Absolute testing of flats decomposed to even and odd functions

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ABSTRACT

This paper describes a method for measuring the absolute flatness of flats. A function in a Cartesian coordinate system can be expressed as the sum of even-odd, odd-even, even-even, and odd-odd functions. Three flats are measured at eight orientations; one flat is rotated 180°, 90°, and 45° with respect to another flat. From the measured results the even-odd and the odd-even functions of each flat are obtained first, then the even-even function is calculated. All three functions are exact. The odd-odd function is difficult to obtain. For the points on a circle centered at the origin, the odd-odd function has a period of 180° and can be expressed as a Fourier sine series. The sum of one half of the Fourier sine series is obtained from the 90° rotation group. The other half is further divided into two halves, and one of them is obtained from the 45° rotation group. Thus, after each rotation, one half of the unknown components of the Fourier sine series of the odd-odd function is obtained. The flat is approximateed by the sum of the first three functions and the known components of the odd-odd function. In the simulation, three flats (each is an OPD map obtained from a Fizeau interferometer) are reconstructed. The theoretical derivation and the simulating results are presented.

I. Introduction

In a Fizeau interferometer, two flats face each other and form a cavity. The interference fringes detected reveal the flatness of the cavity. In the traditional three-flat method,[1, 2] the flats are compared in pairs. By rotating the flats with respect to each other, the exact profiles along several diameters of each flat are obtained. A method with more flats and more combinations has been proposed.[3] However, with both methods, only the profiles along some straight lines can be solved, and the relationship of these profiles on a flat has not been defined. Several methods[4-7] have been proposed to measure the flatness of the entire surface. All these methods involve tremendous calculations in the least squares sense. Thus, the fine structure of the surface tends to disappear.

In this paper, we modify the three-flat method and use the properties of the odd and the even functions, especially when the function is rotated or flipped, to calculate the entire profile of a flat. Every point of a flat is obtained without using the least squares method.

II. Theory

A function F(x,y) in a Cartesian coordinate system can be expressed as the sum of an even-odd, an odd-even, an even-even, and an odd-odd function as follows.

$$F(x,y) = F_{ee} + F_{oo} + F_{oe} + F_{eo}, \tag{1}$$

where

$$\begin{aligned} F_{ee}(x,y) &= (F(x,y) + F(-x,y) + F(x,-y) + F(-x,-y))/4, \\ F_{oo}(x,y) &= (F(x,y) - F(-x,y) - F(x,-y) + F(-x,-y))/4, \\ F_{eo}(x,y) &= (F(x,y) + F(-x,y) - F(x,-y) - F(-x,-y))/4, \\ F_{oe}(x,y) &= (F(x,y) - F(-x,y) + F(x,-y) - F(-x,-y))/4. \end{aligned} \tag{2}$$

Because the flats are facing each other, one flat is flipped. If two flats are F(x,y) and G(x,y), and G(x,y) is flipped in x, then the measured OPD is equal to F(x,y)+G(-x,y). For convenience, we define two operators $[\ ]^x$ and $[\ ]^\theta$:

$$\begin{aligned} &\text{Flip in x} && [F(x,y)]^x = F(-x,y), \\ &\text{Rotate } \theta && [F(x,y)]^\theta = F(x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta). \end{aligned} \tag{3}$$

Thus, $[F(x,y)]^{180^\circ} = F(-x,-y)$. From Eq. (1),

$$\begin{aligned} [F(x,y)]^{180^\circ} &= F_{ee} + F_{oo} - F_{oe} - F_{eo}, \\ [F(x,y)]^x &= F_{ee} - F_{oo} - F_{oe} + F_{eo}. \end{aligned} \tag{4}$$

Figure 1 shows the three flats, A(x,y), B(x,y), and C(x,y), of a front view and a rear view. The coordinate systems indicate the orientations of the flats. Figure 2 shows the eight configurations and the corresponding measurements. In each configuration, the flat above is of a front view, and the one below is flipped in x and is of a rear view. In some configurations, one flat is rotated 180°, 90°, or 45° with respect to another flat. The equations of the eight configurations are

$$\begin{aligned} M_1 &= A + B^x, & M_2 &= A^{180^\circ} + B^x, \\ M_3 &= A^{90^\circ} + B^x, & M_4 &= A^{45^\circ} + B^x, \\ M_5 &= A^{180^\circ} + C^x, & M_6 &= B + C^x, \\ M_7 &= B^{90^\circ} + C^x, & M_8 &= B^{45^\circ} + C^x. \end{aligned} \tag{5}$$

