Measurement of the inhomogeneity of a window

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Abstract. We describe three methods to measure the inhomogeneity of a window material. The first method immerses the window in a liquid between two planes. However, this method is inconvenient for some applications. The second method measures the optical figure of the front surface and then measures the return wavefront that transmits through the window and reflects from the rear surface of the window. The advantage of this method is that it can remove the contributions of both the surface figures and the return fiat plus the system error of the interferometer. The disadvantage is that a small wedge must be fabricated between the two surfaces to eliminate spurious interference. The third method derives the inhomogeneity of the window material by measuring the optical figure of the front surface of the window and then flipping the mirror to measure the back surface. The advantage of this method is that it is not necessary to have a wedge between the two surfaces. The disadvantage of the window-flipping method is that the contribution of system error can increase.

Subject terms: optical window inhomogeneity; optical testing; index of refraction.

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CONTENTS

1. INTRODUCTION

The deformation of a transmitted wavefront through a window is due to a combination of the inhomogeneity of the window material and the figures of both surfaces of the window. Several methods have been proposed to measure the inhomogeneity. [1-5] In theory, if both surfaces are much better than the amount of the homogeneity of interest, any deformation of the wavefront is due to the inhomogeneity of the window material.

Opticians, however, want to know the inhomogeneity of a window material before precision polishing is done. Adachi et al.' immersed a fine ground optical material into a cavity filled with a liquid of the same refractive index as the window. They were able to eliminate the contribution of both surfaces and measure the inhomogeneity of the window. The disadvantage of this method is that the use of a liquid is inconvenient for some applications. However, if both surface figures of the window are known, it is possible to derive the inhomogeneity of the window material. With the aid of a digital interferometer, it is possible to measure the figures of the two surfaces and store this information for later use. By mathematically manipulating the data, we can subtract the contribution of both surfaces from the transmitted wavefront.

Schwider et a1.[5] measured the contribution of the rear surface with the wavefront transmitted through the window and reflected by the rear surface. Thus, they were able to remove the contributions of both surfaces, the return flat, and the system error of the interferometer with four measurements. Since the transmitted wavefront is used, we call this the transmission method. One disadvantage of this method is that a wedge has to be fabricated between the two surfaces to eliminate the spurious reflection. On the other hand, it is a straightforward procedure to measure the rear surface of the window by flipping the window and then removing its contribution to the transmitted wavefront. We call this the window-flipping method. However, the contribution of the system error can increase with this method.

In the following sections we compare the mathematical derivations for the transmission method and the window-flipping method and show the experimental results of the three methods. We also discuss the error analysis in details.

2. THREE MEASUREMENT METHODS

2.1. Liquid immersion method

The liquid immersion method uses two optical flats to form a cavity filled with a liquid. A window is dipped into a liquid that has the same refractive index as the window material. The liquid must be stable to perturbations and be harmless. The transmitted wavefront through the cavity reflects off a flat, and the return wavefront is measured with a digital interferometer. The contribution of the cavity is removed by taking the difference of two measurements with and without the window in the cavity.

2.2. Transmission method

In this section, we summarize the procedure given by Schwider et a1.[5] An interferometer measures the return wavefront of light after it is transmitted through a window, is reflected by a flat, and is returned. The return wavefront is the sum of the contri-

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butions of the inhomogeneity of the window material, the figures of both surfaces of the window, and the return flat. Because both surfaces and the return flat are not absolutely flat, the errors due to the surfaces must be removed to determine the inhomogeneity. The procedure, shown in Fig. 1, is as follows:

1. Remove the window and adjust the return flat to obtain a reflection from the return flat C.
2. Insert the window. Adjust the return flat to get a reflection from the return flat C through the window.
3. Measure the wavefront reflected from the front surface *A* of the window.
4. Adjust the window to obtain the reflection from the rear surface B.

