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Rough Surface Interferometry Using a CO_2 Laser Source

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It is well known[1] that the specular component of the scattered field increases with increasing wavelength for a random rough surface. For a sufficiently long wavelength, optically rough surfaces behave similarly to smooth surfaces that can be measured by interferometric techniques. The CO_2 laser with a wavelength of 10.6 µm offers the necessary increase in wavelength, while it retains sufficient accuracy to make useful measurements on optically rough surfaces.

In a Twyman-Green interferometer, the fringes of equal thickness are localized in the vicinity of the surface being measured.* A relationship between the fringe contrast and the surface roughness can be calculated by considering the average intensity at each point when wavefronts reflected from a smooth surface and a rough surface are added together.

Surface irregularities, which are small compared with the wavelength used, introduce only phase errors in the wavefront. For normal incidence, the magnitude of the error is $2k\xi$, where $k = 2\pi/\lambda$, is the height of the rough surface measured from a zero mean, and the factor of 2 is required to account for the double path on reflection.

The total amplitude when the wavefronts are added is

$$A(x) = A_1 + A_2 \exp\{ik[2\xi(x) + \alpha x]\}, \tag{1}$$

where α is a small angle between the plane surface and the mean of the rough surface. The resulting intensity is

$$I(x) = A_1^2 + A_2^2 + 2A_1A_2 [\cos(k\alpha x) \cos(2k\xi) - \sin(k\alpha x) \sin(2k\xi)]. \tag{2}$$

When the correlation width $\xi(x)$ is small compared with the fringe spacing,

$$\Delta x = \lambda/\alpha, \tag{3}$$

the intensity can be replaced with its statistical average. A gaussian surface height distribution with standard deviation σ has a probability distribution function,

$$p(\xi) = \{1/[\sigma(2\pi)^{\frac{1}{2}}]\} \exp[-\xi^2/2\sigma^2], \tag{4}$$

which gives

$$\overline{\cos(2k\xi)} = \exp[-2k^2\sigma^2], \tag{5}$$

and

$$\overline{\sin(2k\xi)} = 0. \tag{6}$$

The average intensity at a point is then

$$\overline{I(x)} = A_1^2 + A_2^2 + 2A_1A_2 \cos(k\alpha x) \exp[-2k^2\sigma^2]. \tag{7}$$

The contrast of fringes is defined as

$$C \equiv (I_{max} - I_{min})/(I_{max} + I_{min}) = C_a C_\sigma, \tag{8}$$

where

$$C_a = 2A_1A_2/(A_1^2 + A_2^2), \tag{8a}$$

and

$$C_\sigma = \exp[-8\pi^2\sigma^2/\lambda^2]. \tag{8b}$$

Figure 1 is a plot of fringe contrast as a function of the scale roughness given by σ/λ when the two amplitudes are equal.

A sample glass was ground with aluminum oxide with an average particle size of 14.5 µm. The standard deviation of the surface was found' to be **0.5** µm. Figure 1 shows that the theoretical contrast of fringes for this sample is greater than 0.8 for a wavelength of 10.6 µm. Since the theory predicts a reasonably high contrast of fringes, a CO_2 laser was constructed to conduct interference experiments on optically rough surfaces.

As an initial experiment, a Fresnel mirror arrangement was chosen because of its simplicity. Figure 2 is a schematic of the experimental configuration. A spheriral mirror was used to diverge the beam onto the Fresnel mirror combination at nearly

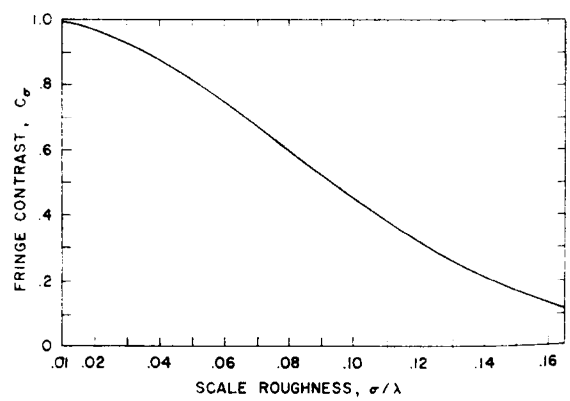

Fig. 1. Fringe contrast as a function of scale roughness.

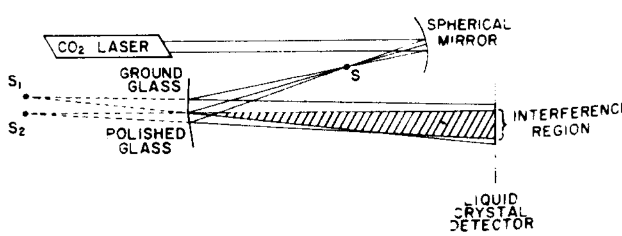

Fig. 2. Schematic of the Fresnel mirror experiment.

(a)

(b)

Fig. 3. Interference pattern obtained from (a) a ground glass surface (σ = 0.5 pm) and (b) a polished surface using a 10.6-μm laser.

normal incidence. The Fresnel mirrox consisted of the ground glass surface described above and a polished surface inclined at a small angle θ. From the geometry of the arrangement and Eq. (3), the fringe spacing is

$$\Delta x = \lambda D/2\theta A, \quad (9)$$

where D is the distance from the detector to the point image S and A is the distance from S to the intersection of the Fresnel mirror.

Figure 3 is a black and white photograph of a colored fringe pattern that appeared on the heat sensitive detector. The detector was a liquid crystal of the cholesteric type supported on a thin membrane.[6] This type of detector reflects different colors of light depending on the temperature. The fringe spacing is 0.25 cm, which corresponds to a mirror angle of 0.6".

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