



**College of Optical Sciences** 

# 9.0 Special Interferometric Tests for Aspherical Surfaces



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  - 9.3.11 Stitching Interferograms





- Aspheric surfaces are of much interest because they can provide
  - Improved performance
  - Reduced number of optical components
  - Reduced weight
  - Lower cost
- Since many aspherics are either conics or similar to conics, we will first say a few words about conics



#### **9.1.1 Conics**



# A conic is a surface of revolution defined by means of the equation

$$s^2 - 2rz + (k+1)z^2 = 0$$

Z axis is the axis of revolution. k is called conic constant. r is the vertex curvature.

$$s^2 = x^2 + y^2$$





#### **Sag for Conic**

$$z = \frac{r - \sqrt{r^2 - s^2 (k+1)}}{(k+1)} \left( \frac{r + \sqrt{r^2 - s^2 (k+1)}}{r + \sqrt{r^2 - s^2 (k+1)}} \right)$$
$$= \frac{r^2 - r^2 + s^2 (k+1)}{(k+1)r + (k+1)\sqrt{r^2 - s^2 (k+1)}}$$

$$z = \frac{s^2 / r}{1 + \left[1 - (k+1)(s / r)^2\right]^{1/2}}$$

$$s^2 = x^2 + y^2$$



# **Types of Conics**





- K > O Oblate ellipsoid (ellipsoid rotated about minor axis)
- K=0 Sphere
- -1 < K < O Prolate ellipsoid (ellipsoid rotated about major axis)
  - K= -1 Paraboloid
  - K< -1 Hyperboloid





#### 9.1.2 Sag for Aspheres

$$z = \frac{s^2 / r}{1 + \left[1 - (k+1)(s/r)^2\right]^{1/2}} + A_4 s^4 + A_6 s^6 + \cdots$$
$$s^2 = x^2 + y^2$$

# k is the conic constant r is the vertex radius of curvature A's are aspheric coefficients





- Slope of the aspheric departure from a spherical surface determines the difficulty of the test.
- For sufficiently large slopes, the light from the surface under test will not go back into the test setup.
- In the case of an interferometric test, the wavefront slope determines the spacing of the fringes.



# Wavefront Departure and Slope versus Radius









#### Null Tests - Perfect optics give straight equally spaced fringes

- 9.2.1 Conventional null optics
- 9.2.2 Holographic null optics
- 9.2.3 Computer generated holograms





A null interferometer strives to create a wavefront that matches the test surface and is normally incident everywhere. The reflected light retraces the incident path, and a null interferogram results.







# 9.2.1 Conventional Null Optics





#### Hubble Pictures (Before and After the Fix)









## **Offner Null Compensators**



Single-mirror compensator with field lens.



# **Testing of Hyperboloid**









# **Meinel Hyperboloid Test**









#### **Null Tests for Conics**



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## **Hindle Test**



**Testing convex paraboloid** 





#### **Modified Hindle Tests**



Simpson-Oland-Meckel Test





# **Testing Concave Parabolic Mirrors**







# **Testing Elliptical Mirrors**







# 9.2.2 Holographic Null Optics







# 9.2.3 Computer Generated Holograms

#### **CGH Interferometer**







# **<u>Computer Generated Hologram</u>**





# **Light in Spatial Filter Plane**









### **CGH Used as Null Lens**



- Can use existing commercial interferometer
- Double pass through CGH, must be phase etched for testing bare glass optics
- Requires highly accurate substrate





# **CGH Optical Testing Configurations**



**Zone plate interferometer** 

**CGH** test plate



# **CGH Test Plate Configuration**





Configuration for CGH test plate measurement of a convex asphere.



Alternate configuration for CGH test convex aspheres.



