Basic Interference and Classes of Interferometers

- **Basic Interference**
  - –Two plane waves
  - –Two spherical waves
  - –Plane wave and spherical wave
- **Classes of Interferometers**
  - –Division of wavefront
  - –Division of amplitude

Optical Detectors Respond to Square of Electric Field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_1\hat{a}_1 e^{i(\omega_1 t+\alpha_1)} + E_2\hat{a}_2 e^{i(\omega_2 t+\alpha_2)}$$

$$\begin{aligned}\bar{I} &= \text{Constant}\left|\vec{E}_1 + \vec{E}_2\right|^2 \\ &= \text{Constant}\left[\mathrm{E}_1^2 + E_2^2 + 2E_1E_2(\hat{a}_1\cdot\hat{a}_2)\cos[(\omega_1-\omega_2)t+\alpha_1-\alpha_2]\right]\end{aligned}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1\cdot\hat{a}_2)\cos[(\omega_1-\omega_2)t+\alpha_1-\alpha_2]$$

Irradiance at each point varies cosinusoidally with time at the difference frequency

Interference Fringes

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1 \cdot \hat{a}_2)\cos[(\omega_1 - \omega_2)t + \alpha_1 - \alpha_2]$$

Let $\omega_1 = \omega_2$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1 \cdot \hat{a}_2)\cos(\alpha_1 - \alpha_2)$$

Bright interference fringe

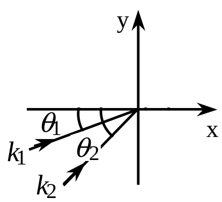
$$\alpha_1 - \alpha_2 = 2\pi m$$

Dark interference fringe

$$\alpha_1 - \alpha_2 = 2\pi\left(m + \frac{1}{2}\right)$$

Optics 505 - James C. Wyant Page 3 of 26

Interference of Two Plane Waves



$$\vec{E}_1 = E_1 e^{i\left(\vec{k}_1 \cdot \vec{r} - \omega t + \phi_1\right)}\hat{a}_1$$
$$\vec{E}_2 = E_2 e^{i\left(\vec{k}_2 \cdot \vec{r} - \omega t + \phi_2\right)}\hat{a}_2$$

$$\vec{k}_1 = k\left(\cos\theta_1\,\hat{i} + \sin\theta_1\,\hat{j}\right)$$
$$\vec{k}_2 = k\left(\cos\theta_2\,\hat{i} + \sin\theta_2\,\hat{j}\right)$$
$$\vec{r} = x\,\hat{i} + y\,\hat{j}\,, \quad k = \frac{2\pi}{\lambda}$$

$$\alpha = \alpha_1 - \alpha_2 = \vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r} + \phi_1 - \phi_2$$

Let $\phi_1 = \phi_2$

Bright fringe

$$\alpha = 2\pi m$$
$$= k[x(\cos\theta_1 - \cos\theta_2) + y(\sin\theta_1 - \sin\theta_2)]$$

Dark fringe

$$\alpha = 2\pi\left(m + \frac{1}{2}\right)$$

Optics 505 - James C. Wyant Page 4 of 26

Fringe Spacing

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1 \cdot \hat{a}_2)\cos(\alpha_1 - \alpha_2)$$

Bright fringe

$$\alpha = \alpha_1 - \alpha_2 = 2\pi m = k[x(\cos\theta_1 - \cos\theta_2) + y(\sin\theta_1 - \sin\theta_2)]$$

