Mathematica items needed

System Evaluation

SE-1

We are using a lateral shear interferometer to measure the MTF of an f/4 optical system for a spatial frequency of 100 l/mm and a wavelength of 500 nm. It can be shown that

$$\frac{\text{MTF}[\nu]_{\text{actual}}}{\text{MTF}[\nu]_{\text{max theoretical}}} = f[\phi_{\text{rms}}]$$

where $f[\phi_{\text{rms}}]$ is a function involving the rms of the wavefront difference function for a shear corresponding to a spatial frequency of 100 l/mm. What is $f[\phi_{\text{rms}}]$?

Solution

$$\text{PupilFunction} = P[x, y]\, e^{i\,\phi[x,y]}$$

We will assume that over open portion of the aperture the intensity transmittance is unity.

$$\frac{\text{MTF}[\nu]}{\text{MTF}[\nu]_{\max}} = \frac{\text{Abs}\left[\int\int P\left[x+\frac{s}{2}\right] P\left[x-\frac{s}{2}\right] e^{i\,\phi\left[x+\frac{s}{2},y\right]}\, e^{-i\,\phi\left[x-\frac{s}{2},y\right]}\, dx\, dy\right]}{\text{Abs}\left[\int\int P\left[x+\frac{s}{2}\right] P\left[x-\frac{s}{2}\right] dx\, dy\right]}$$

$$= \frac{\text{Abs}\left[\int\int_{\text{AreaOfOverlap}} e^{i\,\phi\left[x+\frac{s}{2},y\right]}\, e^{-i\,\phi\left[x-\frac{s}{2},y\right]}\, dx\, dy\right]}{\text{AreaOfOverlap}}$$

Let $\phi_p = \phi\left[x+\frac{s}{2}, y\right] - \phi\left[x-\frac{s}{2}, y\right]$ = wavefront difference function for shear s.

If ϕ_p is small then

$$e^{i\,\phi_p} \approx 1 + i\,\phi_p + \frac{1}{2}\left(i\,\phi_p\right)^2 + \cdots$$

$$\frac{\text{MTF}[\nu]}{\text{MTF}[\nu]_{\max}} = \text{Abs}\left[1 + i\,\overline{\phi_p} - \frac{1}{2}\,\overline{\phi_p{}^2}\right] \approx 1 - \frac{1}{2}\left(\overline{\phi_p{}^2} - \left(\overline{\phi_p}\right)^2\right)$$

$$= f[\phi_{\text{rms}}] = 1 - \frac{1}{2}\left(\phi_{p,\text{rms}}\right)^2$$

SE-2

a) An f/10 optical system is defocused $\lambda/2$. Using only geometrical optics, calculate the MTF at spatial frequencies of 10, 30, and 50 lines/mm. Assume the wavelength is 0.5 μm.

b) A target having an intensity distribution of the form

$$b\left(1+\frac{15}{23}\sum_{n=0}^{2}\frac{1}{2\,n+1}\,\text{Cos}[2\,\pi\,(2\,n+1)\,\nu_o\,x]\right)$$

where ν_o = 10 lines/mm is imaged by an optical system which over the spatial frequency region of interest has an OTF which can be approximated by the equation

$$\text{H}[\nu]=\left(1-\left(\frac{\nu}{\nu_{oo}}\right)^2\right)\text{Exp}\left[i\,2\,\pi\,5\left(\frac{\nu}{\nu_{oo}}\right)^2\text{Sign}[\nu]\right]$$

where ν_{oo} = 100 lines/mm. Give an analytical expression for the resulting image and give the physical distance each frequency component is shifted.

c) What wavefront error is present in an optical system which has an OTF given by

$$\text{H}[\nu] = [\text{Diffraction limited MTF}]\ e^{i\,2\,\pi\,(\nu/\nu_{oo})}$$

where ν_{oo} is a constant?

