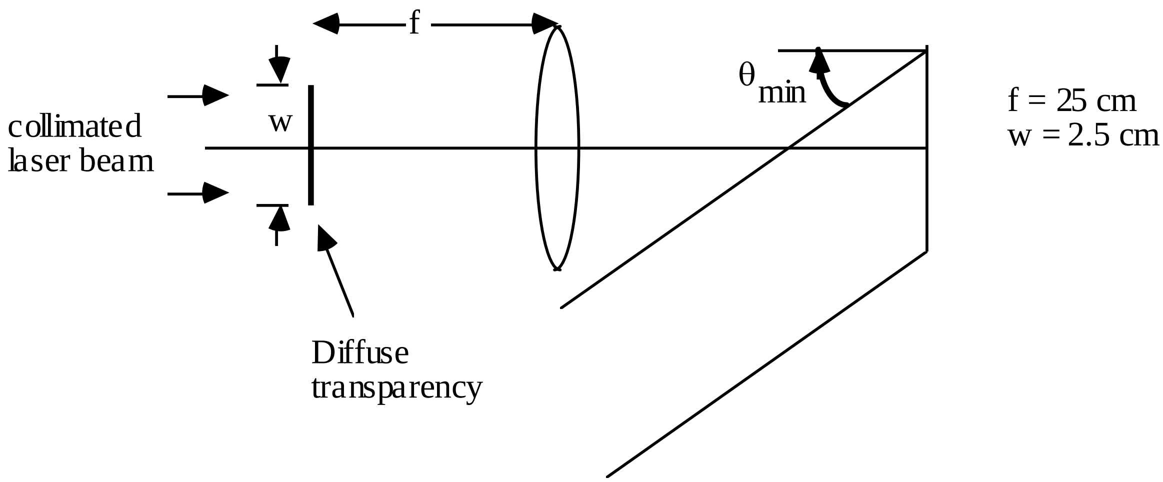
Holography and Speckle

HS - 1

A hologram is made using the geometry shown below. What is the minimum reference angle (θ_{min}) to insure separation of orders if a) the self-interference term can be neglected, and b) the self-interference term can not be neglected? Neglect effects of film non-linearity.



Solution

The rays from the object will be incident on the hologram for angles between ± ArcTan$\left[\frac{w}{2f}\right]$. The angle in degrees is given by

```
angles = ±(ArcTan[50, 2.5] / Degree)
±2.86241
```

- **a**

Ignoring intermodulation terms to separate orders we need to make sure we have no closed fringes. Thus, no part of the object beam can be parallel to the reference beam.

$\theta_{\text{min}} = 2.86°;$

- **b**

Considering self-interference terms the minimum angle in degrees is given by

```
θmin = ArcSin[3 Sin[2.8624 °]] / Degree
8.61607
```

HS - 2

A hologram is produced using a spherical wave of 50 cm radius of curvature for the object wave (x, y, z coordinates of 0, 10, -50) and a spherical wave of 50 cm radius of curvature for the reference wave (x, y, z coordinates of 0, -10, -50). The wavelength is 633 nm. The reconstructing wavefront is a spherical wave of 25 cm radius of curvature (x, y, z coordinates of 0, 0, -25) and 633 nm wavelength.

a) What is the shape of the interference fringes making up the hologram?

b) Give the x,y,z coordinates of the primary and conjugate images.

Solution

- **a**

The fringes will be essentially straight fringes parallel to the x-axis.

- **b**

Since there is no power in the hologram (straight fringes) both primary and conjugate images will be in the plane of the reconstructing source. The primary image will be on one side of the reconstructing source and the conjugate image will be on the other side. The y-separation of two beams forming hologram was 20 at a z distance of 50. Therefore, the image coordinates are given by