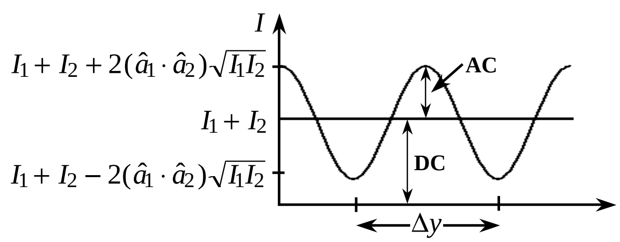
Straight equi-spaced fringes

Look in x=0 plane

Fringe spacing

$$\Delta y = \frac{\lambda}{\sin\theta_1 - \sin\theta_2}$$

Fringe Visibility

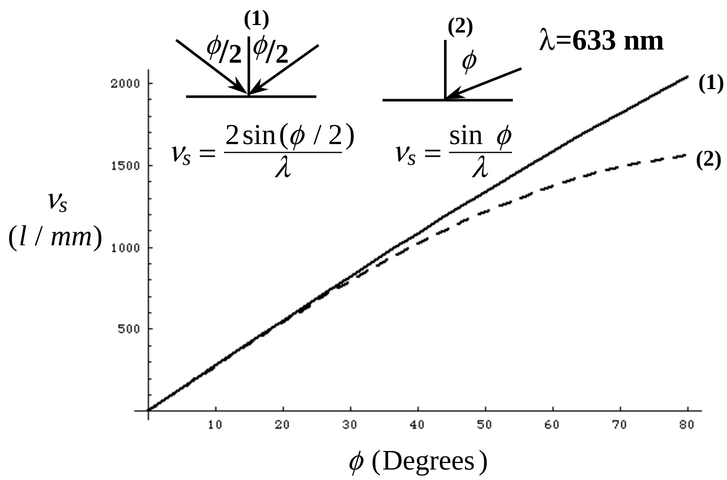


Fringe Visibility $= V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

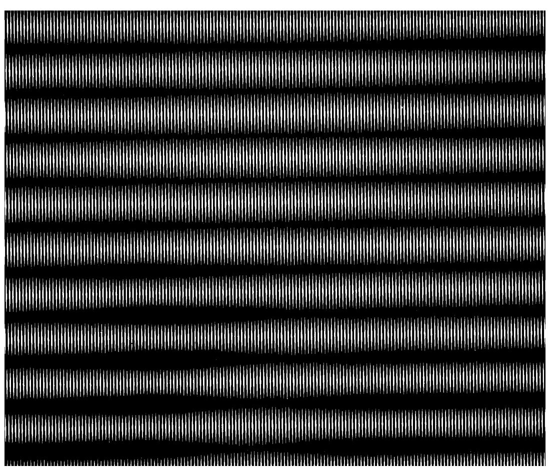
if $\hat{a}_1 \cdot \hat{a}_2 = 1$

$$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{AC}{DC}$$

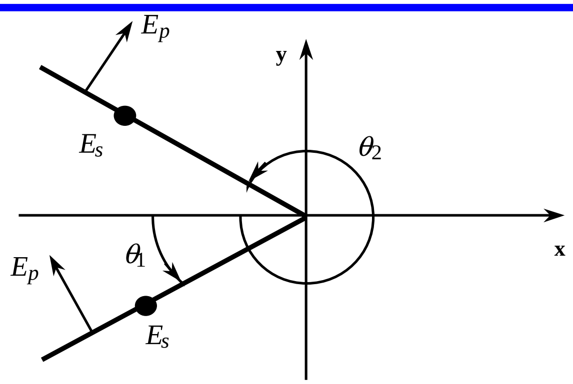
Fringe Spatial Frequency



Moiré Pattern - Two Plane Waves



Effect of Polarization Direction



Dependence of $\hat{a}_1 \cdot \hat{a}_2$ on angle for s and p polarization

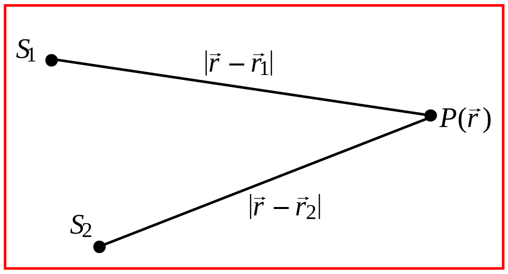
s polarization: $\hat{a}_1 \cdot \hat{a}_2 = 1$ for all angles

p polarization: $\hat{a}_1 \cdot \hat{a}_2$ depends upon angle

Optics 505 - James C. Wyant

Page 9 of 26

Interference of Two Spherical Waves



$$\vec{E}_1 = \hat{a}_1 \frac{B_1}{|\vec{r} - \vec{r}_1|} e^{i[k|\vec{r}-\vec{r}_1| - \omega t + \phi_1]}$$

$$\vec{E}_2 = \hat{a}_2 \frac{B_2}{|\vec{r} - \vec{r}_2|} e^{i[k|\vec{r}-\vec{r}_2| - \omega t + \phi_2]}$$

