Special Interferometric Tests for Aspherical Surfaces

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Aspheric Surfaces

Aspheric surfaces are of much interest because they can provide

- **Improved performance**
- **Reduced number of optical components**
- **Reduced weight**
- **Lower cost**

Conics

A conic is a surface of revolution defined by means of the equation

$$\boxed{s^2 - 2rz + (k+1)z^2 = 0}$$

Z axis is the axis of revolution. k is called conic constant. r is the vertex curvature.

$$s^2 = x^2 + y^2$$

Sag for Conic

$$z = \frac{s^2 / r}{1 + [1 - (k+1)(s/r)^2]^{1/2}}$$

$$s^2 = x^2 + y^2$$

Sag for Asphere

$$\boxed{z = \frac{s^2/r}{1+[1-(k+1)(s/r)^2]^{1/2}} + A_4 s^4 + A_6 s^6 + \ldots}$$

$$s^2 = x^2 + y^2$$

k is the conic constant

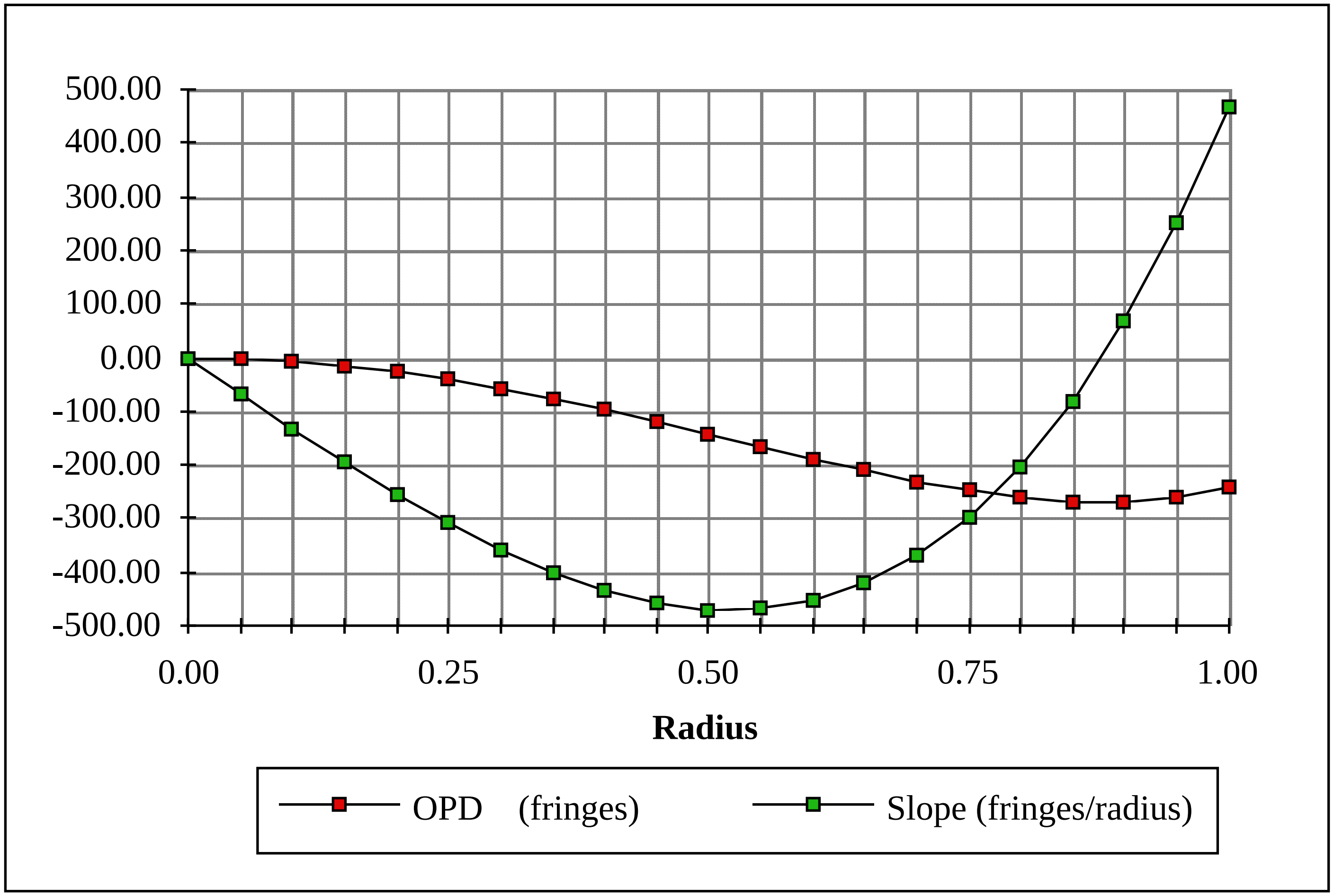
r is the vertex radius of curvature

A's are aspheric coefficients

Difficulty of Aspheric Test

Slope of aspheric departure determines difficulty of test

Wavefront Departure and Slope versus Radius



Aspheric Testing Techniques

- **Null Tests - Perfect optics give straight fringes**
  - **Conventional null optics**
  - **Holographic null optics**
  - **Computer generated holograms**

- **Non-null Tests - Even perfect optics do not give straight fringes**
  - **Lateral shear interferometry**
  - **Radial shear interferometry**
  - **High-density detector arrays**
  - **Sub-Nyquist interferometry**
  - **Long-wavelength interferometry**
  - **Two-wavelength holography**
  - **Two-wavelength interferometry**

9.2) Null Tests

If the asphere is perfect, perfectly straight fringes will be produced.

9.2.1 Refractive Null Optics

Ref: Chapter 12 of Malacara
Abe Offner, Appl. Opt. P. 153, 1963

The third-order spherical aberration introduced by a parabola can be balanced only by a combination of third and higher order aberration if balancing is done at any position other than at the parabola, where the corrector would have to be as large as the parabola itself. To get around this problem we add a field lens to image the lens onto the parabola. We can move the field lens slightly away from the center of curvature to match the required aberration.

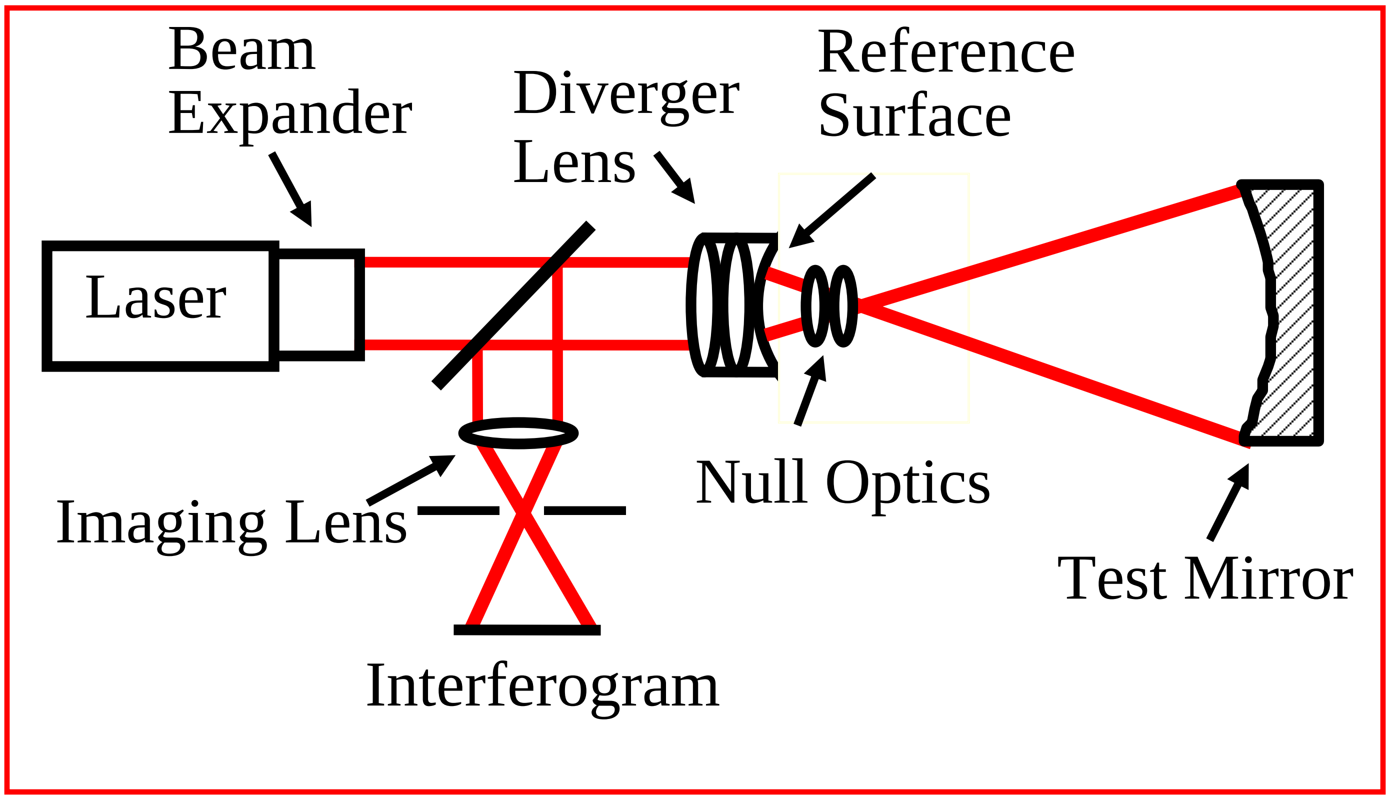
9.2.2 Reflective Null Optics

Ref: Chapter 12 and Appendix 2 of Malacara

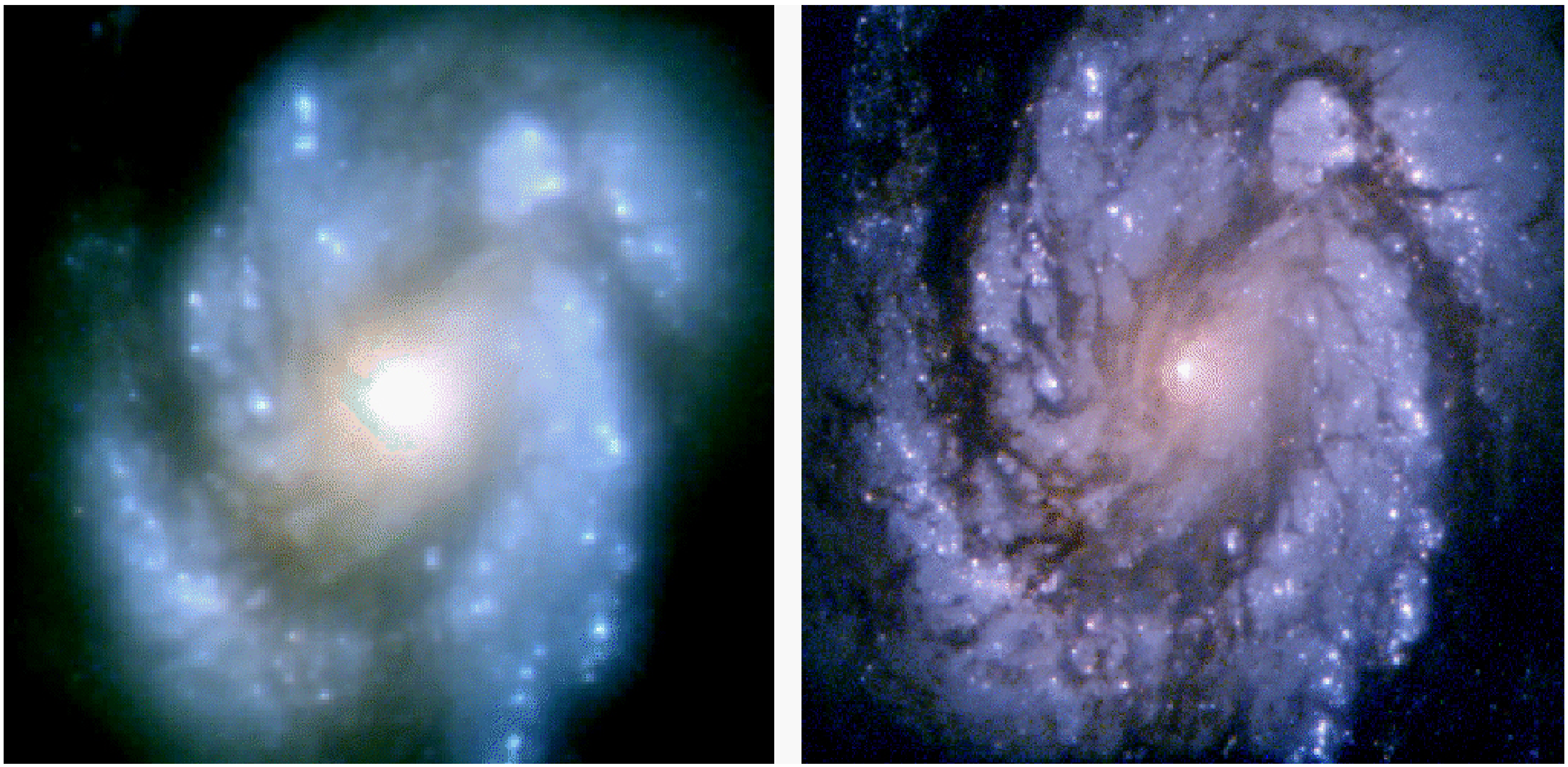
Note that the Hindle test is the same as the Ritchey-Common test except the test is performed on axis.

A problem with the Hindle test is that it requires a large spherical mirror. A method for eliminating the requirement for having such a large sphere is to make a concave test plate the same size as the convex hyperboloid, and test the concave surface in the Hindle test. The test plate can be tested using a spherical mirror not much larger than the test plate, then by use of a Fizeau interferometer the test plate can be used to test the convex hyperboloid. This test technique is called the Silvertooth test, after Bud Silvertooth who first suggested the idea.

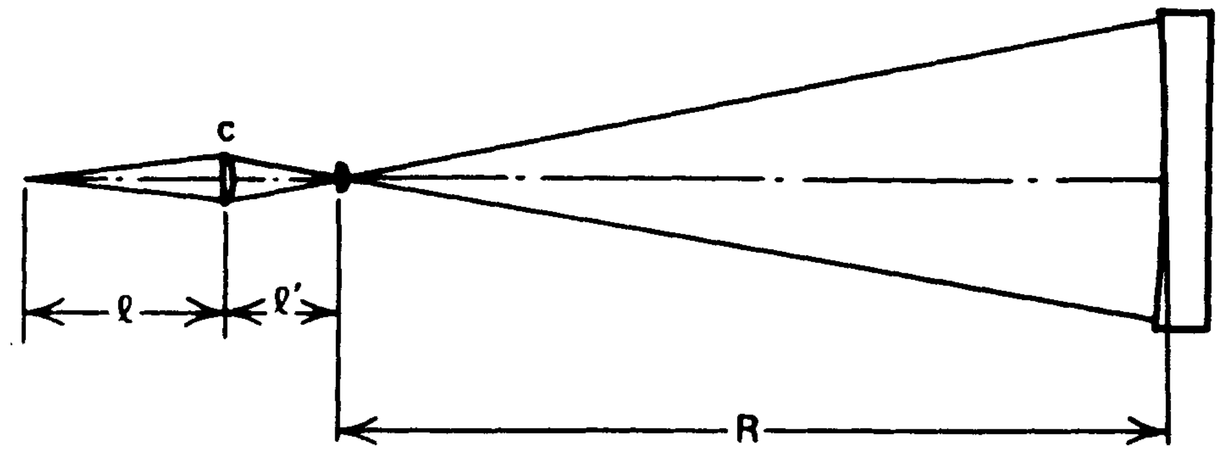
Conventional Null Optics



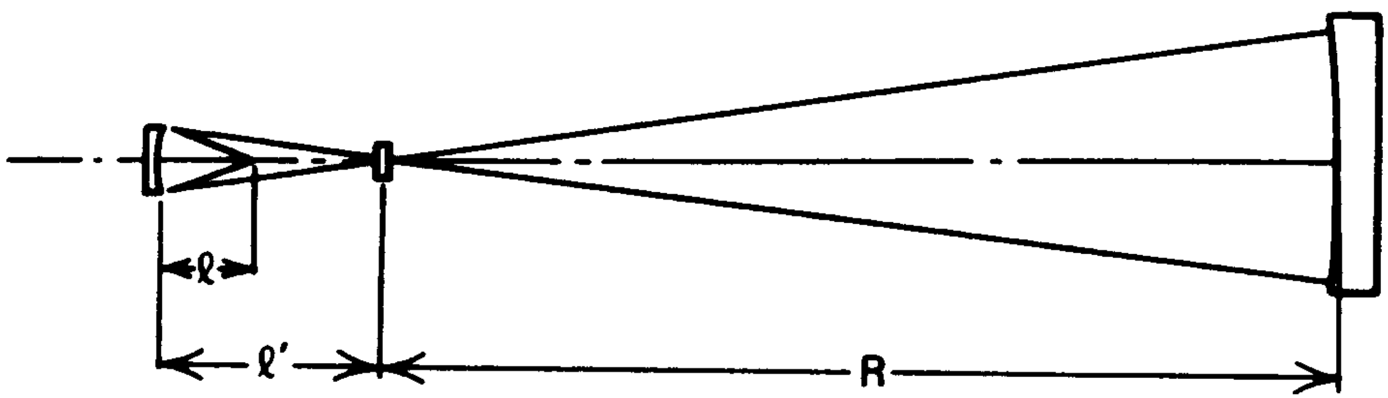
Hubble Pictures
(Before and After the Fix)



Offner Null Compensators

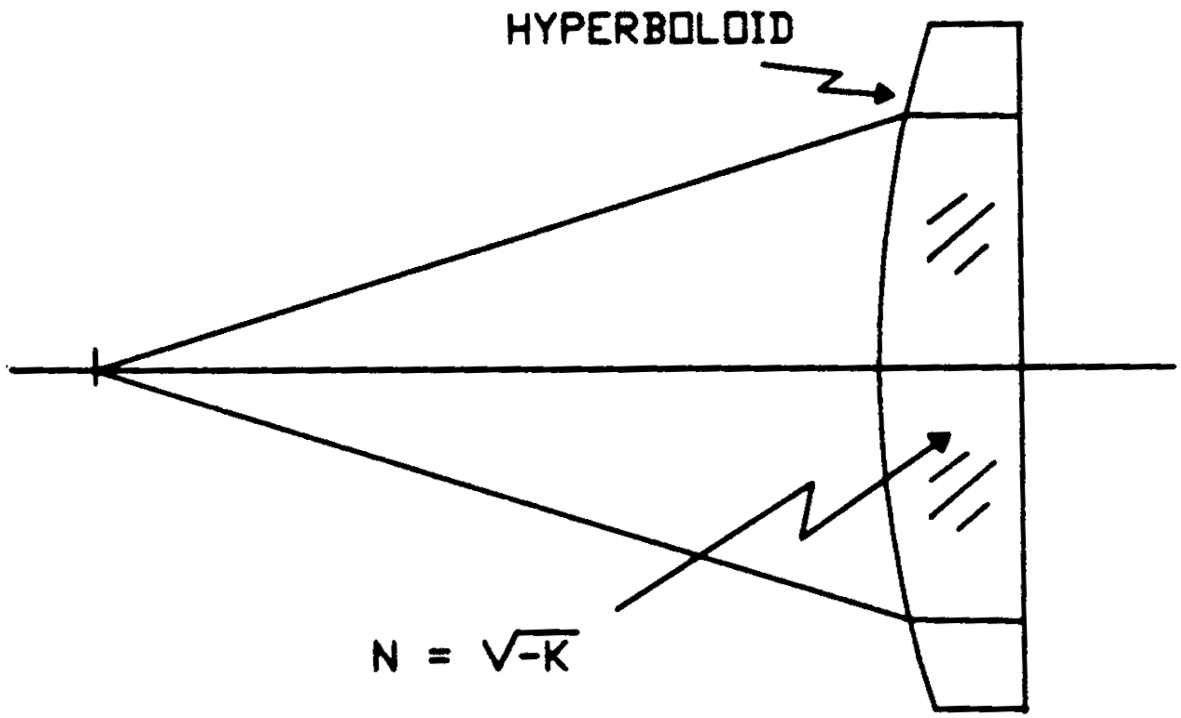


Refracting compensator with field lens.

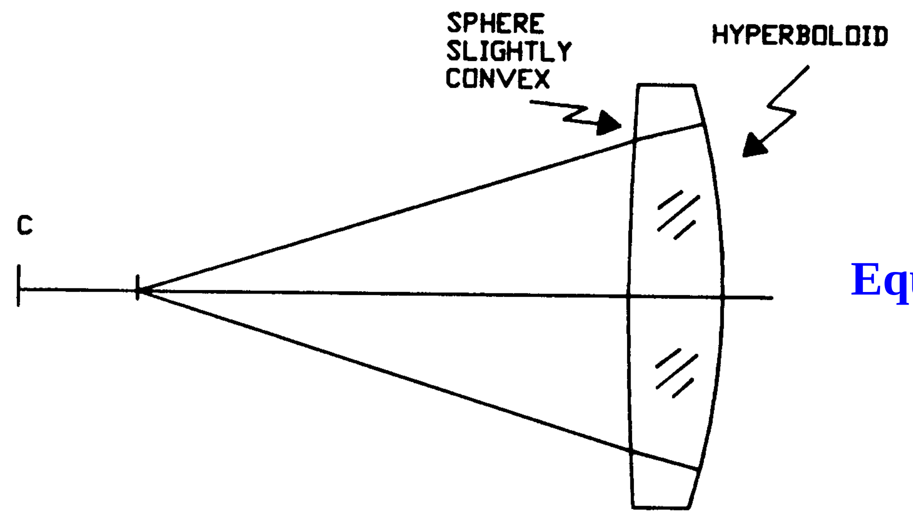


Single-mirror compensator with field lens.