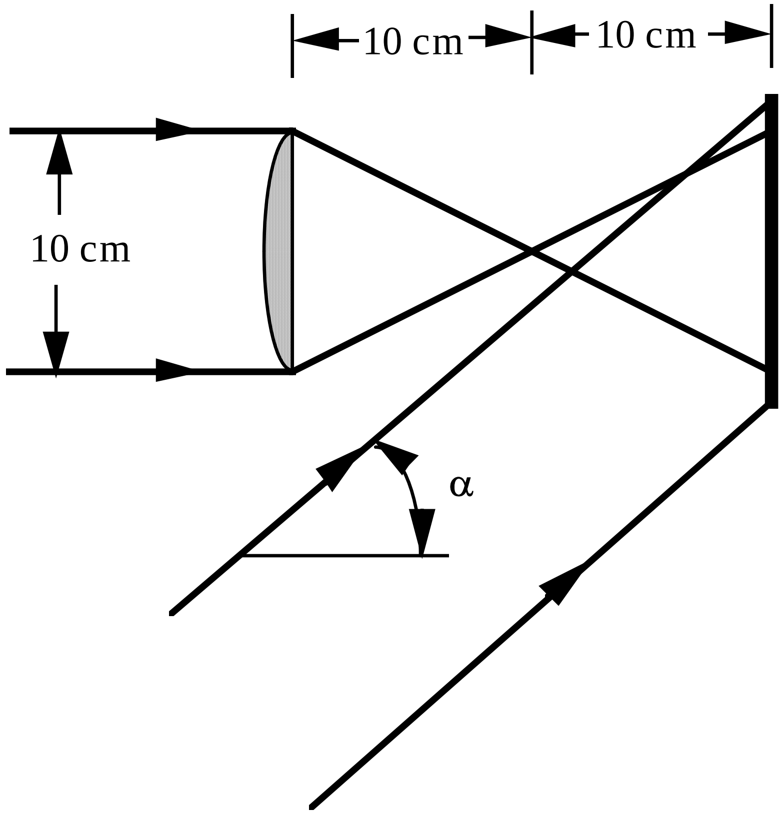
Image coordinates: (0, -10, -25) and (0, 10, -25).

HS - 3

A hologram of a spherical wave is made using the geometry shown below. If film non-linearities are neglected,

a) What is the minimum reference angle (αmin) to insure separation of the zero and first orders?

b) Why can self-interference terms be neglected?



Solution

- **a**

```
α = N[ArcTan[10 cm / 20 cm] / Degree]
26.5651
```

- **b**

No self-interference terms because object consists of only one point.

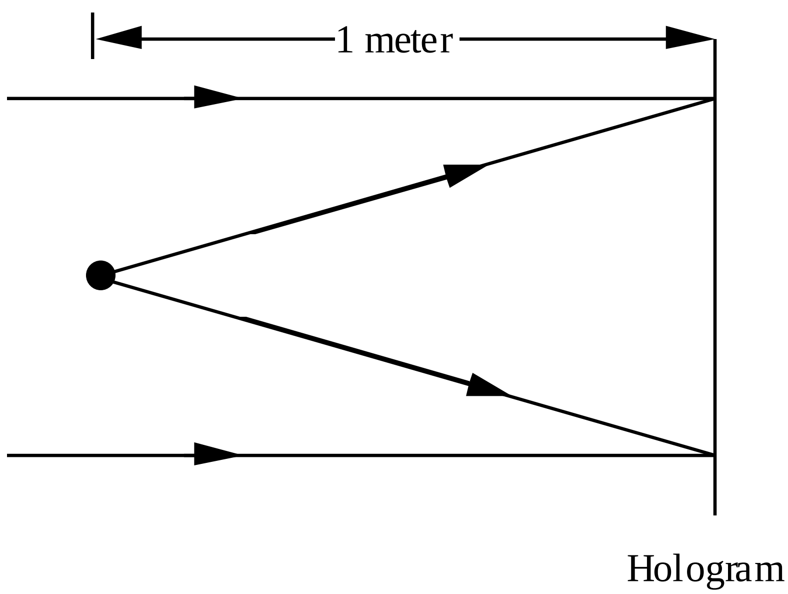
HS - 4

A hologram is made using the setup shown below with a wavelength of 633 nm.

a) If the reconstruction wavelength is 500 nm, where will the primary and conjugate orders be focused?

b) If the reconstruction wavelength is 633 nm, where will the primary and conjugate orders be focused if the hologram is illuminated with a diverging spherical wave having a 1.5 meter radius of curvature?

c) What would you do in the reconstruction process to separate the zero, primary, and conjugate orders? Comment on the self-interference terms.