Expressed mathematically,

$$\begin{aligned} M_1(x,y) &= 2C + 2S + k_1 \ , \\ M_2(x,y) &= 2(1 - n_0)A + 2(n_0 - 1)B + 2C \\ &\quad + 2\Delta + 2S + k_2 \ , \\ M_3(x,y) &= 2A + 2S + k_3 \ . \\ M_4(x,y) &= 2(1 - n_0)A + 2n_0B + 2\Delta + 2S + k_4 \ . \end{aligned} \tag{1}$$

Here M_1, M_2, M_3, and M_4 are the measured wavefronts in each of the above steps; *A*, *B*, and C are the surface errors of the front surface, the rear surface, and the return flat, respectively; and A is the wavefront deviation due to the inhomogeneity of the window, i.e., material contribution. The average or nominal refractive index is n_0 and $k_{1\text{-}4}$ are arbitrary constants because only the relative phase is measured in each measurement. Finally, S is the contribution of the system error. From Eq. (l), multiplying (M_2 - M_1) and (M_4 - M_3) by two factors n_0 and n_0 - 1, respectively, then

$$\begin{aligned} n_0(M_2 - M_1) - (n_0 - 1)(M_4 - M_3) \\ = 2n_0[(n_0 - 1)(B - A) + \Delta] - 2(n_0 - 1) \\ \times [n_0(B - A) + \Delta] + \text{constant} \\ = 2\Delta + \text{constant} \ . \end{aligned} \tag{2}$$

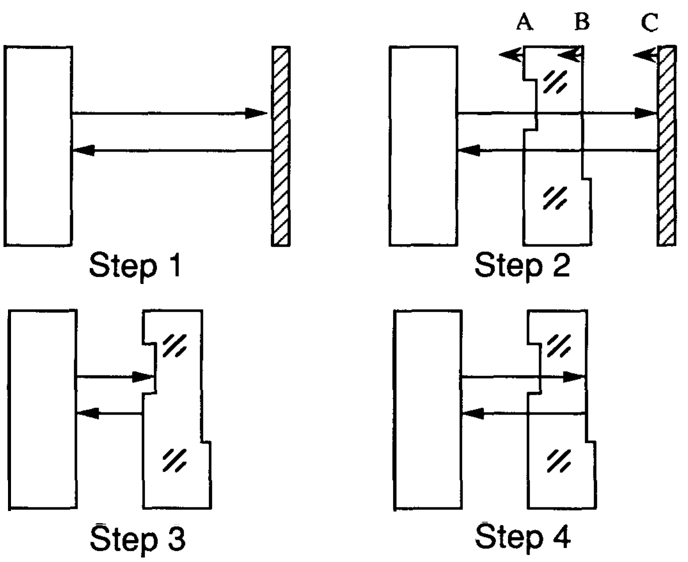


Fig. 1. The measurement procedure for the transmission method.

Therefore,

$$\Delta(x,y) = [n_0(M_2 - M_1) - (n_0 - 1) \times (M_4 - M_3)]/2 + \text{constant}' \ . \tag{3}$$

Thus the wavefront deviation A due to the inhomogeneity can be obtained. When the thickness of the window is given, the variation of the refractive index can be calculated. The constant in the equation affects only the bias of the variation, which is not important with respect to the inhomogeneity. It should be noted that the system error S is eliminated in Eq. (2), in addition to the errors due to the front surface *A*, the rear surface *B*, and the return flat C.

2.3. Window-flipping method

The general practice is to measure the two surfaces of the window separately. The front surface is easy to measure. However, for the rear surface, we must flip the window about an axis, e.g., the x axis, and make a measurement, as shown in Fig. 2, step 4'. This requires a flipping mechanism to ensure that the images on the detector overlap before and after flipping the window. After obtaining the measurement wavefront, M'_4 in step 4', we flip M'_4 mathematically about the x axis to obtain $\mathbf{M'_4}$. Thus,

$$\begin{aligned} M_4'(x,y) &= -2\mathbf{B} + 2S + k_4' \ , \\ \mathbf{M}_4'(x,y) &= -2B + 2\mathbf{S} + k_4' \ . \end{aligned} \tag{4}$$

The bold letters denote the transformation of the array flipping 180 degrees about then axis, i.e., $\mathbf{M'_4(x,y)}$ = M'_4(x,-y), **B(x,y)** = **B(x,-y),** and **S(x,y)** = S(x,-y). It should be noted that there is a minus sign with B due to the window flipping.

