that

$q = 2\ p$

$q - p = 5$

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Solve[{q == 2 p, q == p + 5}, {p, q}]
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{{p → 5, q → 10}}
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$$f = \frac{1}{\frac{1}{5} + \frac{1}{10}}\ cm = 3.33\ cm$$

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2) (10 Pts)

An Abbe refractometer having a reference prism of refractive index 1.6 is used to measure the refractive index of a solid sample. During the measurement the maximum angle the light is from the normal in the reference prism is 76°.

a) What is the refractive index of the solid sample being measured?
b) What is the minimum refractive index of the index matching fluid that can be used for this test?

▪ Solution

▪ a)

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n = 1.6 Sin[76 °]
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1.55247
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▪ b)

The refractive index of the index matching fluid must be greater than the index of the sample being measured. Therefore, the index must be greater than 1.55247.

3) (30 Pts)

A table of Zernike polynomials is attached to aid in answering this question.

a) (5 Pts) An optical system has aberration of the form $2\lambda\, \rho^2 x$, where $-1 \leq x \leq 1$. What is the optimum amount of tilt (magnitude and direction) to maximize the Strehl ratio?

b) (10 Pts) A wavefront is measured to have an aberration of the form $2\rho^2 + 4\rho^4$. Rewrite this wavefront in terms of three Zernike terms that are orthogonal in a continuous fashion over the interior of a unit circle.

c) (5 Pts) At a wavelength of 633 nm a wavefront gives a Strehl ratio of 0.9. Give the approximate rms of the wavefront in units of microns.

d) (10 Pts) I want to maximize the Strehl ratio by balancing third-order and fifth-order spherical aberrations with defocus. What are the optimum amounts of third-order spherical and defocus if I have 8λ of fifth-order spherical?

- **Solution**
- **a)**

The Zernike of interest is $(3\rho^2 - 2)$ x. Therefore to minimize the rms and maximize the Strehl ratio we need

$-\frac{4}{3}\lambda$ of x - tilt

- **b)**

$$2\rho^2 + 4\rho^4 = \frac{2}{3}(6\rho^4 - 6\rho^2 + 1) + 3(2\rho^2 - 1) + \frac{7}{3}(1)$$

- **c)**

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ans = Solve[0.9 == e^(-rms^2), rms]; rms (0.633 μm)/(2 π) /. ans[[2]]
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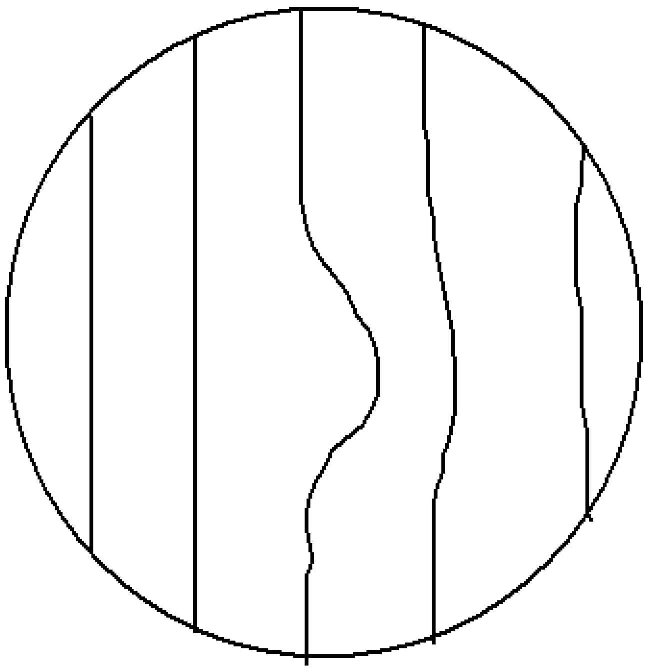
0.0327011 μm

- **d)**

We have 8 waves of fifth-order spherical. Therefore, we need $\frac{8}{20}(-30) = -12$ waves of third-order spherical and $\frac{8}{20}(12) = \frac{24}{5}$ waves of defocus.

4) (25 Pts)

The following interferogram was obtained using a two beam interferometer to test a sample at a wavelength of 500 nm.



If the sample being tested is a flat mirror, what is maximum departure from flatness in units of nm if

a) (5 Pts) the sample is tested at normal incidence in a Twyman-Green interferometer?

b) (5 Pts) the sample is tested at normal incidence in a laser-based Fizeau interferometer?

c) (10 Pts) the sample is tested using a Mach-Zehnder interferometer where the sample is used as one of the mirrors in the interferometer? Sketch the interferometer setup.

If the sample being tested is a plane parallel plate of refractive index 1.5,what is the maximum thickness variation if

d) (5 Pts) the sample is tested in a Mach-Zehnder interferometer?

