

Absolute Measurement of Spherical Surfaces

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ABSTRACT

The testing of spherical surfaces using the three-measurement technique outlined by Jensen requires very precise alignment of the sphere relative to the interferometer. An easier technique for the absolute measurement of spherical surfaces has been developed which does not require the precise alignment of the Jensen technique and uses only two measurements. As long as the test surface does not contain any aberrations with odd symmetry, these aberrations can be subtracted from the measurement and an absolute measurement of the test surface can be obtained. This paper describes and compares these two techniques and shows results of testing a $\lambda/12$ P-V (peak-to-valley) sphere (N.A.=0.4) using both techniques with a phase-measuring Fizeau interferometer. These measurement techniques are repeatable to ± 0.01 waves P-V.

1. INTRODUCTION

The absolute measurement of spherical surfaces is important with optics specified to be as least as good as $\lambda/10$ peak-to-valley (P-V), where λ is the test wavelength. A number of techniques have been described in the literature.¹⁻⁵ A technique widely used with phase-measuring interferometry was first described by Jensen,² and then further discussed by Bruning,³ Truax,⁴ and Elssner et al.⁵ This technique has the advantage of giving the absolute shape of the sphere under test independent of the reference surface and diverger optics as long as the test surface is aligned correctly. The main disadvantage of this technique is the requirement that the test surface be aligned so that it can be rotated and keep the fringe pattern unchanged. A simpler technique has been developed which does not require this precise alignment and requires only two measurements. The result is not exactly an absolute measurement, but as long as the test surface has even symmetry, the test can be absolute.

2. THREE-POSITION ABSOLUTE MEASUREMENT TECHNIQUE

The technique of absolute measurement of spherical surfaces as described by Jensen² requires three separate measurements of the surface under test. These three measurements are depicted in Fig. 1. The first measurement is with the test surface at the focus of the diverger lens (also known as the catseye position). The second measurement is with the test surface positioned such that its center of curvature is at the focus of the diverger lens. The third measurement is taken after rotating the test surface 180° about the optical axis. Mathematically, these three measurements can be written as

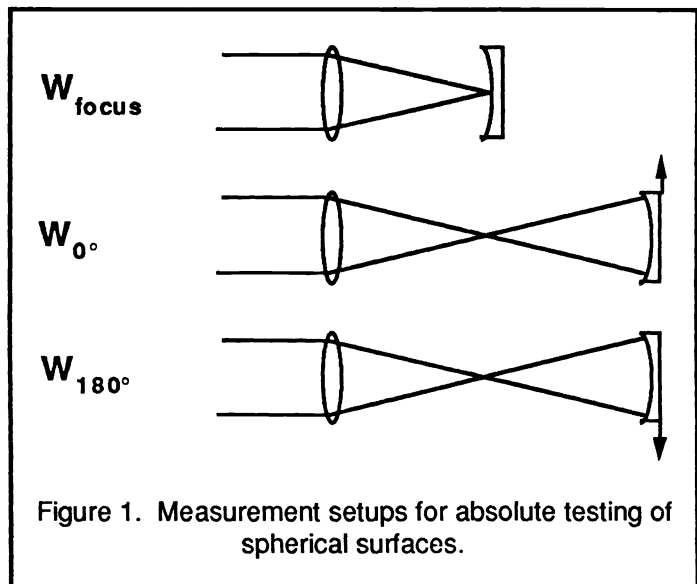


Figure 1. Measurement setups for absolute testing of spherical surfaces.

$$W_{\text{focus}} = W_{\text{ref}} + \frac{1}{2} \left[W_{\text{div}} + \overline{W}_{\text{div}} \right] \quad (1)$$

$$W_{0^\circ} = W_{\text{surf}} + W_{\text{ref}} + W_{\text{div}} \quad (2)$$

$$W_{180^\circ} = \overline{W}_{\text{surf}} + W_{\text{ref}} + W_{\text{div}} \quad (3)$$

where W refers to a wavefront, surf refers to the test surface, ref refers to the optics in the reference arm of the interferometer and the reference surface, and div refers to the optics in the test arm of the interferometer minus the test surface including the diverger lens. A bar over a wavefront indicates a 180° rotation of that wavefront. These three measurements can then be used to solve for the test surface using

$$W_{\text{surf}} = \frac{1}{2} \left[W_{0^\circ} + \overline{W}_{180^\circ} - W_{\text{focus}} - \overline{W}_{\text{focus}} \right] \quad (4)$$

which is simply calculated with additions, subtractions, and 180° rotations of the three measurements. If a large number of similar spheres are to be tested, then the aberrations in the interferometer and errors due to the reference surface can be obtained by calculating

$$W_{\text{ref}} + W_{\text{div}} = \frac{1}{2} \left[W_{0^\circ} - \overline{W}_{180^\circ} + W_{\text{focus}} + \overline{W}_{\text{focus}} \right] \quad (5)$$