From Eqs. (1) and (4)

$$\begin{aligned} M_2 - M_1 + (n_0 - 1)(M_3 + \mathbf{M}_4') \\ = 2[(1 - n_0)(A - B) + \Delta] + 2(n_0 - 1) \\ \times [(A - B) + S + \mathbf{S}] + \text{constant} \\ = 2\Delta + 2(n_0 - 1)(S + \mathbf{S}) + \text{constant}. \end{aligned} \tag{5}$$

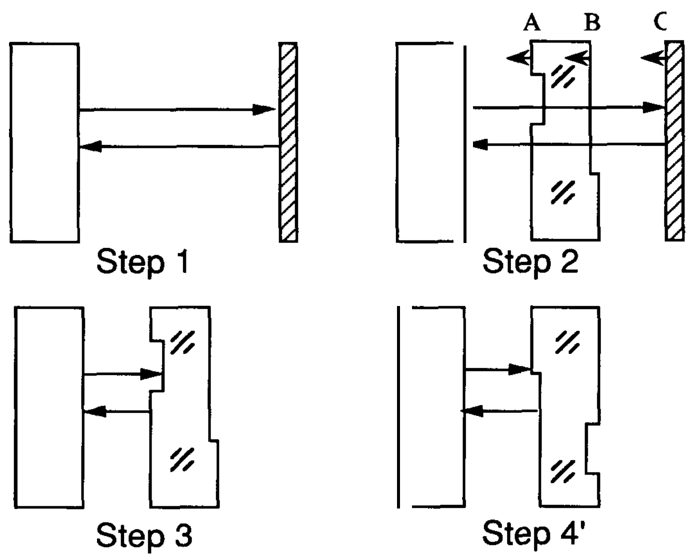


Fig. 2. The measurement procedure using the window flipping method.

Thus,

$$\Delta = [M_2 - M_1 + (n_0 - 1)(M_3 + \mathbf{M}_4')]/2 \\ - (n_0 - 1)(S + \mathbf{S}) + \text{constant}' \ . \tag{6}$$

It is clear that the system error does not vanish unless it is antisymmetric about the axis of flipping (e.g., the x axis). Because of the window flipping, the system error is not canceled, but can actually increase.

3. EXPERIMENT

We measured a round window of SFL57, 10 mm thick and 50 mm in diameter, and a rectangular window of BK7, 63 mm long by 38 mm wide by 50 mm thick using a digital interferometer. The windows were measured in the direction along the thickness. The peak-to-valley (p-v) values of the surface figures of both windows are between 0.75 wave and 1.50 waves. Figure 3 shows the interference fringes of the front and the rear surfaces and the return wavefront reflected by the return flat with the SFL57 window. Figures 4(a), 4(b), and 4(c) are the measured inhomogeneity of glass SFL57 obtained with the three methods. The figure clearly shows that there is a delta distribution in the homogeneity. Figure 5 is another measurement result with the transmission method, where the window was rotated by an angle of 30 deg. Mathematically rotating Fig. 5, and comparing it with Fig. 4(a), the difference is 0.010 wave rms, as shown in Fig. 6. Because of the size of the BK7 window, it was not tested with the liquid immersion method. Figures 7(a) and 7(b) show the measured inhomogeneity of glass BK7 with the transmission method and with the window-flipping method. The figure clearly shows that there is a cylindric distribution along the longest dimension in the homogeneity. From Figs. 4 and 7, the wavefront deviations due to the inhomogeneity of SFL57 and BK7 are about 0.75 wave, where the interval cycle of the isometric contour is 0.007 wave. Thus, the variation of the refractive index is about 5×10^{-5} and 1×10^{-5} for SFL57 and BK7, respectively.

Figures 4(a) and 4(b) are very similar to each other, but compared to Fig. 4(c) they have a greater power, i.e., more fringes. The reason why Fig. 4(c) has fewer fringes is that the liquid used in the liquid immersion method has a refractive index approximately equal to 1.785, at 589.3 nm, which does not match that of the window material. Figures 4 and 7 clearly show that there is a delta distribution in the homogeneity in the SFL57 window and a cylindric distribution in the BK7 window. We believe that this delta distribution occurred while the sample was prepared, and that the cylindric distribution occurred in the melting and/or annealing process. Figure 6 is the difference between two measurements; one of them is rotated mathematically. The figure shows the footprint of the three-chuck mount. Thus, the measurement accuracy in this experiment is about 0.010 wave rms, and is limited by the mounting mechanism. If the measurement is performed carefully, 0.005 wave rms can be achieved. The similarity among the results using all three methods shows that the transmission method works very well.

