Coherence and Source Requirements

- Coherence Time
- Coherence Length
- Partial Coherence
- Temporal Coherence
- Spatial Coherence

James C. Wyant

Coherence

- Temporal Coherence
  - A source is never strictly monochromatic
- Spatial Coherence
  - A source is never truly a point source

James C. Wyant

Coherence Time

Maximum transit time difference for good contrast fringes

$$E = A\cos(kx - 2\pi\nu t)$$

$$\Delta\nu = \text{Frequency Bandwidth}$$

For good contrast fringes

$$\left|2\pi\bar{\nu}\Delta t - 2\pi(\bar{\nu} + \Delta\nu)\Delta t\right| < 2\pi$$

$$\Delta t < \frac{1}{\Delta\nu}$$

Coherence time

$$\Delta t = \tau_c = \frac{1}{\Delta\nu}$$

James C. Wyant

Coherence Length

Distance light travels during coherence time

Coherence length

$$l_c = c\tau_c = \frac{c}{\Delta\nu} = \frac{\bar{\lambda}^2}{\Delta\lambda}$$

James C. Wyant

Partial Coherence

$$I = \langle (E_1 + E_2)(E_1 + E_2)^* \rangle$$
$$= \langle |E_1|^2 + |E_2|^2 + E_1 E_2^* + E_1^* E_2 \rangle$$
$$= I_1 + I_2 + 2\,\mathrm{Re}\langle E_1 E_2^* \rangle$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| e^{i\phi_{12}(\tau)}$$

Let

$$\phi_{12}(\tau) = \alpha_{12}(\tau) - \overline{\omega}\tau$$

Then

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma_{12}(\tau)| \cos(\alpha_{12}(\tau) - \overline{\omega}\tau)$$

James C. Wyant

Degree of Coherence

$|\gamma_{12}(\tau)|$ is called degree of coherence

Effect of $|\gamma_{12}(\tau)|$ is to reduce fringe contras

$|\gamma_{12}(\tau)| = 1,$ complete coherence

$0 < |\gamma_{12}(\tau)| < 1,$ partial coherence

$|\gamma_{12}(\tau)| = 0,$ complete incoherence

Effect of $\phi_{12}(\tau)$ is to shift the fringes

James C. Wyant

Fringe Visibility

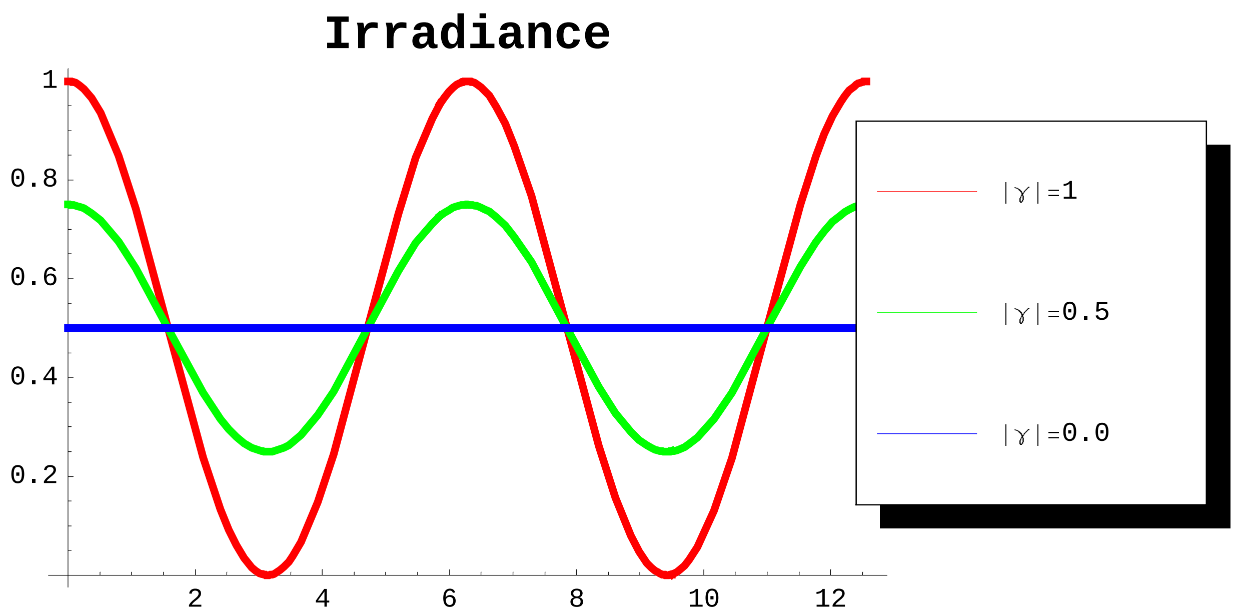
$i = i1 + i2 + 2\sqrt{i1\,i2}\,\text{Abs}[\gamma12[\tau]]\,\text{Cos}[\alpha12[\tau] - \bar{\omega}\,t];$