This reference wavefront can then be subtracted from measurements of subsequent test spheres as long as the radii of curvature are similar. If there is a large difference in radii of curvature, a new reference wavefront must be measured. This technique will work with both Twyman-Green and Fizeau interferometers.

The necessary alignments to perform this procedure have been outlined by Elssner et al.⁵ First of all, the optical axis is defined by the first measurement in the catseye position with the fringes nulled. The detector in the interferometer needs to be aligned so that it is centered on the optical axis. Next, the test surface needs to be aligned relative to the optical axis. This is the tricky part. In order to rotate the test surface by 180° without altering the fringe pattern, the vertex of the sphere must lie on the optical axis, the axis of rotation defined by the rotation stage must coincide with the optical axis, and the center of curvature of the test surface must lie at the focus of the diverger lens. Figure 2 shows a drawing of the possible misalignments for testing a sphere in a Fizeau interferometer, and Fig. 3 shows the test and reference surfaces after alignment. Elssner et al. state that a mount with a minimum of eight degrees of freedom is required to do this alignment as long as the test surface has been centered in its mount. We have found that six degrees of freedom is sufficient in order to rotate the test surface by 180° and keep the fringe pattern within 2 fringes of being nulled. Figure 4 shows a mount containing eight degrees of freedom to test a sphere. The sphere is mounted to an x-y stage which is used to center the sphere on the rotation axis. A five-axis mount is used to align the axis of rotation with the optical axis of the interferometer. The tip-tilt of the five axis is not always necessary. It ensures that the sphere is being tested at its center. In addition, it is advantageous to have a separate mount with a flat for the measurement at the catseye position. This makes it easier to take all the data quickly once the sphere has been aligned relative to the interferometer. The flat needs tip-tilt and z translation for fine positioning.

To align the sphere so that it can be rotated 180° without changing the fringe pattern requires a high-quality rotation stage. Stages with aluminum races and ball bearings do not repeatably return to the same location after rotation. A simple means of alignment involves looking at the rotation of the return spot from the sphere in a focal plane compared to the return spot from the reference surface as the sphere is rotated. Figure 5 illustrates the location of the two return spots as the sphere is rotated. This procedure starts with the two spots on top of one another. After a 180° rotation, the sphere x-y position is adjusted to bring the sphere return spot halfway back to the position of the reference spot. The sphere is then rotated back 180° and 5-axis x-y position is adjusted to line up the two spots. This procedure is continued until there is no noticeable movement of the spot as the sphere is rotated. At this point, the fringe pattern can be observed and the same

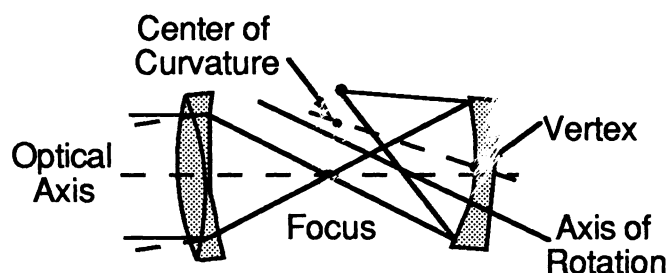


Figure 2. Possible misalignments include position of reference surface, sphere vertex, sphere center of curvature, and axis of rotation, all relative to the optical axis, and the location of the sphere along the optical axis.