4. DISCUSSION

Here we discuss the error sources and the measurement accuracy. The first error source is the interferometer random noise, as explained below. The contribution of the window to the transmitted wavefront is

$$\text{OPD}(x,y) = [n(x,y) - 1]T(x,y) \tag{7}$$

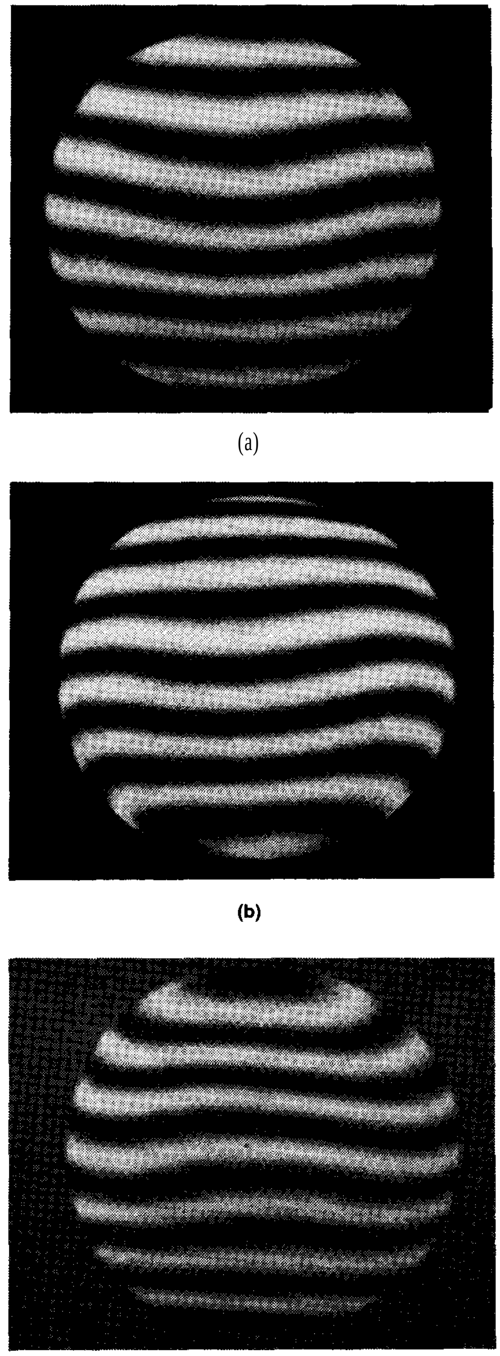


Fig. 3. Interference fringes of an SFL57 window. (a) Front surface, (b) rear surface, (cl return wavefront reflected by a return flat.

where n(x,y) is the refractive index, and T(x,y) is the window thickness. Both are functions of x and y. For simplicity, only x is used. Therefore, the variation of the optical path difference (OPD) over the pupil of the window is

$$\begin{aligned} \text{OPD}(x) - \text{OPD}_0 &= [n(x) - 1]T(x) - (n_0 - 1)T_0 \\ &= [n_0 + \eta(x) - 1][T_0 + t(x)] - (n_0 - 1)T_0 \\ &\approx (n_0 - 1)t(x) + T_0\,\eta(x) \end{aligned} \tag{8}$$

where n_0 is the average refractive index, T_O is the nominal or average thickness, and OPD_0 is the average OPD equal to

$(n_0 - 1)T_0$. The refractive index variation n(x) equals $n(x) - n_0$. The thickness variation t(x), equal to $T(x) - T_0$, is equal to the difference of the front and rear surfaces plus the wedge of window, if it exists. However, the effect of the wedge is equivalent to the tilt of a wavefront, which is not important and usually not measured. Hence, $(n_0 - 1)t(x)$ equals the wavefront deviation due to the thickness variation, and $T_0\eta(x)$ is equal to the wavefront deviation due to the inhomogeneity. In Eq. (8), the cross term, $\eta(x)t(x)$, is dropped because it is too small compared to the other two terms.