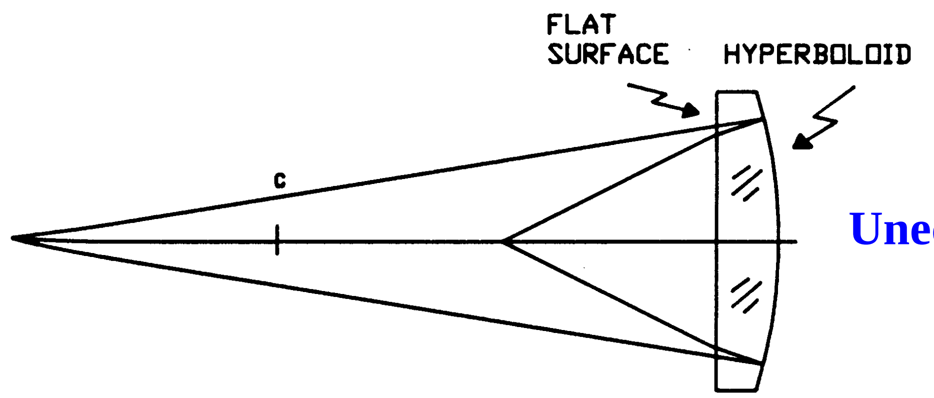
Testing of Hyperboloid



Meinel Hyperboloid Test

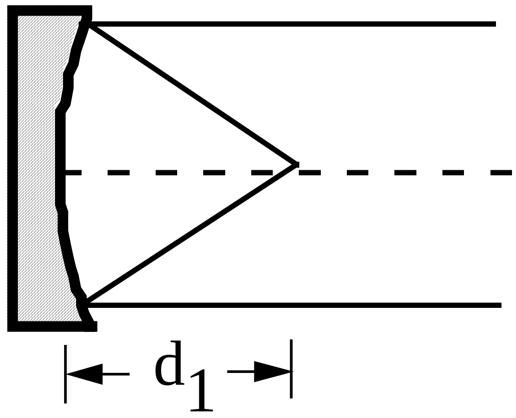


Equal conjugates.

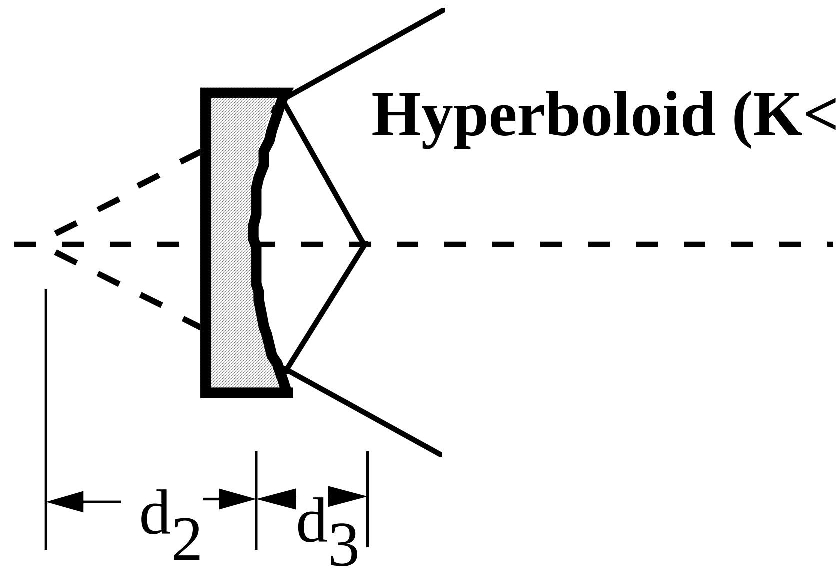


Unequal conjugates.

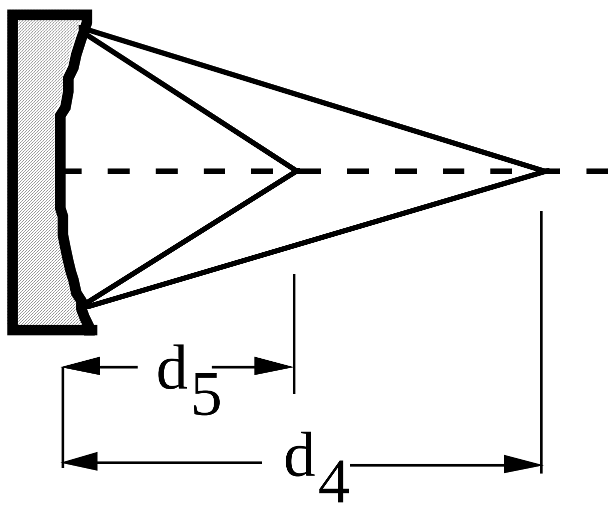
Null Tests for Conics



Parabola (K=-1)



Hyperboloid (K<-1)



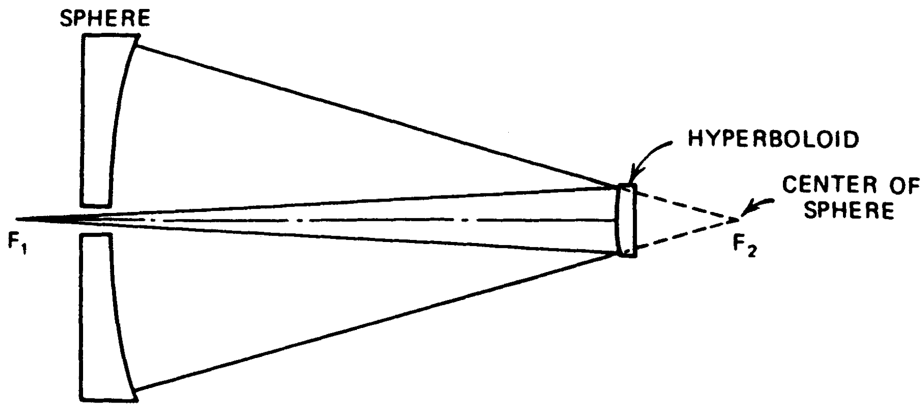
Ellipsoid (-1< K<0)

$$d_1 = r/2$$

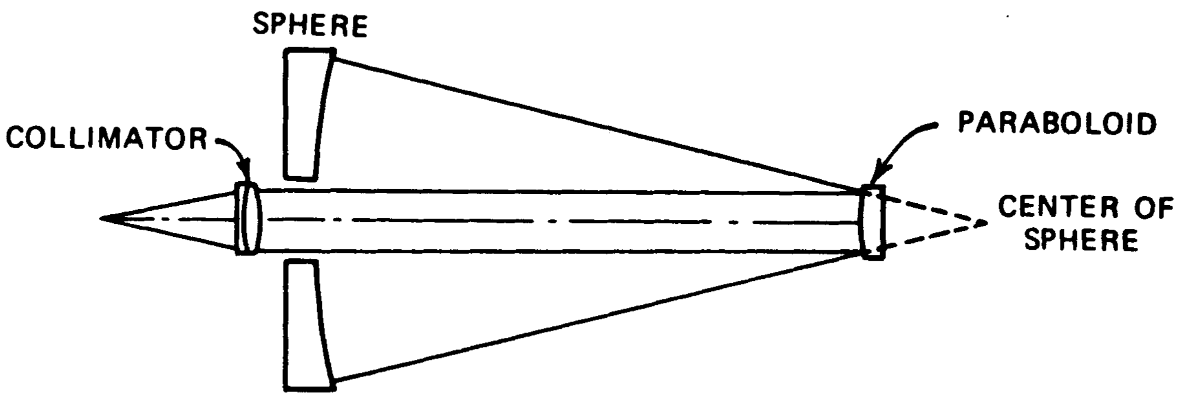
$$d_2, d_3 = \frac{r}{K+1}\left(\sqrt{-K} \pm 1\right)$$

$$d_4, d_5 = \frac{r}{K+1}\left(1 \pm \sqrt{-K}\right)$$

Hindle Test

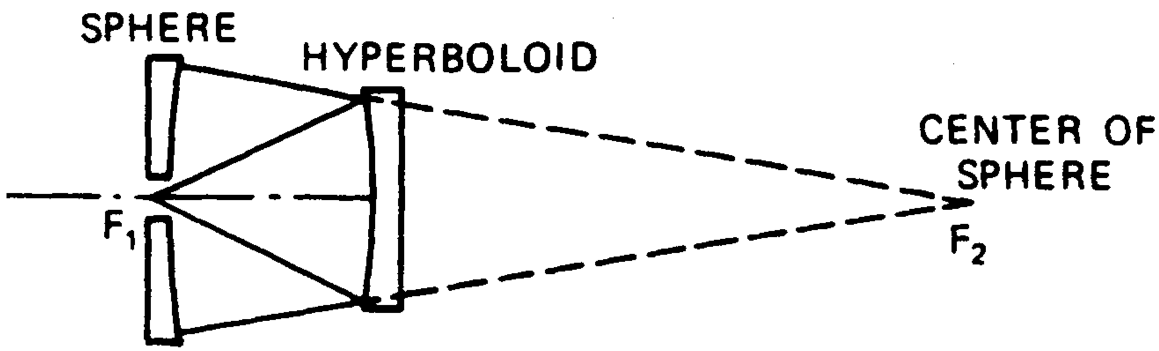


Testing convex hyperboloid.

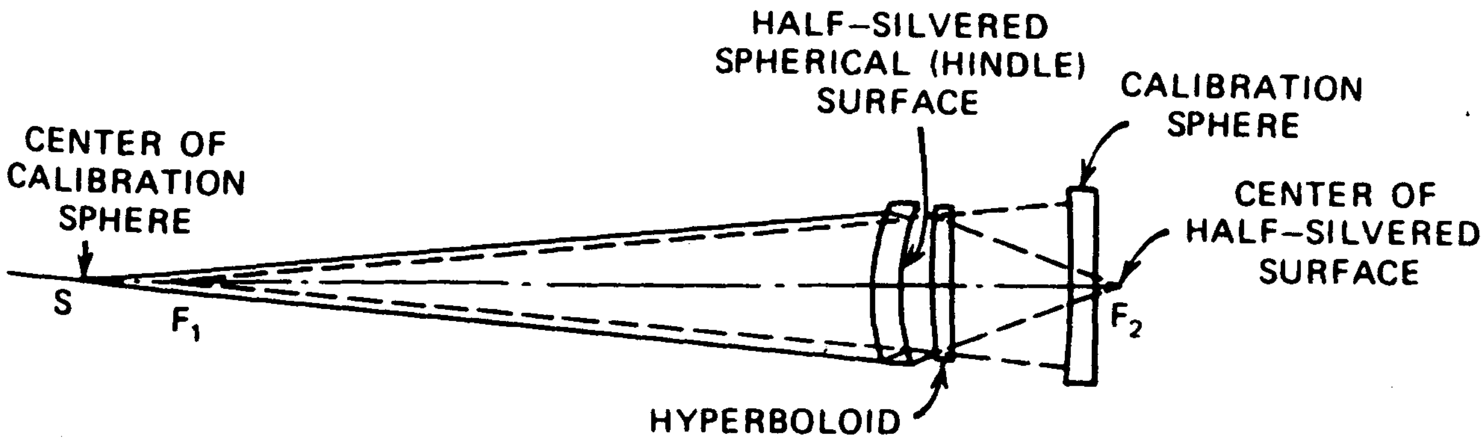


Testing convex paraboloid.

Modified Hindle tests

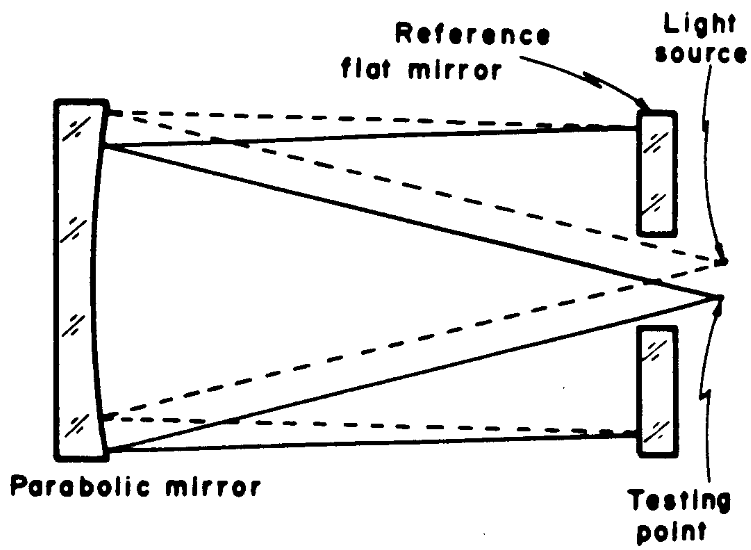


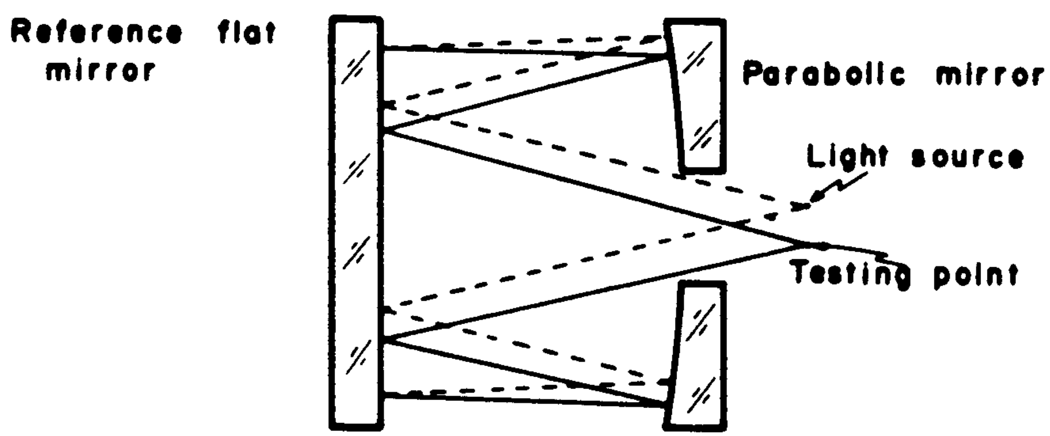
Silvertooth Test (concave hyperboloid can be used as test plate to test convex hyperboloids).



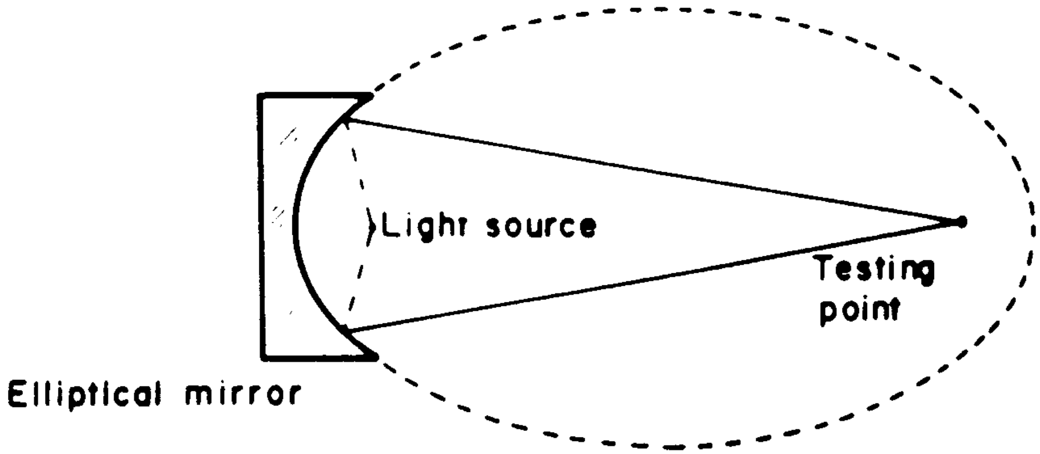
Simpson-Oland-Meckel Test.