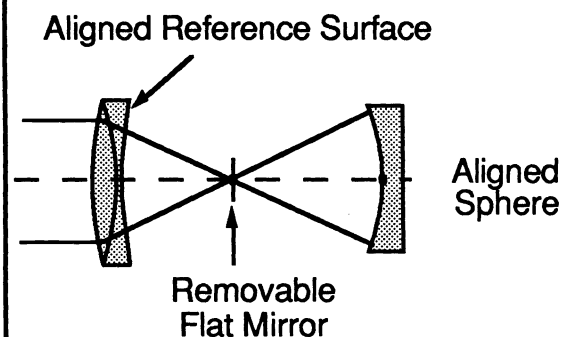


Figure 3. Sphere aligned with reference flat.

procedure followed until the fringe pattern is stationary as the sphere is rotated. With a good rotation stage, this alignment procedure is sufficient to measure spherical surfaces with NA's (numerical apertures) of 0.5 or less to $\lambda/20$ P-V. To perform a high accuracy measurement, good optics (at least $\lambda/10$ P-V) are necessary so that the rays transverse the same path back through the interferometer after reflecting from the sphere.

3. TWO-POSITION ABSOLUTE MEASUREMENT TECHNIQUE

Because the alignment gets much more difficult as the NA gets larger, a simpler technique was developed. This technique only requires two measurements. These two measurements are given by eqs. (1) and (2). When measurement 1 is subtracted from measurement 2, the result is the wavefront due to the surface plus an error term due to the diverger,

$$W_{0^\circ} - W_{\text{focus}} = W_{\text{surf}} + \frac{1}{2} \left[W_{\text{div}} - \overline{W_{\text{div}}} \right]. \quad (6)$$

For aberrations with even symmetry, such as defocus, spherical, and astigmatism, the error term is zero because these

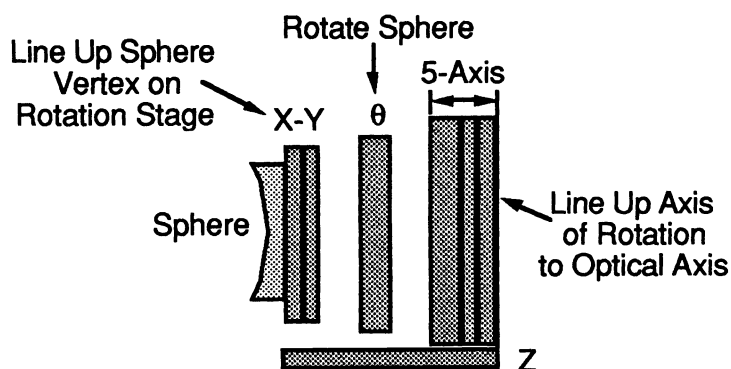


Figure 4. Mount with necessary degrees of freedom to align sphere to rotate 180° and keep fringes.

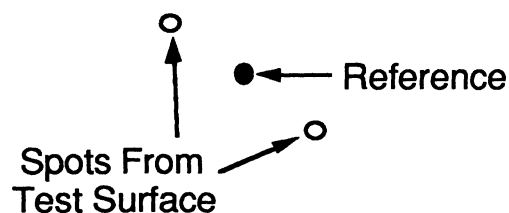


Figure 5. Return spots from reference surface and test sphere as test sphere is rotated.

aberrations cancel out ($W_{\text{div}} = \overline{W_{\text{div}}}$). The difference of the two measurements then becomes

$$W_{0^\circ} - W_{\text{focus}} = W_{\text{surf}} \quad (7)$$

For aberrations with odd symmetry such as coma, the error term is not zero ($W_{\text{div}} \neq \overline{W_{\text{div}}}$). Because the odd aberrations add, the difference between the two measurements will be

