Extended Range Two-Wavelength Interferometry

James C. Wyant
Optical Sciences Center
University of Arizona
Tucson, AZ
85721

Abstract

Two-wavelength phase-shifting interferometry is a powerful technique for extending the range of single-wavelength interferometry. This paper discusses three techniques for extending the dynamic range of phase shifting interferometry without losing the precision of a single short-wavelength measurement. The techniques are derived using basic interferometric principles and the tradeoffs between the different techniques are discussed.

Introduction

Interferometry is an extremely powerful tool that provides measurement capabilities ranging from Angstroms to millions of miles. While the basic principles of interferometry have been known more than 100 years, the addition of modern electronics, computers, and software has made interferometry an extremely useful technique for solving numerous metrology problems. With the addition of phase-shifting interferometry techniques[1,2] interferometric data can be transferred to computer memory and sophisticated analysis of the data can be performed.

One of the largest advantages of optical interferometry is also one of its largest disadvantages. Due to the short wavelength of the light, the sensitivity of a single wavelength interferometric measurement is very high, but also due to the short wavelength of the light the dynamic range is limited unless additional information is available. In a single wavelength interferometric measurement the phase of the light, and hence the phase of the interference pattern, repeats itself at distance intervals equal to the wavelength. Thus, an optical path difference of δ gives the same interferometric measurement as $(n \lambda + \delta)$, where n is an integer.

One good way of eliminating integer wavelength ambiguities in distance or height measurements is to use a white light scanning interferometer. If a white light source is used in an interferometer the best contrast interference fringes are obtained only when the two paths in the interferometer are equal. Thus, if an interferometer is made such the path length of the sample arm of the interferometer is varied, distances or height variations across the sample can be determined by looking at the mirror or sample positions for which the fringe contrast is a maximum. In this measurement there are no height ambiguities and since in a properly adjusted interferometer the sample is in focus when the maximum fringe contrast is obtained, there are no focus errors in the measurement of surface microstructure.[3–5] The major drawback of this type of scanning interferometer measurement is that only a single distance or surface height is being measured at a time and a large number of measurements and calculations are required to determine a large range of distance or surface height values. Also, if the height information is solely determined by looking at the scan position for maximum fringe contrast, the measurement has less accuracy than can be obtained by looking at the phase of the interference fringes.

A second excellent solution to solving the ambiguities in the measurement of the integer number of wavelengths present in a distance measurement is to perform the measurement at more than one wavelength and compare the measurement results for the different wavelengths to determine the true distance. Performing an interferometric measurement at two or more wavelengths and comparing the phases of the interference fringes for the different wavelengths has been described by several authors during the last 100 years.[6–12] By combining two-wavelength interferometry with phase-shifting techniques very powerful commercial interference microscopes have been made.

The purpose of this paper is to use basic interferometric principles to describe three ways of using phase-shifting interferometry and two-wavelength interferometry to increase the dynamic range of an interferometric measurement. The techniques are not limited to phase-shifting measurements, but phase-shifting provides a good way of performing the measurements to high accuracy so two-wavelength measurements can be practical for large path differences.

Two-wavelength interferometry and phase-shifting interferometry

In phase-shifting interferometry three or more measurements of the irradiance of the interference fringes are made for different phase differences between the two interfering beams. Numerous algorithms can be used for the calculation of the phase. Of importance to this discussion is that the tangent of the phase difference between the two interfering beams is determined. Furthermore, the sign of the cosine of the phase and the sign of the sine of the phase are determined, so the phase is determined modulo 2π. As long as the phase difference between adjacent detector points is less than π (opd less than $\lambda/2$) discontinuities in the measured phase for adjacent data points can be removed. One goal of this paper is to describe methods for increasing the allowable phase difference between adjacent data points.