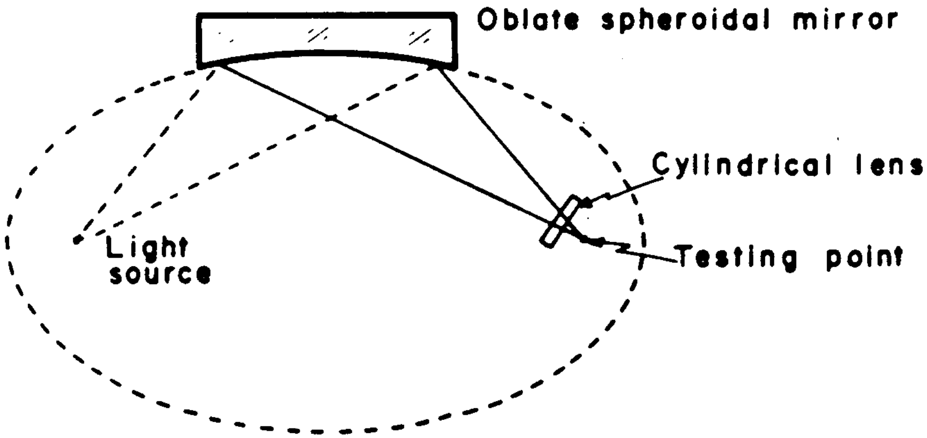
Testing Concave Parabolic Mirrors



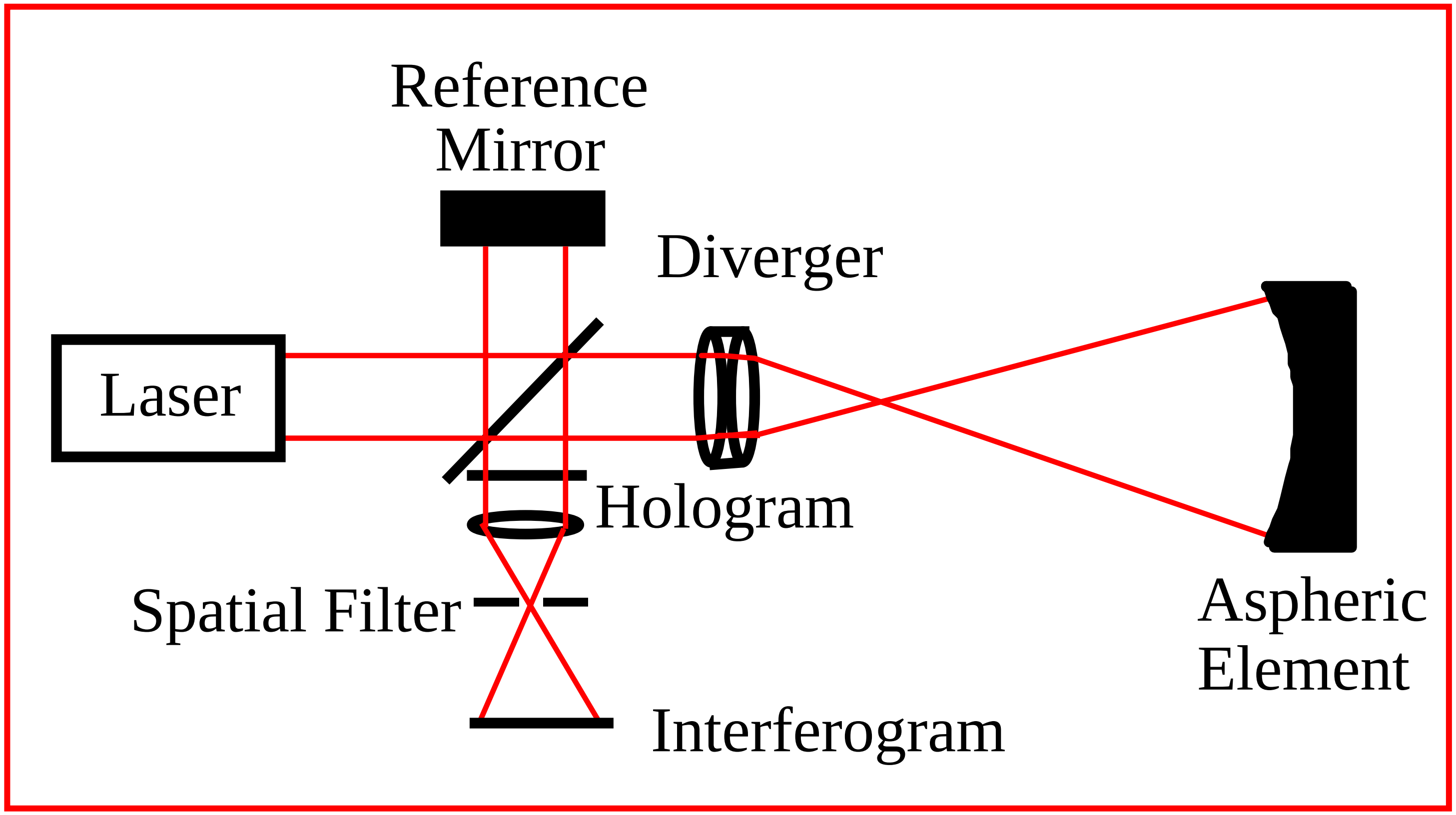


Testing Elliptical Mirrors

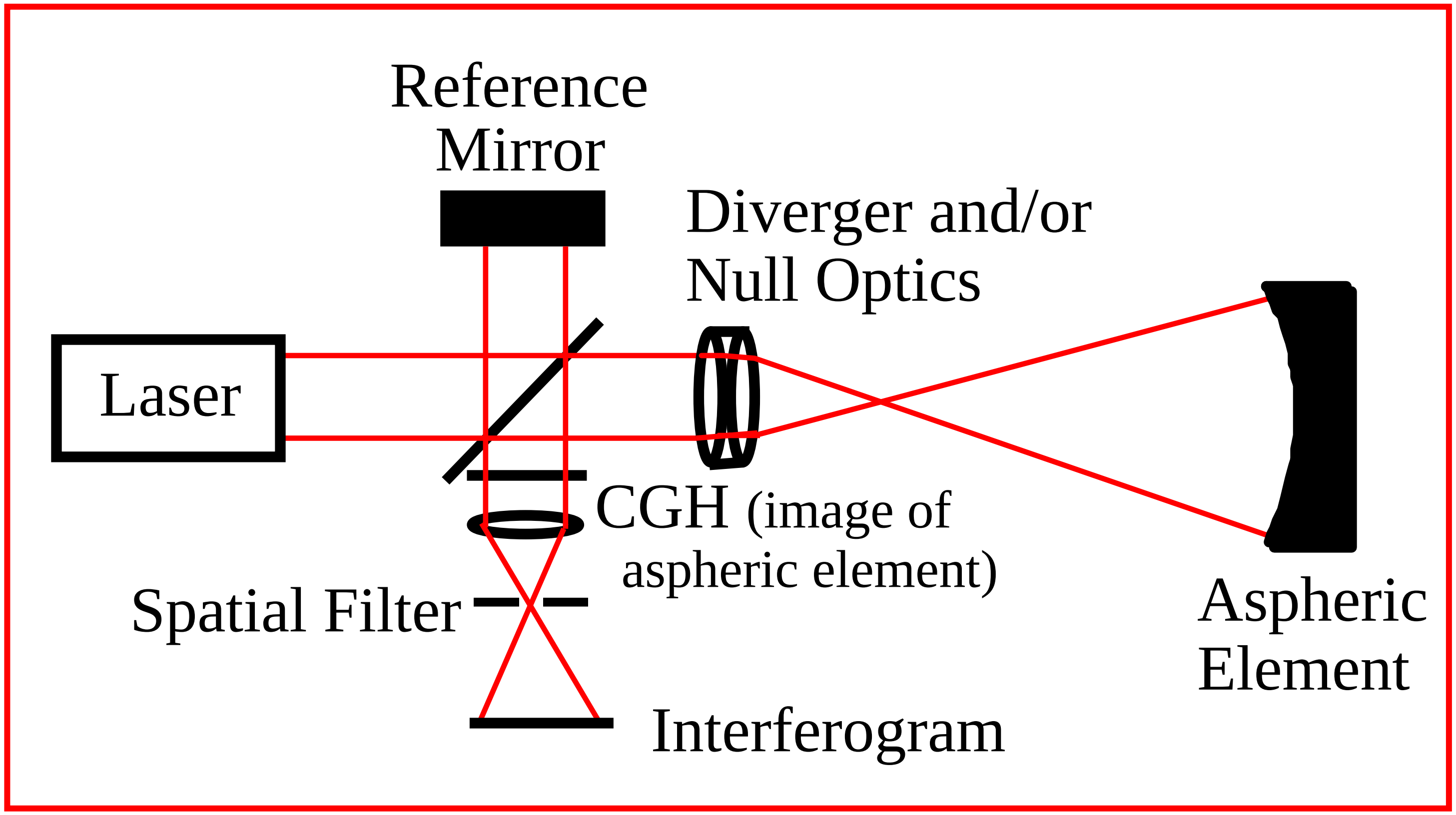




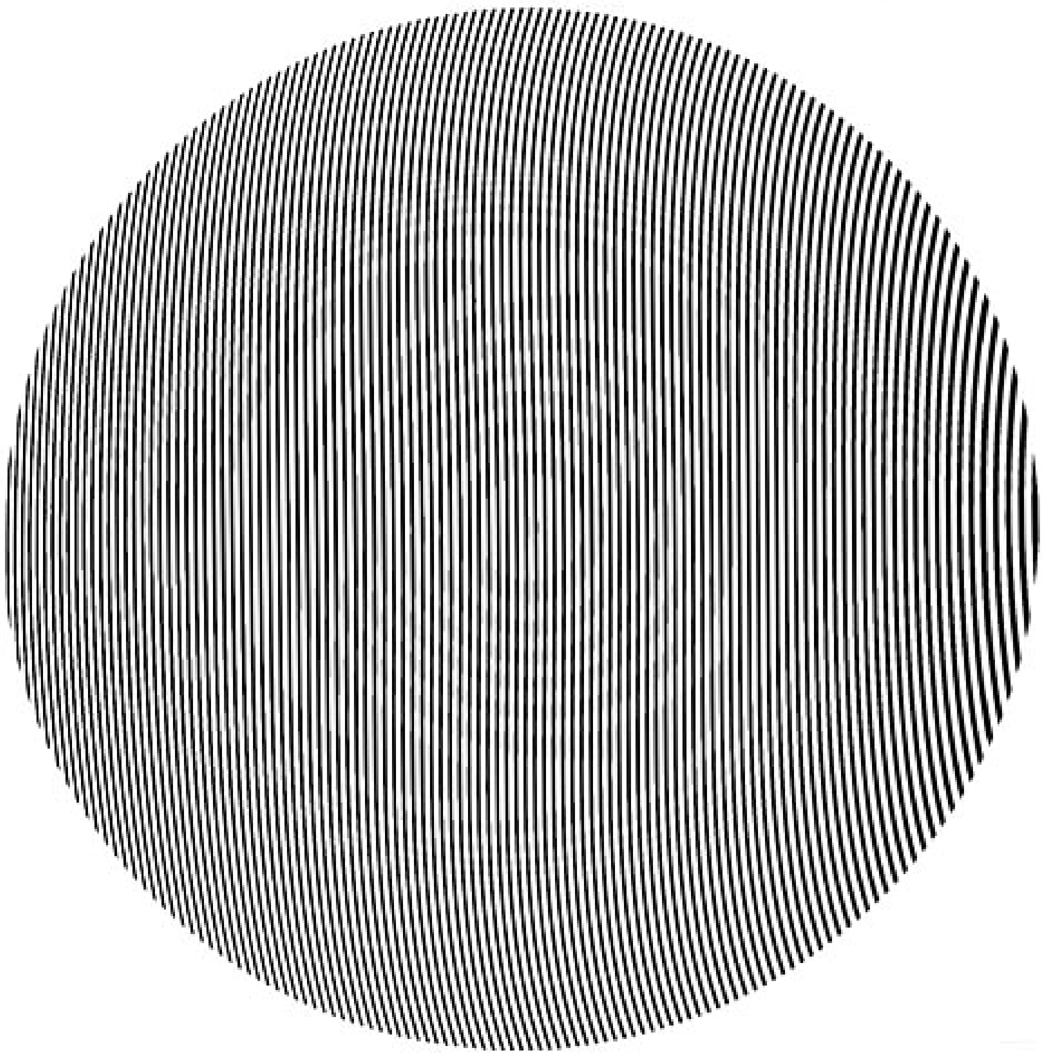
Holographic Null Optics



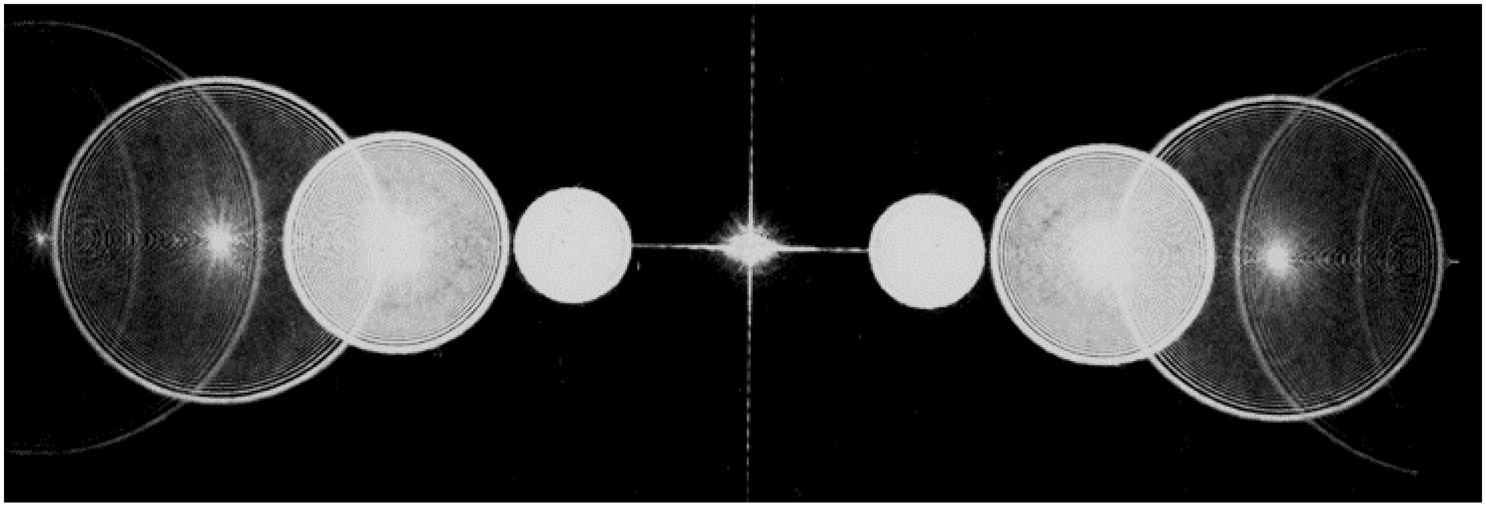
CGH Interferometer



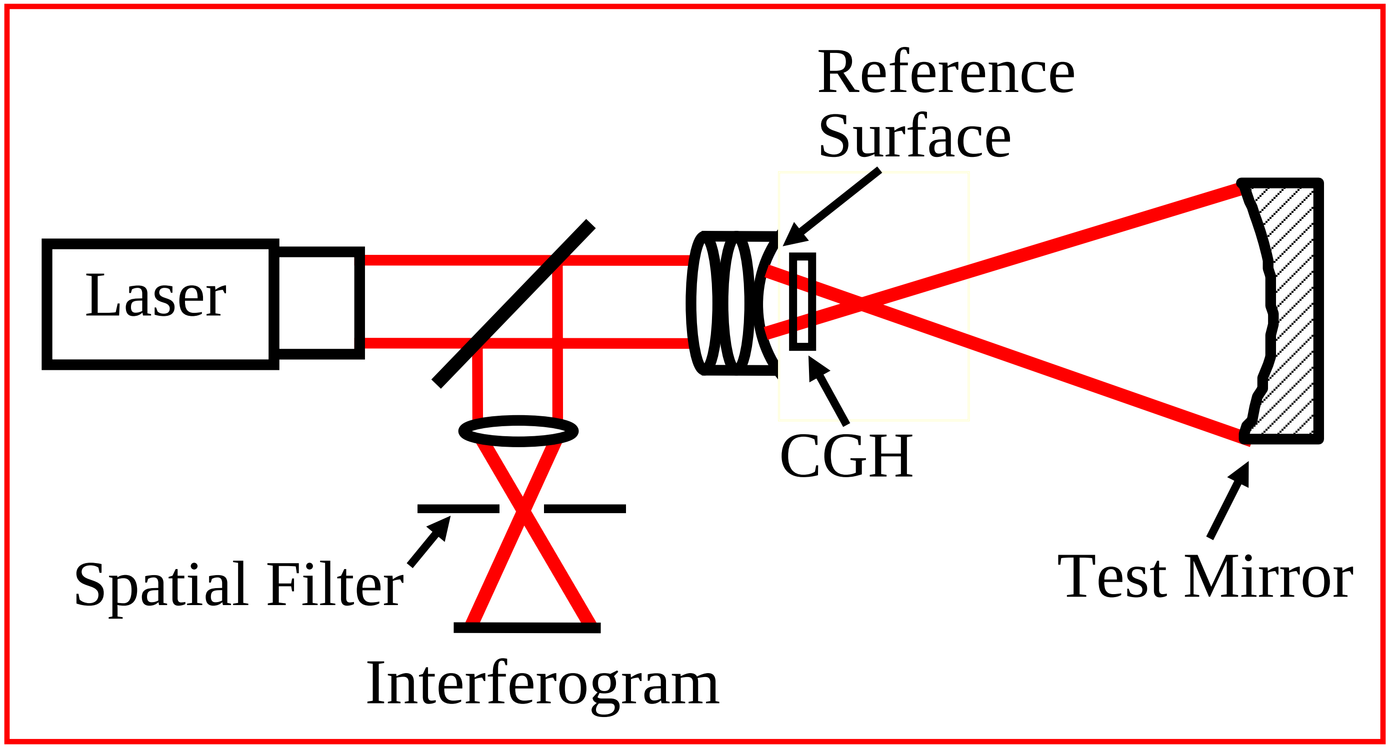
Computer Generated Hologram



Light in Spatial Filter Plane

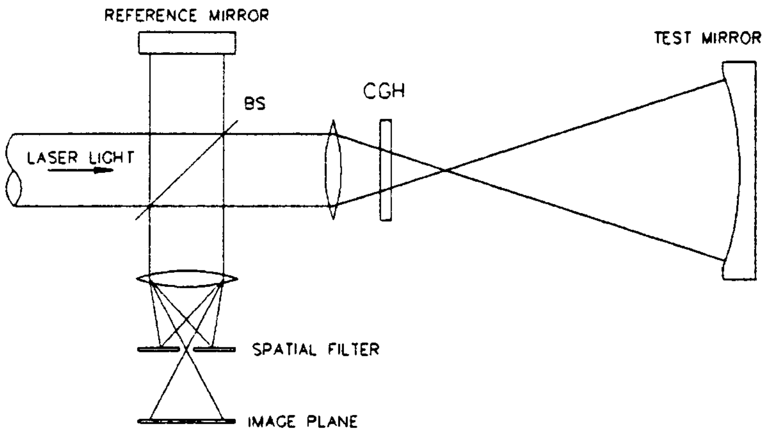


CGH Used as Null Lens

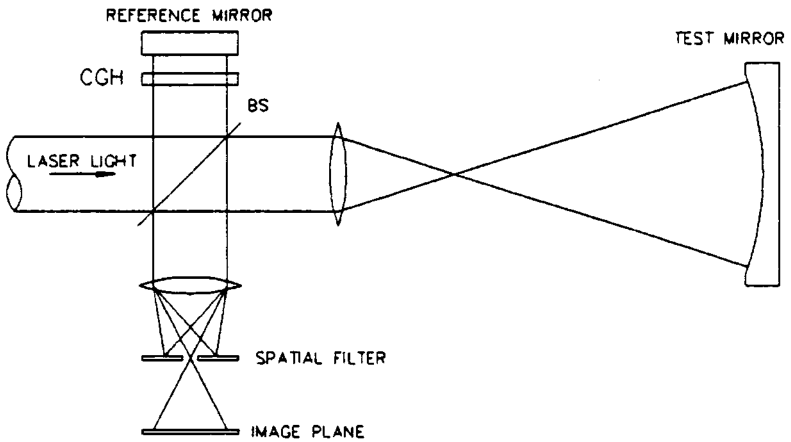


- **Can use existing commercial interferometer**
- **Double pass through CGH, must be phase etched for testing bare glass optics**
- **Requires highly accurate substrate**

