Basic Classical Interferometers

- Plane Parallel Plate
- Fizeau
- Michelson
- Twyman-Green
- Mach-Zehnder
- Lateral Shear
- Radial Shear

Plane Parallel Plate - Point Source

Non Localized fringes

By symmetry, fringes in plane parallel to plate are circular about normal SN
**Plane Parallel Plate - Extended Source (Path Difference)**

**Optical path difference**

\[ \Delta l = (AB + CB)n - n'AD \]

\[ AB = CB = \frac{d}{\cos \theta} \]

\[ AD = AC \sin \theta'; AC = 2d \tan \theta \]

\[ = 2d \tan \theta \sin \theta' \]

\[ n' \sin \theta' = n \sin \theta \]

\[ \Delta l = \frac{2nd \cos \theta}{\cos \theta} - 2n' \frac{\sin \theta \sin \theta}{\cos \theta} \]

\[ = \frac{2nd}{\cos \theta} [1 - \sin^2 \theta] \]

**Plane Parallel Plate - Extended Source (Haidinger Fringes)**

**Fringes localized at infinity**

\[ \delta = \frac{2\pi}{\lambda} 2nd \cos \theta \pm \pi \]
Haidinger Fringes

Plane Parallel Plate - Extended Source
(Transmitted Light)

Low reflectance surfaces give low visibility fringes. Transmitted and reflected fringe patterns are complimentary.
Fizeau Fringes - Point Source
(1862)

\[ \delta = \frac{2\pi}{\lambda} 2nd \cos \theta \pm \pi \]

- \( d \) is film thickness (function of position)
- \( \theta \) is angle within film (function of position)

Non localized fringes

Fizeau Fringes - Broad Source
(1862)

Fringes localized near film

Near the film rays from source points see approximately same \( d \)

Variations in \( \cos \theta \) reduced if
- a) camera has small aperture focused on film
- b) if \( \theta \approx 0 \), \( \cos \theta \approx 1 \) for moderate spread in \( \theta \)
Classical Fizeau Interferometer

![Diagram of Classical Fizeau Interferometer]

Typical Interferogram obtained using Fizeau Interferometer

![Typical Interferogram Image]
Relationship between Surface Height Error and Fringe Deviation

Surface height error = \( \frac{\lambda}{2} \left( \frac{\Delta S}{S} \right) \)

Fizeau Fringes

For a given fringe the separation between the two surfaces is a constant.

Height error = \( \frac{\lambda}{2} \left( \frac{\Delta S}{S} \right) \)
Newton’s Rings

For m\th dark fringe from center

\[ d_m = m\frac{\lambda}{2} \quad \rho_m \approx \sqrt{m\lambda R} \]

Soap Bubbles and Oil Films

For bright fringe

\[ \frac{2\pi}{\lambda} 2nd \cos \theta + \pi = m2\pi \]

If \( d > > \lambda \), \( m \) varies greatly for change in \( \lambda \).
If \( d = \text{few} \lambda \), \( m \) varies slowly with \( \lambda \).

Therefore, with thin films see color fringes.
Color changes with variations in thickness and \( \theta \).
**Michelson Interferometer (1881)**

[Image of Michelson Interferometer diagram]

**Michelson Interferometer (Fringes of Equal Inclination - Haidinger)**

[Image of Michelson Interferometer diagram with equations]

Bright fringe when \(2d \cos \theta = m\lambda\)
Michelson Interferometer
(Fringes of Equal Thickness)

Michelson Interferometer Fringes

Upper row - Fringes of Equal Inclination
Lower row - Fringes of Equal Thickness
Path differences increases outward from the center
White Light Fringes

Twyman-Green Interferometer
(Flat Surfaces)
**Mach-Zehnder Interferometer**

Testing samples in transmission

**Lateral Shear Interferometry**

Measures wavefront slope

Shear Plate

Source

Collimator Lens

Interferogram

Shear $= \Delta x$
### Lateral Shear Fringes

\[ \Delta W(x, y) \text{ is wavefront being measured} \]

Bright fringe obtained when

\[ \Delta W(x + \Delta x, y) - \Delta W(x, y) = m\lambda \]

\[
\left( \frac{\partial \Delta W(x, y)}{\partial x} \right)_{\text{Average over \ shear distance}} \cdot (\Delta x) = m\lambda
\]

**Measures average value of slope over shear distance**

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### Collimation Measurement

- **No wedge in shear plate**
  - Not collimated
  - Collimated (one fringe)

- **Vertical wedge in shear plate**
  - Not collimated
  - Collimated
Radial Shear Interferometry

Wavefront is interfered with expanded version of itself

\[
R = \frac{S_1}{S_2}
\]

Analysis of Radial Shear Interferograms

Wavefront being measured

\[
\Delta W(\rho, \theta) = W_{020}\rho^2 + W_{040}\rho^4 + W_{131}\rho^3 \cos \theta + W_{222}\rho^2 \cos^2 \theta
\]

Expanded beam can be written

\[
\Delta W(R\rho, \theta) = W_{020}(R\rho)^2 + W_{040}(R\rho)^4 + W_{131}(R\rho)^3 \cos \theta + W_{222}(R\rho)^2 \cos^2 \theta
\]

Hence, a bright fringe is obtained whenever

\[
\Delta W(\rho, \theta) - \Delta W(R\rho, \theta) = W_{020}\rho^2(1 - R^2) + W_{040}\rho^4(1 - R^4) + W_{131}\rho^3(1 - R^3) \cos \theta + W_{222}\rho^2(1 - R^2) \cos^2 \theta
\]

Same as Twyman-Green if divide each coefficient by \((1 - R^n)\)
Radial Shear Interferogram

- Variable Sensitivity Test
- Large shear - results same as for Twyman-Green
- Small shear - Low sensitivity test