Solution

■ **a**

The hologram has a focal length that goes as

$$\pm 1\,\text{m}\,\frac{633\,\text{nm}}{\lambda_{\text{reconstruction}}}\ /.\ \{\lambda_{\text{reconstruction}} \to 500\,\text{nm}\}\ //\ N$$

$1.266\,\text{m}\,(\pm 1.)$

The two reconstructed first orders are located 1.266 m to the left of the hologram and 1.266 m to the right of the hologram

■ **b**

The focal length is now ± 1 m.

$$\frac{1}{f} = \frac{1}{\text{imageDistance}} + \frac{1}{\text{reconstructionDistance}}$$

```
ans = Solve[1/f == 1/imageDistance + 1/reconstructionDistance, imageDistance]
```

$$\left\{\left\{\text{imageDistance} \to -\frac{f\ \text{reconstructionDistance}}{f - \text{reconstructionDistance}}\right\}\right\}$$

```
ans /. {f -> 1 m, reconstructionDistance -> 1.5 m}
{{imageDistance → 3. m}}

ans /. {f -> -1 m, reconstructionDistance -> 1.5 m}
{{imageDistance → -0.6 m}}
```

The images are located 0.6 meters to the left of the hologram and 3 m to the right of the hologram.

- **c**

We could block half of the hologram so there are no closed fringes.
If we can change the direction of the plane wave we could bring the plane wave in at an angle such that there are no rays in the spherical beam that are parallel to the plane wave.

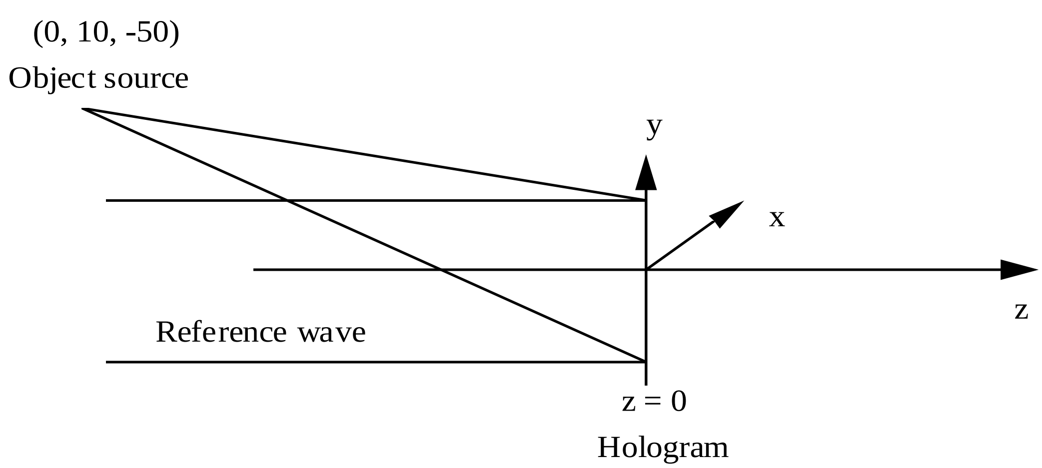
There will be no self interference terms because each beam originates from a single point source.

HS - 5

A hologram is produced using a spherical wave of 50 cm radius of curvature for the object wave (x, y, z coordinates of 0, 10, -50) and a collimated plane wave incident at normal incidence for the reference wave. The wavelength is 633 nm. The reconstructing wavefront is also a plane wave at normal incidence and 633 nm wavelength.

a) Give the x, y, z coordinates of the primary and conjugate images.

b) Where are the self interference terms located?



Solution

- **a**

Primary Image

Since the reconstructing wavefront is identical to the reference wavefront the primary image is identical to the original object. The image coordinates are (0, 10, -50).

