Curved Surface Testing

CS-1

The interferogram shown below was obtained using the Twyman-Green interferometer to test a cheap camera lens in double pass. The camera lens is f/1.7 with a focal length of 50 mm.

a) Draw a schematic diagram and carefully explain the alignment procedure you would use to test the lens.

b) How good is the lens? What are the predominant aberrations? What is the approximate magnitude of the lens aberration? Comment upon whether you think the aberrations are in the design or fabrication errors. Be sure to give your reasons.

c) If the test arm of the interferometer is shortened, the fringes move up in the interferogram. What are the signs of the aberrations - i.e., for various zones is the optical path through the lens too large or too small?

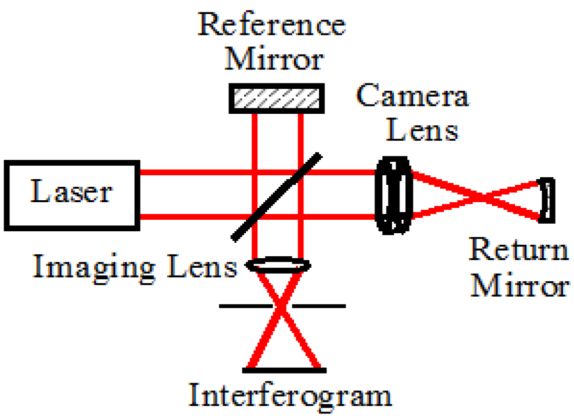
d) Make a rough calculation as to the geometrical blur size for a point object.

e) What f/number should be used to obtain no more than 1/4 wave of aberration?

f) Is this an acceptable lens? Explain.

Solution

a)



In the final setup the center of curvature of the return mirror is at the focus of the lens under test. The return mirror can be concave or convex. The reflections from the camera lens can be used to properly orient the camera lens. The reference mirror should initially be adjusted to retroreflect the beam incident upon it. To achieve this we place the return mirror at the focus of the lens and the reference mirror is adjusted to obtain a minimum number of fringes which means the reference mirror is retroreflecting the incident light. Now when the return mirror is placed in the position shown in the drawing it is properly adjusted to retroreflect the incident light when a minimum number of fringes is obtained. The reference mirror can now be adjusted to give the desired number of tilt fringes. The fringes should be made as straight as possible by adjusting the distance between the camera lens and the return mirror.

b)

The P-V of the lens tested in double pass is about 4 fringes. Thus for single pass the P-V OPD error is about 2 fringes, or 1.2 microns. The predominate aberration is spherical, although a little coma is present. The coma is probably due to fabrication, and the spherical is probably a design error. My guess is that to keep the cost down, the number of elements was minimized.

c)

The center of the lens is too thick relative to the outer zones.

d)

Maximum slope is approximately 25 fringes per radius.

$$\text{BlurDiameter} = 2\,\frac{R}{h}\,\frac{\partial \Delta W}{\partial \rho} = 2\,\frac{(50\ \text{mm})}{14.7\ \text{mm}}\,(25\,(0.3\ \mu\text{m})) = 50\ \mu\text{m}$$

e)

f/number for ¼ wave aberration approximately (53mm/30mm)1.7 = 3.

f)

The lens is probably acceptable. Stop the lens down 1 or 2 f stops, and it is a good lens.

CS-2

In the optics shop we are making some aspheric collimating lenses. If we use a LUPI (Twyman-Green) to test these lenses in double pass we have to worry about off-axis aberrations introduced by tilting the lens during the test.

a) We claim that we can reduce the effect of misalignment by subtracting coma in the data reduction. However, if we subtract coma, we should also subtract some astigmatism. Why?

b) If we have a 30 cm diameter, 1.2 m focal length BK7 lens bent for minimum spherical aberration and we subtract 2 waves of coma, approximately how much astigmatism should we subtract? The wavelength is 633 nm.

c) What if only 0.2 waves of coma are present?

Solution

a)

Both coma and astigmatism are off-axis aberrations and if a lens is tilted both coma and astigmatism will be produced.

b)

For minimum spherical

$$\Delta W_{coma} = \frac{h\,U}{16\,(f_{no})^2}\left(\frac{1}{n+2}\right)$$

$$\Delta W_{ast} = \frac{h\,U^2}{4\,(f_{no})}$$

U as a function of ΔW_{coma} is given by

$$U = \frac{16\,\Delta W_{coma}\,(f_{no})^2\,(n+2)}{h}$$

Solving for ΔW_{ast} yields

$$\Delta W_{ast} = \frac{h}{4\,(f_{no})}\left(\frac{16\,\Delta W_{coma}\,(f_{no})^2\,(n+2)}{h}\right)^2$$

$$\frac{64\,(2+n)^2\,f_{no}^3\,\Delta W_{coma}^2}{h}$$

$$\Delta W_{ast} \frac{\lambda}{633 \times 10^{-9}\,m} \;/.\; \{n \to 1.523,\, f_{no} \to 4,\, h \to 15 \times 10^{-2}\,m,\, \Delta W_{coma} \to 633 \times 10^{-9}\,m\}$$

0.214535 λ

Therefore we should subtract about .4 λ of astigmatism for double pass.

c)

$$\Delta W_{ast} \frac{\lambda}{633 \times 10^{-9}\,m} \;/.\; \{n \to 1.523,\, f_{no} \to 4,\, h \to 15 \times 10^{-2}\,m,\, \Delta W_{coma} \to 0.1 \times 633 \times 10^{-9}\,m\}$$

0.00214535 λ

Therefore for 0.02 waves of coma we should subtract about .004 λ of astigmatism for double pass.