#### **Reference and Test Beams in CGH Test Plate Setup**







#### **Error Source**



#### Pattern distortion (Plotter errors)

- Substrate surface figure
- Alignment Errors





- The hologram used at m<sup>th</sup> order adds m waves per line;
- CGH pattern distortions produce wavefront phase error:

$$\Delta W(x,y) = -m\lambda \frac{\varepsilon(x,y)}{S(x,y)}$$

 $\epsilon(x,y)$  = grating position error in direction perpendicular to the fringes; S(x,y) = localized fringe spacing;

For m = 1, phase error in waves = distortion/spacing

0.1  $\mu$ m distortion / 20  $\mu$ m spacing ->  $\lambda$ /200 wavefront



#### **Plotters**



#### E-beam

- Critical dimension 1 micron
- Position accuracy 50 nm
- Max dimensions 150 mm
- Laser scanner
  - Similar specs for circular holograms





- Put either orthogonal straight line gratings or circular zone plates on CGH along with grating used to produce the aspheric wavefront
- Straight line gratings produce plane waves which can be interfered with reference plane wave to determine plotter errors
- Circular zone plates produce spherical wave which can be interfered with reference spherical wave to determine plotter errors





- Use direct laser writing onto custom substrates
- Use amplitude holograms, measure and back out substrate
- Use an optical test setup where reference and test beams go through substrate





- Lateral misalignment gives errors proportional to slope of wavefront
- Errors due to longitudinal misalignment less sensitive if hologram placed in collimated light
- Alignment marks (crosshairs) often placed on CGH to aid in alignment
- Additional holographic structures can be placed on CGH to aid in alignment of CGH and optical system under test




# **Use of CGH for Alignment**

Commonly CGH's have patterns that are used for aligning the CGH to the incident wavefront.





Using multiple patterns outside the clear aperture, many degrees of freedom can be constrained using the CGH reference.





# **Projection of Fiducial Marks**



- The positions of the crosshairs can be controlled to micron accuracy
- The patterns are well defined and can be found using a CCD
- Measured pattern at 15 meters from CGH. Central lobe is only 100 µm FWHM



#### CGH Alignment for Testing Off-Axis Parabola







# Holographic test of refractive element having 50 waves of third and fifth order spherical aberration









#### **CGH Test of Parabolic Mirror**



No CGH



CGH



#### Aspheric Testing Using Partial Null Lens and CGH





Partial null lens test without CGH



CGH-partial null lens test



Null lens test





### **CGH Test of Lens**







#### 9.3 Non-Null Tests

# Non-null Tests - Even perfect optics do not give straight equally spaced fringes

- 9.3.1 SCOTS
- 9.3.2 Scanning Pentaprism Test
- 9.3.3 Lateral Shear Interferometry
- 9.3.4 Radial Shear Interferometry
- 9.3.5 High-density detector arrays
- 9.3.6 Sub-Nyquist Interferometry
- 9.3.7 Long-Wavelength Interferometry
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- The goal of a non-null test of aspherics is to not have to rely on a part-specific null lens or CGH.
- Three general requirements for any interferometric non-null test:
  - Must get light back into interferometer
  - Must be able to resolve the fringes
  - Must know precisely the test setup





- With a null test the interferometer creates a wavefront to match the test surface and the reflected wavefront retraces the incident path. This is not true for non-null test.
- If the reflected wavefront does not retrace the incident wavefront, path-dependent induced aberrations (retrace errors) result. To obtain the true surface, the induced aberrations must be removed by reverse raytracing.







- Geometrical tests such as Foucault, wire, or Ronchi tests, can be used as discussed above and we will not discuss them in this section.
- The Shack-Hartmann test can also be used as discussed above and we will not discuss it here, but we will discuss a related test, the SCOTS test.
- Several interferometric tests that can be fairly easily computerized will be discussed.





- SCOTS Software Configurable Optical Test System
- Hartmann test in reverse
- Measures slope
- Accuracies in the range of 100 200 nrad (rms) have been achieved
- Ref: Su, Parks, Wang, Angel, and Burge Appl. Opt., "Software configurable optical test system: a computerized reverse Hartmann test", 49(23), 4404-4412, (2010).
- Ref: Su, Wang, Burge, Kaznatcheev, and Idir, "Non-null full field X-ray mirror metrology using SCOTS: a reflection deflectometry approach", Opt. Express 20(11), 12393-12406 (2012).