Solution

a)

The MTF is the modulus of the Fourier transform of the point spread function. Since we are using geometrical optics the PSF is the geometrical spot due to defocus. If D is the spot diameter due to defocus, δ is the longitudinal defocus, and f_{no} is the f/number of the system

$$\text{D}=\frac{\delta}{f_{no}}$$

The OPD at the edge of the pupil due to defocus is

$$\text{OPD}=\frac{\delta}{8\,(f_{no})^2}$$

Therefore, the spot diameter is given by

$$\text{D} = 8\,f_{no}\,\text{OPD}$$

The MTF is thus given by

$$\text{MTF}=\frac{2\,J_1[\pi\,(8\,f_{no}\,\text{OPD})\,\nu]}{\pi\,(8\,f_{no}\,\text{OPD})\,\nu}$$

or in *Mathematica* terms

$$\textbf{MTF} = \frac{\textbf{2 BesselJ[1, } \pi\,\textbf{(8 } f_{no}\,\textbf{OPD) } \nu\textbf{]}}{\pi\,\textbf{(8 } f_{no}\,\textbf{OPD) } \nu}\textbf{; } f_{no}\textbf{ = 10; OPD = .25 }\left(10^{-3}\right)\textbf{ mm;}$$

MTF /. ν -> 10 / mm

0.951457

MTF /. ν -> 30 / mm

0.616962

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MTF /. ν -> 50 / mm
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0.181192

b)

The target is given by

$$o = b\left(1 + \frac{15}{23}\sum_{n=0}^{2}\frac{1}{2n+1}\,\text{Cos}[2\pi\,(2n+1)\,10\,x]\right)$$

$$b\left(1 + \frac{15}{23}\left(\text{Cos}[20\pi x] + \frac{1}{3}\,\text{Cos}[60\pi x] + \frac{1}{5}\,\text{Cos}[100\pi x]\right)\right)$$

$$H[\nu\_] := \left(1 - \left(\frac{\nu}{100}\right)^2\right)\,\text{Exp}\left[i\,2\pi\,5\left(\frac{\nu}{100}\right)^2\,\text{Sign}[\nu]\right]$$

$$\text{Hmag}[\nu\_] := \left(1 - \left(\frac{\nu}{100}\right)^2\right)$$

$$\text{Hphase}[\nu\_] := 2\pi\,5\left(\frac{\nu}{100}\right)^2\,\text{Sign}[\nu]$$

To find the image we multiply the amplitude of the individual frequency components by Hmag[ν] and we add Hphase[ν] to the phase. The resulting image is given by

$$i = b\left(1 + \frac{15}{23}\sum_{n=0}^{2}\frac{1}{2n+1}\,\text{Hmag}[(2n+1)\,10]\,\text{Cos}[2\pi\,(2n+1)\,10\,x + \text{Hphase}[(2n+1)\,10]]\right)$$

$$b\left(1 + \frac{15}{23}\left(\frac{99}{100}\,\text{Cos}\left[\frac{\pi}{10} + 20\pi x\right] + \frac{91}{300}\,\text{Cos}\left[\frac{9\pi}{10} + 60\pi x\right] - \frac{3}{20}\,\text{Sin}[100\pi x]\right)\right)$$

The physical shift of the individual components is given by the phase of the OTF divided by 2π times the period.

```
TableForm[Table[{(2 n + 1) 10, N[1 / ((2 n + 1) 10)], Hmag[(2 n + 1) 10],
   Hphase[(2 n + 1) 10], N[Hphase[(2 n + 1) 10] / (2 π ((2 n + 1) 10))]}, {n, 0, 2}],
 TableHeadings -> {None, {"ν(1/mm)", "Period(mm)", "MTF", "Phase", "Shift(mm)"}}]
```

ν(1/mm)	Period(mm)	MTF	Phase	Shift(mm)
10	0.1	$\frac{99}{100}$	$\frac{\pi}{10}$	0.005
30	0.0333333	$\frac{91}{100}$	$\frac{9\pi}{10}$	0.015
50	0.02	$\frac{3}{4}$	$\frac{5\pi}{2}$	0.025

Note that for the 50 lines/mm case the shift is a period, plus a quarter of a period. In this case the Cosine for the image distribution is replaced with a minus Sine.

c)

Tilt.