Using Eqs. (1) and (4), M_1, M_2, and M_5 can be written as

$$\begin{aligned} M_1 &= A_{ee} + A_{oo} + A_{oe} + A_{eo} + B_{ee} - B_{oo} - B_{oe} + B_{eo}, \\ M_2 &= A_{ee} + A_{oo} - A_{oe} - A_{eo} + B_{ee} - B_{oo} - B_{oe} + B_{eo}, \\ M_5 &= A_{ee} + A_{oo} - A_{oe} - A_{eo} + C_{ee} - C_{oo} - C_{oe} + C_{eo}. \end{aligned} \tag{6}$$

Therefore, all the odd-even and the even-odd parts of the three flats can be obtained easily as given below:

$$\begin{aligned} A_{oe} + A_{eo} &= (M_1 - M_2)/2, \\ B_{oe} + B_{eo} &= [(M_1 - [M_1]^{180^\circ})/2 - (A_{oe} + A_{eo})]^x, \\ C_{oe} + C_{eo} &= [(M_5 - [M_5]^{180^\circ})/2 - [A_{oe} + A_{eo}]^{180^\circ}]^x. \end{aligned} \tag{7}$$

To cancel all the odd-even and the even-odd parts from M_1, M_5, and M_6, one can rotate the data by 180° using the rotation operation defined in Eg. (4). We define m_1, m_5, and m_6 as

$$m_1 = (M_1 + [M_1]^{180^o})/2 = A_{ee} + A_{oo} + B_{ee} - B_{oo},$$
$$m_5 = (M_5 + [M_5]^{180^o})/2 = A_{ee} + A_{oo} + C_{ee} - C_{oo}, \quad (8)$$
$$m_6 = (M_6 + [M_6]^{180^o})/2 = B_{ee} + B_{oo} + C_{ee} - C_{oo}.$$

It should be noted that m_1, m_5, and m_6 include only even-even and odd-odd functions. All the even-even parts can also be obtained easily as given below:

$$A_{ee} = (m_1 + m_5 - m_6 + [m_1 + m_5 - m_6]^x)/4,$$
$$B_{ee} = (m_1 + [m_1]^x - 2A_{ee})/2, \quad (9)$$
$$C_{ee} = (m_5 + [m_5]^x - 2A_{ee})/2.$$

Now all the odd-even, the even-odd, and the even-even parts of the three flats are obtained. If we subtract the known even-even parts from Eq. (8), it can be shown that the odd-odd parts can not be solved. Fortunately, for the points on a circle centered at the origin, an odd-odd function in a Cartesian coordinate system is an odd function of θ in a polar coordinate system and has a period of 180^o. Thus, $F_{oo}(x,y)$ can be expressed as a Fourier sine series as follows.

$$F_{oo}(x,y) = \sum_{m=1} f_{2m} \sin(2m\theta), \quad (10)$$

$$[F_{oo}(x,y)]^{90^o} = -\sum_{m=odd} f_{2m} \sin(2m\theta) + \sum_{m=even} f_{2m} \sin(2m\theta),$$

where $x^2 + y^2 =$ constant, f_{2m} is the corresponding coefficient, and the indexes are nature numbers. To emphasize that $F_{oo}(x,y)$ has a fundamental frequency of 2 (i.e., a period of 180^o), a subscript 2θ is added to $F_{oo}(x,y)$. Thus, Eq (10) can be rewritten as

$$F_{oo,2\theta} = F_{oo,2odd\theta} + F_{oo,2even\theta}, \quad (11)$$
$$[F_{oo,2\theta}]^{90^o} = -F_{oo,2odd\theta} + F_{oo,2even\theta},$$

where

$$F_{oo,2even\theta} = \sum_{m=even} f_{2m} \sin(2m\theta) = \sum_{m=1} f_{4m} \sin(4m\theta) = F_{oo,4\theta}, \quad (12)$$

$$F_{oo,2odd\theta} = \sum_{m=odd} f_{2m} \sin(2m\theta).$$

The subscripts $2odd\theta$ and $2even\theta$ represent the sum of the odd terms of $F_{oo,2\theta}$ and the sum of the even terms of $F_{oo,2\theta}$, respectively. It should be noted that in Eq. (11) $F_{oo,2odd\theta}$ of $[F_{oo,2\theta}]^{90^o}$ has an opposite sign of that of $F_{oo,2\theta}$. The subscript 4θ of Eq. (12) indicates that $F_{oo,2even\theta}$ has a fundamental frequency of 4 (i.e., a period of 90^o), and can be divided into two groups as follows.