Conjugate Image

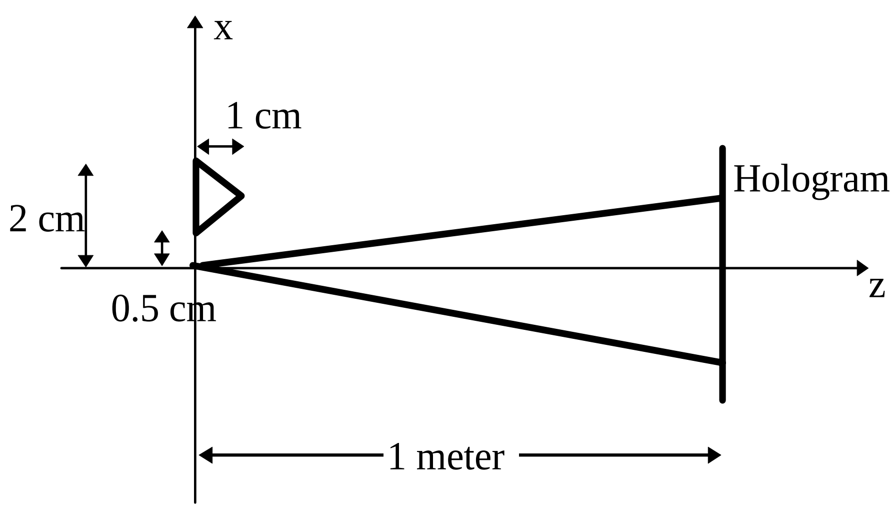
Plane reference and reconstructing wavefront and wavelength same in reconstruction as in making of hologram so coordinates of conjugate image will be (0, 10, 50). That is, the image will be a real image located the same distance to the right of the hologram as the virtual image (primary image) is located to the left of the hologram.

- **b**

There are no self-interference terms from the object since the object is a single spherical wave.

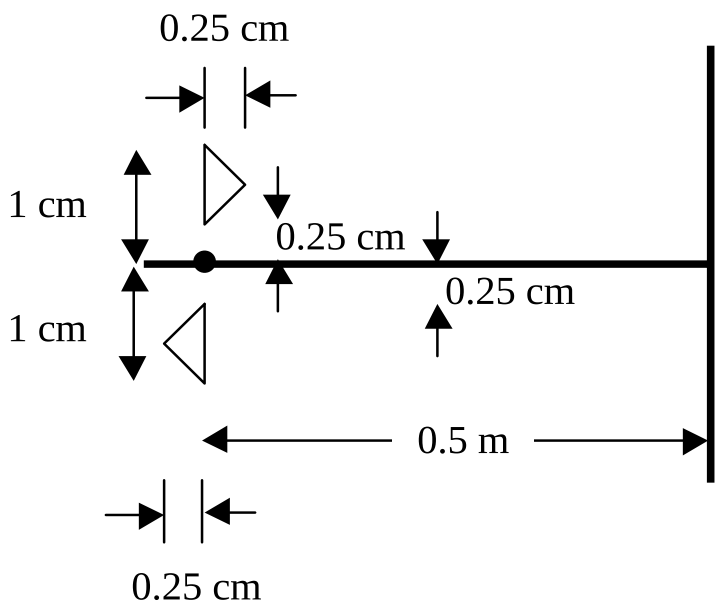
HS - 6

A hologram is made by interfering a point source and a triangular shaped object as shown below. The hologram is reconstructed using a point source 0.5 m from the hologram. Give the x and z coordinates of the primary and conjugate images. Give a sketch to show what the images look like.



Solution

Note that the longitudinal magnification goes as the square of the lateral magnification.