From Fig. 1, $M_2(x) - M_1(x)$ equals the contribution from the window. Also, from Eq. (l),

$$[M_2(x) - M_1(x)]/2 = (n_0 - 1)[B(x) - A(x)] + \Delta(x) + \text{constant} . \quad (9)$$

There is a factor 2 in the denominator, because $M_2(x)$ and $M_1(x)$ are the returned wavefronts, which go through the window and the cavity twice. Comparing Eq. (9) with Eq. (8), we can see that

$$t(x) = B(x) - A(x) ,$$

and (10)

$$\Delta(x) = T_0\eta(x) = T_0[n(x) - n_0] ,$$

(a)

(b)

(c)

Fig. 4. The measured inhomogeneity of glass SFL57 with (a) the transmission method, (b) the window-flipping method. and (c) the liquid immersion method. The interval of the isometric contour is 0.07 wave. (a) p-v = 0.711 wave, rms = 0.166 wave. (b) p-v = 0.793 wave, rms = 0.166 wave. (c) p-v = 0.646 wave, rms = 0.141 wave. The figures clearly show that there is a delta-shaped distribution of the homogeneity.

Fig. 5. The measured inhomogeneity of the SFL57 window with the transmission method where the window was rotated 30 deg. The interval of the isometric contour is 0.07 wave, p-v = 0.725 wave, and rms = 0.164 wave.

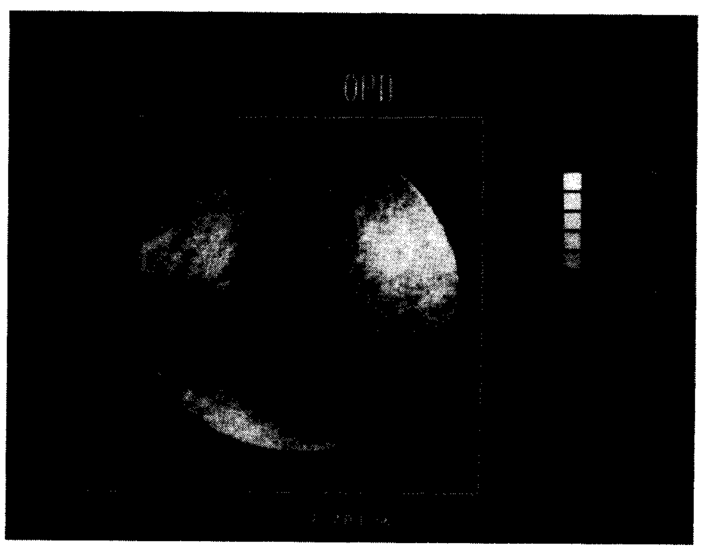


Fig. 6. The difference between Fig. 4(a) and Fig. 5 shows the effect of the three-chuck mount. Figure 5 is mathematically rotated to match the orientation of Fig. 4(a).

where the constant is dropped because only the variation is of interest. Thus the wavefront deviation A(n) due to the inhomogeneity is equal to the refractive index variation $n(x) - n_0$ times the thickness of the window T_0. If the thickness of the window T_0 is given, from Eq. (l0), $n(x) - n_0$ can be obtained as follows:

$$n(x) - n_0 = \Delta(x)/T_0 \tag{11}$$

It is important to know that the peak-to-valley value of Eq. (11) gives the maximum variation of n.

From the right-hand side of Eq. (3), the rms error δ_1 for the measurement of the material contribution due to the random noise is equal to

$$\delta_1 = 0.5\varepsilon[2n_0^2 + 2(n_0 - 1)^2]^{1/2} \tag{12}$$

where ε is the rms error for a phase measurement due to the random noise, i.e., the repeatability of the interferometer. Thus, for $n_0 = 4$, the rms error is 3.5ε, and for $n_0 = 1.6$, the rms error is 1.2ε. In Eq. (11), $\Delta(x)/T_0$ determines the refractive index variation. Similarly, δ_1/T_0 gives the measurement accuracy of the refractive index variation. Because δ_1 is limited by the interferometer, the thicker the window, the more accurate the result. Note that Δ and δ_1 in Eqs. (11) and (12) have the same unit of dimension in waves, and that Δ/T_0 and δ_1/T_0 are dimensionless.