## **SCOTS and Hartmann Test**



#### **Geometry of SCOTS Test**





Use line source to measure either x-slope or y-slope. Sinusoidal fringes can be used instead of line and phase-shifting techniques can be used.





#### **Measurement of Off-Axis Parabola**





#### 8.4 m Giant Magellan Telescope



SCOTS

Interferometry





# 9.3.2 Scanning Pentaprism Test

- Pentaprism relays a collimated beam to the surface under test. The angle of the reflected beam is a measure of the surface slope of the mirror.
- The pentaprism is scanned across the surface to sample the slope error at a number of points.
- Powerful advantage of pentaprism is that it deviates light by a fixed angle, thus measurements are relative insensitive to prism movement errors or alignment errors.
- Pentaprism test often used for measuring flat mirrors and parabolic mirrors.







Ref: Su, Burge, Cuerden, Sasian, and Martin, "Scanning pentaprism measurements of off-axis aspherics", Proc. Of SPIE, Vol. 7018, 70183T, (2008).



#### **Test Results**







## **9.3.3 Lateral Shear Interferometry**



- We have previously discussed, so only need to mention advantages and disadvantages for testing aspheres.
- Advantages
  - Can vary sensitivity by varying the amount of lateral shear
- Disadvantages
  - Two interferograms are required for non-symmetric wavefronts
  - Must know the amount of shear and direction of shear very accurately
  - Helps less with wavefronts having larger slopes





#### We are measuring essentially the slope of the wavefront

 $\Delta W = Ax^{N}$  $\partial \Delta W = NAx^{N-1}\partial x$ 

Therefore

 $\frac{\text{Fringes with LSI}}{\text{Fringes with T-G}} = N \frac{\text{Shear Distance}}{\text{Pupil Radius}}$ 







- We have previously discussed, so only need to mention advantages and disadvantages for testing aspheres.
- Advantages
  - Can vary sensitivity by varying the amount of radial shear
- Disadvantages
  - The shear varies over the pupil with the largest shear at the edge of the pupil, which is generally the location of maximum slope. Thus, we get the least help where we need the most help.





# 9.3.5 High-Density Detector Arrays

- Theoretically need at least two detectors per fringe if we know nothing about the wavefront we are testing. Due to noise, and the fact that each detector is averaging over a part of a fringe, generally 2.5 or 3 detectors per fringe are required. Less than 100% fill factor is desirable, but then more light is required.
- If we have additional information about the wavefront being tested, such as the surface height to within a quarter wavelength, or that the slope is continuous, it is often possible to perform a measurement using fewer than two detectors per fringe.





- Critical item is to know the system accurately enough so it can be ray traced to determine what the desired asphericity is at the detector plane.
- Knowing the asphericity at the location of the test object is not enough. We must know the asphericity at the location where the measurement is being performed, i.e. the detector plane.
- Test system calibration is probably required.



# 9.3.6 Sub-Nyquist Interferometry



- Fewer than two detector elements per fringe required if additional information is known such as the first and second derivatives of the wavefront are continuous.
- This requires the detector to have a fill less than 1 (sparse array). More light is then required.



# Interferogram Unwrapping



Phase changes greater than  $\pi$  allowed, so unwrapping must take into account the local derivative of the wavefront. Unwrapping must begin in a region with no aliasing.





# 9.3.7 Long-Wavefront Interferometry



- We have previously discussed, so only need to mention advantages and disadvantages for testing aspheres.
- Advantages
  - Using CO<sub>2</sub> laser at wavelength of 10.6  $\mu m$  so fewer fringes are obtained.
- Disadvantages
  - Reduced sensitivity
- Germanium or zinc selenide optics and a bolometer must be used.

λ= 0.633 μm







- Means of obtaining visible light to perform interferometric test having sensitivity of test performed using a long-wavelength nonvisible source.
- **Record hologram at wavelength** λ<sub>1</sub>.
- Reconstruct hologram at wavelength λ2.
- Interferogram same as would be obtained in normal interferometric test using wavelength.