$$F_{oo,4\theta} = F_{oo,4odd\theta} + F_{oo,4even\theta}, \quad (13)$$
$$[F_{oo,4\theta}]^{45^o} = -F_{oo,4odd\theta} + F_{oo,4even\theta},$$

where

$$F_{oo,4odd\theta} = \sum_{m=odd} f_{4m} \sin(4m\theta), \tag{14}$$

$$F_{oo,4even\theta} = \sum_{m=even} f_{4m} \sin(4m\theta).$$

Therefore, an odd-odd function can be expressed as the sum of moddθ terms where m = 2, 4, 8, 16, 32, ·····. That is

$$F_{oo,2\theta} = F_{oo,2odd\theta} + F_{oo,4odd\theta} + F_{oo,8odd\theta} + F_{oo,16odd\theta} + \cdots, \tag{15}$$

where the 8oddθ and 16oddθ terms are defined as in Eq. (14). It should be noted that each term includes a very broad spectrum of the Fourier sine series. For example, $F_{oo,2odd\theta}$ includes the components of sin(2θ), sin(6θ), sin(10θ), sin(14θ), etc. For a smooth surface, the odd-odd part can be well represented by the first two terms of Eq. (15).

From the above discussion, the three odd-odd functions, $A_{oo}(x,y)$, $B_{oo}(x,y)$, and $C_{oo}(x,y)$, have a fundamental frequency of 2 and can be expressed as

$$\begin{aligned} A_{oo} &= A_{oo,2\theta} = A_{oo,2odd\theta} + A_{oo,2even\theta}, \\ B_{oo} &= B_{oo,2\theta} = B_{oo,2odd\theta} + B_{oo,2even\theta}, \\ C_{oo} &= C_{oo,2\theta} = C_{oo,2odd\theta} + C_{oo,2even\theta}, \end{aligned} \tag{16}$$

where all 2oddθ and 2evenθ terms are defined in the same way as in Eq. (12). If a flat has only the odd-odd component, the 2oddθ term can be obtained easily by subtracting two measurements which are obtained before and after the flat is rotated 90° with respect to the other flat, such as M_1 and M_3. Because of the subtraction, the contribution of the surface not rotated is removed, the 2evenθ term of the rotated surface is canceled, and only the 2oddθ term remains. Because all the even-even, the even-odd, and the odd-even parts of each flat are obtained, we can subtract them from M_1, M_3, M_6, and M_7, respectively. The difference includes only the odd-odd part. Define m'_1, m'_3, m'_6 and m'_7 as follows:

$$\begin{aligned} m'_1 &= A_{oo,2\theta} - B_{oo,2\theta}, \\ m'_3 &= [A_{oo,2\theta}]^{90^o} - B_{oo,2\theta}, \\ m'_6 &= B_{oo,2\theta} - C_{oo,2\theta}, \\ m'_7 &= [B_{oo,2\theta}]^{90^o} - C_{oo,2\theta}. \end{aligned} \tag{17}$$

All the 2oddθ parts of the three flats are obtained as given below:

$$\begin{aligned} A_{oo,2odd\theta} &= (m'_1 - m'_3)/2, \\ B_{oo,2odd\theta} &= (m'_6 - m'_7)/2, \\ C_{oo,2odd\theta} &= ([m'_7]^{-90^o} - m'_6)/2. \end{aligned} \tag{18}$$

The 2evenθ term of Eq. (16) can be divided into two halves: 4evenθ and 4oddθ terms. The 4oddθ term can be obtained by rotating one flat 45° instead of 90°. Using a similar procedure for deriving Eqs. (17) and (18), we define m''_1, m''_4, m''_6 and m''_8 as

$$
\begin{aligned}
m''_1 &= A_{oo,4\theta} - B_{oo,4\theta}, \\
m''_4 &= [A_{oo,4\theta}]^{45^o} - B_{oo,4\theta}, \\
m''_6 &= B_{oo,4\theta} - C_{oo,4\theta}, \\
m''_8 &= [B_{oo,4\theta}]^{45^o} - C_{oo,4\theta}.
\end{aligned} \tag{19}
$$

Then, all the 4oddθ terms can be obtained as given below:

$$
\begin{aligned}
A_{oo,4odd\theta} &= (m''_1 - m''_4)/2, \\
B_{oo,4odd\theta} &= (m''_6 - m''_8)/2, \\
C_{oo,4odd\theta} &= ([m''_8]^{-45^o} - m''_6)/2.
\end{aligned} \tag{20}
$$