Solution

a)

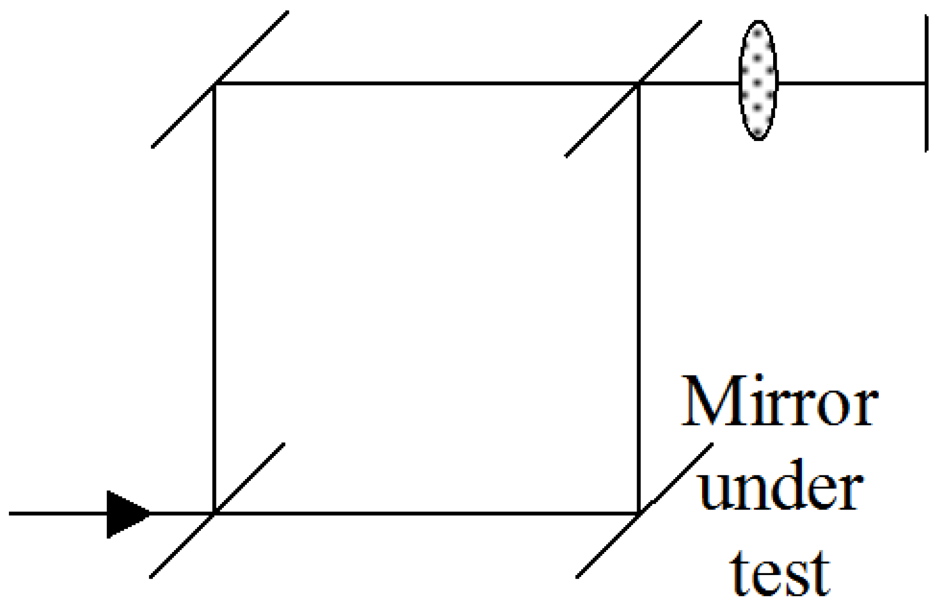
$$\frac{\lambda}{2}\frac{\Delta}{s} = \frac{500\text{ nm}}{2}\frac{1}{2} = 125\text{ nm}$$

b)

The same, 125 nm

c)

$$\frac{\lambda}{2\,\text{Cos}[\theta]}\frac{\Delta}{s} = \frac{500\text{ nm}}{2\,\text{Cos}[45\,°]}\frac{1}{2} = 177\text{ nm}$$



- **d)**

$$\frac{\lambda}{(n-1)}\frac{\Delta}{s} = \frac{500\text{ nm}}{(1.5-1)}\frac{1}{2} = 500\text{ nm}$$

5) (10 Pts)

Give an equation describing the basic shape of the moiré pattern obtained using two patterns whose line positions are determined by the equations $2\rho^4 - 6\rho^2 + 4x = m$ and $-6\rho^4 + \rho^2 + 4x + 2xy = m$, where m is an integer.

- **Solution**

We see the difference.

$2\rho^4 - 6\rho^2 + 4x - \left(-6\rho^4 + \rho^2 + 4x + 2xy\right) = m$, where m is an integer

$8\rho^4 - 7\rho^2 - 2xy = m$

6) (15 Pts)

Phase-shifting interferometry is used with a Twyman-Green interferometer to measure a spherical mirror. The light source wavelength is 633 nm. The interferometer is adjusted to give 6 tilt fringes across the aperture. The detector is a 512 x 512 element CCD read out at a rate of 30 Hz.

a) The phase shifter is incorrectly adjusted so the phase shift between consecutive frames is 86.5 degrees instead of 90 degrees. What is the spatial frequency of the resulting error in the interferogram?

b) If the phase shifting is achieved my translating the reference mirror, how fast should the reference mirror be moving, in units of microns/second, to achieve a phase shift of 90 degrees between consecutive data frames? State any assumptions you are making.

c) What is the maximum surface slope error in units of microns/radius that can be measured?

- **Solution**
- **a)**

Double frequency, therefore 12 cycles across the aperture.

- **b)**

The OPD must change by a quarter wave in $\frac{1}{30}$ second.

$$2\,\text{velocity}\left(\frac{1}{30}\,\text{second}\right) = \frac{0.633\,\mu\text{m}}{4}$$

$$\textbf{velocity} = \frac{\frac{0.633\,\mu\text{m}}{4}}{2\left(\frac{1}{30}\,\text{second}\right)}$$

$$\frac{2.37\,\mu\text{m}}{\text{second}}$$

- **c)**

The surface height must change by less than $\frac{1}{4}$wave between adjacent pixels.

$$\textbf{slope} = \frac{\lambda / 4}{\text{radius} / 256} \;/.\; \lambda \to 0.633\,\mu\text{m}$$

$$\frac{40.512\,\mu\text{m}}{\text{radius}}$$

Exam Grades

```
examGrades = {100, 100, 100, 100, 100, 100, 100, 98, 97, 93, 93, 89,
   83, 82, 80, 75, 75, 72, 71, 71, 66, 65, 62, 60, 59, 57, 55, 53, 49, 46, 42};
```

```
Print["Mean = ", N[Mean[examGrades]]]; Print["Maximum = ", Max[examGrades]];
Print["Median = ", Median[examGrades] // N];
Print["Minimum = ", Min[examGrades]];
Print["Standard Deviation = ", N[StandardDeviation[examGrades]]];
```

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Mean = 77.1935

Maximum = 100

Median = 75.

Minimum = 42

Standard Deviation = 19.0673
```