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}$$











- Measuring OPD
- Wavelength used to record hologram is  $\lambda_1$
- Wavelength used in reconstruction is  $\lambda_2$
- Tilt angle of reference beam used to record hologram is  $\theta_1$
- Tilt angle of reference beam used in reconstruction is  $\theta_2$

**Recording hologram**  

$$I = Abs \left[ e^{i\frac{2\pi}{\lambda_1}OPD} + e^{i\frac{2\pi}{\lambda_1}xSin[\theta_1]} \right]^2$$

$$= 2 + e^{i\frac{2\pi}{\lambda_1}(OPD - xSin[\theta_1])} + e^{-i\frac{2\pi}{\lambda_1}(OPD - xSin[\theta_1])}$$
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**Derivation of Equivalent Wavelength - II** 



 Assume amplitude transmission of hologram proportional to irradiance used to expose the hologram

$$t = \tau_b + \beta I$$

Illuminate hologram

$$I = e^{i\frac{2\pi}{\lambda_2}OPD} + e^{i\frac{2\pi}{\lambda_2}xSin[\theta_2]}$$

 Look at zero order from test beam and diffracted order from reference beam

$$2e^{i\frac{2\pi}{\lambda_2}OPD} + e^{i\frac{2\pi}{\lambda_1}OPD} e^{i\left(\frac{2\pi}{\lambda_2}xSin[\theta_2] - \frac{2\pi}{\lambda_1}xSin[\theta_1]\right)}$$



# **Derivation of Equivalent Wavelength - II**



As long as angle between test and reference beams is large enough the other orders will separate and the resulting interference pattern will be given by

$$i = a + bCos \left[ 2\pi OPD\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) + 2\pi x \left(\frac{Sin[\theta_2]}{\lambda_2} - \frac{Sin[\theta_1]}{\lambda_1}\right) \right]$$

We can write

$$2\pi \left(\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{2}}\right) OPD = \frac{2\pi}{\lambda_{eq}} OPD, \text{ where}$$
$$\frac{1}{\lambda_{eq}} = \left(\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{2}}\right) \text{ or } \lambda_{eq} = \frac{\lambda_{1}\lambda_{2}}{Abs[\lambda_{1} - \lambda_{2}]}$$





# **Advantages and Disadvantages**

#### Advantages

 Wavelength difference can be selected to obtain desired equivalent wavelength and sensitivity

#### Disadvantages

- Since the technique involves finding the difference between two interferograms formed using two different wavelengths, any difference between the two interferograms introduced by chromatic aberration or disturbances such as air turbulence will introduce errors.
- These differences will be scaled by the equivalent wavelength. That is, one fringe error between the two interferograms corresponds to an error of  $\lambda_{eq}$ , not  $\lambda_1$  or  $\lambda_2$ .



#### **Possible Equivalent Wavelengths Obtained with Argon and HeNe Lasers**



λ2	0.4579	0.4765	0.4880	0.4965	0.5017	0.5145	0.6328
λ1							
0.4579	-	11.73	7.42	5.89	5.24	4.16	1.66
0.4765	11.73	-	20.22	11.83	9.49	6.45	1.93
0.4880	7.42	20.22	-	28.50	17.87	9.47	2.13
0.4965	5.89	11.83	28.50	-	47.90	14.19	2.31
0.5017	5.24	9.49	17.87	47.90	-	20.17	2.42
0.5145	4.16	6.45	9.47	14.19	20.17	-	2.75
0.6328	1.66	1.93	2.13	2.31	2.42	2.75	-

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}$$











## Variable Wavelength Interferograms I



















### Variable Wavelength Interferograms II






#### **TWH Test of Aluminum Block**



 $\lambda eq = 10 \text{ mm}$ 





#### **TWH Test of Sesame Street Character**



 $\lambda eq = 2 mm$ 





- Perform measurement at two wavelengths,  $\lambda_1$  and  $\lambda_2$ .
- Computer calculates difference between two measurements.
- Wavefront sufficiently sampled if there would be at least two detector elements per fringe for a wavelength of

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}$$







- Probably not useful for measuring aspherics, but it is a useful technique for measuring many different objects, especially diffuse surfaces.
- Sensitivity of test can be varied by changing spacing of lines (fringes) projected on surface or angle of illumination or angle of viewing.