$\text{visibility} = \frac{\text{imax} - \text{imin}}{\text{imax} + \text{imin}};$

$\text{visibility} = 2\,\frac{\sqrt{i1\,i2}}{i1 + i2}\,\text{Abs}[\gamma12[\tau]];$

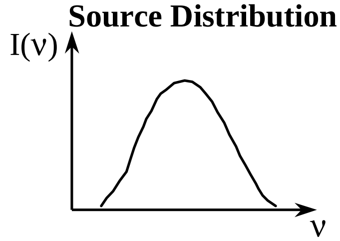
Let $i1 = i2$ then

$\text{visibility} = \text{Abs}[\gamma12[\tau]];$



Temporal Coherence

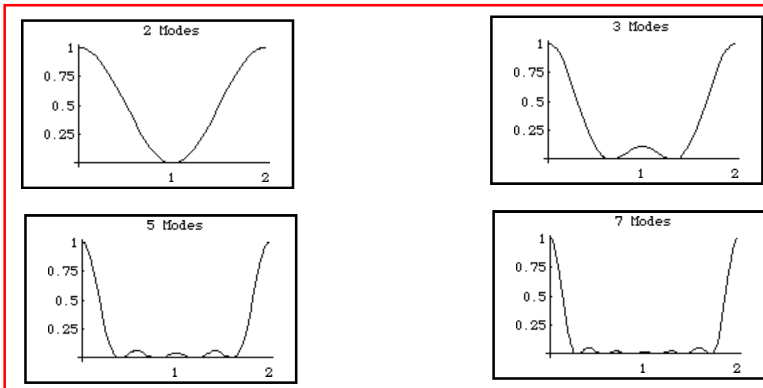
Source Distribution



$$\mathrm{Re}[\gamma(\tau)] = \frac{\int_0^\infty I(\nu)\cos(2\pi\nu\tau)\,d\nu}{\int_0^\infty I(\nu)\,d\nu}$$

James C. Wyant

Fringe Visibility as Function of Path Difference for Laser Having N Longitudinal Modes

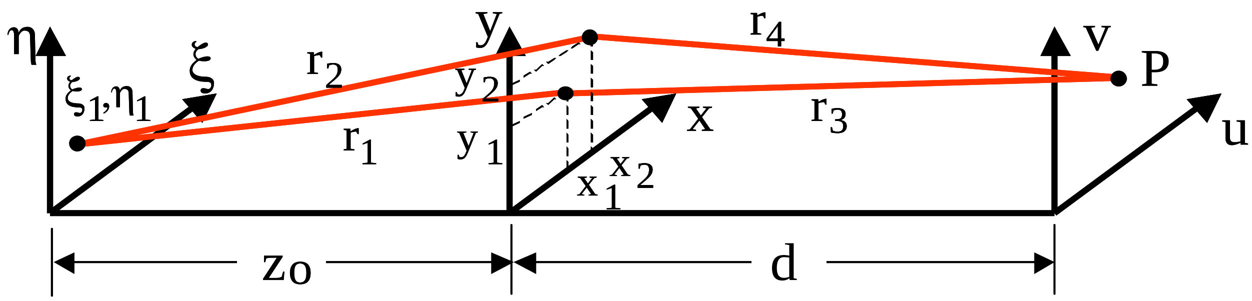


x axis is path difference in units of the laser cavity length

James C. Wyant

5.4.2 Van Cittert ZernikeTheorem

Broad Source - Narrow Spectrum



$$i[p] = i_1 + i_2 + 2\sqrt{i_1 i_2}\,\mathrm{Cos}[k\,(r_1 - r_2) + k\,(r_3 - r_4)]$$