Optics 505 - James C. Wyant

Page 10 of 26

Two Spherical Waves - Fringe Shape

$$\vec{E}_1 = \hat{a}_1 \frac{B_1}{|\vec{r} - \vec{r}_1|} e^{i[k|\vec{r}-\vec{r}_1| - \omega t + \phi_1]} \qquad \sqrt{I_1} = \frac{B_1}{|\vec{r} - \vec{r}_1|} \approx \text{Constant}$$

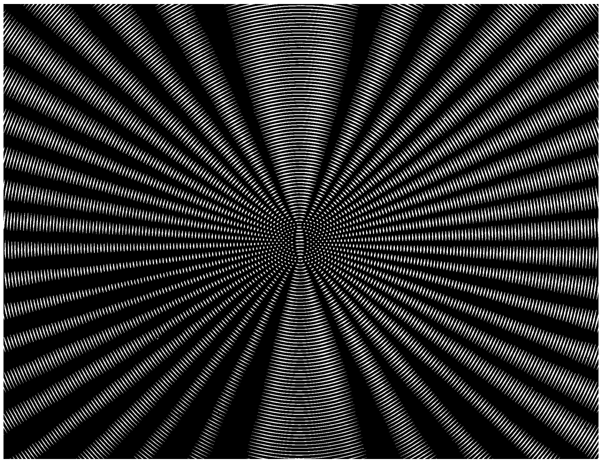
$$\vec{E}_2 = \hat{a}_2 \frac{B_2}{|\vec{r} - \vec{r}_2|} e^{i[k|\vec{r}-\vec{r}_2| - \omega t + \phi_2]} \qquad \sqrt{I_2} = \frac{B_2}{|\vec{r} - \vec{r}_2|} \approx \text{Constant}$$

$$\alpha = \frac{2\pi}{\lambda}\{|\vec{r} - \vec{r}_1| - |\vec{r} - \vec{r}_2|\} + \phi_1 - \phi_2$$
$$= \text{Constant for given fringe}$$

Hyperbolic Fringes

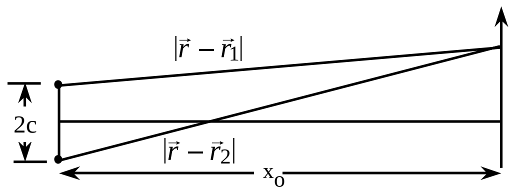
Optics 505 - James C. Wyant Page 11 of 26