CGH Optical Testing Configurations -I

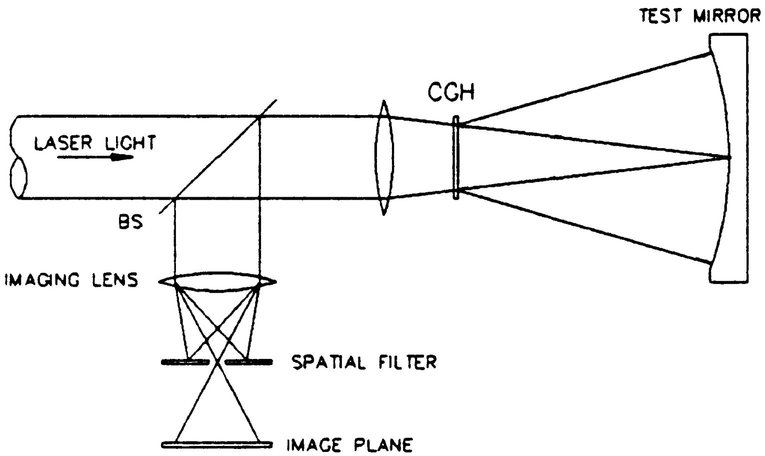


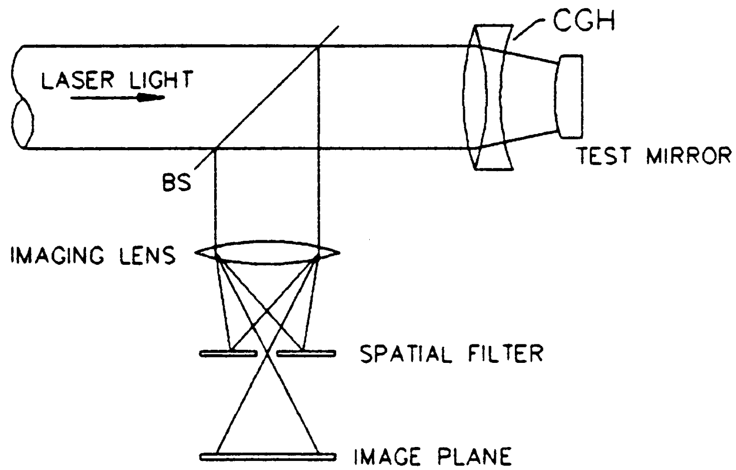
CGH as null lens



CGH in reference arm

CGH Optical Testing Configurations -II

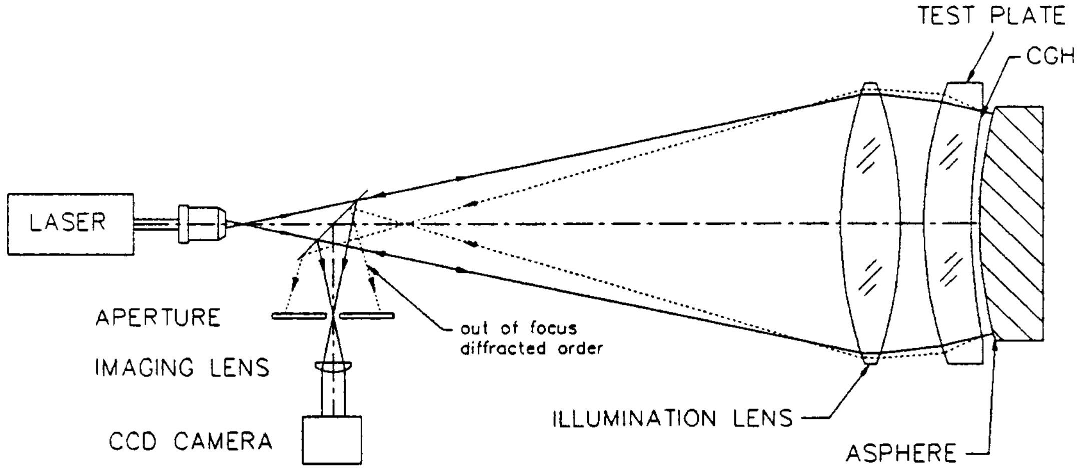




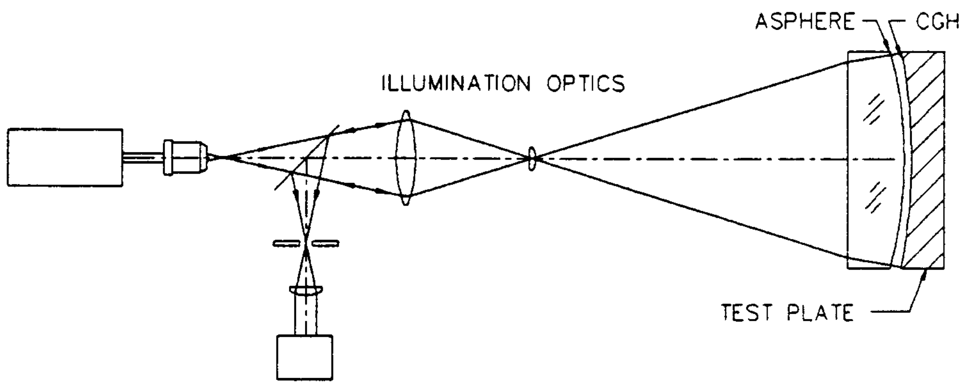
Zone plate interferometer

CGH test plate

CGH Test Plate Configuration

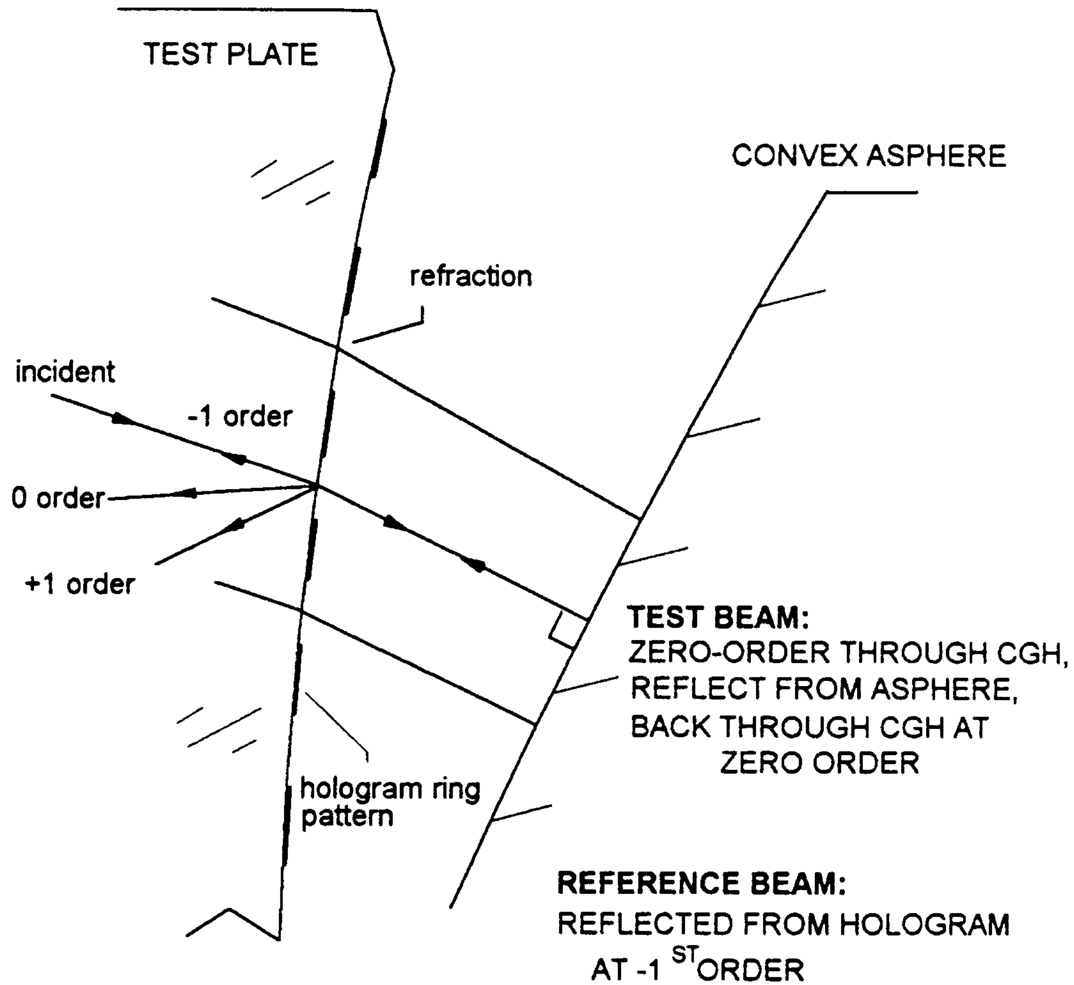


Configuration for CGH test plate measurement of a convex asphere.



Alternate configuration for CGH test convex aspheres.

Reference and Test Beams in CGH Test Plate Setup



Error Source

- **Pattern distortion (Plotter errors)**
- **Alignment Errors**
- **Substrate surface figure**

Pattern Distortion

- **The hologram used at *m*th order adds *m* waves per line;**
- **CGH pattern distortions produce wavefront phase error:**

$$\Delta W(x, y) = -m\lambda \frac{\varepsilon(x, y)}{S(x, y)}$$

$\varepsilon(x,y)$ = grating position error in direction perpendicular to the fringes;

$S(x,y)$ = localized fringe spacing;

For m = 1, phase error in waves = distortion/spacing

0.1 µm distortion / 20 µm spacing -> $\lambda/200$ wavefront

Plotters

- E-beam
  - Critical dimension – 1 micron
  - Position accuracy – 50 nm
  - Max dimensions – 150 mm
- Laser scanner
  - Similar specs for circular holograms

Calibration of Plotter Errors

- Put either orthogonal straight line gratings or circular zone plates on CGH along with grating used to produce the aspheric wavefront
- Straight line gratings produce plane waves which can be interfered with reference plane wave to determine plotter errors
- Circular zone plates produce spherical wave which can be interfered with reference spherical wave to determine plotter errors

Substrate Distortion

- **E-beam written patterns must be fabricated onto standard reticle substrates: thin and flat to only about 1 micron.**
- **These can be printed onto precision substrates, with some loss in accuracy.**
- **For phase etched holograms, you cannot measure the substrate after CGH is recorded and back it out.**

Solving Substrate Distortion Problems

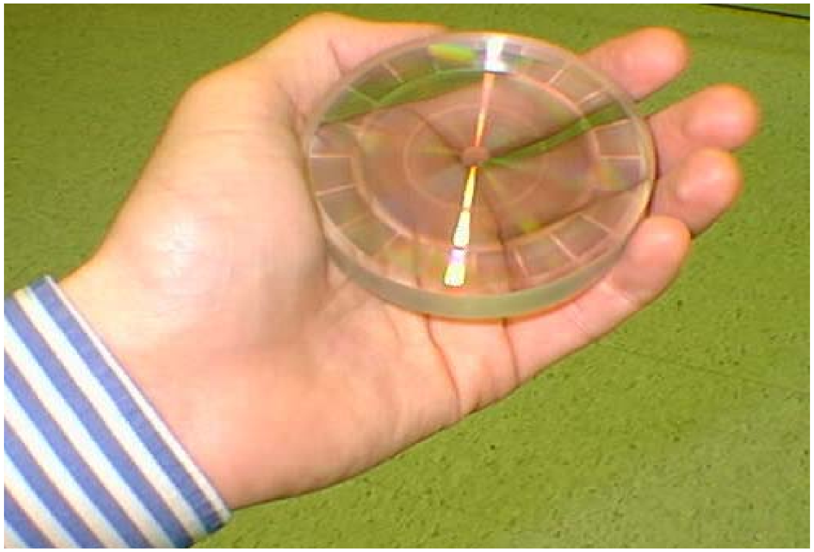
- **Use direct laser writing onto custom substrates**
- **Use amplitude holograms, measure and back out substrate**
- **Use an optical test setup where reference and test beams go through substrate**

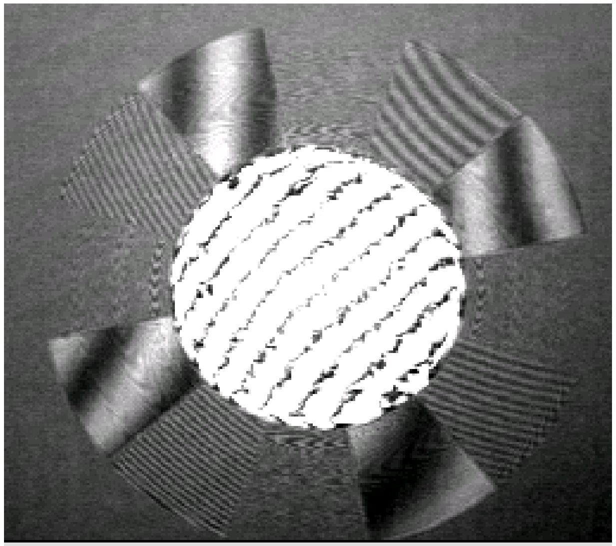
Alignment Errors

- Lateral misalignment gives errors proportional to slope of wavefront
- Errors due to longitudinal misalignment less sensitive if hologram placed in collimated light
- Alignment marks (crosshairs) often placed on CGH to aid in alignment
- Additional holographic structures can be placed on CGH to aid in alignment of CGH and optical system under test

Use of CGH for Alignment

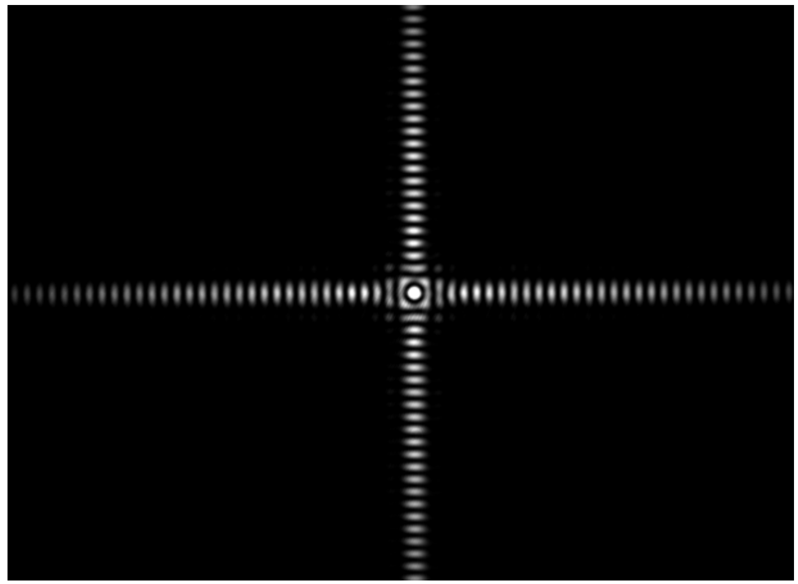
- **Commonly CGH's have patterns that are used for aligning the CGH to the incident wavefront.**





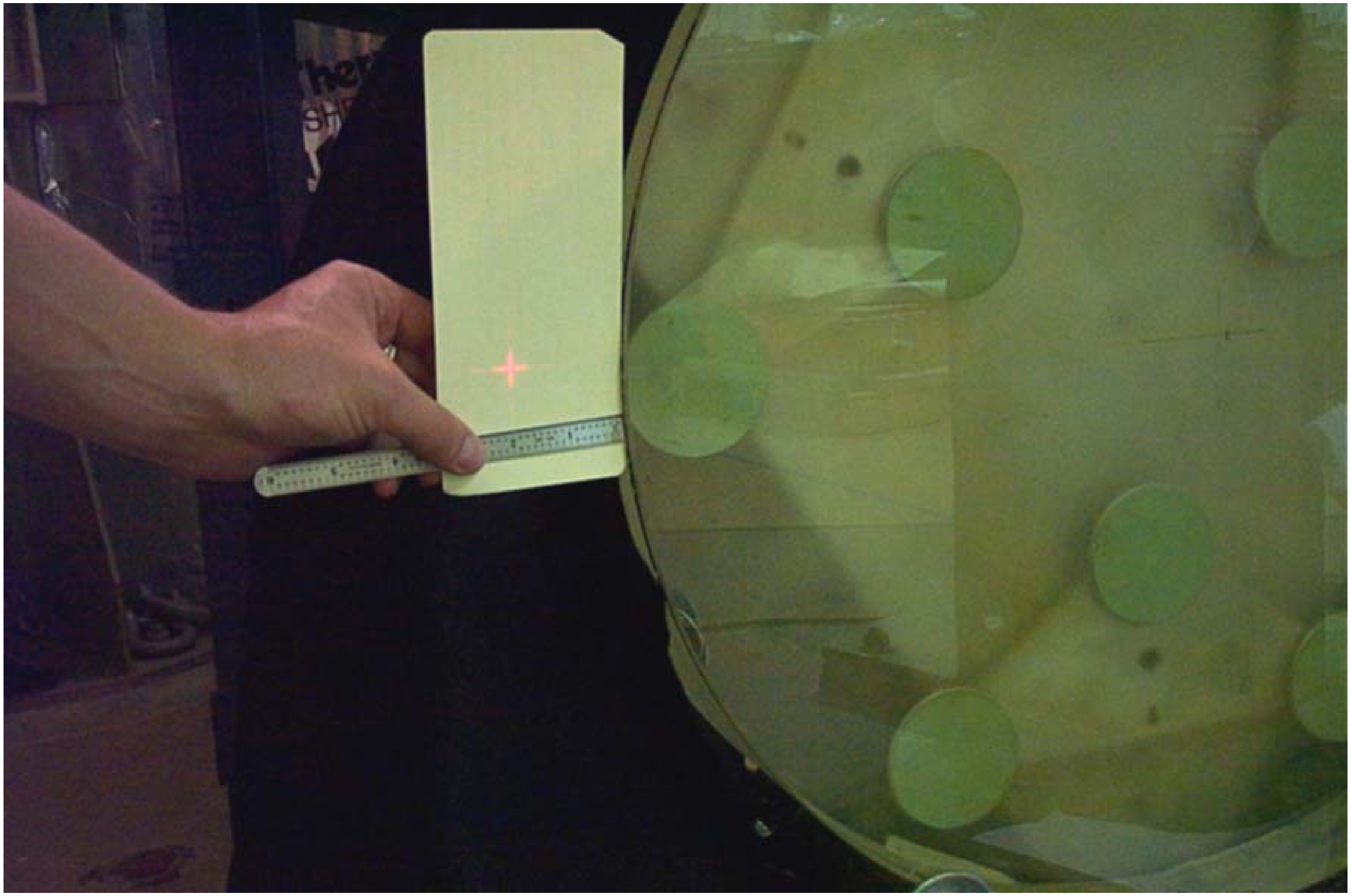
Using multiple patterns outside the clear aperture, many degrees of freedom can be constrained using the CGH reference.

Projection of Fiducial Marks



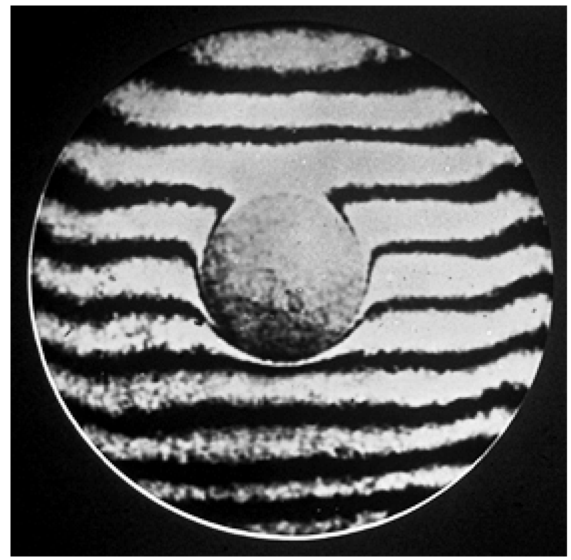
- The positions of the crosshairs can be controlled to micron accuracy
- The patterns are well defined and can be found using a CCD
- Measured pattern at 15 meters from CGH. Central lobe is only 100 µm FWHM

CGH Alignment for Testing Off-Axis Parabola



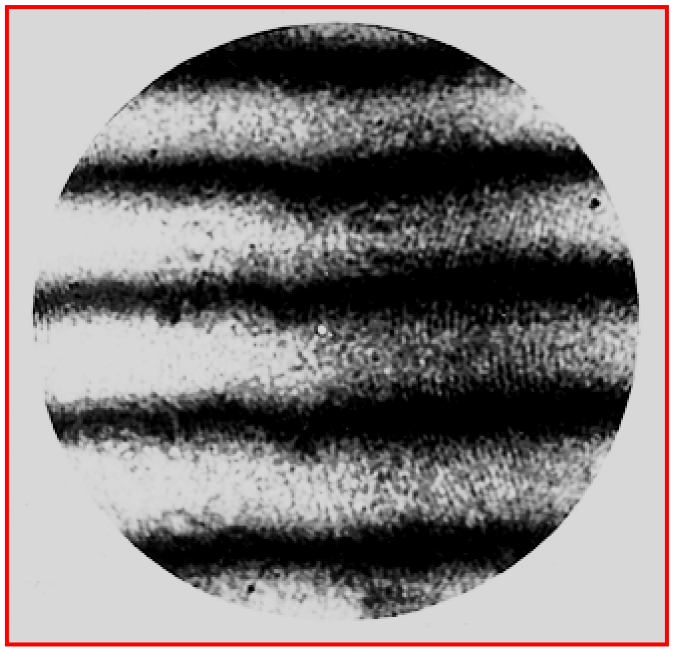
Results for using CGH to test an f/3 Parabolic Mirror

Double pass autocollimation test

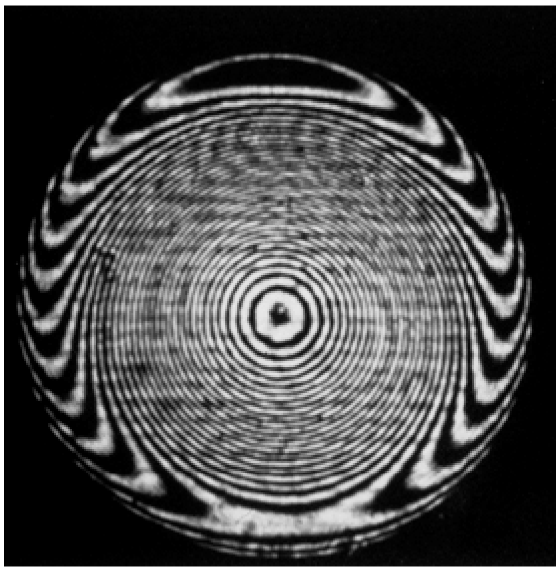


Single pass CGH test

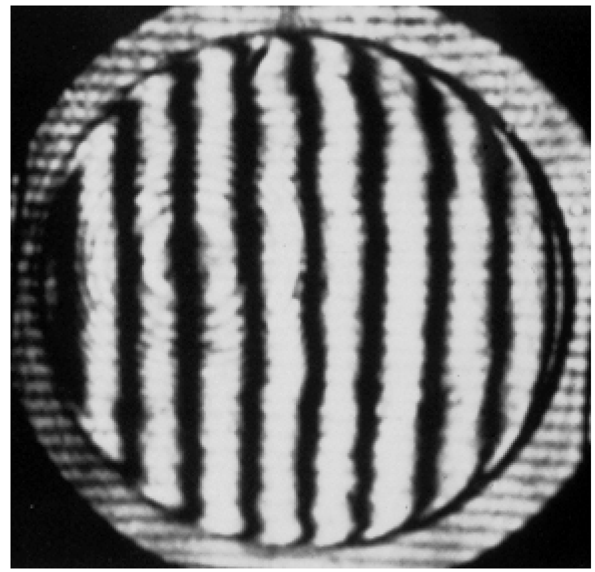
Holographic test of refractive element having 50 waves of third and fifth order spherical aberration



CGH test of parabolic mirror

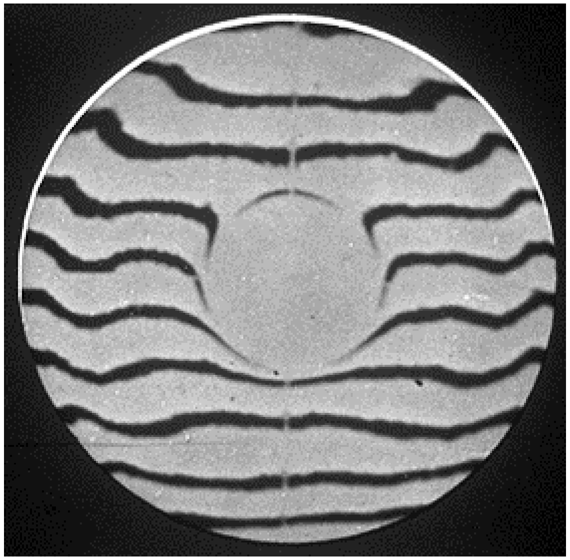


No CGH

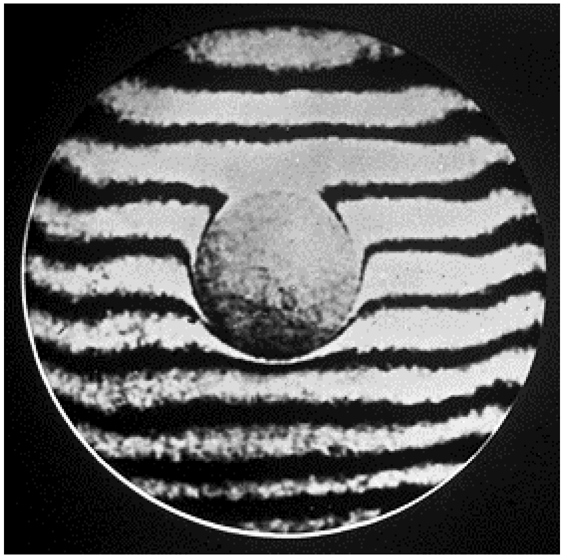


CGH

Results for using CGH to test an f/3 Parabolic Mirror

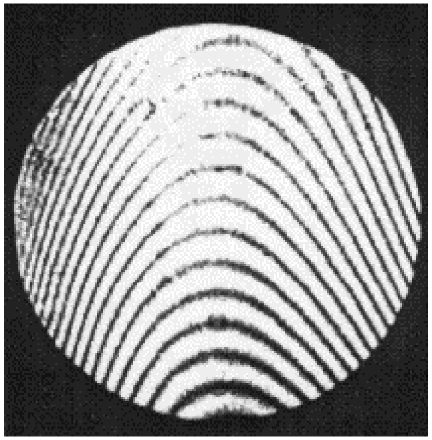


Double pass autocollimation test

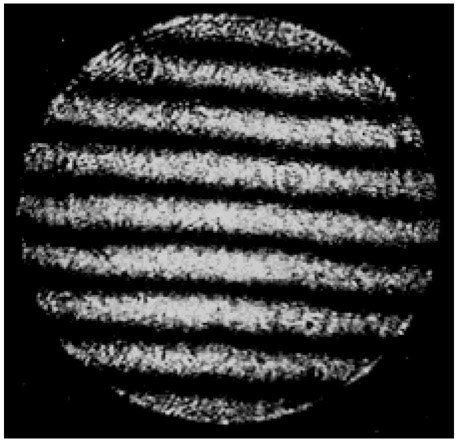


Single pass CGH test

CGH test of aspheric wavefront having 35 waves/radius max slope and 10 waves departure