$$r_1 = \sqrt{(y_1 - \eta_1)^2 + (x_1 - \xi_1)^2 + z_o^2}\,; \qquad r_1 \approx z_o + \frac{(y_1 - \eta_1)^2}{2\,z_o} + \frac{(x_1 - \xi_1)^2}{2\,z_o}$$

$$r_2 = \sqrt{(y_2 - \eta_1)^2 + (x_2 - \xi_1)^2 + z_o^2}\,; \qquad r_2 \approx z_o + \frac{(y_2 - \eta_1)^2}{2\,z_o} + \frac{(x_2 - \xi_1)^2}{2\,z_o}$$

if $\eta_1^2 << z_o\,\lambda$ and $\xi_1^2 << z_o\,\lambda$ then for general source point $\xi,\ \eta$

$$r_1 \approx z_o + \frac{y_1^2 - 2\,y_1\,\eta}{2\,z_o} + \frac{x_1^2 - 2\,x_1\,\xi}{2\,z_o}\,; \qquad r_2 \approx z_o + \frac{y_2^2 - 2\,y_2\,\eta}{2\,z_o} + \frac{x_2^2 - 2\,x_2\,\xi}{2\,z_o}$$

$$i[p] = i_1 + i_2 + 2\sqrt{i_1 i_2}\,\mathrm{Cos}\left[k\left(\frac{x_1^2 - x_2^2}{2\,z_o} - \frac{(x_1 - x_2)\,\xi}{z_o} + \frac{y_1^2 - y_2^2}{2\,z_o} - \frac{(y_1 - y_2)\,\eta}{z_o}\right) + k\,(r_3 - r_4)\right]$$

Let

$$a = \frac{x_1^2 - x_2^2}{2\,z_o} - \frac{(x_1 - x_2)\,\xi}{z_o} + \frac{y_1^2 - y_2^2}{2\,z_o} - \frac{(y_1 - y_2)\,\eta}{z_o}$$

Let $i_1 = i_2$ and have an extended source $i_1 = i[\xi,\ \eta]$

$$i[p] = \left(2\int i[\xi,\ \eta]\,d\xi\,d\eta\right)\left(1 + \frac{\int i[\xi,\ \eta]\,\mathrm{Cos}[k\,a + k\,(r_3 - r_4)]\,d\xi\,d\eta}{\int i[\xi,\ \eta]\,d\xi\,d\eta}\right)$$

$$i[p] = \left(2\int i[\xi,\ \eta]\,d\xi\,d\eta\right)(1 + \mathrm{Re}[\gamma_{12}[\tau]])$$

$$\gamma_{12}[\tau] = \frac{(< e)_1[t]\,e_2{}^*[t + \tau] >}{\sqrt{< e_1\,e_1{}^* > \ < e_2\,e_2{}^* >}} = \frac{\int i[\xi,\ \eta]\,E^{I\,(k\,a + k\,(r_3 - r_4))}\,d\xi\,d\eta}{\int i[\xi,\ \eta]\,d\xi\,d\eta}$$

Let

$$x = x_2 - x_1;\quad y = y_2 - y_1;\qquad \alpha_x = \frac{\xi}{z_o};\qquad \alpha_y = \frac{\eta}{z_o};$$

$$\gamma_{12}[\tau] = E^{Ik\,(r_3 - r_4)}\ \frac{E^{I\,k\,(x_1^2 - x_2^2 + y_1^2 - y_2^2)/2\,z_o}\int i[\alpha_x,\ \alpha_y]\,E^{Ik\,(\alpha_x\,x + \alpha_y\,y)}\,d\alpha_x\,d\alpha_y}{\int i[\alpha_x,\ \alpha_y]\,d\alpha_x\,d\alpha_y}$$

$$\gamma_{12}[\tau] = \text{Abs}[\gamma_{12}[\tau]]\ E^{I\,\phi_{12}[\tau]}$$

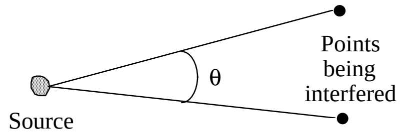
$$\phi_{12}[\tau] = \alpha_{12}[\tau] - \bar{\omega}\,\tau$$

$$\mu_{12} = \gamma_{12}[0] = \frac{E^{I\,k\,(x_1{}^2 - x_2{}^2 + y_1{}^2 - y_2{}^2)/2\,z_o} \int i[\alpha_x,\ \alpha_y]\ E^{\text{I}k\,(\alpha_x\,x + \alpha_y\,y)}\ d\alpha_x\,d\alpha_y}{\int i[\alpha_x,\ \alpha_y]\ d\alpha_x\,d\alpha_y}$$