Moiré Pattern - Spherical Waves



Optics 505 - James C. Wyant Page 12 of 26

Spherical Waves - Special Case #1

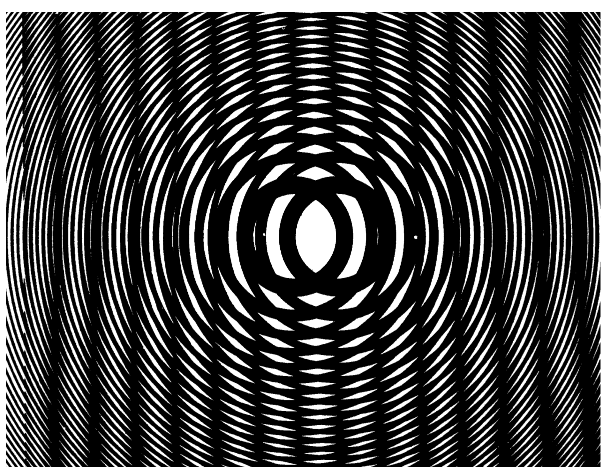


if $x_o >> 2c$ then

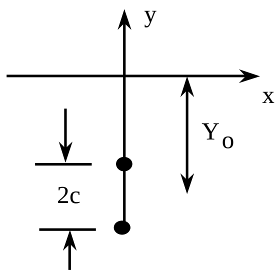
$$m\lambda = \frac{2cy}{x_o}$$

Same result as for two plane waves

Moiré Pattern - Straight Line Fringes



Spherical Waves - Special Case #2



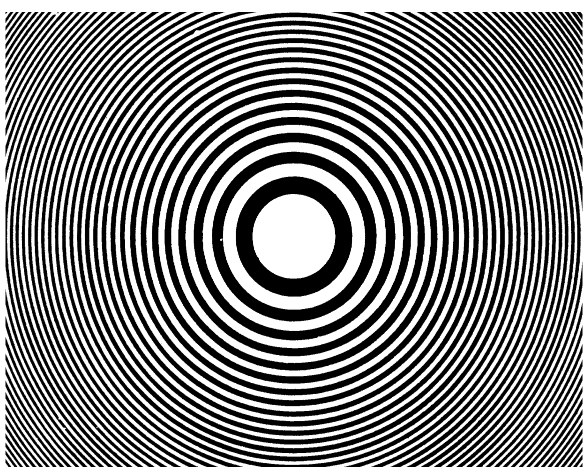
For bright fringe

$$m\lambda = |\vec{r} - \vec{r}_1| - |\vec{r} - \vec{r}_2|$$
$$= 2c - \frac{(x^2 + z^2)}{Y_o^2} c$$

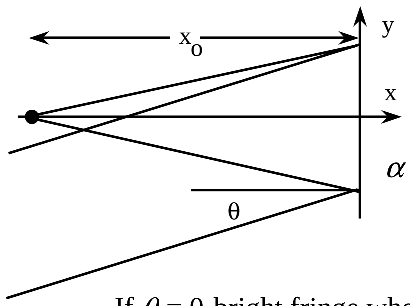
Fringes are concentric circles

$$r = \sqrt{\frac{2c - m\lambda}{c}}\, Y_o \quad \text{spatial frequency} = \frac{1}{\Delta r} = \frac{2c}{y_o^2 \lambda} r$$

Concentric Circular Fringes



Interference of Plane Wave and Spherical Wave



If $|x_o| >> |y|$ and $|z|$ then

$$\alpha = \frac{2\pi}{\lambda}\left\{y\sin\theta - x_o\left[1 + \frac{y^2+z^2}{2x_o^2}\right]\right\}$$

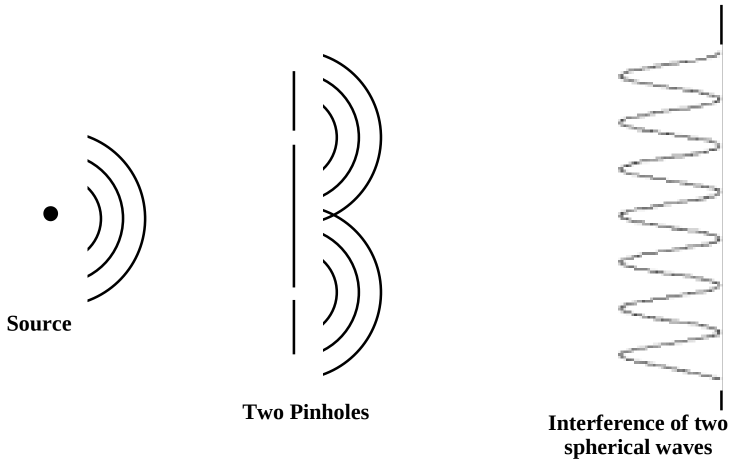
If $\theta = 0$ bright fringe when $\frac{y^2+z^2}{2x_o} = -(m\lambda + x_o) = m'\lambda$