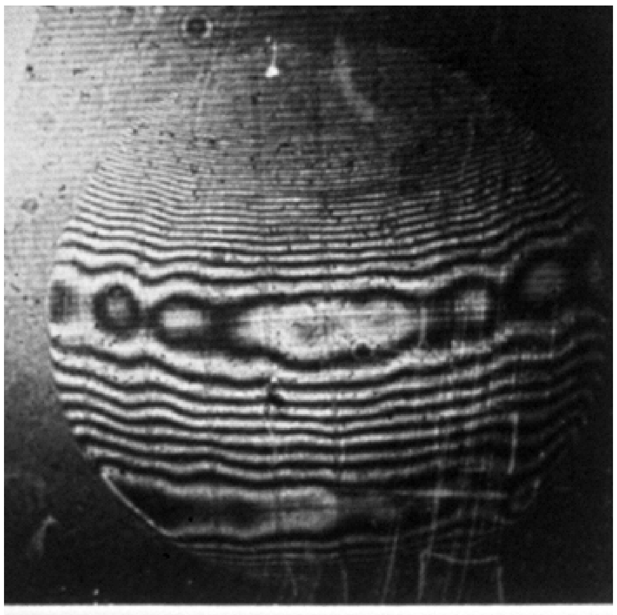


No CGH

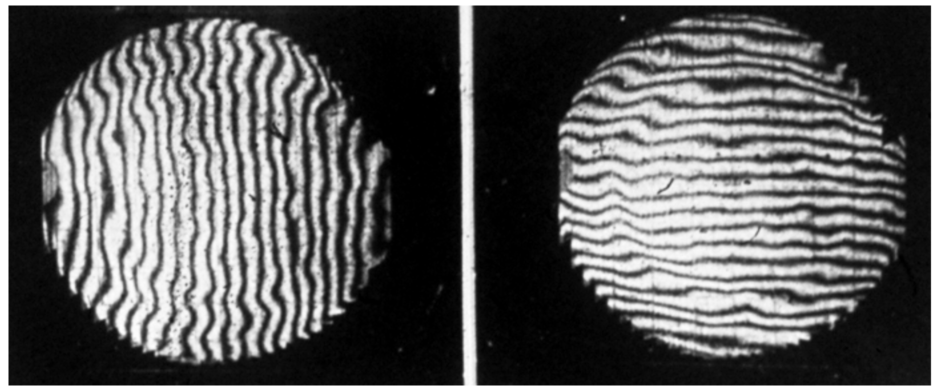


CGH

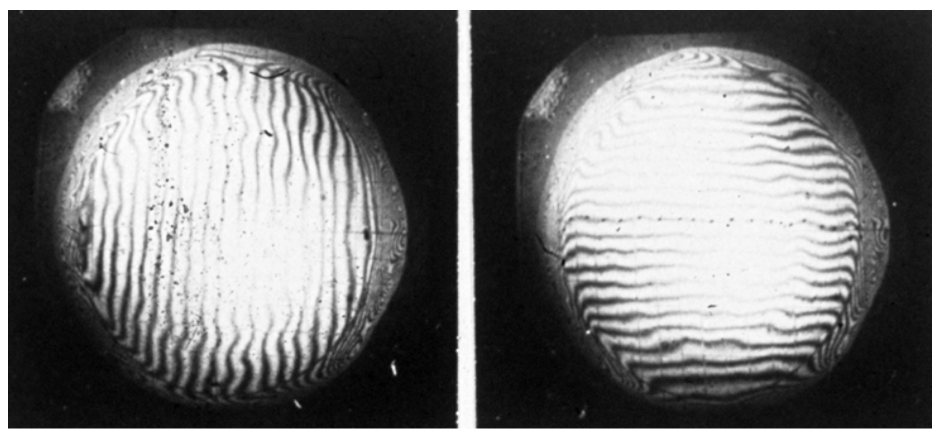
Aspheric Testing Using Partial Null Lens and CGH



Partial null lens test without CGH



CGH-partial null lens test



Null lens test

9.3 Non-null Test

Can use geometrical tests such as Foucault, wire, or Ronchi tests, but we will only discuss interferometric tests. One advantage of interferometric tests is that they are fairly easy to computerize.

9.3.1 Lateral shear interferometry

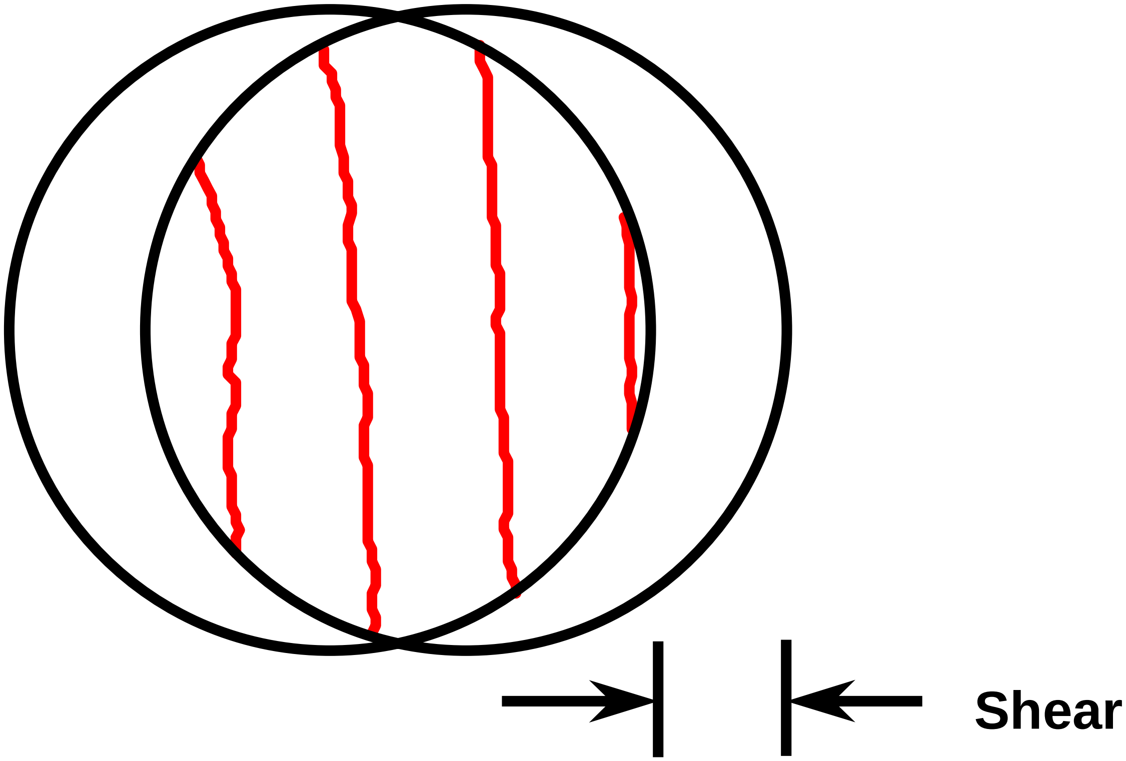
- <u>Advantages</u>
  Can vary the sensitivity by varying the amount of shear.

- <u>Disadvantages</u>
  Two interferograms are required for non-rotationally symmetric wavefronts.
  Must know the amount of shear and direction of shear very accurately.
  Helps less with wavefronts having larger slopes.

$$\frac{\text{Fringes with LSI}}{\text{Fringes with T - G}} = N\,\frac{\text{shear distance}}{\text{pupil radius}}$$

  N is the power of the aberration, i.e. 4 for fourth-order spherical.

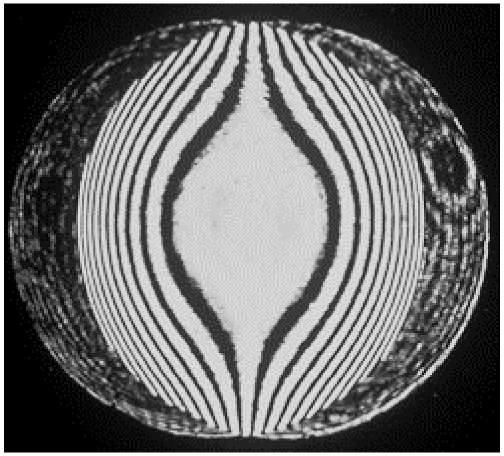
Lateral Shear Interferometry

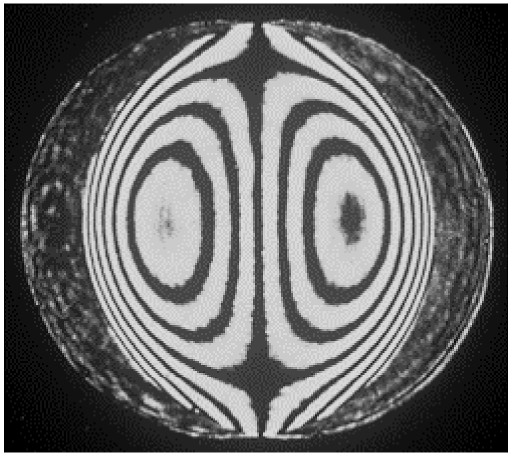


Fringes show loci of constant slope

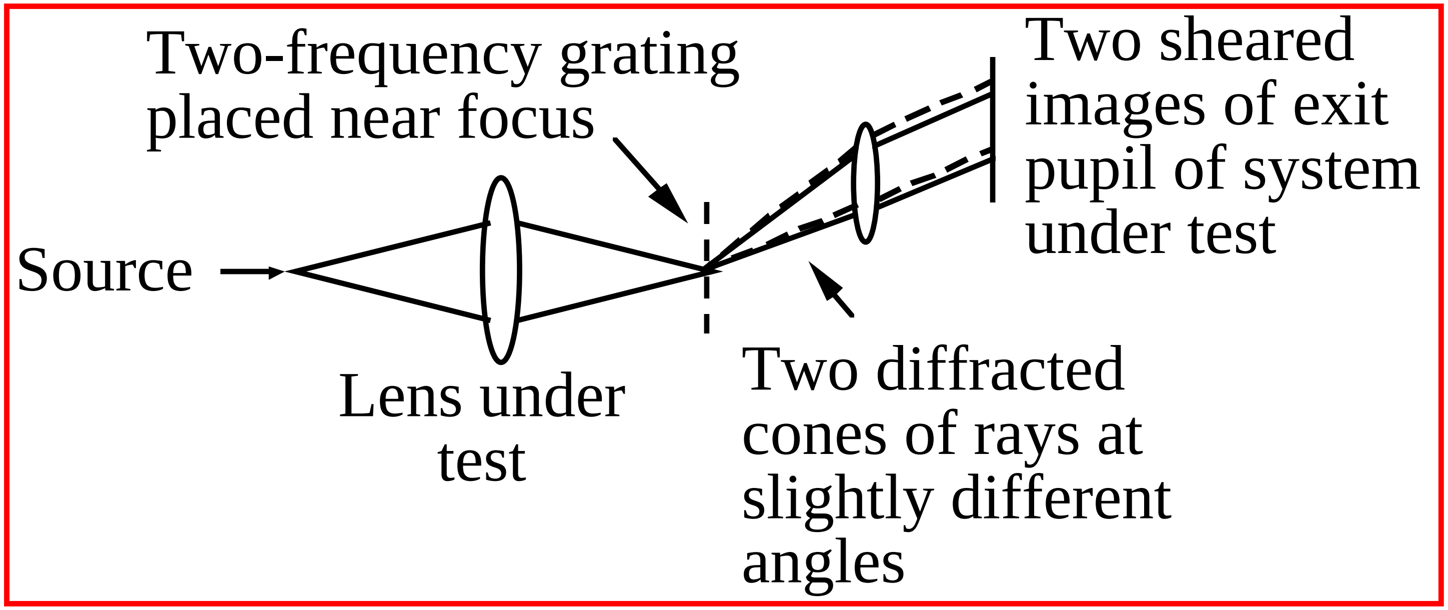
Sensitivity determined by amount of shear

Typical Lateral Shear Interferograms



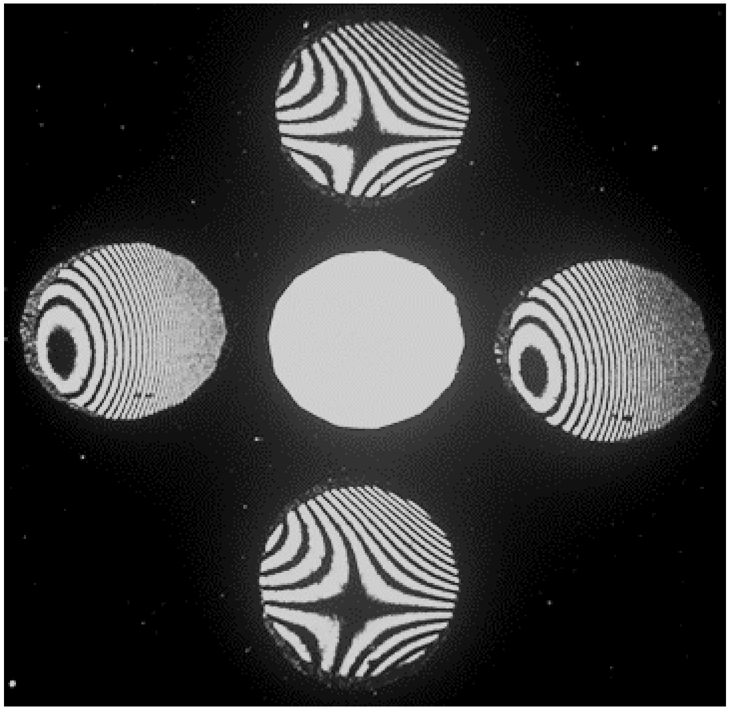


Lateral Shear Interferometer



Measures slope of wavefront, not wavefront shape.

Interferogram Obtained using Grating Lateral Shear Interferometer

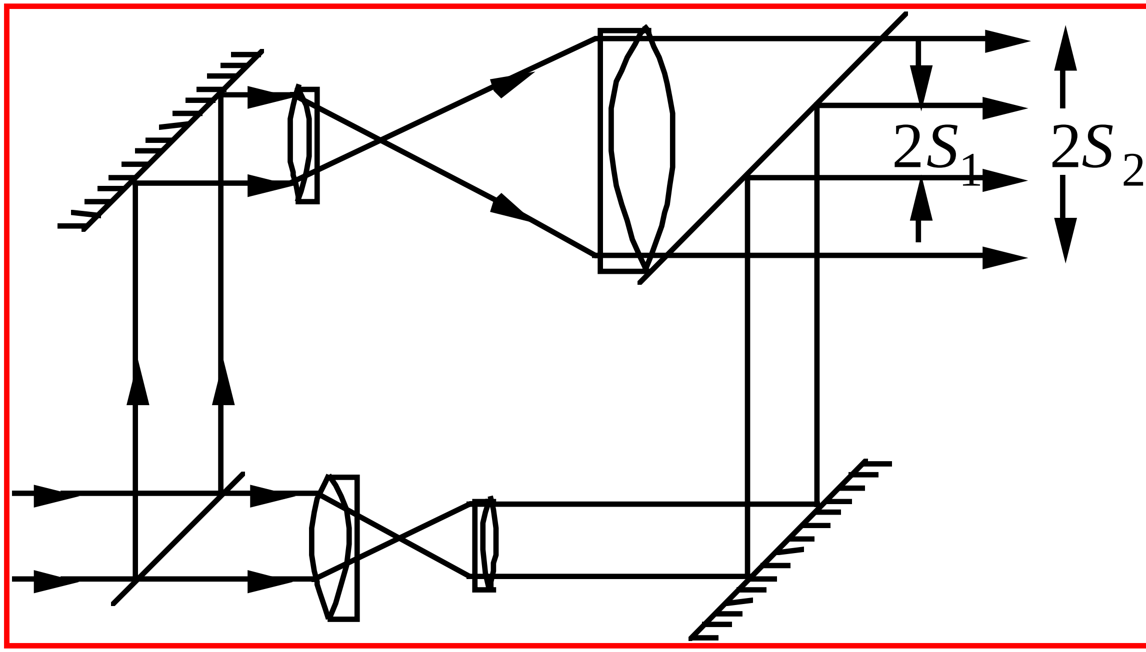


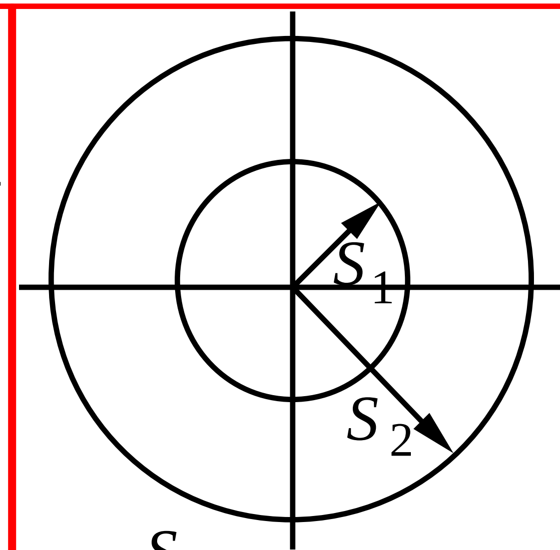
9.3.2 Radial shear interferometry

- <u>Advantages</u>
  Can vary the sensitivity by varying the amount of radial shear.

- <u>Disadvantages</u>
  The shear varies over the pupil with the largest shear at the edge of the pupil, which is generally the location of maximum slope. Thus, we get the least help where we need the most help.

Radial Shear Interferometer





$$R = \frac{S_1}{S_2} = \text{Radial Shear}$$

Measures radial slope of wavefront, not wavefront shape.

9.3.3 High-density detector arrays

Theoretically need at least two detectors per fringe if we know nothing about the wavefront we are testing. Due to noise, and the fact that each detector is averaging over a part of a fringe, generally 2.5 or 3 detectors per fringe required. Less than 100% fill factor is desirable, but then more light is required.

If before performing the test we have additional information about the wavefront being tested, such as the surface height to within a quarter wavelength, or that the slope is continuous, it is often possible to perform a measurement using fewer than two detectors per fringe. This will be covered in the next section.

Critical item is to know the system accurately enough so it can be ray traced to determine what the desired asphericity is at the detector plane. Knowing the asphericity at the location of the test object is not enough. We must know the asphericity at the location where the measurement is being performed, i.e. the detector plane. Calibration is probably required.

High-density detector arrays

- **Must have at least two detector elements per fringe.**

- **Interferogram analysis software can remove desired amount of asphericity.**

- **Must ray trace test setup so correct amount of asphericity is known.**

Sub-Nyquist Interferometry

Require fewer than two detector elements per fringe by assuming first and second derivatives of wavefront are continuous

Long-Wavelength Interferometry

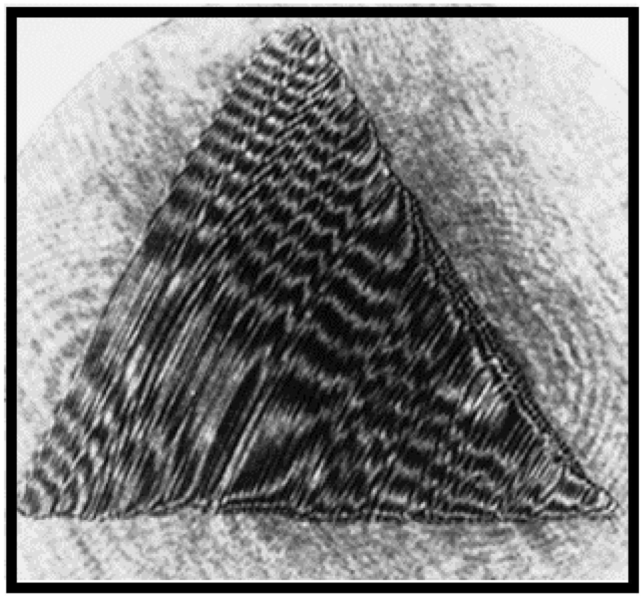
Reduce number of fringes by using a long wavelength source such as a 10.6 micron Carbon Dioxide laser

10.6 Micron Source Interferometer

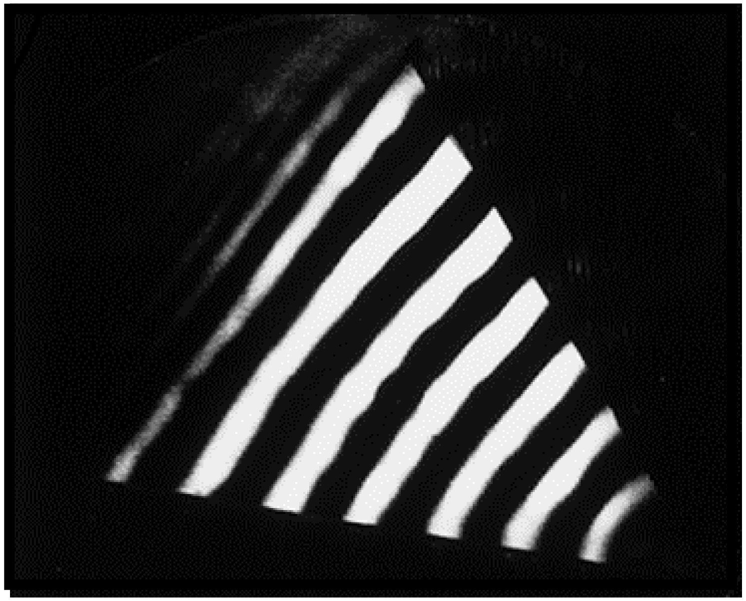
- **Carbon Dioxide Laser**
  - **Excellent coherence properties**
- **Zinc Selenide or Germanium Optics**
- **Pyroelectric Vidicon Detector**

Conventional interferometry techniques work well.