Spatial Coherence

$I(\xi, \eta)$ is source distribution

θ_x and θ_y are angular distance between points being interfered as measured from the plane of the source

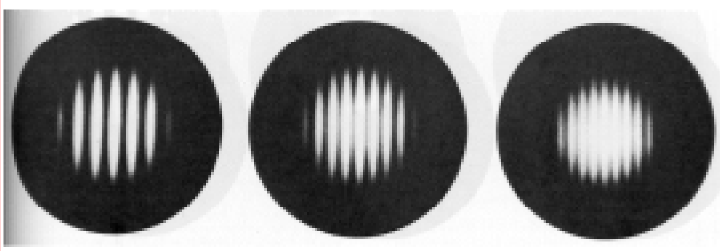


Fringe Visibility

$$V = |\gamma_{12}| = \left| \frac{\iint I(\xi, \eta) e^{ik(\xi\theta_x + \eta\theta_y)} d\xi d\eta}{\iint I(\xi, \eta) d\xi d\eta} \right|$$

James C. Wyant

Interference Fringes obtained Using Partially Coherent Light



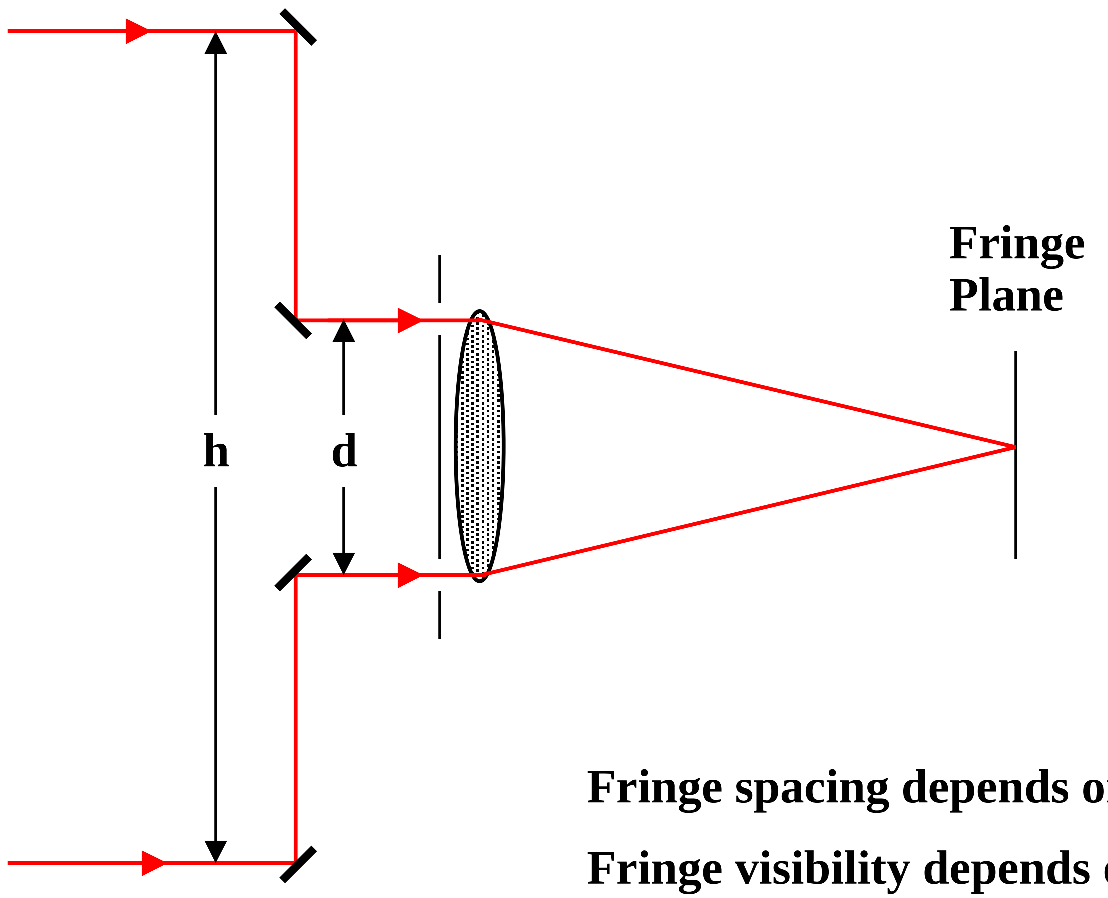
V=0.593 **V=0.361** **V=0.146**

Source size fixed, distance between points interfered varied.