HS - 7

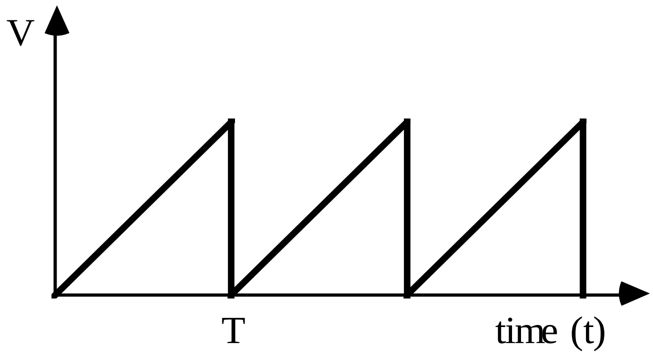
Holographic interferometry is useful for measurement of deformations. Can the surfaces of the objects be rough compared to the wavelength of the light used? Explain.

Solution

Yes. In holographic interferometry for measuring deformations we are measuring changes. Since the surface roughness is the same before and after the deformation it cancels out.

HS - 8

I am using holography to measure the vibration properties of the speakers used in my stereo. The waveform shown below is used to drive the speakers. Assume that the speakers follow the drive signal exactly with a response R(x,y) such that the displacement of the speaker cone is R(x,y)V. Assume the period of the waveform, T, is 10^{-3} second, the exposure time is one second and the photographic process is linear. Give the relative intensity of the hologram reconstruction for two points where RV=(wavelength)/4 and RV=3(wavelength)/2.



Solution

We will solve the problem 2 ways.

Easy way

For the 2 points

$$R\,V = \frac{\lambda}{4};\quad \text{OPD} = \frac{\lambda}{2}$$

$$R\,V = \frac{3\,\lambda}{2};\quad \text{OPD} = 3\,\lambda$$

The fringes will wash out completely for $R\,V = \frac{3\lambda}{2}$ and the resulting intensity = 0.
The fringes will not completely wash out for R $V = \frac{\lambda}{4}$.
Therefore the ratio of the intensities is 0 or ∞.

Hard way

Let the amplitude of the two interfering beams be a and A, a = 1 and A = $e^{i \frac{2\pi}{\lambda} 2 R V}$ where V = α t. The exposing intensity can be written as

$$i = \text{Abs}[A]^2 + a^2 + a\,A^* + a^*\,A$$

The term of interest is $a^*\,A = e^{i \frac{2\pi}{\lambda} 2 R V}$

Since we are integrating over time we are interested in

$$\textbf{ans} = \int_{-T/2}^{T/2} e^{i \frac{2\pi}{\lambda} 2 R \alpha t}\, dt \;\text{// ExpToTrig}$$

$$\frac{\lambda\,\text{Sin}\left[\frac{2\pi R T \alpha}{\lambda}\right]}{2\,\pi\,R\,\alpha}$$

In the above we integrated from -T/2 to T/2 instead of 0 to T to make the integral more symmetric and hence the result simpler.

The intensity of the reconstruction is proportiol to the square of the above or

```
intensity = ans^2 /. α -> V/T
```

$$\frac{T^2 \lambda^2 \mathrm{Sin}\left[\frac{2 \pi R V}{\lambda}\right]^2}{4 \pi^2 R^2 V^2}$$

Thus the resultint intensity is proportional to a Sinc^2 function.

```
intensity /. R V -> λ/4
```

$$\frac{T^2 \lambda^2}{4 \pi^2 R^2 V^2}$$

```
intensity /. R V -> 3λ/2
```

0

Therefore the ratio of the intensities is 0 or ∞.

HS - 9

We have a phase transmission grating produced by interfering two plane waves for which $\nu=3\pi/4$. The grating planes are perpendicular to the surface of the grating. The grating has an average refractive index of 1.52, a thickness of 15 microns, and a grating spacing of 1 micron. The hologram is illuminated with radiation having a wavelength of 514.5 nm.

a) At what angles, measured outside of the hologram, can a plane wave be incident such that the relative diffraction efficiency is greater than or equal to 50%? (Neglect surface reflections.)

b) Repeat part a for a hologram thickness of 10 microns.