Circular fringes of radius $r = \sqrt{2x_o m'\lambda}$

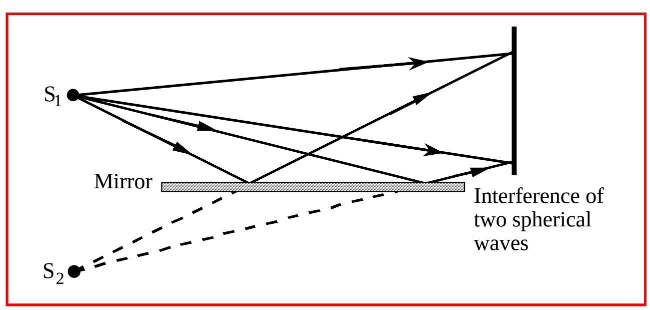
Two Basic Classes of Interferometers

- **Division of Wavefront**
- **Division of Amplitude**

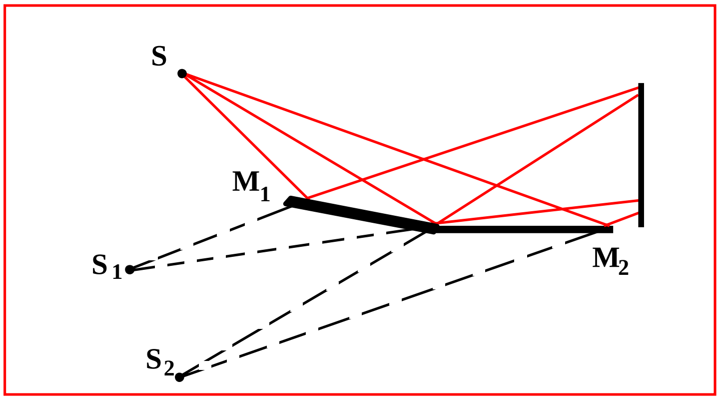
Division of Wavefront (Young's Two Pinholes)



Division of Wavefront (Lloyd's Mirror)



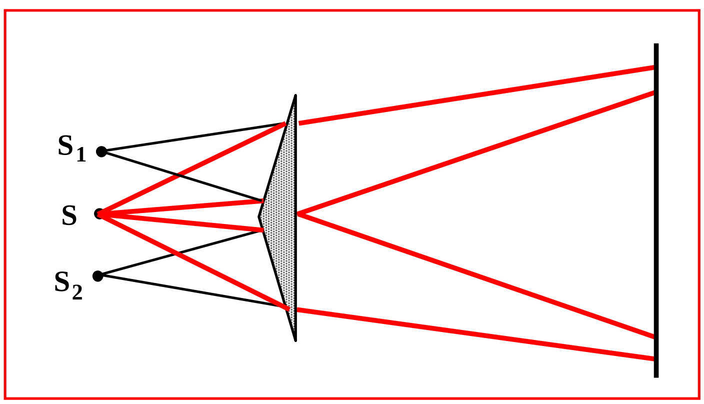
Division of Wavefront (Fresnel's Mirrors)



Optics 505 - James C. Wyant

Page 21 of 26

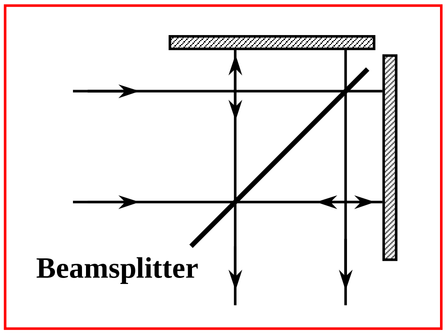
Division of Wavefront (Fresnel's Biprism)



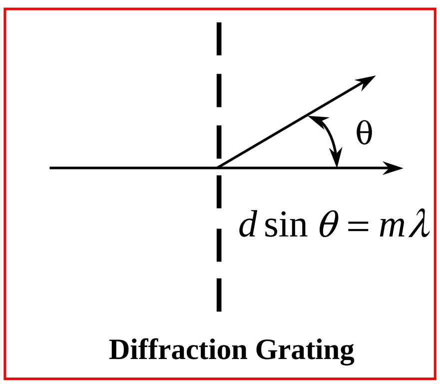
Optics 505 - James C. Wyant

Page 22 of 26

Division of Amplitude (Beamsplitter)



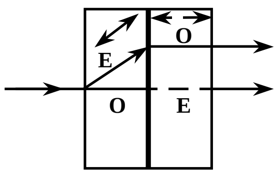
Division of Amplitude (Diffraction)



Division of Amplitude and Division of Wavefront

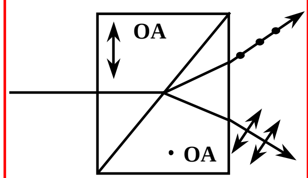
Polarization

Lateral Displacement



Savart Plate

Angular Displacement



Wollaston Prism

Division of Amplitude and Division of Wavefront

Plane Parallel Plate