Reduced Sensitivity Testing



0.633 microns wavelength



10.6 microns wavelength

Testing Rough Surfaces

Assume surface height distribution is Gaussian with standard deviation σ.

The normal probability distribution for the height, h, is

$$\boxed{p(h) = \frac{1}{(2\pi)^{1/2}\sigma} \exp\left(-\frac{h^2}{2\sigma^2}\right)}$$

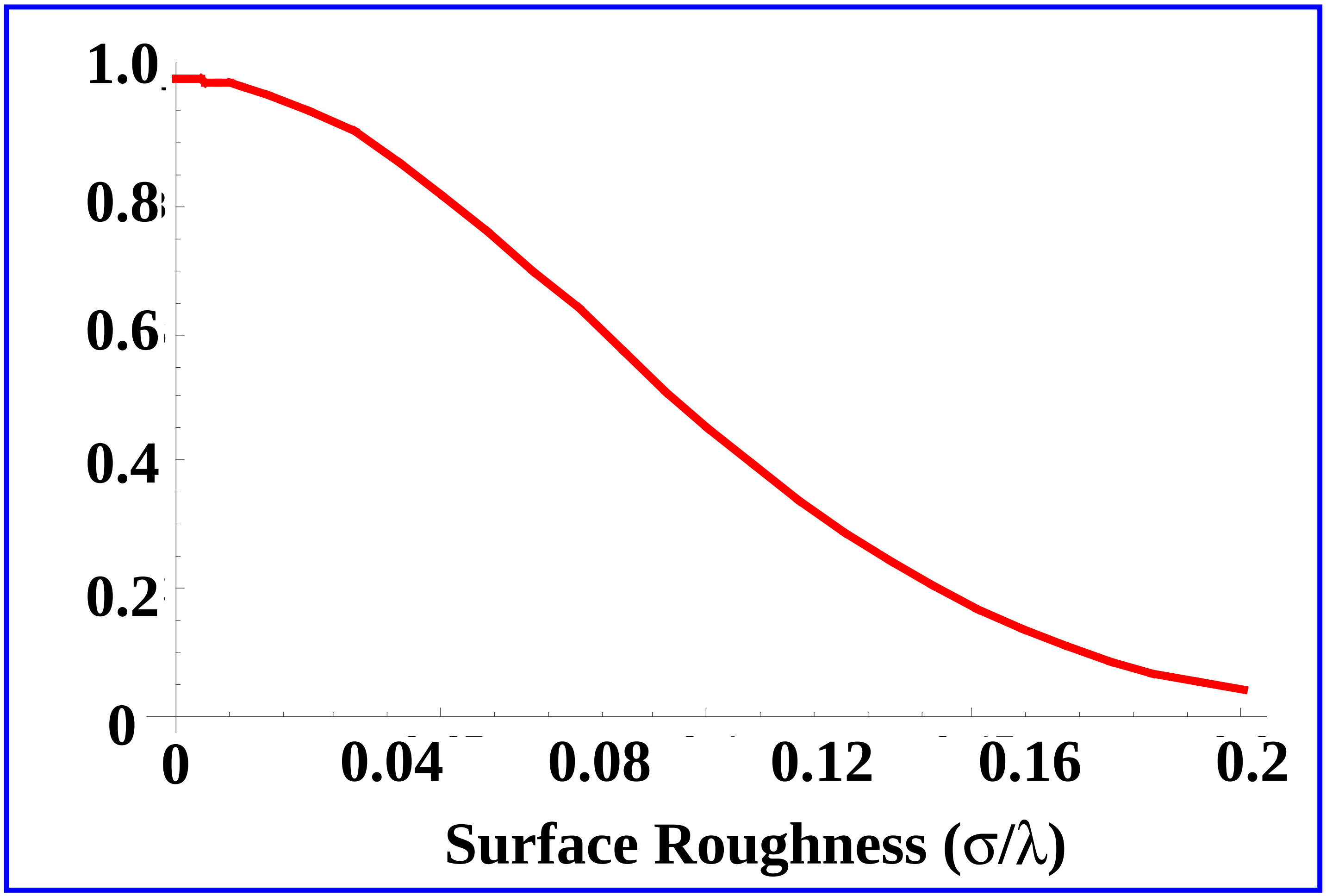
Fringe Contrast Reduction due to Surface Roughness

The fringe contrast reduction due to surface roughness is

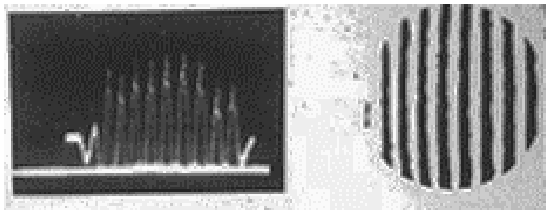
$$\boxed{C = \exp(-8\pi^2\sigma^2/\lambda^2)}$$

Reference: Appl. Opt. 11, 1862 (1980).

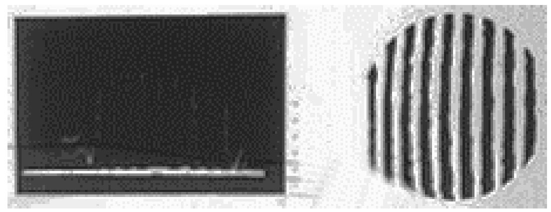
Fringe Contrast versus Surface Roughness - Theory



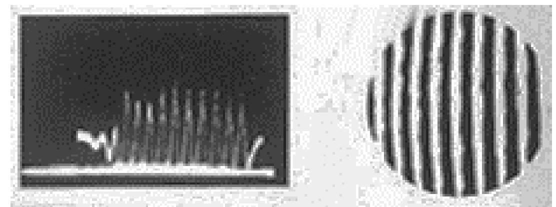
Interferograms Obtained for Different Roughness Surfaces



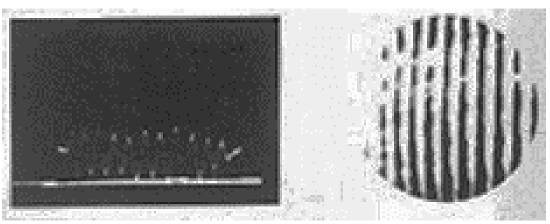
σ = 0 μm, C = 1.0



σ = 0.32 μm, C = 0.93

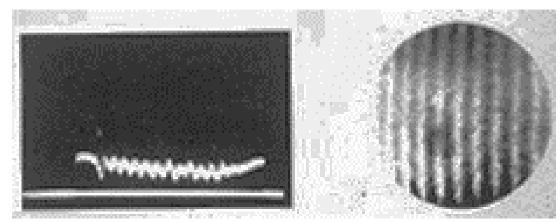


σ = 0.44 μm, C = 0.87



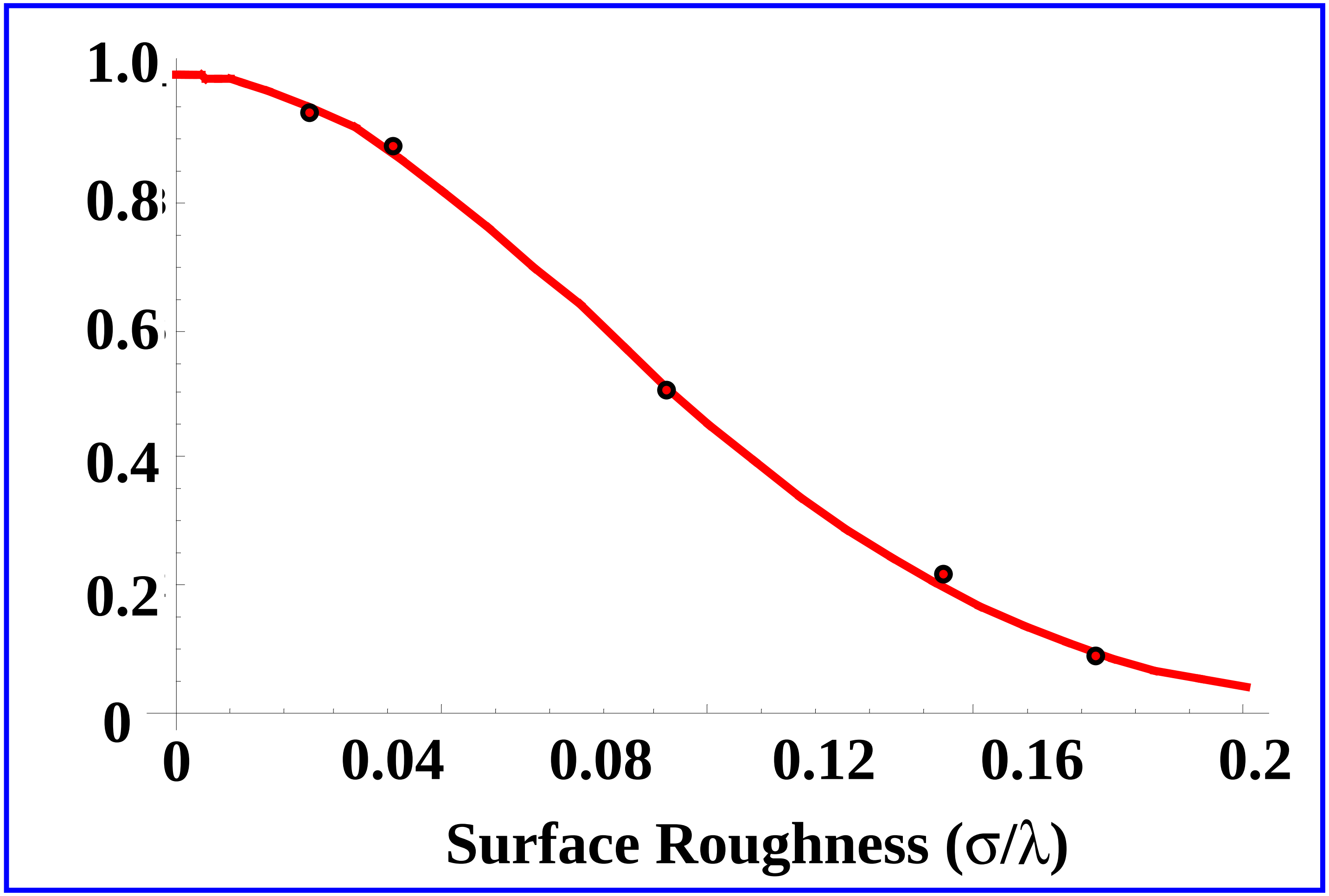
σ = 0.93 μm, C = 0.54

σ = 1.44 μm, C = 0.23

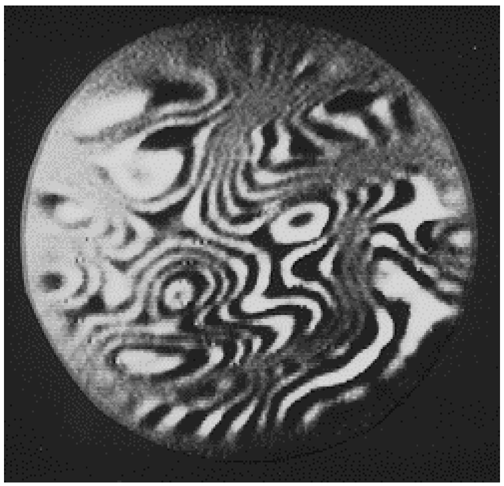
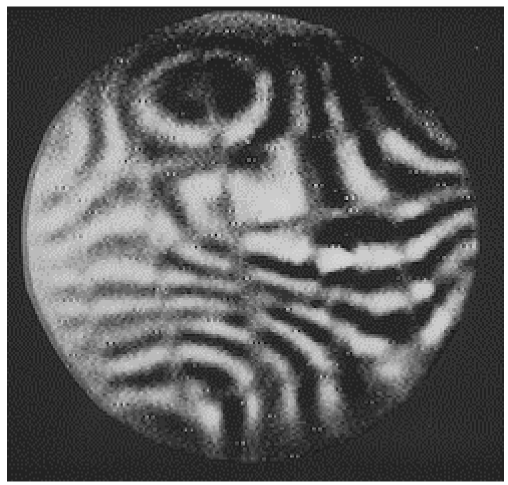
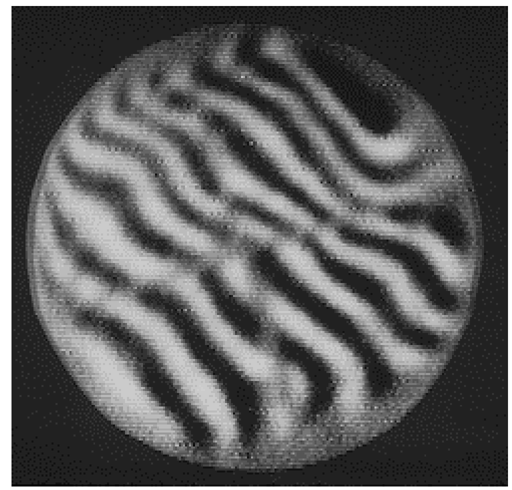


σ = 1.85 μm, C = 0.07

Fringe Contrast versus Surface Roughness - Theory and Experiment



Infrared Interferograms of Off-Axis Parabolic Mirror

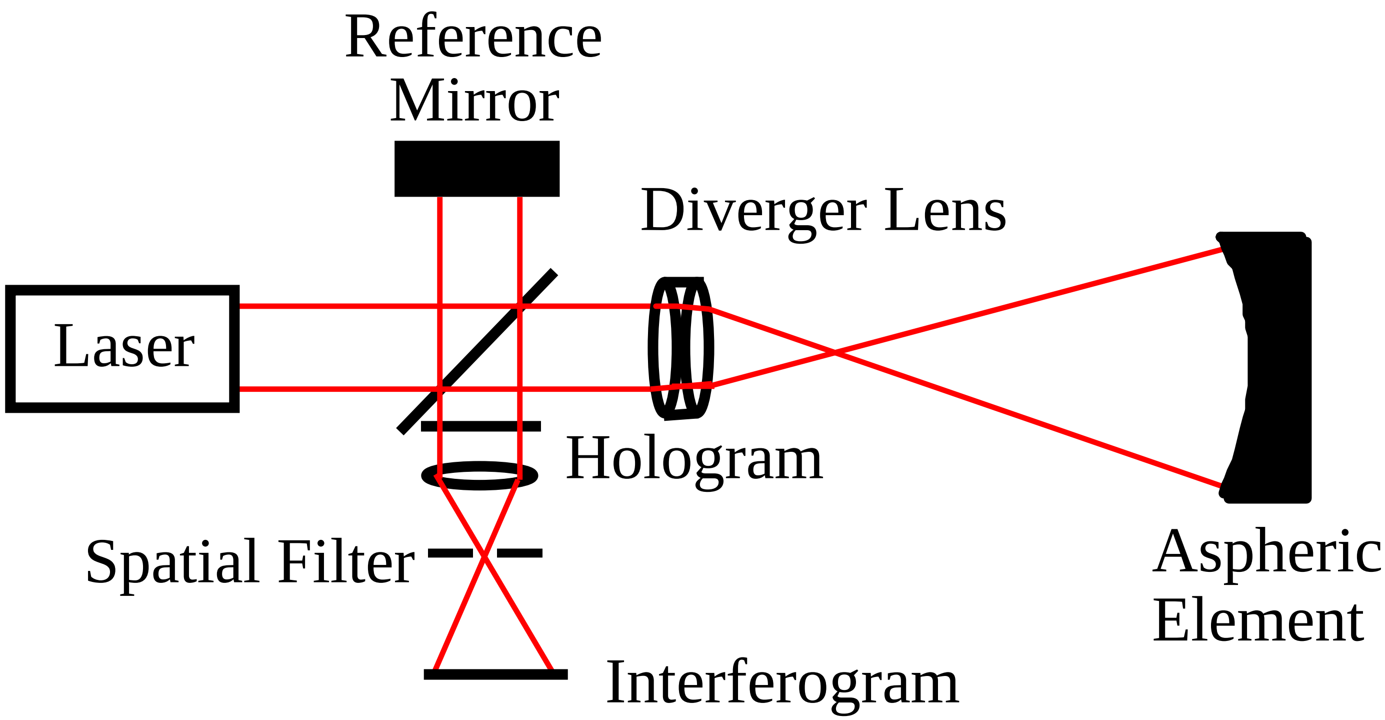
10.6 Micron Wavelength Interferometer

Two-Wavelength Holography

- Means of obtaining visible light to perform interferometric test having sensitivity of test performed using a long-wavelength nonvisible source
- Record hologram at wavelength λ_1
- Reconstruct hologram at wavelength λ_2.
- Interferogram same as would be obtained using wavelength

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}$$

Two Wavelength Holography Interferometer

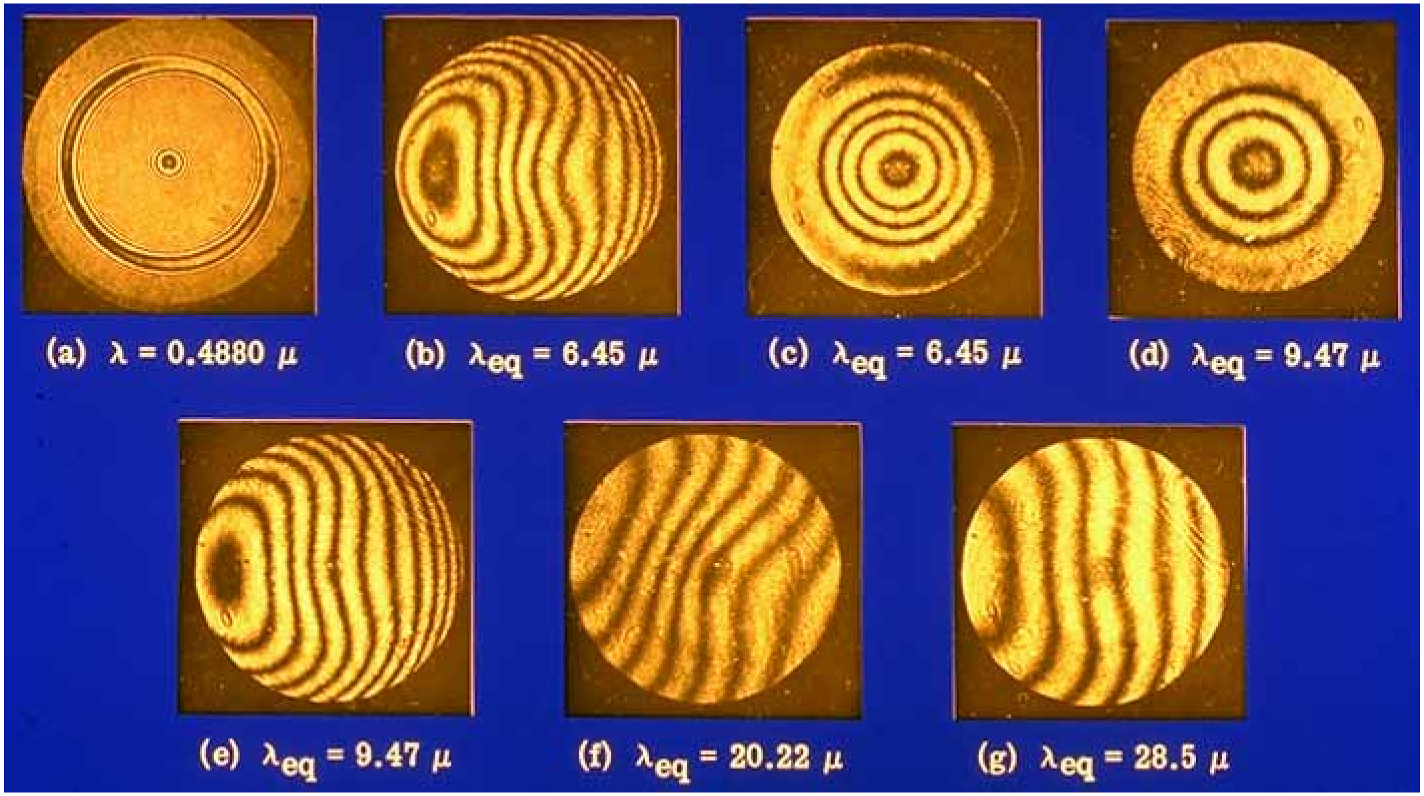


Possible Equivalent Wavelengths obtained with Argon and HeNe Lasers

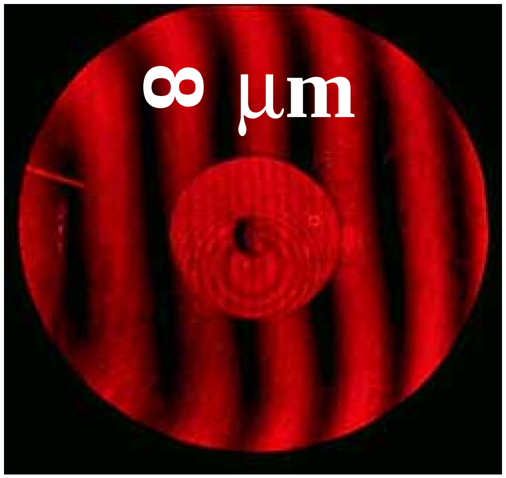
$\lambda 2$ / $\lambda 1$	**0.4579**	**0.4765**	**0.4880**	**0.4965**	**0.5017**	**0.5145**	**0.6328**
0.4579	-	11.73	7.42	5.89	5.24	4.16	1.66
0.4765	11.73	-	20.22	11.83	9.49	6.45	1.93
0.4880	7.42	20.22	-	28.50	17.87	9.47	2.13
0.4965	5.89	11.83	28.50	-	47.90	14.19	2.31
0.5017	5.24	9.49	17.87	47.90	-	20.17	2.42
0.5145	4.16	6.45	9.47	14.19	20.17	-	2.75
0.6328	1.66	1.93	2.13	2.31	2.42	2.75	-