Solution

- **a**

Let η be the diffraction efficiency, λ_a be the wavelength in vacuum, t be the grating thickness, δ be the deviation from the Bragg angle, n_o be the average refractive index, n_1 be the amplitude of the sinusoidal index variation, and θ_o be the angle between the grating planes and the beam, then

$$\eta = \frac{\mathrm{Sin}\left[\sqrt{\xi^2 + \nu^2}\right]^2}{\left(1 + \frac{\xi^2}{\nu^2}\right)};$$

$$\nu = \frac{\pi}{\lambda_a} \frac{n_1 t}{\mathrm{Cos}[\theta_o]};$$

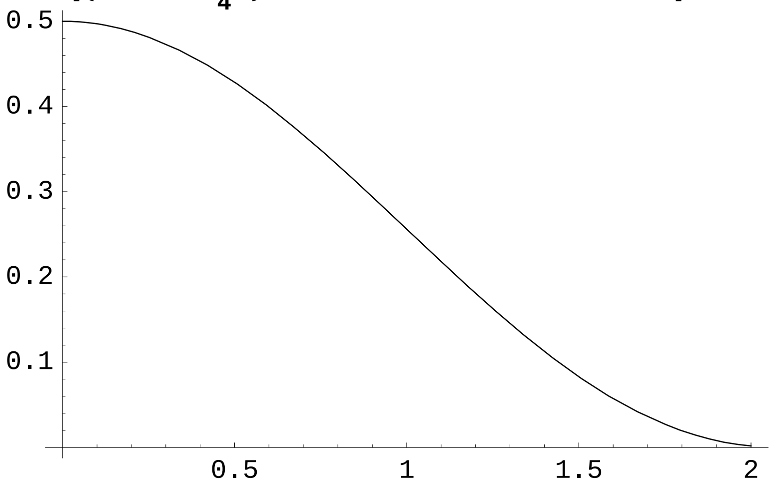
$$\xi = \delta \left(\frac{2 \pi n_o}{\lambda_a}\right) t \,\mathrm{Sin}[\theta_o]$$

Let d be the grating spacing. Since $2\, n_o\, d\, \mathrm{Sin}[\theta_o] = \lambda_a$,

$$\xi = \frac{\delta \pi t}{d}$$

We now need to determine the value of ξ for the diffraction efficiency to drop to one-half the maximum value. An easy way to do this is to plot η as a function of ξ.

```
Plot[{η /. ν → 3π/4}, {ξ, 0, 2}, Background → White];
```



Thus, the diffraction efficiency drops to one-half its maximum value for $\xi \simeq 1$.

Letting $\xi = 1$

```
δ = d/(π t) /. {d → 1, t → 15} // N
0.0212207
```

```
θo = ArcSin[λa/(2 no d)] /. {λa → 0.5145, no → 1.52, d → 1}
0.170062
```

So the angle inside the material ranges between

```
angleInside = {θo - δ, θo + δ}
{0.148841, 0.191283}
```

The angles outside in degrees ranges between

```
angleOutside = ArcSin[no Sin[angleInside]] / Degree /. no → 1.52
{13.0267, 16.7969}
```

■ b

It the thickness, t, is 10 microns

```
δ = d/(π t) /. {d → 1, t → 10} // N
0.031831

angleInside = {θo − δ, θo + δ}
{0.138231, 0.201893}

angleOutside = ArcSin[no Sin[angleInside]] / Degree /. no → 1.52
{12.0897, 17.7459}
```

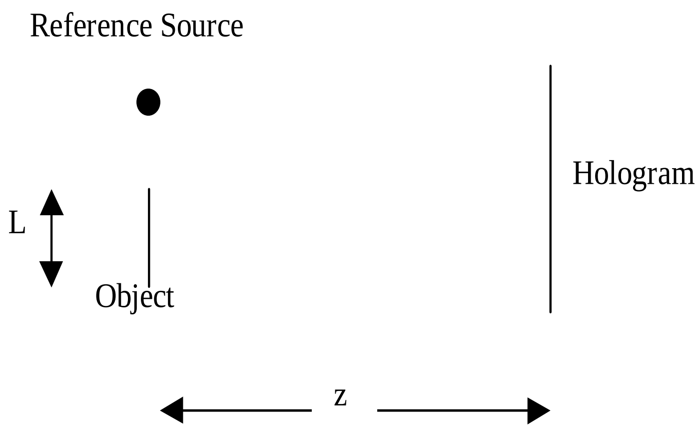
HS - 10

A hologram is made by interfering light from a point source and light from a diffuse transparency of width L as shown below. The distance from the object to the recording plane is z. Assume a linear recording process. The reconstruction wavelength is the same as the recording wavelength. The images are obtained by illuminating the hologram with a plane wave, followed by a positive lens of focal length f.