From B.J. Thompson and E. Wolf, *J. Opt. Soc. Am*. **47**, 895 (1957)

James C. Wyant

Michelson Stellar Interferometer



Fringe spacing depends on d.

Fringe visibility depends on h.

Reprinted from **APPLIED OPTICS,** Vol. 17, page 1853, June 15,1978
Copyright 1978 by the Optical Society of America and reprinted by permission of the copyright owner.

Fringe localization

James C. Wyant

University of Arizona, Optical Sciences Center, Tucson, Arizona 85721.
Received 31 March 1978.
0003-6935/78/0615-1853$0.50/0.
© 1978 Optical Society of America.

The technique for locating the region of fringe localization for an interferometer used with a spatially incoherent source is well known. For each light ray going into an interferometer, two or more rays will emerge. The fringes are localized in the region where these emerging rays derived from a single input ray intersect. The proof most often given for the above result involves a large amount of geometry and algebra.[1,2] The purpose of this short Letter is to point out that the proof follows directly from the van Cittert-Zernike theorem.

The van Cittert-Zernike theorem[3] states that for a quasi-monochromatic spatially incoherent source, the magnitude of the degree of spatial coherence $|\mu_s|$ between two points $P(x_1,y_1)$ and $P(x_2,y_2)$ is given by the magnitude of the normalized Fourier transform of the intensity distribution of the source. That is,

$$|\mu_s| = \left| \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x',y') \exp i\left\{\frac{2\pi}{\bar{\lambda}}[\theta_x x' + \theta_y y']\right\} dx'dy'}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x',y')dx'dy'} \right|, \quad (1)$$

where $I(x',y')$ is the intensity distribution of the source, $\theta_x = (x_2 - x_1/R$, and $\theta_y = (y_2 - y_1)/R$. x_1, y_1, x_2, y_2, and R are illustrated in Fig. 1. In arriving at Eq. (1) it is assumed that the separation R is much greater than both the extent of the source and $[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}$.

It can be easily shown that the magnitude of the coherence function is a maximum when $\theta_x = \theta_y = 0$.[4] If in an interferometer the light at P_1 is interfered with the light at P_2, the fringe visibility is equal to the value of $|\mu s|$ given by Eq. (1). Since for maximum fringe visibility $\theta_x = \theta_y = 0$, the fringe viewing surface for an interferometer should be the surface that is the locus of points of intersection of the rays which originate from one incident ray coming from the source. That is, for maximum fringe visibility, light propagating in the direction of a single ray should interfere with itself. If this surface for ray intersection depends upon the particular ray coming from the source, there may be no region for which good visibility fringes exist. However, if the surface does not depend upon the ray selected, good visibility fringes will exist if the fringes are observed on a surface conjugate to this ray intersection surface.

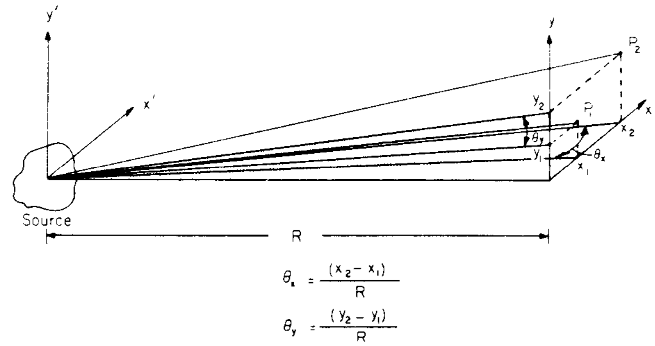


Fig. 1. Illustrating the van Cittert-Zernike theorem.

References

1. M. Born and E. Wolf, *Principles* of *Optics* (Pergamon, New York, 1964), p. 291.
2. M. Francon, *Optical Interferometry* (Academic, New York, 1966), p. 66.
3. R. J. Collier, L. Burckhardt, and L. Lin, *Optical Holography* (Academic, New York, 1971), p. 140.
4. J. D. Gaskill, *Linear Systems, Fourier Transforms and Optics* (Wiley, New York, 1978), p. 174.