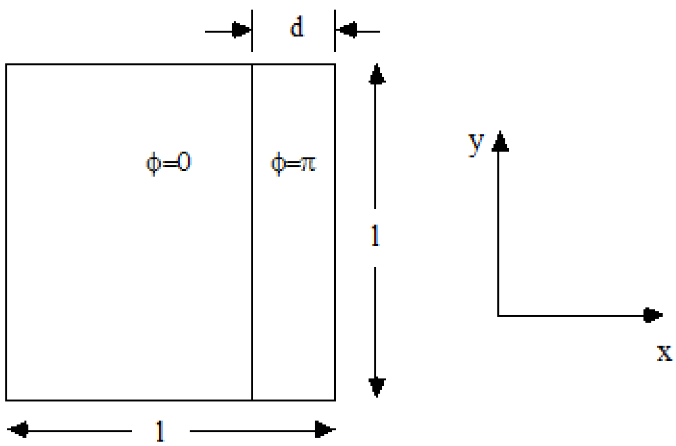
SE-3

An optical system with the square aperture shown below is used to image an incoherently illuminated object. The pupil function has a 180^o phase step as shown in the figure.

a) If l = 0.5 meter, d = 0.1 meter, and the wavelength is 500 nm, what is the cutoff frequency in units of lines/radian in the x and y directions?

b) Give equations for the MTF as a function of spatial frequency in the x and y directions.

c) Sketch the MTF in the x and y directions.

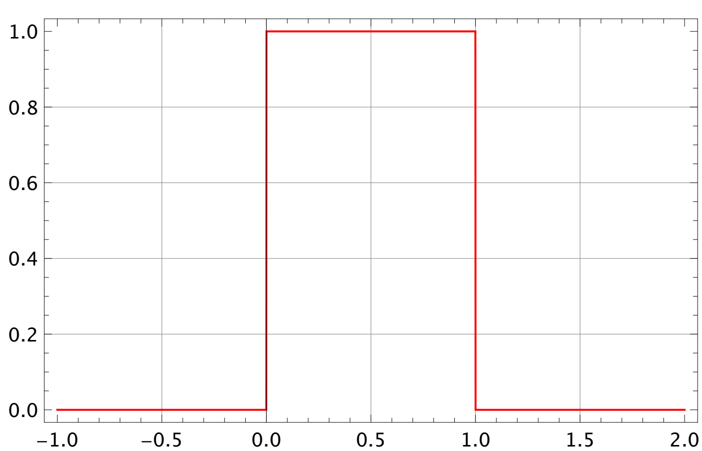


Solution

y direction aperture function

```
fy[y_] := If[0 <= y <= 1, 1, 0]
```

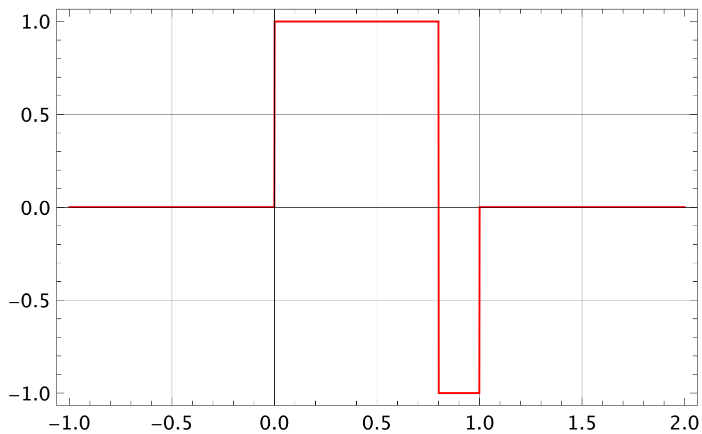
```
Plot[fy[y], {y, -1, 2}, Evaluate[plot2doptions]]
```



x direction aperture function

```
fx[x_] := If[0 <= x <= 0.8 , 1, If[0.8 < x <= 1, -1, 0]]
```

```
Plot[fx[x], {x, -1, 2}, Evaluate[plot2doptions]]
```



a)

$$\text{cutoff} = \frac{1}{\lambda} = \frac{0.5\ \text{m}}{0.5 \times 10^{-6}\ \text{m}} = 10^6 \Big/ \text{radians for both x and y}$$

b)

Frequencies in y:

$$\text{gy}[s\_] := \int_0^1 \text{fy}[y]\ \text{fy}[y+s]\, dy$$

If $y < 1$ MTF $= 1 - \frac{y}{1}$;

If $y > 1$ MTF $= 0$

Frequencies in x:

$$\text{gx}[s\_] := \int_0^1 \text{fx}[x]\ \text{fx}[x+s]\, dx$$

If $x \le d$ MTF $= \frac{1}{1}\left((1-d-x) - x + (d-x)\right) = \frac{1}{1}(1-3x)$

$d < x < 1-d$ MTF $= \frac{1}{1}\left((1-d-x) - d\right) = \frac{1}{1}(1-2d-x)$;