To simplify this discussion we will assume we can neglect phase changes on reflection so we can write

$$\text{phase} = \frac{2\pi}{\lambda}\,\text{opd}.$$

Since the phase is being measured modulo 2π the optical path difference, opd, is being measured modulo λ. We can write

$$\text{opd} = (\text{n} + \phi)\,\lambda.$$

Thus, ϕ is the fractional fringe we are measuring using phase-shifting interferometry and n is an unknown integer that we will determine using two or more measurements at different wavelengths. What we are actually measuring is

```
ϕ[λ_] := FractionalPart[opd / λ]
```

In two-wavelength interferometry we are making two measurements of the fractional fringe using two wavelengths, $\lambda1$ and $\lambda2$, to determine $\phi[\lambda1]$ and $\phi[\lambda2]$. If we knew the integral number of fringes present we could calculate the opd.

```
opdCalculatedλ1 := n λ1 + ϕ[λ1] λ1
```

```
opdCalculatedλ2 := m λ2 + ϕ[λ2] λ2
```

Neglecting measurement errors

```
opdCalculatedλ1 = opdCalculatedλ2
```

Our problem is that we do not know the integers n and m. Below we will see three techniques for determining n and m.

Technique 1, Calculated Distances Equal

The first technique described is the easiest to understand, but it will turn out to be the least accurate. This first technique makes use of the fact that the measured distance is independent of the wavelength used. It follows from equations 0, 0, and 0 that if m = n + Δ

$$n\,\lambda1 + \phi[\lambda1]\,\lambda1 = (n + \Delta)\,\lambda2 + \phi[\lambda2]\,\lambda2$$

where n and Δ are integers.

Solving for n we have

$$n = -\frac{-\Delta\,\lambda2 + \lambda1\,\phi[\lambda1] - \lambda2\,\phi[\lambda2]}{\lambda1 - \lambda2}$$

We must find Δ such that n is an integer. In general there will be more than one Δ that will solve the above equation to within the accuracy to which we know the wavelengths and measured fractional fringe. If the approximate length is known, the range for n and Δ are known. In the absence of any knowledge concerning the range for Δ the best that can be done is to pick the smallest value of Δ for which the above equation is valid. Δ can be obtained from the following if tolerance is an acceptable amount for n to differ from being an integer. In phase shifting interferometry and good environmental conditions a reasonable value for tolerance might be 0.001.

```
Δ = 0;
While[Abs[FractionalPart[ n]] > tolerance, Δ = Δ + 1];
Print[Δ];
```

In the While function Δ will keep increasing by 1 each time through the loop until the fractional part of n differs by less than the tolerance.

Of the three methods described in this paper, this technique has the lowest accuracy because errors in $\phi[\lambda1]$ and $\phi[\lambda2]$ are scaled by $\lambda/(\lambda1 - \lambda2)$, which can be a large number. Thus any small errors in $\phi[\lambda1]$ and $\phi[\lambda2]$ are magnified in the calculation of n.

Technique 2, Fractional Fringes

The second technique described is basically a computerized version of a method called exact fractions described by Michelson and Benoît in 1895[6]. In the exact fractions approach a length is measured by measuring the excess fractional fringes for two or more wavelengths. It is assumed that the length is known to within a few half wavelengths. For our example assume it is known that the length lies between 1209 and 1215 half-wavelengths of the red cadmium line. Let the excess fraction in red light be known to be 0.35. Then we know the actual length is 1209.35, 1210.35,...,1214.35. From this information the number of wavelengths of green light is calculated. A comparison of the calculated number of fractional fringes for green light with the measured number shows which of the calculated lengths is correct. Additional wavelengths can be used to increase the accuracy and reduce the chances of error. (This procedure is described in detail in reference 7). As described in the reference the technique did not work well with two beam interferometers because the fractional fringe could not be determined well enough. The technique worked much better with multiple beam fringes. Fortunately, phase-shifting interferometry provides the precision required to do the measurement well and computers make the calculation easy.

The first step is to calculate the fractional fringe for wavelength λ2 from Equation 0 for opdCalculatedλ1.

$$\text{calculated FractionalFringe}\lambda2 = \text{FractionalPart}\left[\frac{n\,\lambda1 + \phi[\lambda1]\,\lambda1}{\lambda2}\right]$$

If we had no error this should be equal to the measured fractional fringe for wavelength λ2. That is, in the absence of error

$$\phi[\lambda2] = \text{FractionalPart}\left[\frac{n\,\lambda1 + \phi[\lambda1]\,\lambda1}{\lambda2}\right]$$