In summary, the sum of one half of the Fourier sine series (i.e., 2oddθ term) is obtained from the 90° rotation group. The other half is further divided into two halves, and one of them (i.e., 4oddθ term) is obtained from the 45° rotation group. Thus, after each rotation, one half of the unknown components of the Fourier sine series of the odd-odd function is obtained. The higher order terms can be derived by rotating the flat at a smaller angle. For example, the 8oddθ term is determined by rotating 22.5°. If the odd-odd component of the flat can be approximated by the first terms as are those in Eq. (15), the three flats can be approximated by the equations below:

$$
\begin{aligned}
A &= A_{ee} + A_{oe} + A_{eo} + A_{oo,2odd\theta} + A_{oo,4odd\theta}, \\
B &= B_{ee} + B_{oe} + B_{eo} + B_{oo,2odd\theta} + B_{oo,4odd\theta}, \\
C &= C_{ee} + C_{oe} + C_{eo} + C_{oo,2odd\theta} + C_{oo,4odd\theta},
\end{aligned} \tag{21}
$$

where all the terms on the right hand side are obtained from Eqs. (7), (9), (18), and (20).

III. Algorithm

The algorithm of this modified three-flat method is given in the following list.

1. The eight measurements according to Fig. 2.
2. $A_{oe} + A_{eo}$, $B_{oe} + B_{eo}$, and $C_{oe} + C_{eo}$ from Eq. (7).
3. A_{ee} , B_{ee}, and C_{ee} from Eqs. (8) and (9).
4. Define A', B', and C' equal to the sum of the above steps.

$$
\begin{aligned}
A' &= A_{ee} + A_{oe} + A_{eo}, \\
B' &= B_{ee} + B_{oe} + B_{eo}, \\
C' &= C_{ee} + C_{oe} + C_{eo}.
\end{aligned} \tag{22}
$$

5. The 2oddθ term of each flat is obtained as follows.

$$
\begin{aligned}
A_{oo,2odd\theta} &= (M_1 - M_3 - A' + [A']^{90^o})/2, \\
B_{oo,2odd\theta} &= (M_6 - M_7 - B' + [B']^{90^o})/2, \\
C_{oo,2odd\theta} &= ([M_7]^{-90^o} - M_6 - [[C']^x]^{-90^o} + [C']^x)/2.
\end{aligned} \tag{23}
$$

Steps 6 and 7 are similar to Steps 4 and 5 except for 45° rotation, instead of 90°.

6. Define A", B", and C" equal to the sum of the above steps.

$$
\begin{aligned}
A'' &= A_{ee} + A_{oe} + A_{eo} + A_{oo,2odd\theta}, \\
B'' &= B_{ee} + B_{oe} + B_{eo} + B_{oo,2odd\theta}, \\
C'' &= C_{ee} + C_{oe} + C_{eo} + C_{oo,2odd\theta}.
\end{aligned} \tag{24}
$$

7. The 4oddθ term of each flat is obtained as follows.

$$A_{oo,4odd\theta} = (M_1 - M_4 - A'' + [A'']^{45^o})/2,$$
$$B_{oo,4odd\theta} = (M_6 - M_8 - B'' + [B'']^{45^o})/2, \qquad (25)$$
$$C_{oo,4odd\theta} = ([M_8]^{-45^o} - M_6 - [[C'']^x]^{-45^o} + [C'']^x)/2.$$

8. The flats are approximated by the sum of Eqs. (24) and (25).

IV. Simulations

In the simulation, three flats (each is an OPD map obtained from a Fizeau interferometer) are used for the three input flats to generate eight measurements. Figure 3 shows the three flats (A, B, C) and the reconstructed flats. The differences are dominated by quantization errors of the Fizeau interferometer. If synthetic flats which are generated from the first 36 Zernike polynominals[8] are used, the flats can be reconstructed completely, because the highest frequency in θ is 4. This shows that the eight-measurement algorithm recovers the flats as long as the odd-odd parts of the surfaces can be approximated by $2odd\theta$ and $4odd\theta$ terms. If synthetic flats are generated from higher order Zernike polynominals, all the even-even, the even-odd, the odd-even, the $\sin(2m\theta)$, and the $\sin(4m\theta)$ terms, where $m = 1, 3, 5,...$, can still be recovered. But, the coefficients of the reconstructed surfaces corresponding to $\sin(8n\theta)$, $n = 1, 2, 3,...$, are canceled and equal to zero. To recover the higher order terms, more measurements with a smaller rotation angle are needed. It should be noted that the $\cos(2n\theta)$ terms of the Zernike polynominals, where $n = 1, 2, 3,...$, are even-even functions in a Cartesian coordinate system and can be recovered completely.