For example, a window of $T_0 = 10$ mm and $n_0 = 1.6$ is measured at 633 nm. If the maximum wavefront deviation A due to the inhomogeneity is 0.16 wave, peak to valley, from Eq. (ll), the maximum variation of n equals 10×10^{-6}. Typically, glass of good homogeneity[6] has a maximum variation of n_d equal to $\pm 5 \times 10^{-6}$, which are the extremes of the refractive index, and whose difference equals 10×10^{-6}. If the interferometer has a repeatability $\varepsilon = 0.002$ wave rms, then from Eq. (12) the measurement rms error $\delta_1 = 0.0024$ wave, and $\delta_1/T_0 = 0.15 \times 10^{-6}$. Thus, the measurement error is about 1.5%.

Another error source is the error of the refractive index input. In Eq. (2), $(M_2 - M_1)$ and $(M_4 - M_3)$ are multiplied by two factors that are determined by the refractive index. This refractive index usually is the nominal value n_n from a glass catalogue, and is not necessarily equal to the average refractive index of the window material under test. Substitute n_n and $n_n - 1$ for the two factors mentioned above, then

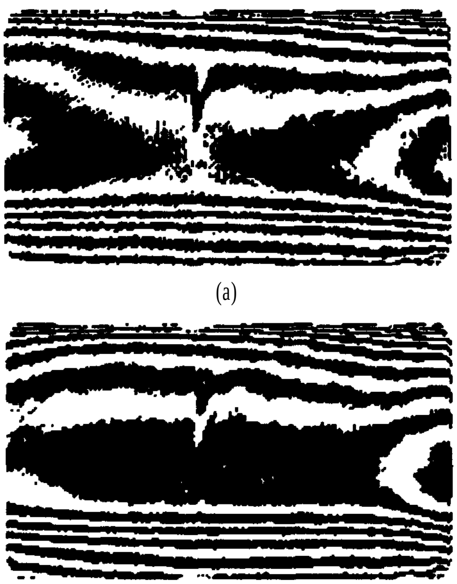
$$\begin{aligned} n_n(M_2 - M_1) &- (n_n - 1)(M_4 - M_3) \\ &= 2n_n[(n_0 - 1)(B - A) + \Delta] - 2(n_n - 1)[n_0(B - A) \\ &\quad + \Delta] + \text{constant} \\ &= 2(n_0 - n_n)(B - A) + 2\Delta + \text{constant} . \end{aligned} \tag{13}$$

Note that if the refractive index input n_n is not equal to the average refractive index *no*, then an error occurs. Therefore, the rms error δ_2 for the measurement of the material contribution due to the error of the refractive index input can be expressed as

$$\delta_2 = |n_0 - n_n|(B - A)_{\text{rms}} \tag{14}$$

For instance, if the discrepancy between the refractive index input and the average refractive index $n_0 - n_n = 0.002$, which is the maximum deviation for a melt from the value stated in a glass catalogue,[6] and the thickness variation $(B - A)_{rms} = 2$ wave, then δ_2 is about 2.004 wave. For the same example above, $\delta_2/T_0 = 0.25 \times 10^{-6}$. From Eq. (14), the smaller the error of the refractive index input, the smaller the measurement error of the refractive index variation. Note that both δ_1 and δ_2 are independent of the average thickness of window.

The typical and maximum values of the variation of the refractive index and the corresponding errors of the homogeneity measurement are summarized and tabulated in Table 1. In the above experiment, for the SFL57 window that is 10 mm thick, the peak-to-valley value of the wavefront deviation is about 0.75 wave. Thus, the maximum variation of n for SFL57 is about 50×10^{-6}. For the BK7 window that is 50 mm thick, the peak-



(a)

(b)

Fig. 7. The measured inhomogeneity of glass BK7 (a) with the transmission method and (b) the window flipping method. The figure clearly shows that there is a cylindric distribution along the longest dimension in the homogeneity. The interval of the isometric contour is 0.07 wave. (a) p-v = 0.722 wave, rms = 0.118 wave. (b) p-v = 0.715 wave, rms = 0.118 wave.