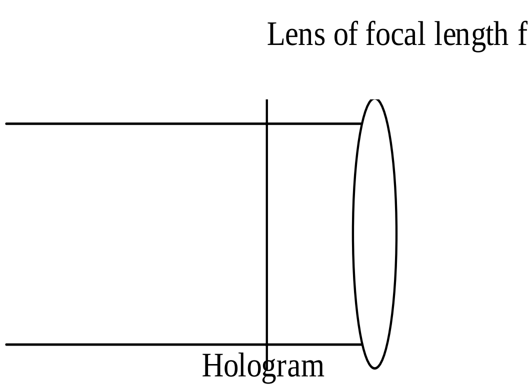
a) What are the positions of the two first-order images relative to the lens?

b) What is the transverse magnification of the two first-order images?

c) How far from the center of the object transparency should the reference point source be placed in order to assure no overlap of the zero-order light with the first-order images?



Hologram Recording



Hologram Reconstruction

Solution

- **a**

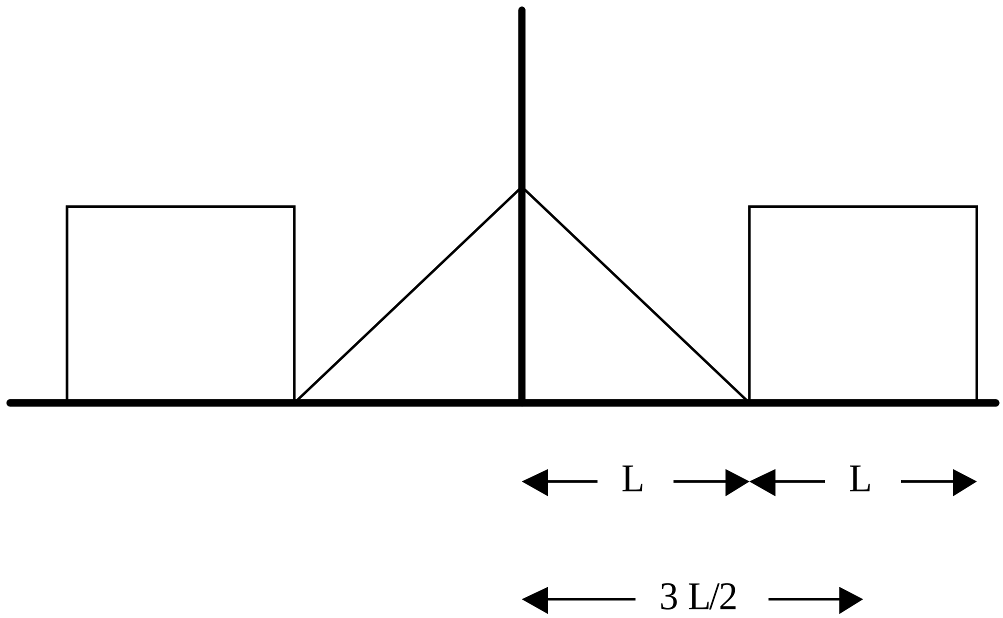
In the focal plane to the right of the lens. The hologram has no power.

- **b**

The magnitude of the transverse magnification is $\frac{f}{z}$.

- **c**

As long as the object is separated from the reference point the zero order and the first order will not overlap, i.e. $(\frac{L}{2})$ from the center of the object. To separate the self-interference terms from the first order the reference beam must be a distance L away from the edge of the object, i.e. $(\frac{3L}{2})$ from the center of the object.



