Aspherical mirror testing using a CGH with small errors

Akira Ono and James C. Wyant

A method for reducing errors in aspherical mirror testing using a computer-generated hologram (CGH) is described. By using a modified filtering method the carrier frequency in the CGH can be reduced by two-thirds, and the resulting error due to distortion is only one-half of that of a conventional CGH. By adopting a Fizeau-type optical setup, only the surface quality of the reference affects the measured results.

I. Introduction

Computer-generated holograms (CGH) are very useful for testing aspherical lenses or mirrors. CGHs can be in-line[1] or off-axis.[2,3] The in-line CGH has an advantage of allowing the compensation of stronger aspheric contributions, but filtering of the spurious diffraction beams and optimizing the system are more difficult than for the off-axis CGH.[4] For these reasons, the off-axis CGH is more commonly used than the on-axis CGH.

In using the off-axis CGH for testing, a distortion in the CGH is one of the most significant error sources.[5–8] The maximum distortion is almost proportional to the maximum spatial frequency in the CGH. To eliminate spurious diffraction orders, in the commonly used testing setup, the CGH must have three times higher carrier frequency than the maximum spatial frequency f of the object wave.[5] If the carrier frequency can be lower, testing errors would be reduced.

Optical element aberration or distortion is another significant error source. Figure 1 shows a typical optical setup using the off-axis CGH for testing an aspherical mirror. If a beam splitter, a reference mirror, and a diverger lens have wave front distortions, ΔW_b, ΔW_r, and ΔW_d, respectively, a total wave distortion can be written statistically as

$$\Delta W = (2\Delta W_b)^2 + \Delta W_r^2 + \Delta W_d^2. \tag{1}$$

From Eq. (1) it is seen that each optical element (especially the beam splitter) has to have better quality than that of a mirror under test.

This paper is concerned with improving those problems by changing filtering means and adapting a Fiz eau-type interferometric optical setup.

II. Optical Setup

In the conventional CGH setup shown in Fig. 1, the 0th-order diffracted beam from the object wave, which contains the aberrations from the object under test, and the 1st-order diffracted beam from the reference wave are passed through the spatial filter. Figure 2 shows spatial frequency distributions of the diffracted beams from the reference wave by the CGH. Since the width of the sidelobe of the second-order diffracted beam is twice that of the 1st-order diffracted beam, for the filter to block the second-order beam the carrier frequency f_c has to be at least three times larger than the sidelobe f of the 1st-order beam.[5,7] The maximum frequency in the CGH will be $f_c + f = 4f$.

If the 1st-order diffracted beam of the reference wave has a wave front of $W(x,y)$, the Nth-order diffracted beam has the wave front of $NW(xy)$, and the aspherical mirror also can be tested using interference between the Nth-order beam of the reference wave and the $N-1$st-order beam of the object beam. Figure 3 shows a spatial frequency distribution of the diffracted beams from the objective wave. When a 0th-order beam of the reference wave and a -1st-order beam of the object wave are considered, the sidelobe width of both beams is almost zero, and adjacent diffracted beams have a width of f. In this case f is sufficient for the carrier frequency f_c in the CGH. The carrier frequency can be one-third, and the maximum frequency $(f_c + f)$ in the CGH can be one-half compared to the filtering method shown in Fig. 1. Consequently, a distortion in the CGH can be one-half of the former method.

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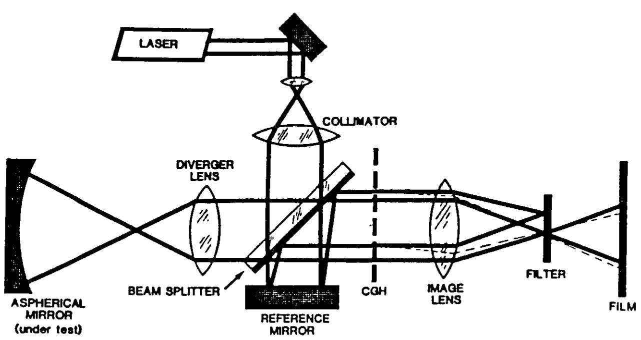

Fig. 1. Typical optical setup using an off-axis CGH for testing an aspherical mirror.