V. Discussion and Conclusion

Using the properties of odd and even functions, the flat is decomposed to four components: odd-even, even-odd, even-even, and odd-odd. The first three are obtained very easily. The odd-odd one is divided into several groups from the Fourier sine series. Because the higher frequency components of the Fourier series are small in general, the odd-odd function can be well approximated by the $2odd\theta$ and the $4odd\theta$ terms. It should be noted that because both $2odd\theta$ and $4odd\theta$ terms include a very broad spectrum of the Fourier sine series, the fine structure on a test flat can be seen in the reconstructed surface. However, since the entire spectrum of the Fourier series is not included, the reconstructed fine structure may have errors. Moreover, in the eight-measurement algorithm, the rotation operation $[\ ]^{+/-45^o}$ in Eq. (25) requires interpolation for the points not on the nodes of a square grid array. This may also introduce small errors. Besides the two error sources, the long period of time for the multiple measurements may be another error source. In experiments, the eight measurements have different tilts and pistons. Because the tilt and the piston are not of interest, they can be subtracted from Eqs. (5) or (7). This does not affect the rest of the equations. In conclusion, the profiles along the four diameters of a flat are exact: 0°, 45°, 90°, and 135°. The relationship among these profiles is also defined exactly, unlike the traditional three-flat method. Because the entire surface of a flat is approximated by Eq. (21), the area between two adjacent diameters is missing $\sin(8m\theta)$ components. These higher order terms can be derived by rotating the flats at smaller angles.

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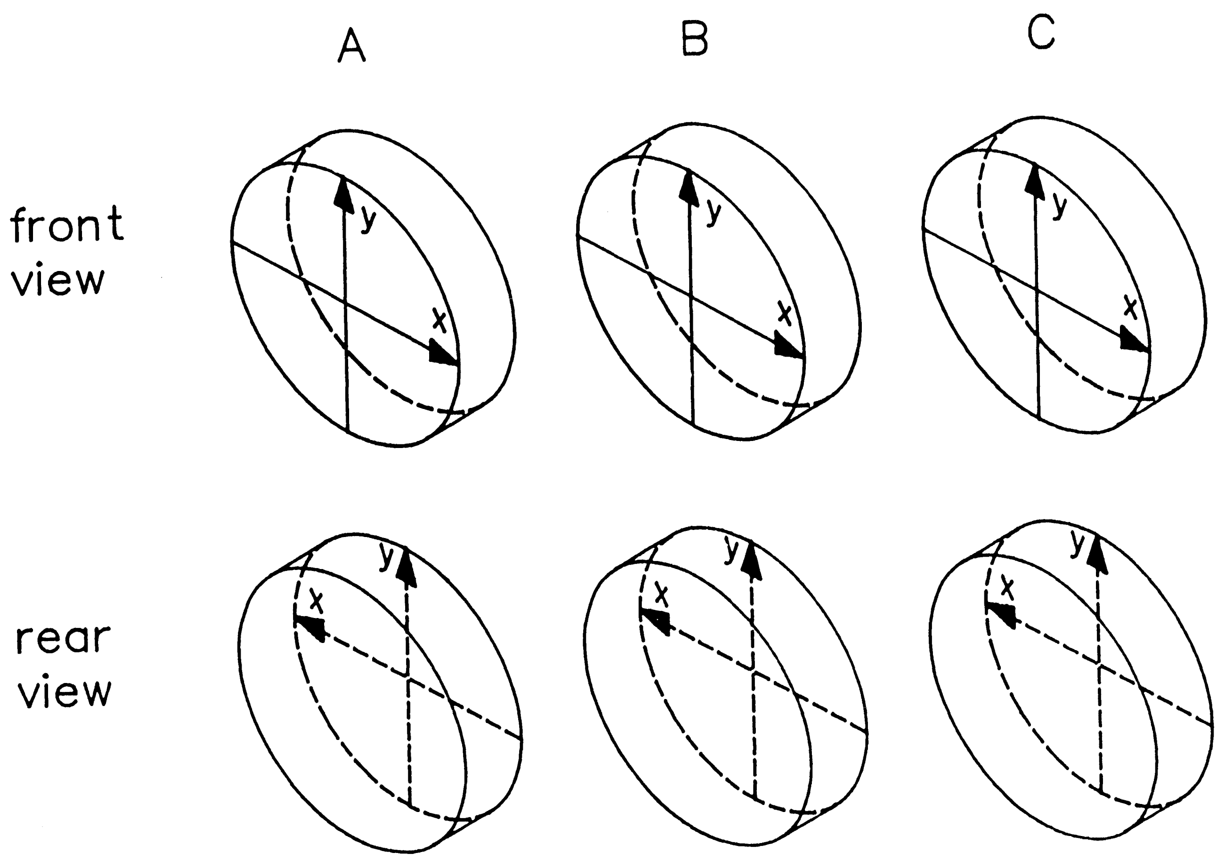


Fig. 1 Three flats, A(x,y), B(x,y), and C(x,y), of a front view and a rear view. The coordinate systems indicate the orientations of the flats.

