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Optical frequency shifter for heterodyne interferometers using multiple rotating polarization retarders

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Methods have been proposed for optical frequency shifting for heterodyne interferometers based on rotating phase retarders inside or in front of the interferometer cavity.[1,2] The frequency shift available by these methods is limited to twice the rotation rate of the rotating element as in Ref. 1, or four times the rotation rate of the rotating element as in Ref. 2.

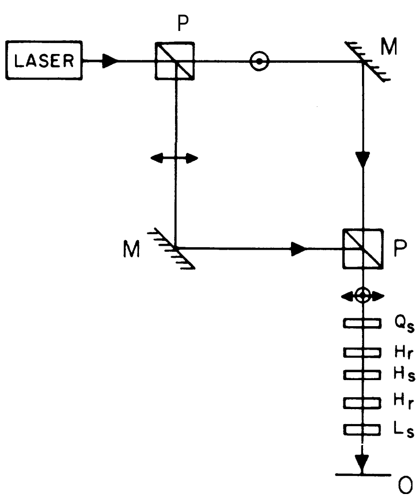

Fig. **1.** Typical interferometer configuration utilizing frequency shifter (see text for nomenclature). A Mach-Zehnder configuration is shown but may be adapted to other inteferometers such as the Twyman-Green.

This Letter describes a method by which the frequency shift equals 4N times the rotation rate, where N is the number of rotating components available. The advantage of this method over previously described methods is that a higher heterodyne frequency is possible than with the previously described techniques.

Let the light entering an interferometer, shown in Fig. 1, be separated into two orthogonal linear polarizations using, for example, a polarization beam splitter P. Each component travels a separate path through the interferometer cavity seeing a different optical phase retardation. Upon recombination the light passes through the frequency shifter and onto a detector plane, where the temporally varying optical signal has a phase equal to the net phase difference between the two paths inside the interferometer.

The frequency shifter, which we shall describe using Jones calculus[3] and the complex wave representation of light, consists of a stationary quarterwave plate (Q_s) followed by a series of rotating halfwave plates (H_r) separated by stationary halfwave plates (H_s), followed by a stationary linear polarizer (L_s). The following analysis demonstrates a frequency shift of eight times the rotation rate of the halfwave plates.

We begin by treating the horizontal (X) component, which we represent by

$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \exp[i(\phi_1 - \omega t)], \tag{1}$$

where ϕ_1 is the phase retardation introduced by one arm of the interferometer, and ω is the angular frequency of the transmitted light.

This component is transmitted through a stationary quarterwave plate (Q_s) whose fast axis is oriented 45° to the x axis, yielding right-hand circularly polarized light:

$$Q_s X = \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \exp[i(\phi_1 - \omega t)] = \begin{bmatrix} 1 \\ -i \end{bmatrix} \exp[i(\phi_1 - \omega t)]. \tag{2}$$

Transmitting the beam through a halfwave plate rotating with angular frequency ω' yields

$$H_rQ_sX = -i\begin{bmatrix}\cos 2\omega' t & \sin 2\omega' t\\ \sin 2\omega' t & -\cos 2\omega' t\end{bmatrix}\begin{bmatrix}1\\ -i\end{bmatrix}\exp[i(\phi_1-\omega t)]$$

$$= -i\begin{bmatrix}1\\ i\end{bmatrix}\exp\{i[\phi_1-(\omega+2\omega')t]\}, \tag{3}$$

which is left-handed circularly polarized light with angular frequency $\omega + 2\omega'$.

Introducing a stationary halfwave plate whose fast axis lies on the x axis retransforms the polarization state to the left-hand circular:

$$H_sH_rQ_xX = -i\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}(-i)\begin{bmatrix}1\\ i\end{bmatrix}\exp\{i[\phi_1-(\omega+2\omega')t]\}$$

$$= -\begin{bmatrix}1\\ -i\end{bmatrix}\exp\{i[\phi_1-(\omega+2\omega')t]\}. \tag{4}$$

The beam passes through a second rotating halfwave plate rotating in the same direction, which results in left-hand circular polarization with angular frequency $\omega + 4\omega'$.

$$H_rH_sH_rQ_sX = -i\begin{bmatrix}\cos 2\omega' t & \sin 2\omega' t\\ \sin 2\omega' t & -\cos 2\omega' t\end{bmatrix}(-1)\begin{bmatrix}1\\ -i\end{bmatrix}$$

$$\times \exp\{i[\phi_1-(\omega+2\omega')t]\}$$

$$= i\begin{bmatrix}1\\ i\end{bmatrix}\exp\{i[\phi_1-(\omega+4\omega')t]\}. \tag{5}$$

Finally, by passing the beam through a horizontal linear polarizer, we obtain

$$A_1(t) = L_sH_r\cdot H_s\cdot H_r\cdot Q_s\cdot$$

$$X = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} i\begin{bmatrix}1\\ i\end{bmatrix}\exp\{i[\phi_1-(\omega+4\omega')t]\}$$

$$= i\begin{bmatrix}1\\ 0\end{bmatrix}\exp\{i[\phi_1-(\omega+4\omega')t]\}, \tag{6}$$

where the resultant beam is plane polarized but upshifted an amount equal to $4\omega'$ from the original angular frequency.

We repeat the above procedure to the vertical polarization state from the interferometer, which is represented by

$$Y = \begin{bmatrix}0\\ 1\end{bmatrix}\exp[i(\phi_2-\omega t)], \tag{7}$$

where ϕ_2 is the optical phase retardation in the second arm of the interferometer.

Thus by applying the identical transformation, as in Eqs. (3)-(6),

$$A_2(t) = L_s\cdot H_r\cdot H_s\cdot H_r\cdot Q_s\cdot Y = \begin{bmatrix}1\\ 0\end{bmatrix}\exp\{i[\phi_2-(\omega-4\omega')t]\}, \tag{8}$$

where the angular frequency is downshifted an amount $4\omega'$.

By taking the square magnitude of the sum of the amplitudes of the two transmitted components, we have

$$I(t) = |A_1(t) + A_2(t)|^2 = 1 - \sin(\phi_2 - \phi_1 + 8\omega' t). \tag{9}$$

Thus the resulting irradiance distribution has a temporal frequency equal to eight times the original rotation rate of the halfwave plates. The above analysis can be extended for higher frequencies by adding additional halfwave plates. Note that, in practice, the stationary or rotating halfwave plates may lie in any orientation, since the net effect of a constant angular error in orientation is to impart a constant phase shift into the wavefronts transmitted by the frequency shifter, which is generally of little consequence.

References

1. R. Crane, Appl. Opt. 8,538 (1969).
2. G. E. Sommargren, J. Opt. Soc. Am. 65,960 (1975).
3. R. C. Jones, J. Opt. Soc. Am. 31,488 (1941).