11.5.3 Use of lateral shear interferometer to measure the OTF

H. H. Hopkins was the first to show that a lateral shearing interferometer can be used to measure the OTF of an optical system. (Ref: H. H. Hopkins, Opt. Acta 2, 23 (1955).)

The one-dimensional OTF of an optical system $H[f_x]$ is given by the autocorrelation of the pupil function

$$H[f_x] = \frac{1}{c}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\left[x + \frac{s}{2}, y\right] P^*\left[x - \frac{s}{2}, y\right] dx\, dy$$

If z_i is the image distance, D is the pupil diameter, and λ is the wavelength

$$f_x = \frac{s}{\lambda z_i} = \frac{s}{D}\frac{1}{\lambda}\frac{D}{z_i} = \frac{s}{D}\frac{1}{\lambda f^\#}$$

For simplicity let the two interfering beams in a lateral shear interferometer have the same intensity. Let δ be the phase difference between the two sheared interfering wavefronts due to path difference in the interferometer. Generally δ is made to vary linearly with time, i.e. $\delta = \omega t + \phi_o$.

Then the total flux in the interference pattern is

$$F[\delta, s] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{Abs}\left[P\left[x + \frac{s}{2}, y\right] + P\left[x - \frac{s}{2}, y\right] e^{-i\delta}\right]^2 dx\, dy$$

$$= 2\int\int \mathrm{Abs}\left[P\left[x + \frac{s}{2}, y\right]\right]^2 dx\, dy +$$

$$\int\int P\left[x + \frac{s}{2}, y\right] P^*\left[x - \frac{s}{2}, y\right] e^{i\delta} dx\, dy + \text{complex conjugate}$$

$$= 2c\,(1 + \mathrm{Abs}[H[f_x]]\,\mathrm{Cos}[\delta - \Theta[f_x]])$$

2c is the average amount of flux in the two interfering wavefronts.