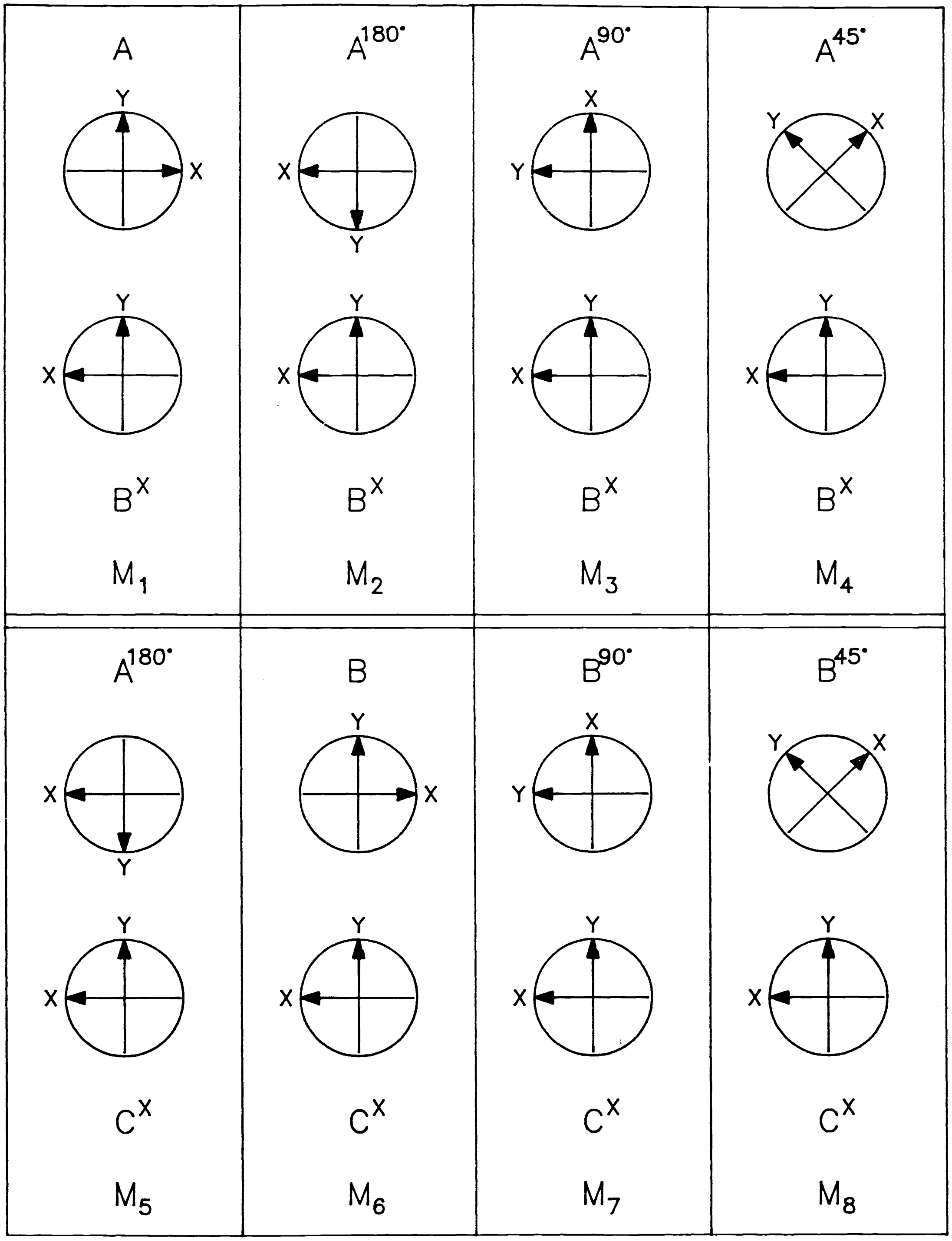
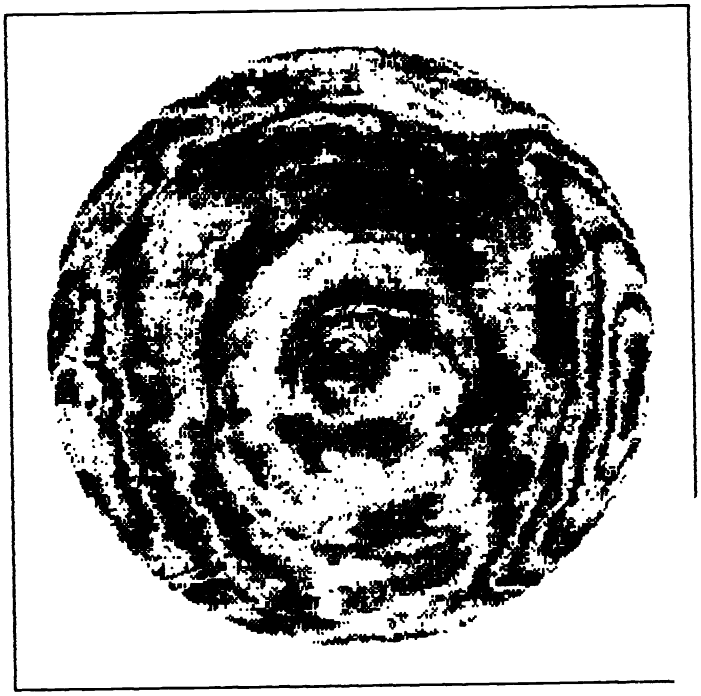