#### **Intersection of Fringes on Surface**



Ref: A. MacGovern, Appl. Opt. 11, 2972 (1972)





Surface height described by z = f[x, y]

Loci of fringes or lines project on surface given by

 $y = zTan[\alpha] + nd$ , then  $y = f[x, y]Tan[\alpha] + nd$ 

# If surface is viewed at angle $\beta$ , then it will appear as though the fringe is intersecting the surface at

$$y = f[x, y]Tan[\alpha] + nd + f[x, y]Tan[\beta]$$
  
or  
$$y = f[x, y](Tan[\alpha] + Tan[\beta]) + nd$$



# **Derivation of Equivalent Wavelength - II**



If surface were tested in a Twyman-Green interferometer using a wavelength  $\lambda$  and plane reference wavefront tilted an angle  $\gamma$  a bright fringe fringe would be obtained whenever

$$2f[x,y] - ySin[\gamma] = -n\lambda$$

**or** 
$$y = \frac{2f[x,y]}{Sin[\gamma]} + \frac{n\lambda}{Sin[\gamma]}$$

Comparing equations for projected lines and T-G interferogram we

See 
$$\frac{2}{Sin[\gamma]} = Tan[\alpha] + Tan[\beta]$$
 and  $\frac{n\lambda_{eq}}{Sin[\gamma]} = nd$  or  $\frac{2}{Sin[\gamma]} = \frac{2d}{\lambda_{eq}}$   
Thus we can write  $\lambda_{eq} = \frac{2d}{Tan[\alpha] + Tan[\beta]}$ 

### **Projected Fringe Contouring**





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## **Projected Fringe Contouring Setup**







#### Can









#### Hand







#### Foot









#### **Foot Scanner**







- Perform sub-aperture test of aspheric and stitch together interferograms.
- Trade-off between overlap between interferograms and number of interferograms required.
- Much easier to describe than to obtain accurate results.



# Subaperture Stitching Interferometry (SSI®)



- What is it?
  - 6-axis motion system
  - "Standard" interferometer
  - Automatic collection of multiple subaperture measurements
  - Magnified, locally nulled subapertures reduce "aspheric" fringe density
  - Compensation of systematic errors
- SSI extends standard interferometry
  - Fast & Large parts
  - Aspheres (up to ~200  $\lambda$ )
- And also can *improve*:
  - Accuracy & Resolution







#### **Extending the SSI to ASI®**

- Variable Optical Null (VON) extends aspheric departure capture range
- Counter-rotating optical wedges
  - Varying the wedge angle and tilt produces astigmatism & coma

SSI

**Plane-parallel** 

ASI (VON)





SOLNA

# SSI/ASI: Summary

- What is it good for?
  - Flexible no dedicated nulls
  - High departure
  - Large NA or CA
  - High vertical resolution
  - High lateral resolution
  - Compensation of systematic errors
- What are its key limitations?
  - Inflection points
  - High slope deviations
  - 3<sup>rd</sup> order spherical uncertainty







- Q<sup>bfs</sup> polynomial form defines surface characterized by rms slope departure of asphere from best-fit sphere. The rms slope can be easily calculated.
- Q<sup>con</sup> form defines surface characterized by sag departure of asphere from base conic. This formulation allows designers to determine need for a particular term by inspection.
- Aspheric terms are orthogonal over pupil.

Ref: G. Forbes, "Shape specification for axially symmetric optical surfaces," Opt. Express 15, 5218-5226 (2007).





- Must know precisely how optics in test setup change aspheric wavefront.
- Must know effects of misalignments, so errors due to misalignments can be removed.





- Must get light back into the interferometer
- Must be able to resolve the fringes
- Must know precisely the optical test setup

This is the most serious problem

