Measurement of OTF -- sinusoidal grating and slit

Can measure the OTF by imaging sinusoidal gratings of different frequencies and scanning the images with a slit or image slit and scan the image using sinusoidal gratings.

One-dimensional case

\[ i[x'] = \int_{-\infty}^{\infty} h[x] \cdot o[x - x'] \, dx \]

\( o[x] \) = intensity distribution of incoherently illuminated object
\( h[x] \) = image of line source
\( i[x'] \) = image

To eliminate factor of magnification, the ideal image of the object is considered as the object.

Let object be \( o[x'] = b_o \left( 1 + \cos(2 \pi \nu x') \right) \)

\[ i[x'] = b_o \int_{-\infty}^{\infty} h[x] \left( 1 + \cos(2 \pi \nu (x' - x)) \right) \, dx \]

\[ = b_o \int_{-\infty}^{\infty} h[x] \left( 1 + \cos(2 \pi \nu x) \cos(2 \pi \nu x') + \sin(2 \pi \nu x) \sin(2 \pi \nu x') \right) \, dx \]

From the definition of the OTF we know

\( I[\nu] = H[\nu] \cdot O[\nu] \)

Let

\[ H_c[\nu] = \int_{-\infty}^{\infty} h[x] \cos(2 \pi \nu x) \, dx \]

\[ H_s[\nu] = \int_{-\infty}^{\infty} h[x] \sin(2 \pi \nu x) \, dx \]

\[ H[\nu] = \int_{-\infty}^{\infty} h[x] \, e^{-i \cdot 2 \pi \nu x} \, dx = H_c[\nu] - i \cdot H_s[\nu] \]

Then

\[ i[x'] = b_o \left( \int_{-\infty}^{\infty} h[x] \, dx + \cos(2 \pi \nu x') \cdot H_c[\nu] + \sin(2 \pi \nu x') \cdot H_s[\nu] \right) \]

If

\[ T[\nu] = \sqrt{H_c[\nu]^2 + H_s[\nu]^2} \]

\[ \theta[\nu] = \text{ArcTan} \left[ \frac{H_s[\nu]}{H_c[\nu]} \right] \]

\[ i[x'] = b_o \left( \int_{-\infty}^{\infty} h[x] \, dx + T[\nu] \cos(2 \pi \nu x' - \theta[\nu]) \right) \]

Let
\[ \int_{-\infty}^{\infty} h[x] \, dx = 1, \]

then average illumination of image of grating is equal to that of the object

\[ i[x'] = b_o \left( 1 + T[\nu] \cos(2 \pi \nu x' - \theta[\nu]) \right) \]