Fig. 2 Eight configurations and the corresponding measurements. In each configuration, the flat below is flipped in x and is of a rear view.

Flat A



Reconstructed
Flat A

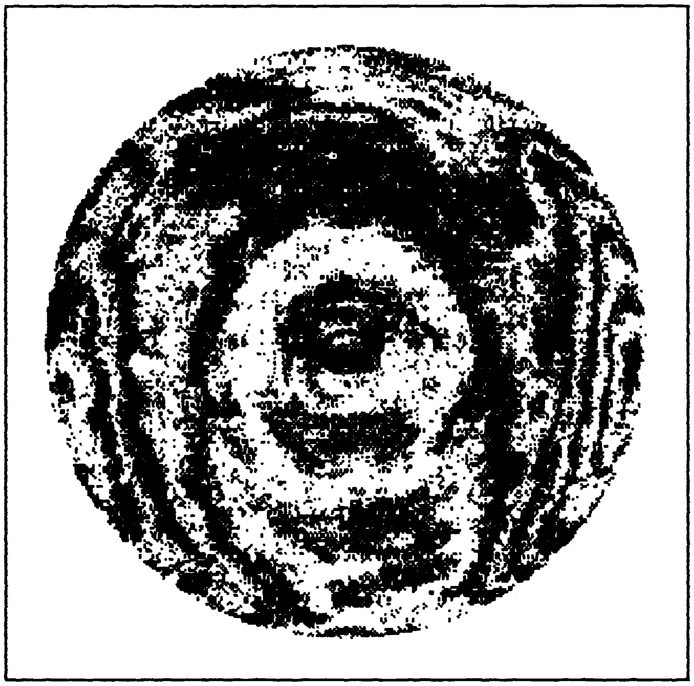
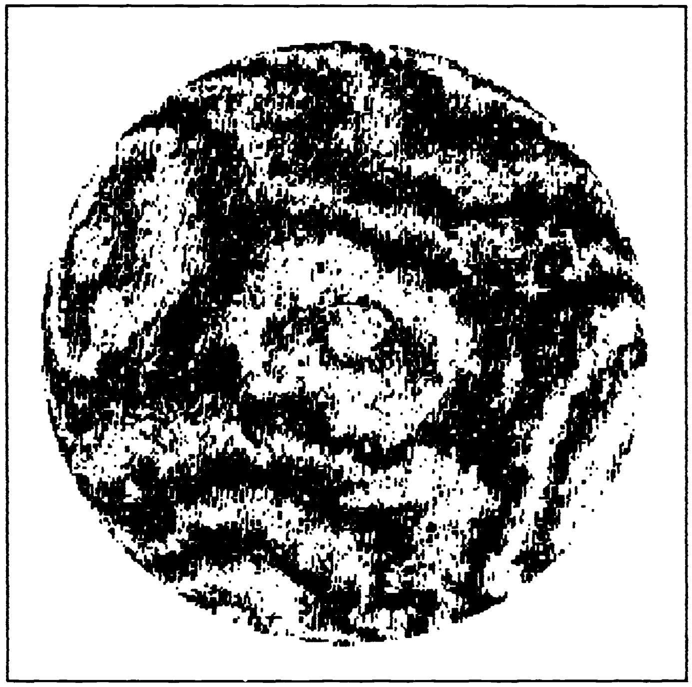


Fig. 3(a) Isometric contour of Flat A (top) and its reconstructed flat (bottom).
The interval is 0.005 wave.