This equation can be solved for n, and the opd can be calculated. In general there will be more than one n that will solve the above equation to within the accuracy to which we know the wavelengths and measured fractional fringe. If an approximate length is known, the range for n is known. Picking the smallest value of n for which the above equation is valid and letting tolerance be the acceptable difference between the calculated fractional fringe and the measured fractional fringe we could write

$$n = 0;$$

$$\text{While}\left[\text{Abs}\left[\text{FractionalPart}\left[\frac{n\,\lambda 1 + \phi[\lambda 1]\,\lambda 1}{\lambda 2}\right] - \phi[\lambda 2]\right] > \text{tolerance},\ n = n + 1\right];$$

$$\text{length} = n\,\lambda 1 + \phi[\lambda 1]\,\lambda 1;\ \text{Print}[n,\ \text{length}];$$

In the While function n will keep increasing by 1 each time through the loop until the calculated and fractional fringe for $\lambda 2$ differ by less than the tolerance.

Technique 3, Equivalent Wavelength

The third technique is equivalent to what was done above, except now the path lengths are determined in terms of a quantity called the equivalent wavelength, λeq, which is calculated from the two wavelengths $\lambda 1$ and $\lambda 2$, used in the measurement.

If we take the difference between the phases for the two wavelengths we have

$$\text{phase}\lambda 1 - \text{phase}\lambda 2 = 2\,\pi\left(\frac{1}{\lambda 1} - \frac{1}{\lambda 2}\right)\text{opd}$$

It is convenient to write this as

$$\text{phase}\lambda 1 - \text{phase}\lambda 2 = \frac{2\,\pi}{\lambda\text{eq}}\,\text{opd}$$

where

$$\boldsymbol{\lambda\text{eq} = \frac{\lambda 1\,\lambda 2}{\text{Abs}[\lambda 2 - \lambda 1]};}$$

We can write the fractional fringe difference for the two wavelengths as

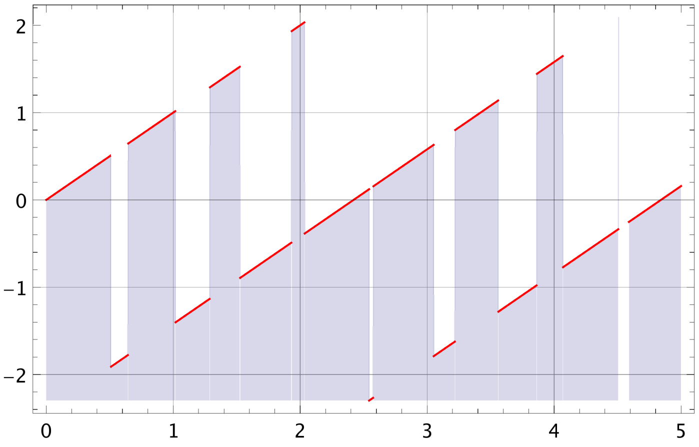
$$\phi[\lambda 1] - \phi[\lambda 2] = \frac{\text{opd}}{\lambda\text{eq}}$$

We can calculate

$$\boldsymbol{\text{opdInitial} := \lambda\text{eq}\,(\phi[\lambda 1] - \phi[\lambda 2])}$$

The plot of opdInitial shown below shows there are discontinuities.

```
λ1 = 0.5085824;      λ2 = 0.6438472;
Plot[opdInitial, {opd, 0, 5}, Filling → Bottom, Evaluate[plot2doptions]]
```



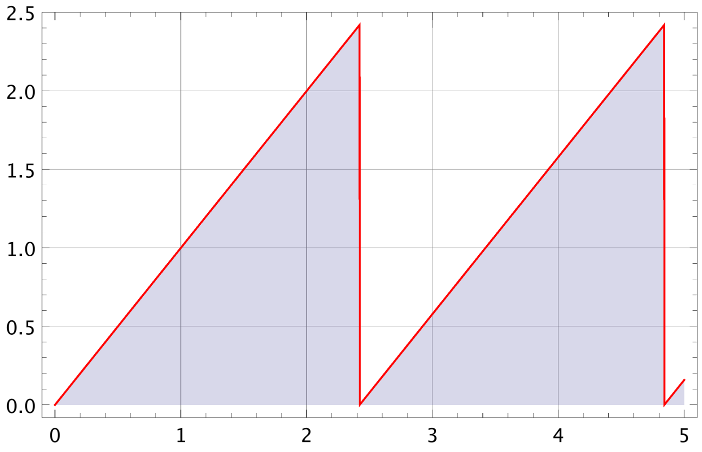
For this example

```
λeq = 2.42;
```

To take care of the discontinuities λeq must be added or subtracted to the appropriate data points. Since for this example $\lambda 2 > \lambda 1$ we will add λeq whenever the difference becomes negative.

```
opdCalculated := If[opdInitial < 0, opdInitial + λeq, opdInitial]
```

```
Plot[opdCalculated, {opd, 0, 5}, Filling → Bottom, Evaluate[plot2doptions]]
```