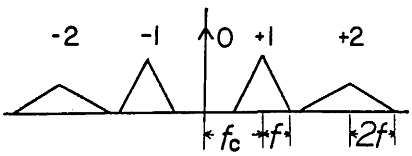

Fig. 2. Spatial frequency distribution of the diffracted beams from the reference wave produced by a CGH.

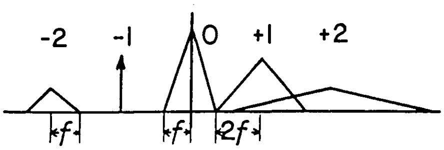

Fig. 3. Spatial frequency distribution of the diffracted beams from the object beam.

Figure 4 shows the optical setup. A spherical surface is used for a reference surface instead of a plane mirror. A filter passes the -1st-order diffracted beam from the object wave and the 0th-order diffracted beam from the spherical reference surface.

The optical setup shown in Fig. 4 is of the Fizeau interferometer type. Since in the Fizeau interferometer an object wave and a reference wave go through almost the same position at each optical element, aberrations of the beam splitter and a diverger lens do not have to be considered. Only the reference surface is required to have good quality. In this paper, a spherical surface having quality of better than $\lambda/10$ was used for the reference. Ichioka and Lohmann[1] also suggested the Fizeau-type optical setup for the CGH. The difference between their optical setup and the optical setup shown in Fig. 4 is not only that the latter uses an off-axis CGH but also that the CGH is located in the collimated beams. If the CGH is located in diverging beams like Ichioka's optical setup, the alignment error of the CGH positioning affects the testing result seriously.

By adopting the Fizeau-type optical setup, an additional advantage of taking a high visibility interferogram is obtained. In the optical setup shown in Fig. 4, the intensity of the -1st-order diffracted beam of the object wave I_o and intensity of the 0th-order diffracted beam of the reference wave I_r are written as

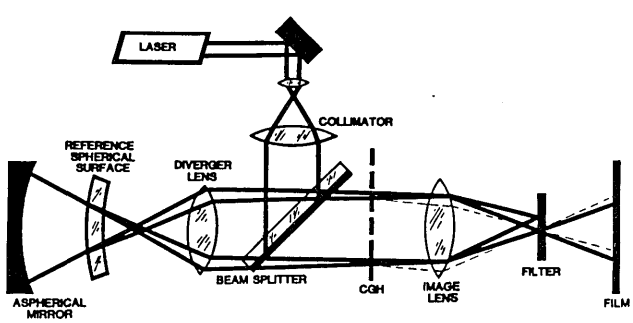

Fig. 4. Optical setup.

$$I_o = C \cdot R_0 \cdot (T_r)^2 \cdot D_{-1} \qquad I_r = C \cdot R_r \cdot D_0, \tag{2}$$

where C is a coefficient, R_0 and R_r are reflective ratios of an aspherical mirror under test and a reference surface, respectively, T_r is a transparent ratio, and D_0 and D_{-1} are CGH diffraction ratios of 0th-order and -1st-order, respectively. If there is no absorption in the reference surface, T_r is written as

$$T_r = 1 - R_r. \tag{3}$$

Since the best visibility in the interferogram can be obtained under the condition of $I_o = I_r$, from Eqs. (2) and (3) an optimum value of R_r can be written as

$$R_r = 1 + \frac{\frac{D_0}{D_{-1}}\sqrt{\left(\frac{D_0}{D_{-1}}\right)^2 + 4R_0\left(\frac{D_0}{D_{-1}}\right)}}{2R_0}. \tag{4}$$

D_0 and D_{-1} are defined by the procedure of making the CGH and usually $R_0 \simeq 1$.

For the results in this paper, the CGH was drawn using a plotter (Hewlett Packard 7225A) connected to a desk-top computer (Hewlett Packard model 85) and photoreduced using a conventional camera with a close-up lens. This is one of the easiest procedures for making the CGH, but diffraction efficiency is very low because a zero level in binary data is represented by a gray tone instead of black (see Sec. V). For example, the value of D_0/D_{-1} was 24 in our CGH. Substituting $D_0/D_{-1} = 24$ and $R_0 = 1$ into Eq. (4) yields

$$R_r = 0.038. \tag{5}$$

Since the reflective ratio of glass for a normal incident beam is almost the same value as Eq. (5), good visibility interference fringes could be obtained by using a noncoated spherical glass surface for the reference surface.