Table 1. The typical and maximum values of the variation of the refractive index and their corresponding wavefront deviation and errors of the homogeneity measurement.

	n_0-n_n (1)	δ_2 (2)	δ_1 (3)
maximum	±0.0020	0.040 wave	-----
typical	±0.0005	0.008 wave	0.0024 wave
	$n(x)$-n_0 (4)	$\Delta(x)$ (5)	$\Delta(x)/T_0$
maximum	2×10^{-4} (= $\pm1\times10^{-4}$)	3.20 wave	200×10^{-6}
typical	1×10^{-5} (= $\pm5\times10^{-6}$)	0.16 wave	10×10^{-6}

$T_0 = 10$ mm, 1 wave = 633 nm.

(1) The refractive index deviation for a melt from the value n_n stated in a glass catalogue[6], where n_0 is the average refractive index.

(2) δ_2 is due to the error of the input refractive index, where (B-A),, = 2 waves.

(3) δ_1 is due to the interferometer repeatibilty, and depends on no, not n_0-n_n. For $\varepsilon = 0.002$ wave rms and n_0=1.6, from Eq. (12) δ_1 equals 1.2ε.

(4) The degree of the refractive index variation over the pupil.

(5) The peak-to-valley value of the wavefront deviation Δ (x) due to inhomogeneity for a window 10 mm thick. $\Delta(x) = T_0[n(x)-n_0]$.

to-valley value of the wavefront deviation is also about 0.75 wave. Because the BK7 window is much thicker than the SFL57 window, the maximum variation of n for BK7 is much smaller than that of SFL57, approximately equal to 10×10^{-6}. Therefore, the refractive index of BK7 is much more uniform than that of SFL57. It is obvious but important that the peak-to-valley value of the wavefront deviation is also dependent on the size of the window and not just the thickness. In brief, for the same window material, the larger and thicker the piece, the more difficult it is to obtain a uniform refractive index.

When the window-flipping method is used, from Eq. (6), the two errors mentioned above still exist. Besides those, we see that the system error could contribute the greatest part of the measure error, if $S + \mathbf{S}$ is not equal to zero. For the same window of $T_0 = 10$ mm and $n_0 = 1.6$ measured at 633 nm, if the system error $S_{rms} = 0.01$ wave, δ is on the order of 0.01 wave. Thus, $\delta / T_0 = 0.6 \times 10^{-6}$.

5. CONCLUSION

The advantage of the oil immersion method is that the surface only needs to be fine ground. The disadvantage of this method is that the use of a liquid is inconvenient for some applications. It should be noted that the refractive index of oil must be as close as possible to that of the window material. However, oil of a higher refractive index is difficult to obtain.

The two major error sources for the transmission method are the random noise of the interferometer system and the discrepancy between the refractive index input and the refractive index averaged over the pupil of the test window. The contributions of the two error sources have approximately the same magnitude. The contribution from the first error source is linearly proportional to the interferometer repeatability. For the second error source, the resulting error is linearly proportional to the error of the refractive index input. Both error sources are independent of the average window thickness. Therefore, the measurement accuracy of the refractive index variation is inversely proportional to the thickness. Hence, in order to obtain the most accurate measurement, we should use the thickest window plate, the least noisy interferometer, and the best estimated refractive index.

The advantages of this transmission method are as follows: (1) The contributions due to the figure errors of both surfaces of the window, the return flat, and the system error are removed completely. (2) It is easy to mathematically process the data because only the operations of multiplication and differencing are necessary. (3) It is easy to take a measurement because neither a flipping mechanism nor liquid is needed. The disadvantage of this method is that it requires both surfaces to be polished within a few fringes, and a small wedge must be fabricated between the two surfaces to eliminate the spurious interference .

For the window-flipping method, the system error contributes the greatest part of the measurement error, because the system error is not canceled. The two error sources mentioned above still exist for this method. The advantage of this method is that because the two surfaces are measured in reflection completely independently, the spurious reflection from the back surface can be blocked by spreading a coating on the measured surface. Therefore, it can test a window that has no wedge between the two surfaces.

6. ACKNOWLEDGMENTS

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