Now the phase difference repeats at intervals equal to λeq and we can measure path differences as large as λeq/2 without any ambiguities. In conventional single wavelength phase shifting interferometry the opd between adjacent data points must be less than $\lambda/2$ for no wavelength ambiguities in the opd. Using the equivalent wavelength approach the dynamic range is now increased such that now the opd between adjacent data points can be as large as λeq/2 for no wavelength ambiguities.

As pointed out in reference 12, while the above results are correct, and it is often convenient to use the concept of an λeq, this concept does not use all the known information since not only is $\phi[\lambda 1]$-$\phi[\lambda 2]$ known, but both $\phi[\lambda 1]$ and $\phi[\lambda 2]$ are known modulo 1. The following is one way to make use of this information.

Next we will calculate our first estimate of the integer number of wavelengths present for wavelength $\lambda 1$.

We can calculate

$$\texttt{n}\lambda\texttt{1Estimate} = \texttt{IntegerPart}\left[\frac{\texttt{opdCalculated}}{\lambda 1}\right];$$

Then our first guess of the opd is

$$\texttt{opdEstimate} = (\texttt{n}\lambda\texttt{1Estimate} + \phi[\lambda 1])\ \lambda 1$$

The correct value for the integer number of wavelengths present for λ1 is

$$\texttt{n}\lambda\texttt{1Correct} = \texttt{IntegerPart}\left[\frac{\texttt{opdCalculated} + \texttt{n}\ \lambda\texttt{eq}}{\lambda 1}\right]\ \text{where}\ \texttt{n} = \texttt{0, 1, 2, 3, ...}$$

and

$$\texttt{opdCorrect} = \left(\texttt{IntegerPart}\left[\frac{\texttt{opdCalculated} + \texttt{n}\ \lambda\texttt{eq}}{\lambda 1}\right] + \phi[\lambda 1]\right)\ \lambda 1$$

The problem is that we do not know the value of n. However, n can be found since the fractional fringe order number for λ2 calculated using the above length must be equal to the measured fractional fringe number. That is

$$\texttt{FractionalPart}\left[\frac{(\texttt{IntegerPart}[(\texttt{opdCalculated} + \texttt{n}\ \lambda\texttt{eq}) / \lambda 1] + \phi[\lambda 1])\ \lambda 1}{\lambda 2}\right] = \phi[\lambda 2]$$

The problem now is to determine n such that the above is correct. The following shows one way of doing this.

$$\begin{aligned}
&\texttt{n} = \texttt{0};\\
&\texttt{While}\Big[\texttt{Abs}\Big[\\
&\quad\texttt{FractionalPart}\left[\frac{(\texttt{IntegerPart}[(\texttt{opdCalculated} + \texttt{n}\ \lambda\texttt{eq}) / \lambda 1] + \phi[\lambda 1])\ \lambda 1}{\lambda 2}\right] - \phi[\lambda 2]\Big] >\\
&\ \ \texttt{tolerance, n} = \texttt{n} + \texttt{1}\Big];\ \texttt{Print[n]};
\end{aligned}$$

Thus, if the noise is sufficiently low the opd difference between adjacent data points can be larger than λeq/2. However, if the noise is low enough to increase the dynamic range between adjacent data points to more than λeq/2, it should be possible to use different wavelength pairs to increase λeq and obtain the same increased dynamic range.

Summary

Several techniques exist for increasing the dynamic range of an interferometric test by using two or more wavelengths while still keeping the accuracy of a single wavelength. While these techniques have been known for some time, their usefulness depend upon the low noise, high accuracy, measurements made possible using modern electronics, computers, and software.

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