$$W_{0^\circ} - W_{\text{focus}} = W_{\text{surf}} + W_{\text{div}} \quad (8)$$

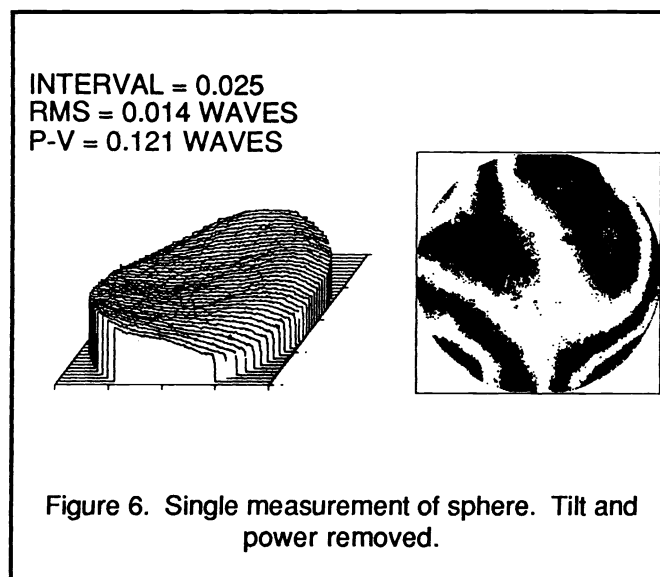
Since most spherical surfaces do not have coma in them, and because a misalignment of the spherical test surface would not induce coma into the measurement, it can be assumed that any coma in the measurement should be due to interferometer or the diverger lens. As long as the coma is assumed to be in the interferometer and not in the test surface, it can be subtracted from the measurement to yield the test surface independent of the interferometer. For higher-order aberrations, those with even symmetry will cancel while those with odd symmetry will not cancel and should be subtracted as long as they are not in the test surface.

If it can not be assumed that there is no coma in the test surface, it may be found by additional simple calculations. Assuming only third-order aberrations, the coma due to both the interferometer and the test surface is found by rotating the data set by 180° , and subtracting this from the original data set. This causes the defocus, spherical, and astigmatism to cancel and leaves twice the coma. It should be noted that the coma due only to the test surface can only be found by including a third measurement as in the Jensen technique. However, for quick and easy to set up measurements yielding surface shape in the range of $\lambda/10$ to $\lambda/15$ P-V, this technique is very useful. If greater accuracy is required, the Jensen technique is better to use. It should be noted that for $\lambda/20$ P-V measurements, the quality of the optics in the interferometer becomes critical.

4. RESULTS

To compare the two techniques, a 0.4 NA sphere was tested in a Fizeau interferometer with a diverger lens having an F/1.1 spherical reference surface. The source was a Helium-Neon laser operating at $0.6328 \mu\text{m}$. Both algorithms were implemented using phase-measurement interferometry techniques using a CCD-TV camera and a 68030-based computer. A single measurement of the sphere is shown in Fig. 6 with an rms of 0.014 waves and a P-V of 0.121. Tilt and power have been subtracted from this measurement because they are functions of the alignment which are not part of the test surface. All measurement results have been evaluated over 95% of the aperture, and tilt and power have been subtracted. A flat placed at the focus of the diverger lens in the catseye position shows that there is 0.522 waves P-V of coma present in the interferometer as seen in Fig. 7a. With third-order coma subtracted from this measurement, there is a noticeable odd aberration having a three-point symmetry with a P-V of 0.121 waves. This aberration can be expressed in polynomial form using the 9th and 10th Zernike polynomials which have a functional form given by

$$\rho^3 \cos 3\theta \text{ and } \rho^3 \sin 3\theta ,$$



INTERVAL = 0.10
RMS = 0.076 WAVES
P-V = 0.522 WAVES

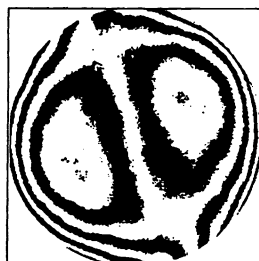


Figure 7a. Catseye measurement. Tilt and power removed.

INTERVAL = 0.05
RMS = 0.027 WAVES
P-V = 0.243 WAVES

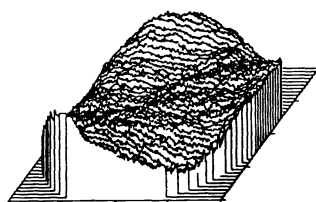


Figure 7b. Catseye measurement. Tilt, power and coma removed.

where ρ is the normalized radius and θ is the azimuthal angle. Using the three-position measurement technique of Jensen (Eqs. (4) and (5)), the errors in the interferometer showing the quality of the collimating lens, the diverger lens, and the reference surface are 0.084 waves P-V as seen in Fig. 8a. This means that the interferometer optics are good to $\lambda/12$ P-V. The spherical test surface is shown in Fig. 8b and has 0.081 waves ($\lambda/12$) P-V. Using the two-position measurement technique described in this paper, a measurement with tilt, power, and third-order coma subtracted is shown in Fig. 9. It has a P-V of 0.200 waves and has an error present with the same noticeable three-point symmetry seen in the measurement taken at the catseye position. This error is obviously not in the test surface and can be subtracted. Figure 10 shows a wavefront generated from the Zernike 9 and 10 polynomial coefficients of the wavefront shown in Fig. 9. This error term has a P-V of 0.102 waves. Once this error term is subtracted from the two-position absolute measurement of Fig. 9, the test sphere has a P-V of 0.089 waves as shown in Fig. 11. This compares very favorably to the three-position measurement. The orientation of the test surface is the same for both measurements. Notice the roll-off in the lower left corner of both results. Both techniques show that the sphere is better than $\lambda/10$ P-V and even though the numbers are not exactly the same, gross errors on the surface are the same in both measurements. These measurements are repeatable to ± 0.01 waves P-V.