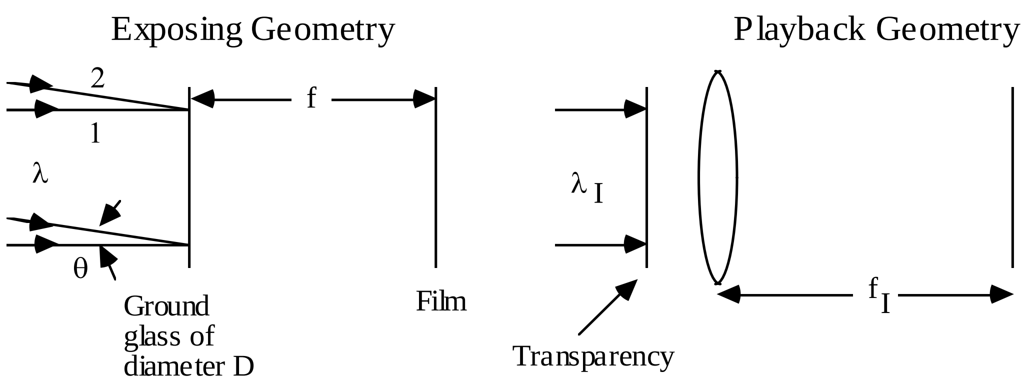
HS - 11

A piece of ground glass of diameter D is illuminated normally with a monochromatic collimated beam of wavelength λ, and the resulting speckle pattern is recorded on film a distance f away. A second recording of the resulting speckle pattern is made for the collimated beam incident at an angle θ. Assume that after processing, the amplitude transmittance of the film is proportional to the exposing irradiance. The resulting film transparency is illuminated with a collimated plane wave of wavelength λ_I. A lens of focal length f_I is placed after the transparency and the light distribution in the focal plane of the lens is observed. In answering the questions, you can assume small angles, if you wish.

a) What is the relationship between the fringe spacing observed in the focal plane of lens f_I and θ, D, f, λ , λ_I , f_I , and other pertinent quantities?

b) How many bright fringes can be observed in the focal plane? What is the minimum value of θ such that a bright fringe is observed at the center and edges of the pattern?

c) The results of this question imply that as far as resolution is concerned, all astronomical telescopes can be replaced with ground glass optics. What is wrong with this statement?



Solution

Two identical speckle patterns are recorded separated a distance = f θ.

The maximum spatial frequency recorded = $\nu_{\max} = \frac{d}{\lambda f}$.

The spread of diffracted light in the focal plane of lens f_I is $\pm f_I \lambda_I \nu_{\max} = \pm \frac{f_I \lambda_I d}{\lambda f}$.

▪ a

Fringe spacing

Considering the speckles as Young's double pinholes, or using the shift theorem gives the fringe spacing as

$$\text{fringeSpacing} = \frac{\lambda_I f_I}{\text{speckleDisplacement}}$$

$$\textbf{fringeSpacing} = \frac{\lambda_I f_I}{f \theta};$$

▪ b

Number of fringes

1 bright fringe in center, plus other fringes spaced $\frac{\lambda_I f_I}{f \theta}$.

$$\text{numberFringes} = 1 + \frac{\text{spread of diffracted light}}{\text{fringe spacing}}$$

$$\textbf{numberFringes} = \frac{\frac{2 \lambda_I f_I d}{f \lambda}}{\frac{\lambda_I f_I}{f \theta}}$$

$$\frac{2 d \theta}{\lambda}$$

For a bright fringe at the center and edge, numberFringes = 3.

```
Solve[3 == 1 + (2 d θmin)/λ, θmin]
{{θmin → λ/d}}
```

Note that this is normal resolution for aperture of diameter d.

■ c

To obtain above resolution we must have good contrast speckles. To obtain good contrast speckles, the OPD's must be less than the coherence length of the source. Thus for ground glass telescope optics we would need long coherence lengths, and hence narrow spectral bandwidths and we would end up with little light for most applications. In addition, the light would be spread over a large area, and again there would be a problem with light intensity.