Problem HS-11

HS-11)

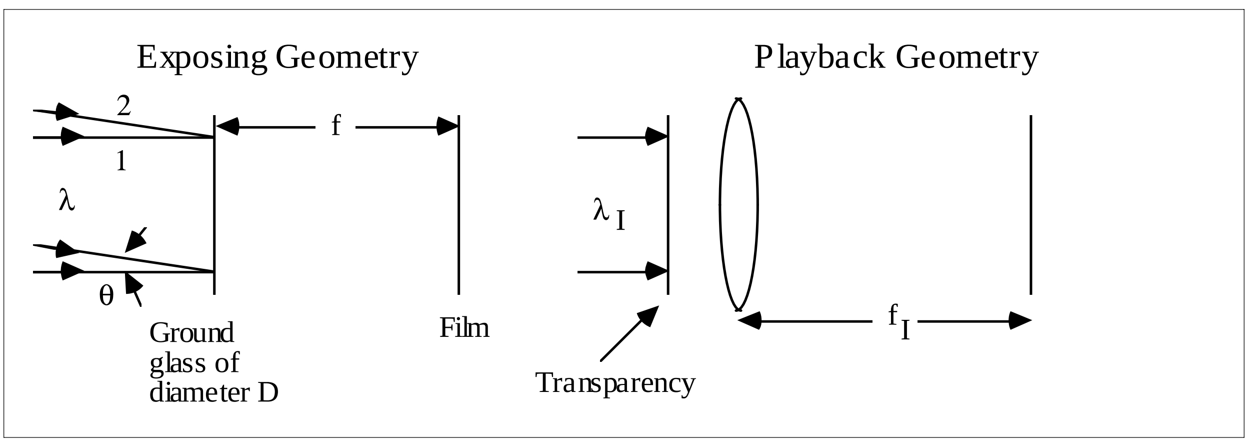
Question

A piece of ground glass of diameter D is illuminated normally with a monochromatic collimated beam of wavelength λ, and the resulting speckle pattern is recorded on film a distance f away. A second recording of the resulting speckle pattern is made for the collimated beam incident at an angle θ. Assume that after processing, the amplitude transmittance of the film is proportional to the exposing irradiance. The resulting film transparency is illuminated with a collimated plane wave of wavelength λ_I. A lens of focal length f_I is placed after the transparency and the light distribution in the focal plane of the lens is observed. In answering the questions, you can assume small angles, if you wish.

a) What is the relationship between the fringe spacing observed in the focal plane of lens f_I and θ, D, f, λ , λ_I, f_I, and other pertinent quantities?

b) How many bright fringes can be observed in the focal plane? What is the minimum value of θ such that a bright fringe is observed at the center and edges of the pattern?

c) The results of this question imply that as far as resolution is concerned, all astronomical telescopes can be replaced with ground glass optics. What is wrong with this statement?



Solution

Two identical speckle patterns are recorded separated a distance = f θ.

The maximum spatial frequency recorded = $\nu_{max} = \frac{d}{\lambda f}$.

The spread of diffracted light in the focal plane of lens f_I is $\pm f_I \lambda_I \nu_{max} = \pm \frac{f_I \lambda_I d}{\lambda f}$.

a

Fringe spacing

Considering the speckles as Young's double pinholes, or using the shift theorem gives the fringe spacing as

$$\text{fringeSpacing} = \frac{\lambda_I f_I}{\text{speckleDisplacement}}$$

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fringeSpacing = λI fI / (f θ);
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b

Number of fringes

1 bright fringe in center, plus other fringes spaced $\frac{\lambda_I f_I}{f \theta}$.

numberFringes = 1+ $\frac{\text{spread of diffracted light}}{\text{fringe spacing}}$

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numberFringes = (2 λI fI d / (f λ)) / (λI fI / (f θ))
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$$\frac{2 d \theta}{\lambda}$$

For a bright fringe at the center and edge, numberFringes = 3.

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Solve[3 == 1 + 2 d θmin / λ, θmin]
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$$\left\{\left\{\theta_{min} \to \frac{\lambda}{d}\right\}\right\}$$

Note that this is normal resolution for aperture of diameter d.

c

To obtain above resolution we must have good contrast speckles. To obtain good contrast speckles, the OPD's must be less than the coherence length of the source. Thus for ground glass telescope optics we would need long coherence lengths, and hence narrow spectral bandwidths and we would end up with little light for most applications. In addition, the light would be spread over a large area, and again there would be a problem with light intensity.