III. Experiment

For an application of the Fizeau-type CGH optical setup with modified filtering method, a $f/3$ 204-mm diam parabolic mirror was tested. Figure 5 shows the CGH pattern. The object wave from the parabolic mirror has an optical phase difference of $W_4 = 2\pi \times 23.44$ from the reference wave at the edge of the mirror. When adding a defocusing of $W_2 = -1.5W_4 = -2\pi \times 35.16$, the maximum spatial frequency f (the same as the sidelobe f) in the object wave is written as

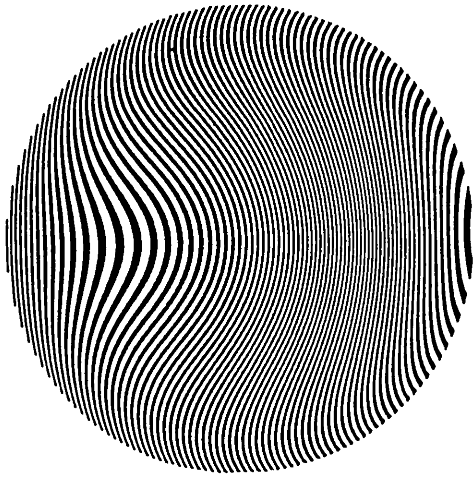

Fig. 5. CGH pattern used for aspherical mirror testing.

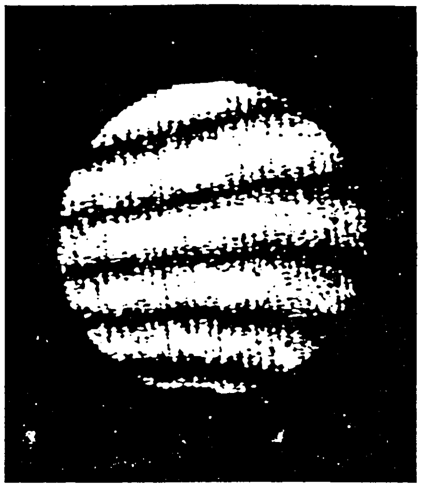

Fig. 6. Interferogram obtained measuring CGH distortion using the interferometric method.

$$f = 23.44/r, \tag{6}$$

where r is the radius of the CGH.[6] In ideal cases, the carrier frequency f_c can be equal to f, as mentioned above. Considering that the sidelobe might be broadened by distortions of diverger and image lens and the photofilm, $f = 36/r$ ($\simeq 1.5f$) was selected for filtering the spurious diffracted beam successfully. The maximum frequency is then $F + f_c = 59.44/r$. In the usual CGH system shown in Fig. 1, the value of f_c is bigger than $3f$, and the maximum frequency is bigger than $4f$ ($=93.76/r$). This means that the CGH distortion shown in Fig. 4 is $\sim$3/5 of the CGH distortion shown in Fig. 1

The CGH distortion was measured using the interferometric method[9,10] in advance of testing the parabolic mirror. In this method a grating was drawn and reduced to a photofilm with the same plotter and camera used to make the CGH; then the distortion was measured from an interferogram between $+N$th- and $-N$th-order diffracted beam by the grating. A wave distortion ΔW in the -1st-order diffracted beam by the CGH can be estimated as

$$\Delta W = \frac{f_m}{f_g} \cdot \frac{\Delta P}{2N} \cdot \lambda, \tag{7}$$

where f_m is the maximum spatial frequency in the CGH, f_g is a spatial frequency in the grating, ΔP is the maximum fringe distortion in the interferogram, and λ is the wavelength of the laser. Figure 6 shows a resultant interferogram between +3rd- and -3rd-order diffracted beam by a grating with a frequency of $75/r$. Since Fig. 5 has the maximum fringe distortion of $\sim$3/4 fringe, from Eq. (7) the -1st-order diffracted beam by the CGH was considered to have a maximum wave distortion of 0.1λ.