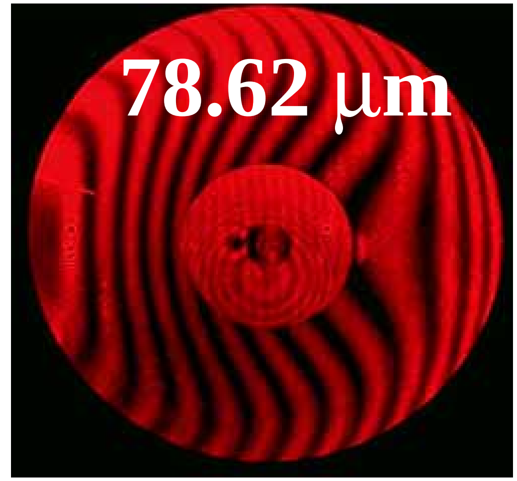
$$\lambda_{\mathbf{eq}} = \frac{\lambda_{\mathbf{1}}\lambda_{\mathbf{2}}}{|\lambda_{\mathbf{1}} - \lambda_{\mathbf{2}}|}$$

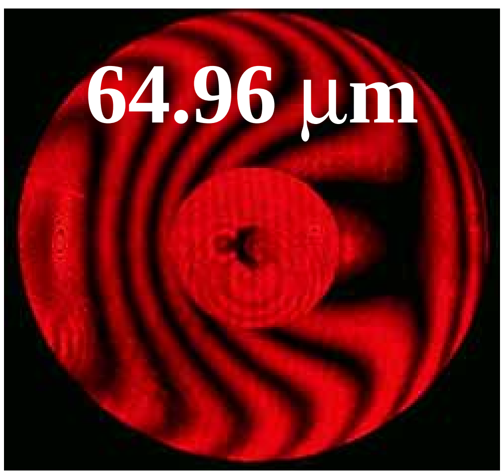
Two Wavelength Holography Interferograms

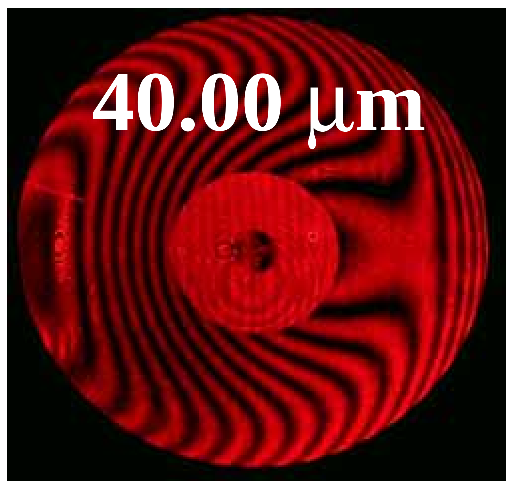


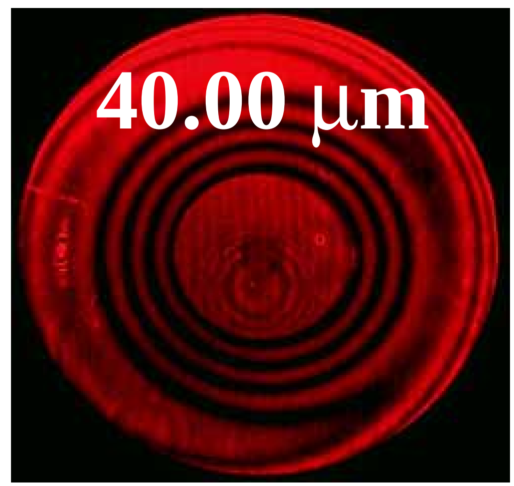
Dye Laser Interferograms I



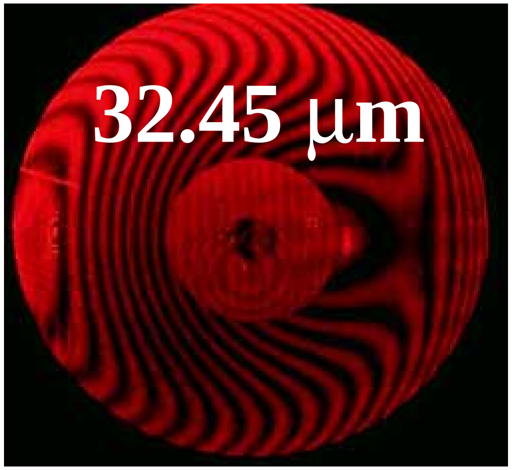


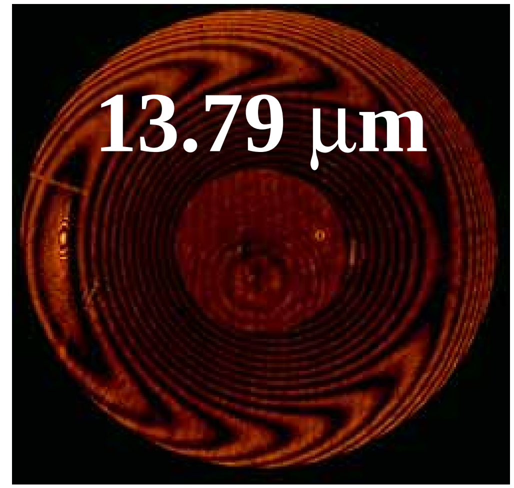


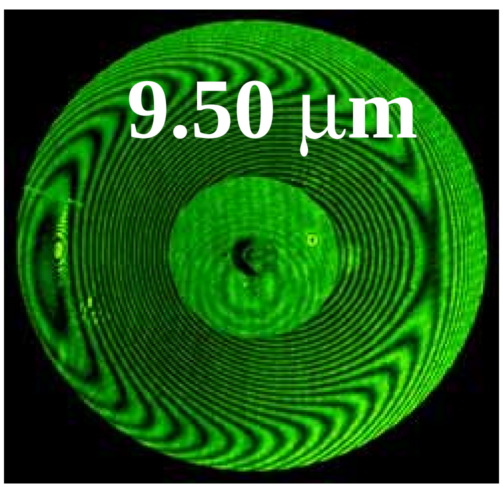


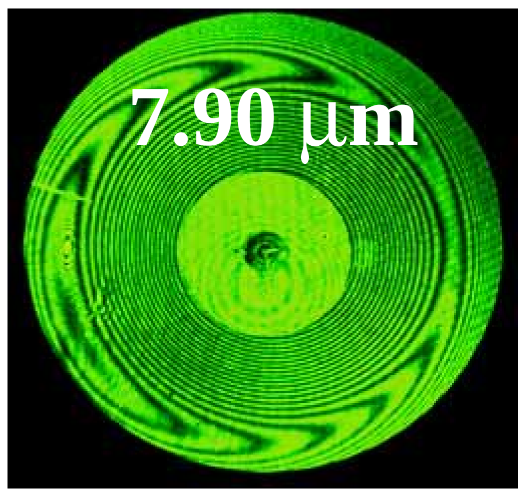


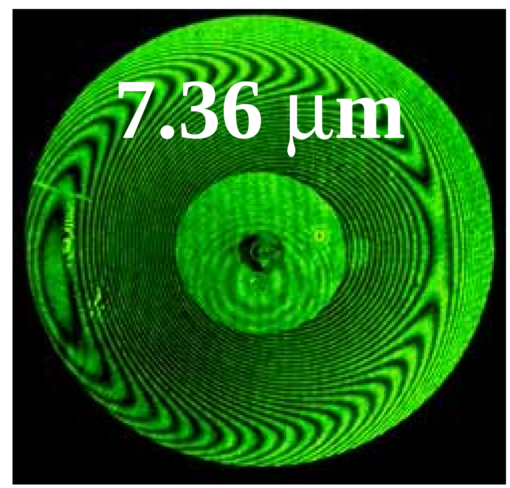
Dye Laser Interferograms II



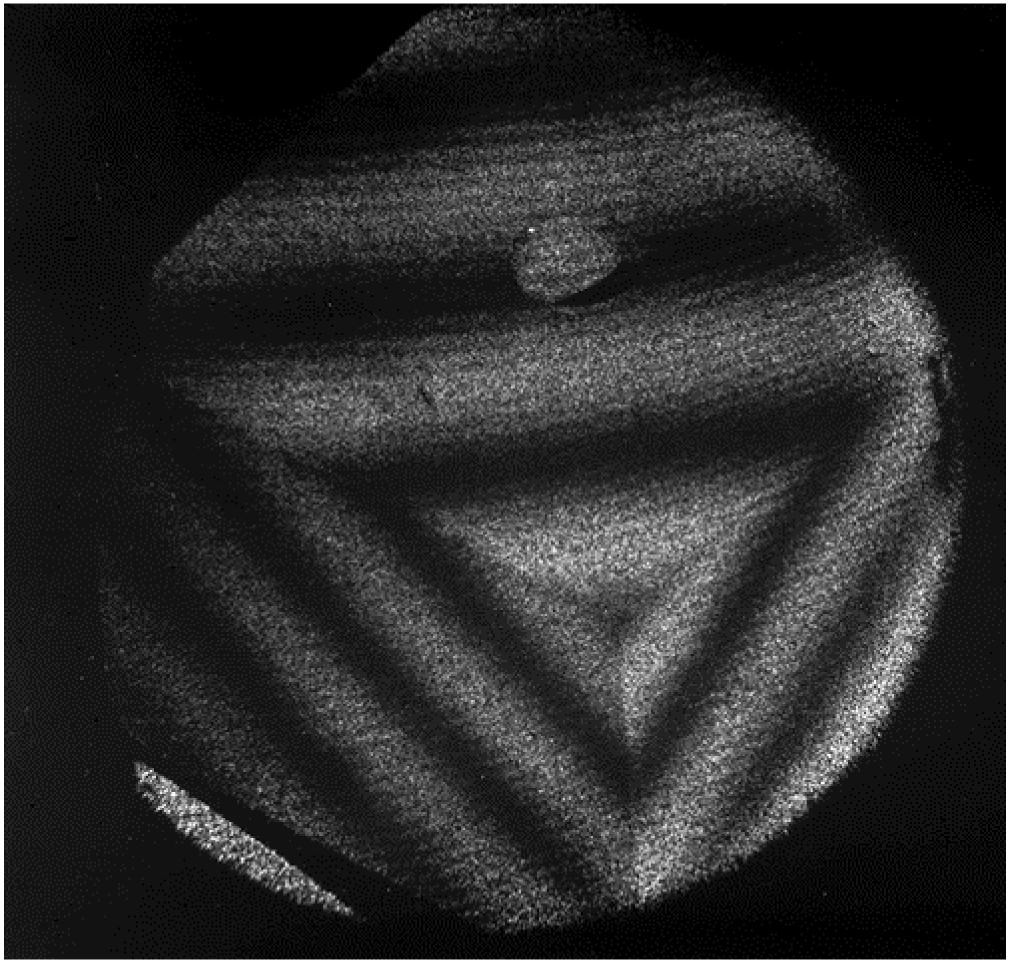






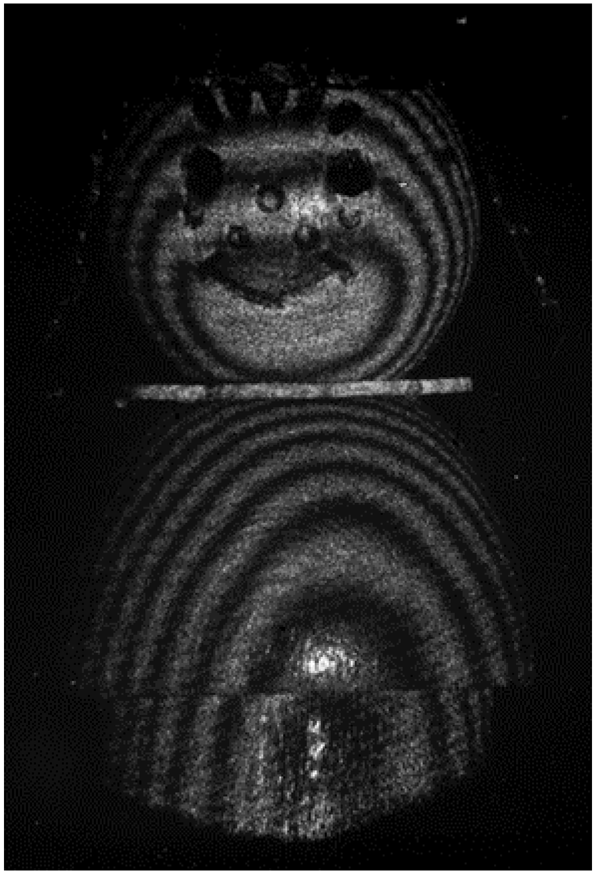


TWH Test of Aluminum Block



λeq = 10 mm

TWH Test of Seaseme Street Character



λ**eq = 2 mm**

Two-Wavelength Interferometry

Perform measurement at two wavelengths, λ_1 and λ_2.

Computer calculates difference between two measurements.

Wavefront sufficiently sampled if there would be at least two detector elements per fringe for a wavelength of

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}$$

12.5. MOIRÉ INTERFEROMETRY

Moiré interferometry, which can be regarded as a form of holographic interferometry, is a complement to conventional holographic interferometry, especially for testing optics to be used at long wavelengths. Although TWH can be used to contour surfaces at any longer-than-visible wavelength, visible interferometric environmental conditions are required. Moiré interferometry can be used to contour surfaces at any wavelength longer than 10 μm (with difficulty) or 100 μm with reduced environmental requirements and no intermediate photographic recording setup. For nondestructive testing, holographic interferometry has a precision of a small fraction of a micrometer and is useful over a deformation amplitude of a few micrometers, whereas moire interferometry has a precision ranging from 10-100 μm to millimeters, with a correspondingly increased useful range of deformation amplitude.

125.1. Basic Principles

Although moiré techniques have been used for many years, only recently has the full potential of moiré interferometry been realized (Brooks and Heflinger 1969, Takaski, 1970, 1973, MacGovern 1972, Benoit et al. 1975). If parallel equispaced planes or fringes are projected onto a nonplanar

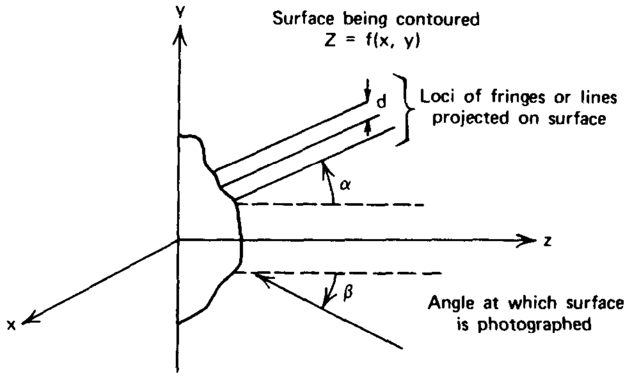


Figure 12.14. Fringes projected on surface Z =f(x, y) at angle a **and viewed at angle** b.

surface and the surface is viewed at an angle different from that at which the fringes are projected, curved fringes are seen. It can be shown that a photograph of these fringes is equivalent to a hologram made of the surface using a long wavelength light source (MacGovern 1972). If a surface described by the function Z=f(x,y) is illuminated and viewed as shown in Fig. 12.14, a photograph of the projected fringes shows contour lines of the surface relative to a plane surface, where the contour interval C is given by

$$C = \frac{d}{\tan\alpha + \tan\beta}. \tag{12.7}$$

The sign convention used for the angles is shown in the figure.
The moire pattern of the photograph of the projected fringes, as compared with a straight line pattern, is equivalent to changing the tilt of the reference surface. The moiré pattern of two photographs of projected fringes for two different objects gives the difference between the two objects, for example, a master optical surface and another supposedly identical optical surface. Likewise, deformation measurements can be made.

12.52. Experimental Setups

Several experimental setups can be used to perform moire interferometry, of which three are illustrated in Figs. 12.15 to 12.17.

In Fig. 12.15a a grating is projected onto the surface being contoured. There is no requirement that the light be coherent or even monochromatic. Both the camera and the grating projector should be telecentric systems so that the angles of projection and view are well defined. The surface being

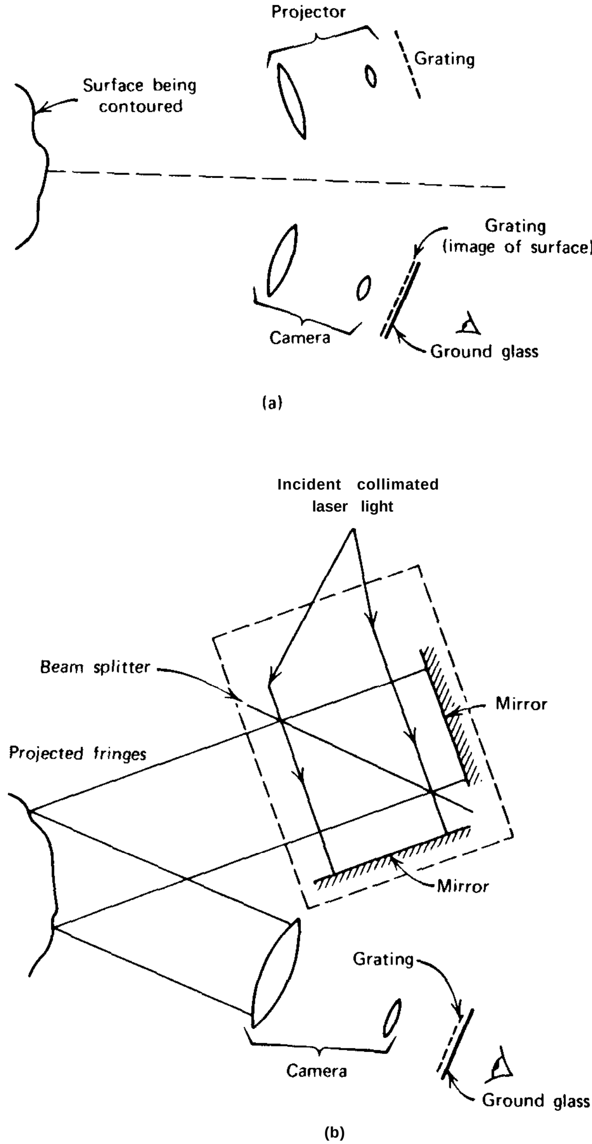


Figure 12.15. Experimental setups for moiré interferometry. (a) Projecting grating on surface. (6) Projecting fringes on surface.

contoured is imaged onto a grating so as to select the desired tilt of the reference plane. If ground glass is placed next to the second grating, the moire pattern can be viewed directly. The moire pattern can be photographed by replacing the ground glass with a sheet of film. This technique has the disadvantage that the relatively high frequency fringes must be transferred through an optical system with attendant loss of contrast. In addition, the projector has a limited depth of focus.

To meet this objection, the grating projector can be replaced with an interferometer, as shown in Fig. 12.15b. In this case a coherent laser beam is used, and a beam splitter with one mirror slightly tilted produces nonlocalized interference fringes, which fall on the surface to be contoured. This method has the advantage that, since the lines projected on the surface are nonlocalized fringes resulting from the interference of two collimated beams, there are no depth-of-focus problems in the projection system.

The higher frequency (carrier frequency) will be displayed on the final photograph unless some effort is made to avoid it. One technique to eliminate the carrier frequency is to use spatial filtering, as illustrated in Fig. 12.16. A second technique is to raise the carrier frequency above the resolution limit of the film. For instance, Polaroid film has a resolution limit of 22 to 28 line pairs per millimeter; since the moire pattern is created before the film plane, only the relatively coarse moire will be recorded if the carrier frequency exceeds about 22 line pairs per millimeter.