INTERVAL = 0.025
RMS = 0.010 WAVES
P-V = 0.084 WAVES

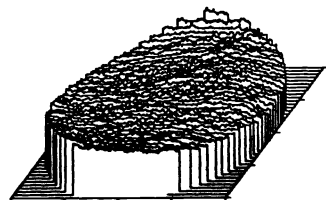


Figure 8a. Three-position absolute reference showing errors in reference and diverger. Tilt and power removed.

INTERVAL = 0.025
RMS = 0.011 WAVES
P-V = 0.081 WAVES

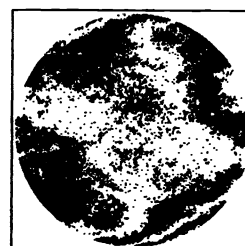
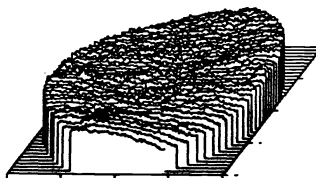


Figure 8b. Three-position absolute measurement of sphere. Tilt and power removed.

INTERVAL = 0.050
RMS = 0.024 WAVES
P-V = 0.200 WAVES

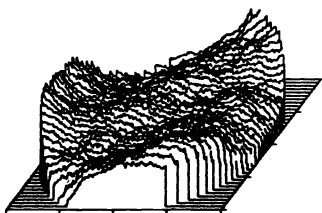


Figure 9. Two-position measurement with tilt, power, and coma removed.

INTERVAL = 0.025
RMS = 0.018 WAVES
P-V = 0.102 WAVES

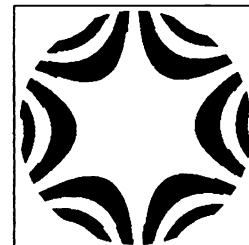
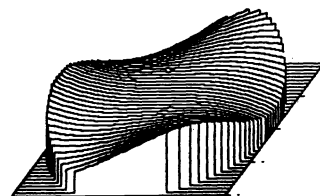


Figure 10. Zernike 9 and 10 coefficients of two-position measurement.

5. CONCLUSION

The three-position measurement technique for absolute measurement of spherical surfaces requires critical alignment of the test surface and a very good rotation stage. It theoretically has a very high precision and accuracy, but is hard to do. A faster and simpler technique for the absolute measurement of spherical surfaces has been introduced which does not require the precise alignment of the Jensen technique. It requires only two measurements instead of three, and a complex mount for rotating the test object and retaining fringes is not required. The test assumes that there is no coma (or higher order aberrations with odd symmetry) introduced by the test surface so that odd aberrations may be subtracted from the measurement. This is not strictly an absolute test, but for the measurement of high NA surfaces which need to be at least $\lambda/10$ P-V, it is sufficient in most cases. It is easy to measure spherical surfaces to $\lambda/10$ P-V. $\lambda/20$ P-V can be done, but it must be done with care and high quality diverging optics (better than $\lambda/10$ P-V) must be used.

6. REFERENCES

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INTERVAL = 0.025
RMS = 0.011 WAVES
P-V = 0.089 WAVES

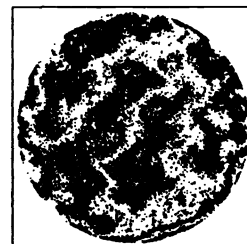
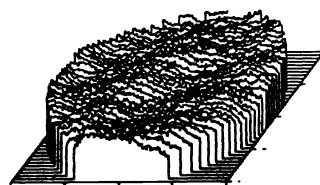


Figure 11. Two-position measurement with tilt, power, coma, and Zernike 9 and 10 removed.