The CGH radius r is determined as

$$r = \frac{l_d}{2l_m} \cdot r_m, \tag{8}$$

where l_d and l_m are focal lengths of the diverger lenses and the aspherical mirror, respectively, and r_m is a radius of the mirror. A radius error Δr of the CGH causes a wave distortion of ΔW_r written as

$$\Delta W_r \simeq \Delta r \cdot f \cdot \lambda. \tag{9}$$

From Eqs. (6), (8), and (9), ΔW_r is rewritten as

$$\Delta W \simeq 23.44 \frac{2\Delta r \cdot l_m \cdot \lambda}{l_d \cdot r_m} . \tag{10}$$

Equation (10) shows that not only the CGH radius error Δr but also focal length l_d affect the wave distortion. It is difficult to know precisely the value of l_d for a commercial lens. A CGH with suitable radius can be selected from among many CGHs where each has a different radius from others, so that spherical aberration in an interferogram may be smallest. Here ten CGHs with an average radius of 8 mm and a radius difference of 0.05 mm between each were made. From Eq. (9) ΔW was estimated as 0.073λ. Consequently, in this experiment, mirror testing is considered to have an accuracy of 0.17λ even including a distortion of 0.05λ caused by a CGH alignment error. The accuracy of 0.17λ is fairly good taking account of the procedure for making CGHs and the quality of the optical elements used in this experiment.

Figure 7 shows an interferogram of testing a parabolic mirror mentioned above using the optical setup shown in Fig. 4. The maximum fringe distortion in the interferogram is 0.2 fringe. Considering testing accuracy, a wave reflected by this mirror has a distortion of $<0.2\lambda$. While the CGH was not bleached and the interference pattern was not taken by a high contrast film, the interferogram has good fringe visibility.

IV. Conclusion

For reducing errors in aspherical mirror testing using the CGH, a Fizeau-type optical setup and a modified filtering method were presented. By adopting a Fizeau-type optical setup, aberrations of a beam splitter and a diverger lens do not have to be considered, and high visibility interferograms can be obtained. By using the modified filtering method, the CGH can have

Fig. 7. Resultant interferogram for testing a parabolic mirror using the optical setup shown in Fig. 4.

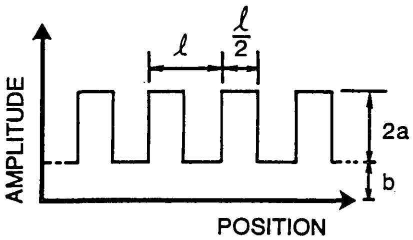

Fig. 8. Amplitude distribution for a gray tone rectangular grating.

about half of the usual CGH distortion. For an application, a parabolic mirror with a 204-mm diameter and 612-mm focal length was tested. Testing results with an error of $<0.17\lambda$ were obtained even using the easy process of the CGH and inexpensive optical elements

V. Appendix

The CGH taken into a photofilm has very low diffraction efficiency. The diffraction efficiency of a sinusoidal amplitude grating with a dc component was estimated in Ref. 11. From this, the diffraction efficiency of a rectangular amplitude grating with gray tone is estimated as follows. Figure 8 shows amplitude distribution in a rectangular grating. On a screen located far from the grating (in the Fourier domain), the amplitude distribution of a diffracted beam is written as

$$g(x) = b\delta(x) + a\ \mathrm{sinc}\frac{x}{2}\sum_{n=-\infty}^{\infty}\delta(x - n\pi). \tag{11}$$

From Eq. (11) intensities of the 0th- and 1st-order diffracted beams are obtained as

$$I(0) = (a+b)^2 = a^2 + b^2 + 2ab, \tag{12}$$
$$I(1) = a^2\left(\mathrm{sinc}\frac{\pi}{2}\right)^2 = 0.405a^2;\ \text{1st order.}$$

For example, assuming that intensities from the transparent and dark parts in the grating are 0.8 and 0.2, respectively, a and b are written as

$$a = \frac{\sqrt{0.8} - \sqrt{0.2}}{2} = 0.223, \qquad b = \sqrt{0.2} = 0.447. \tag{13}$$

By substituting Eq. (13) into Eq. (12), $I(0)$ and $I(1)$ are obtained as $I(0) = 0.45$ and $I(1) = 0.02$.

The intensity of 1st order is ~0.04 times weaker than that of the 0th order. These values show that a little transmitted beam from the dark part reduces diffraction efficiency a great deal. So, for obtaining high fringe visibility in the testing using the CGH, a little overexposure in the photoreduction process is recommended.

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When this work was done Akira Ono was a visiting scholar at the Optical Sciences Center of the University of Arizona.