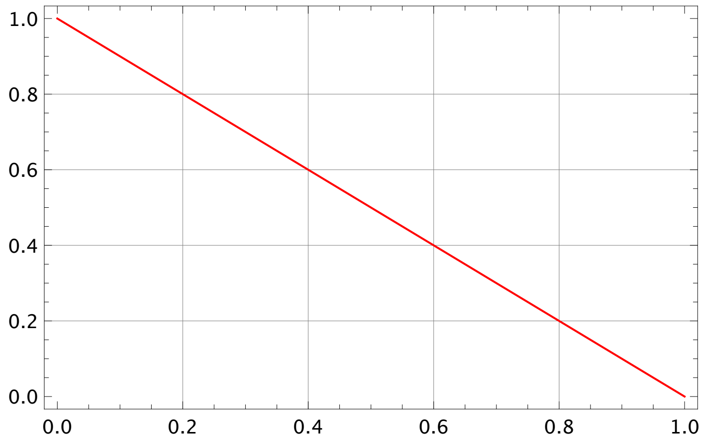
$1-d < x < 1$ MTF $= \frac{1}{1}(x-1)$;

$x > 1$ MTF $= 0$;

c)

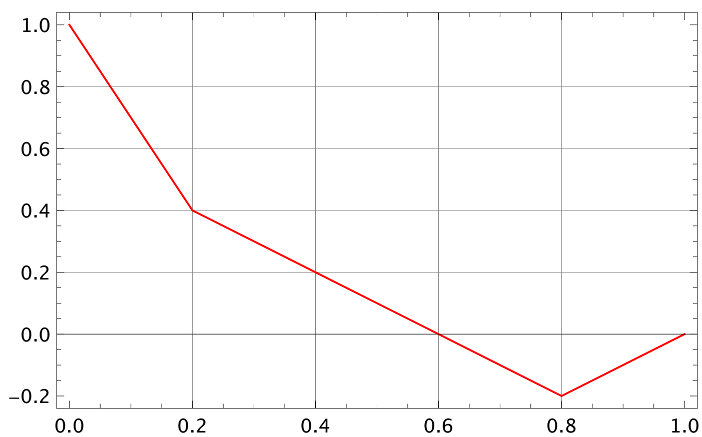
Frequencies in y:

```
Plot[gy[s], {s, 0, 1}, Evaluate[plot2doptions]]
```



Frequencies in x:

```
Plot[gx[s], {s, 0, 1}, Evaluate[plot2doptions]]
```



SE-4

A lateral shear interferometer is used to measure the MTF of a lens having a 100 mm focal length operating at a wavelength of 500 nm. The lens is used to image a target located at infinity. How much shear should be introduced to measure the MTF for a frequency of 50 lines/mm if

a) the lens has a 20 mm diameter?

b) the lens has a 30 mm diameter?

Solution

a)

For Young's two pinholes interference

$$\Delta \frac{x}{L} = \lambda$$

where Δ is the distance between the two pinholes, x is the fringe spacing, and L is the distance from the two pinholes to the fringes. We need to find Δ.

$$\Delta = \frac{\lambda L}{x} = .5 \times 10^{-3}\ \text{mm}\ (100\ \text{mm})\ \frac{50}{\text{mm}} = 2.5\ \text{mm}$$

b)

Same as part a) since result does not depend upon lens diameter.

SE-5

A 20 mm diameter lens having a 200 mm focal length operating at a wavelength of 500 nm is used to image a target.

a) What is the cutoff frequency of the modulation transfer function in image space for incoherent illumination if

i) the target is at infinity?

ii) the target is 400 mm from the lens?

b) A lateral shear interferometer is used to measure the MTF of the lens for a target at infinity. How much shear should be introduced to measure the MTF for a frequency of 50 lines/mm?

Solution

a)

i)

$$\nu_c = \frac{1}{\lambda f\#} = \frac{1}{\left(.5 \times 10^{-3}\ \text{mm}\right)\ 10} = \frac{200\ \text{lines}}{\text{mm}}$$

ii)

$$\nu_c = \frac{1}{\lambda f\#} = \frac{1}{\left(.5 \times 10^{-3}\ \text{mm}\right)\ 20} = \frac{100\ \text{lines}}{\text{mm}}$$

b)

We are measuring the MTF at 1/4 the cutoff frequency so the shear should be 1/4 the aperture diameter, or 5 mm.