Measurement of OTF -- sinusoidal grating and slit

Can measure the OTF by imaging sinusoidal gratings of different frequencies and scanning the images with a slit or image slit and scan the image using sinusoidal gratings.

One-dimensional case

$$i[x'] = \int_{-\infty}^{\infty} h[x]\, o[x' - x]\, dx$$

o[x] = intensity distribution of incoherently illuminated object
h[x] = image of line source
i[x′] = image

To eliminate factor of magnification, the ideal image of the object is considered as the object.

Let object be $o[x'] = b_o\,(1 + \mathrm{Cos}[2\pi\nu x'])$

$$i[x'] = b_o \int_{-\infty}^{\infty} h[x]\,(1 + \mathrm{Cos}[2\pi\nu\,(x' - x)])\, dx$$

$$= b_o \int_{-\infty}^{\infty} h[x]\,(1 + \mathrm{Cos}[2\pi\nu x]\,\mathrm{Cos}[2\pi\nu x'] + \mathrm{Sin}[2\pi\nu x]\,\mathrm{Sin}[2\pi\nu x'])\, dx$$

From the definition of the OTF we know

$$I[\nu] = H[\nu]\, O[\nu]$$

Let

$$H_c[\nu] = \int_{-\infty}^{\infty} h[x]\,\mathrm{Cos}[2\pi\nu x]\, dx$$

$$H_s[\nu] = \int_{-\infty}^{\infty} h[x]\,\mathrm{Sin}[2\pi\nu x]\, dx$$

$$H[\nu] = \int_{-\infty}^{\infty} h[x]\, e^{-i\,2\pi\nu x}\, dx = H_c[\nu] - i\, H_s[\nu]$$

Then

$$i[x'] = b_o \left(\int_{-\infty}^{\infty} h[x]\, dx + \mathrm{Cos}[2\pi\nu x']\, H_c[\nu] + \mathrm{Sin}[2\pi\nu x']\, H_s[\nu] \right)$$

If

$$T[\nu] = \sqrt{H_c[\nu]^2 + H_s[\nu]^2}$$

$$\theta[\nu] = \mathrm{ArcTan}\left[\frac{H_s[\nu]}{H_c[\nu]}\right]$$

$$i[x'] = b_o \left(\int_{-\infty}^{\infty} h[x]\, dx + T[\nu]\,\mathrm{Cos}[2\pi\nu x' - \theta[\nu]] \right)$$

Let

$$\int_{-\infty}^{\infty} h[x]\, dx = 1,$$

then average illumination of image of grating is equal to that of the object

$$i[x'] = b_o\,(1 + T[\nu]\,\mathrm{Cos}[2\,\pi\,\nu\,x' - \theta[\nu]])$$