A third possible setup is shown in Fig. 12.17. In this case the same grating is used for both projecting and viewing. This setup has the advantage that the camera does not have to resolve the higher frequency grating lines, and must be capable of resolving only the moiré. This, in principle, yields a higher contrast moire pattern. Another advantage is that the grating may be freely translated (but not rotated) in its plane without changing the perceived moire pattern. If the grating is slowly moved during the recording exposure, it will not appear on the photograph; only the stationary moire pattern will be recorded. The perceived sensitivity (fringes per unit deformation) may be varied easily by rotating the grating; it goes to zero when the grating lines lie parallel to the light source-camera

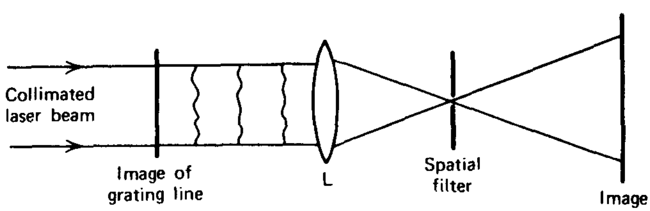


Figure 12.16. Use of spatial filtering to eliminate carrier frequency.

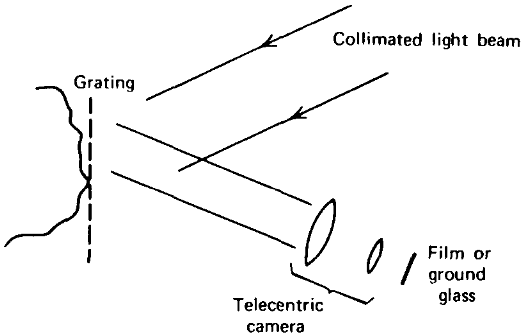


Figure 12.17. Use of single grating for projection and viewing.

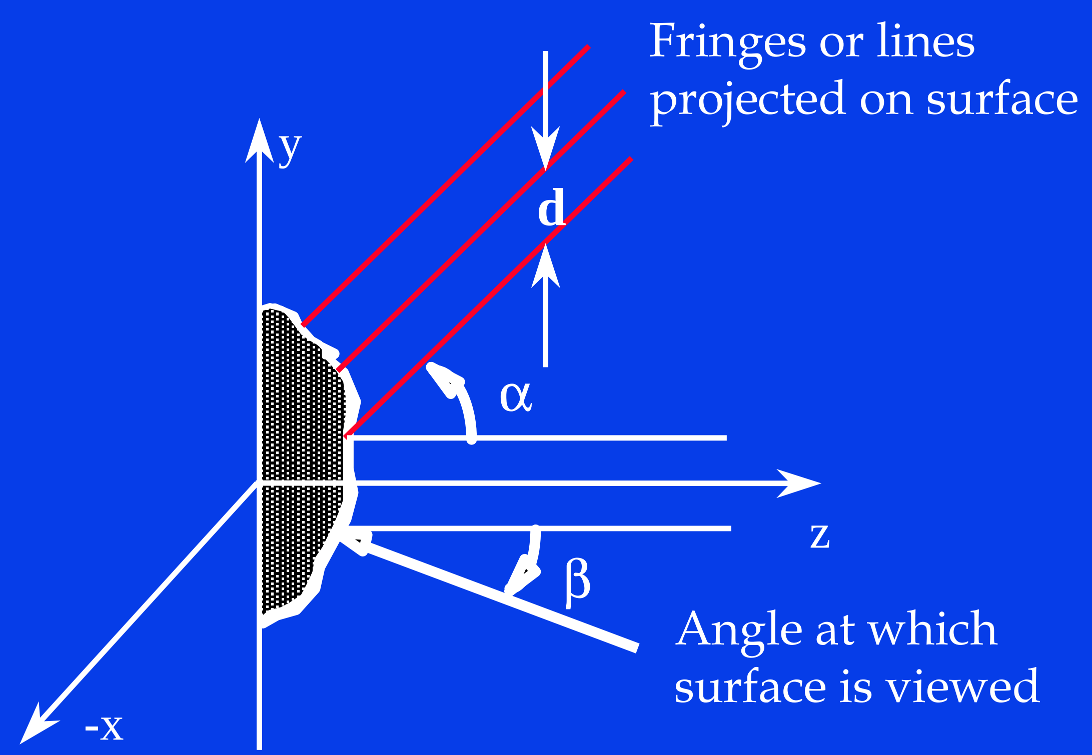
plane. Finally, the contouring may be performed with white light, where projector and camera are telecentric and have a large relative aperture. This technique has the disadvantage that the grating must be reasonably near the surface being contoured. This requirement is relaxed, however, as the light source becomes better collimated, the camera lens goes to larger f numbers, and the carrier frequency decreases.

12.53. Experimental Results

Figure 12.18 shows results obtained testing a spherical surface in the setup shown in Fig. 12.17. The equivalent wavelength in this instance was 200 µm. As stated above, moire interferometry is definitely a complement to conventional holography and should be of particular use in testing components for longer wavelength optical systems.

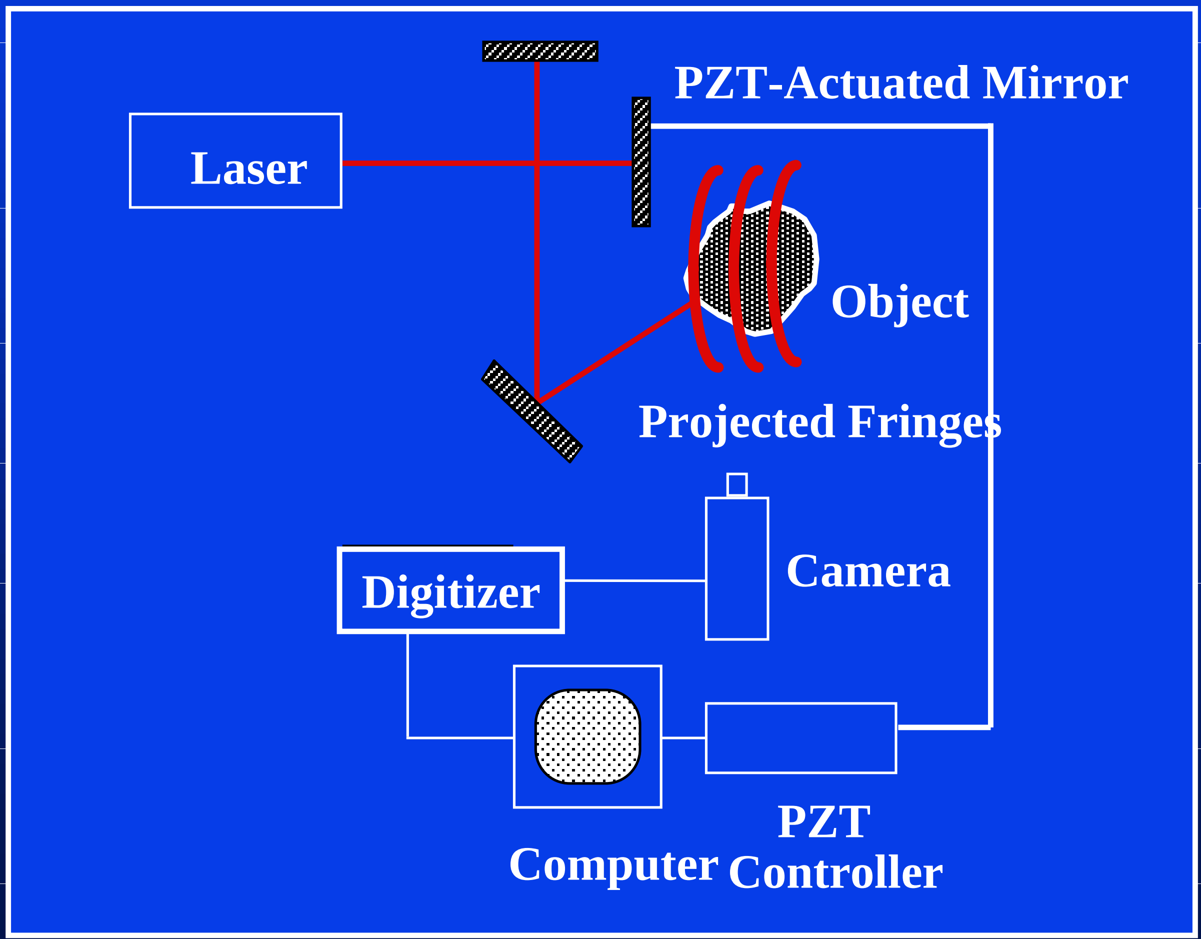
Figure 12.18. Moiré interferogram obtained when testing *a* spherical mirror (λ_{eq} = 200 µm).

Projected Fringe Contouring

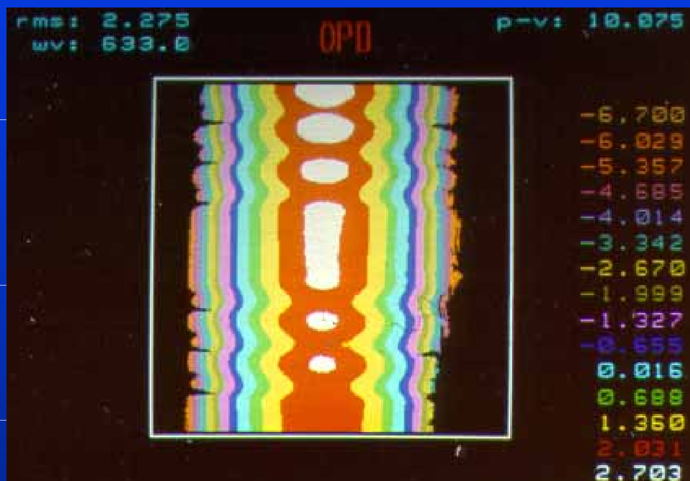


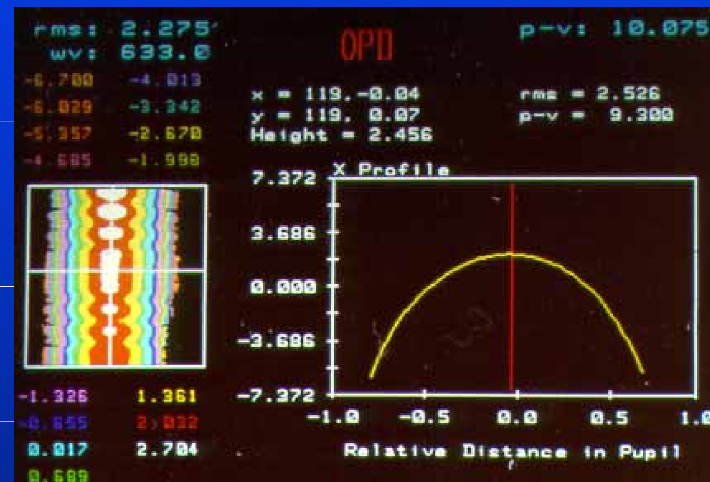
$$\lambda_{eq} = \frac{2d}{\tan\alpha + \tan\beta}$$

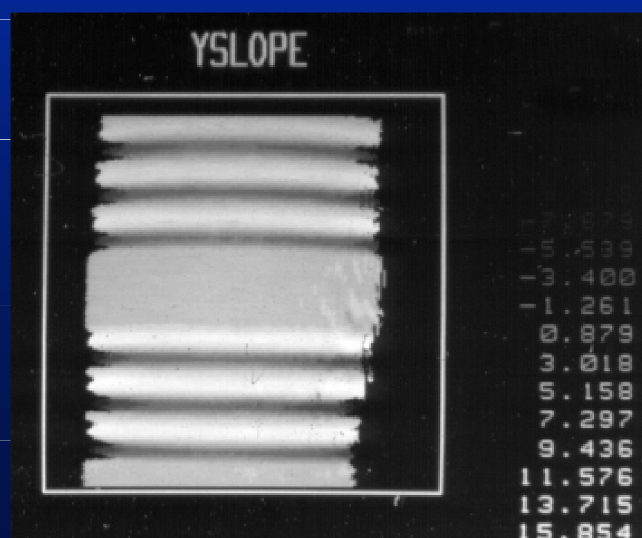
Projected Fringe Contouring Setup



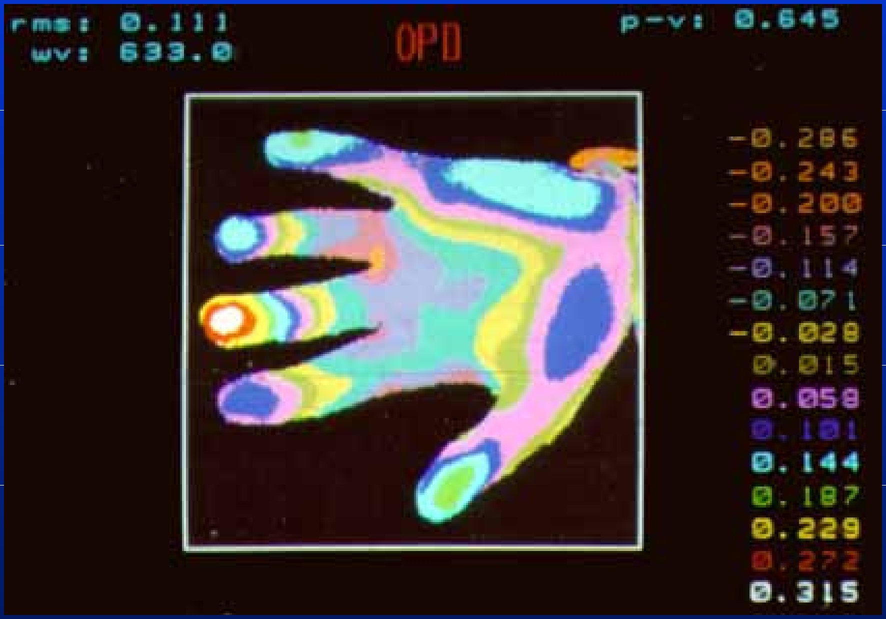
Can


rms: 2.275
wv: 633.0
OPD
p-v: 10.075
-6.700
-6.029
-5.357
-4.685
-4.014
-3.342
-2.670
-1.999
-1.327
-0.655
0.016
0.688
1.360
2.031
2.703

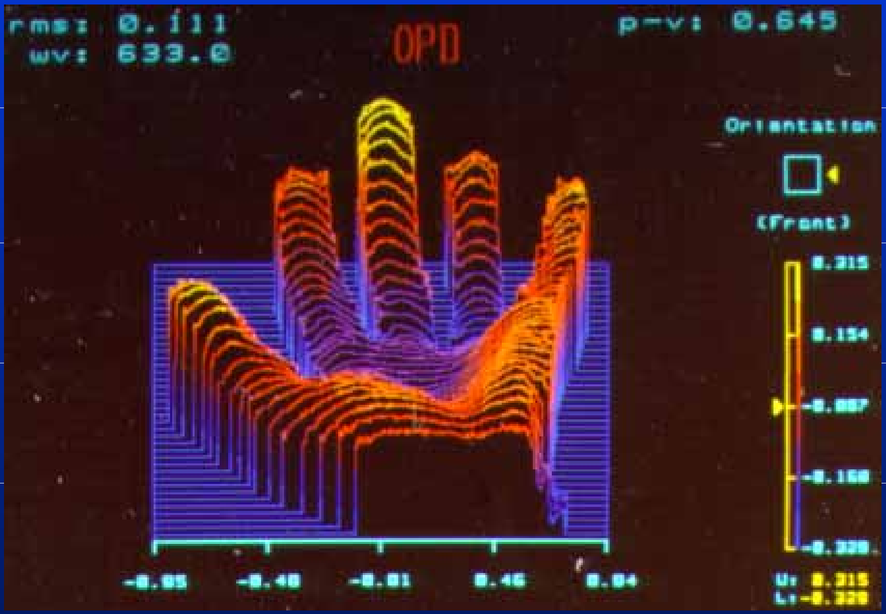

rms: 2.275
wv: 633.0
OPD
p-v: 10.075
x = 119, -0.04
y = 119, 0.07
Height = 2.456
rms = 2.526
p-v = 9.300
X Profile
7.372
3.686
0.000
-3.686
-7.372
-1.0
-0.5
0.0
0.5
1.0
Relative Distance in Pupil
-1.326
1.361
-0.655
2.032
0.017
2.704
0.689


YSLOPE
-5.539
-3.400
-1.261
0.879
3.018
5.158
7.297
9.436
11.576
13.715
15.854

Hand

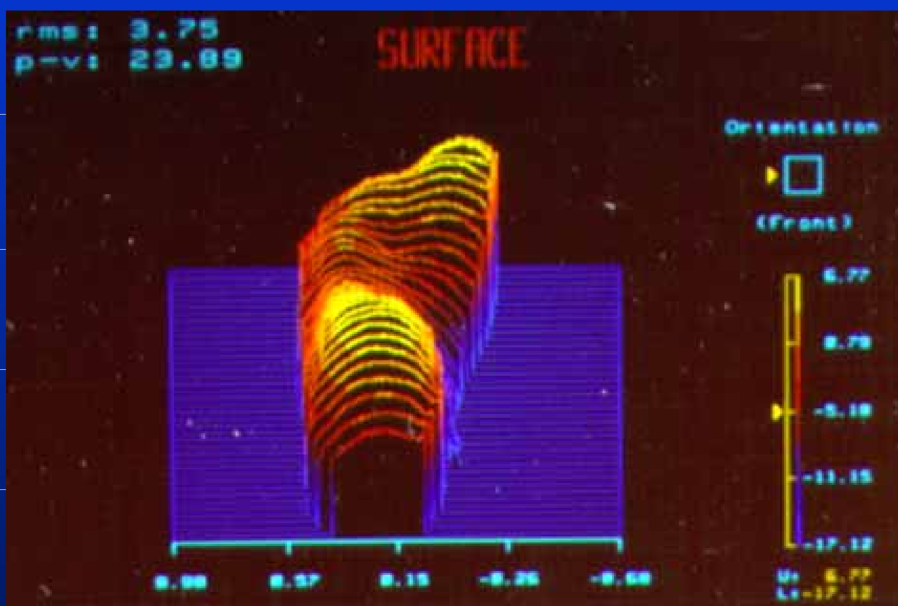


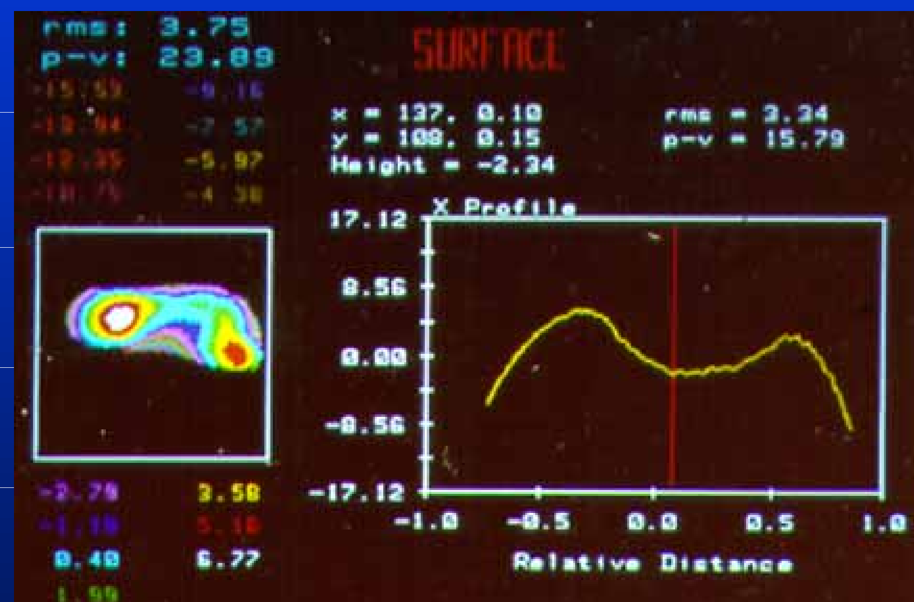
rms: 0.111
wv: 633.0
OPD
p-v: 0.645
-0.286
-0.243
-0.200
-0.157
-0.114
-0.071
-0.028
0.015
0.058
0.101
0.144
0.187
0.229
0.272
0.315



rms: 0.111
wv: 633.0
OPD
p-v: 0.645
Orientation
(Front)

Foot


rms: 3.75
p-v: 23.89
SURFACE
Orientation
(Front)


rms: 3.75
p-v: 23.89
SURFACE
x = 137, 0.10
y = 108, 0.15
Height = -2.34
rms = 3.24
p-v = 15.79
X Profile
17.12
8.56
0.00
-8.56
-17.12
-1.0
-0.5
0.0
0.5
1.0
Relative Distance

Foot Scanner

Lens Analysis Software

- **Must know precisely how optics in test setup change aspheric wavefront.**
- **Must know effects of misalignments, so errors due to misalignments can be removed.**

Basic Limitations of Aspheric Testing

- **Must get light back into the interferometer**
- **Must be able to resolve the fringes**
- **Must know precisely the optical test setup**

This is the most serious problem