Flat B



Reconstructed
Flat B

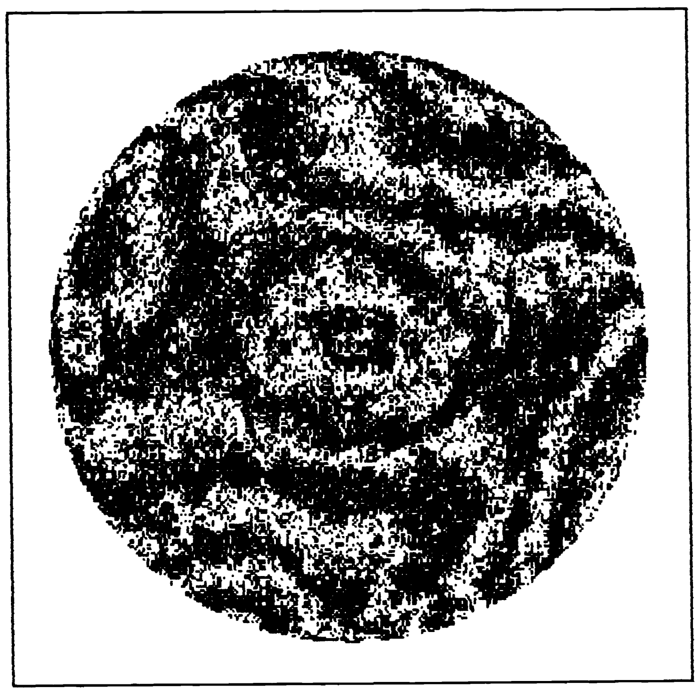
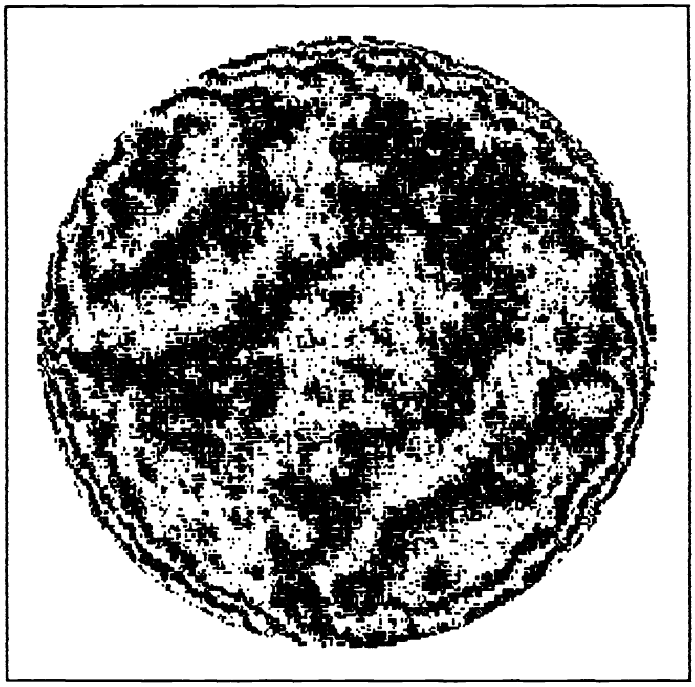


Fig. 3(b) Isometric contour of Flat B (top) and its reconstructed flat (bottom).
The interval is 0.005 wave.

Flat C



Reconstructed
Flat C

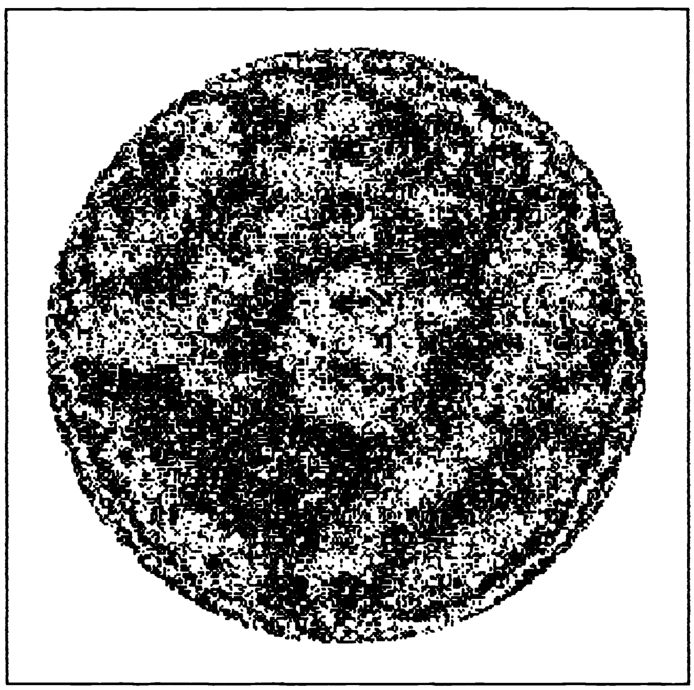


Fig. 3(c) Isometric contour of Flat C (top) and its reconstructed flat (bottom).
The interval is 0.004 wave.