CS-3

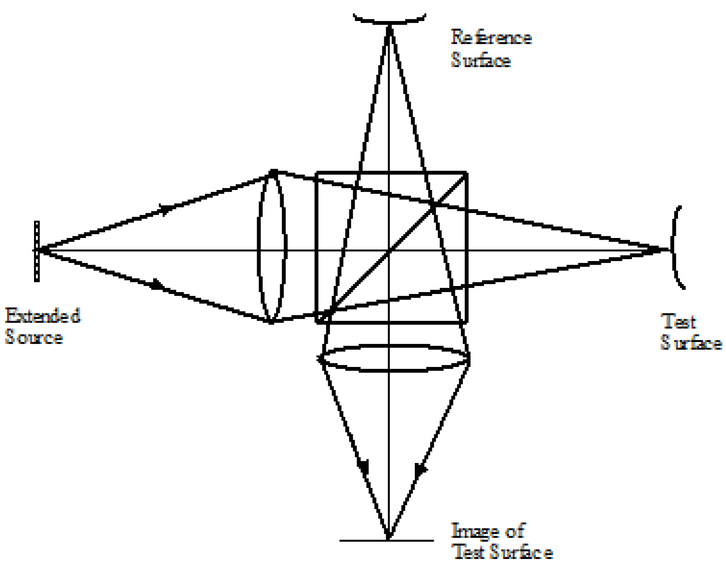
In the normal LUPI test of a convex spherical mirror the quality of the diverger lens is very important.

a) Why is the quality of the diverger lens less important if the arrangement shown below is used?

b) Why is the quality of the beamsplitter less important if we use an extended source and image the source onto the test surface?

c) How will aberrations in the beamsplitter influence the fringes obtained?

d) What additional requirements are imposed upon the beamsplitter if a white light source is used with the interferometer?



Solution

a)

To first order aberrations in the lens cancel because they are in both beams.

b)

Each point in the interference pattern will see an average of the aberrations across the beam splitter.

c)

Aberrations in the beamsplitter will reduce the fringe contrast.

d)

The paths must be matched. Due to dispersion we want the same amount of glass in the two paths.

CS-4

A Fizeau interferometer having a HeNe laser as the light source is used to test flat surfaces. Let the reference surface in the Fizeau be perfect, but the 10 cm diameter collimator lens used in the interferometer has 2 waves of third-order spherical aberration. Give the approximate maximum error in the test results if the separation between the reference surface and the test surface is

a) 1 cm
b) 1 meter.

Solution

$$\Delta w = 2\,\rho^4; \quad \text{slope} = \theta = 8\,\rho^3\,\frac{\lambda}{5\ \text{cm}}$$

$$\theta = 8\,\rho^3\,\frac{\lambda}{5\ \text{cm}}\ /.\ \{\rho \to 1,\ \lambda \to 0.633\times 10^{-4}\ \text{cm}\}$$

0.00010128

If we were to introduce -3 waves of defocus we would reduce the maximum slope by a factor of 4, so

$$\theta_{\text{withDefocus}} = \theta\,/\,4$$

0.00002532

Let L be the distance between the flats.

a)

$$L = 1\ \text{cm}\,\frac{1\,\lambda}{0.633\times 10^{-4}\ \text{cm}};$$

Error without defocus

error = 2 L (1 - Cos[θ])

0.000162048 λ

Error with defocus

error = 2 L (1 - Cos[$\theta_{withDefocus}$])

0.000010128 λ

b)

$$L = 1\,m\,\frac{1\,\lambda}{0.633\times 10^{-6}\,m};$$

Error without defocus

error = 2 L (1 - Cos[θ])

0.0162048 λ

Error with defocus

error = 2 L (1 - Cos[$\theta_{withDefocus}$])

0.0010128 λ

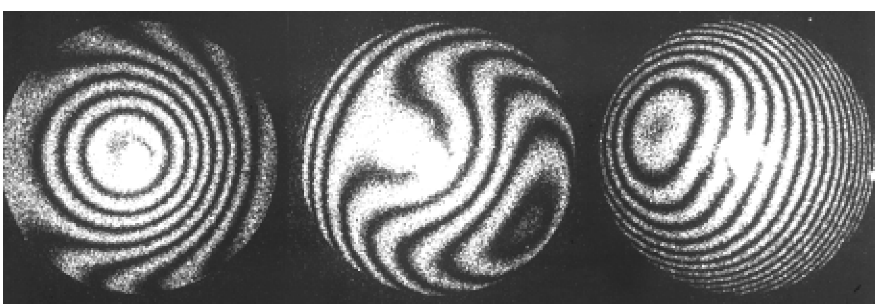
CS-5

The three scatterplate interferograms shown below were made of a 12 inch (30 cm) diameter parabolic mirror tested at the center of curvature.

a) How was the interferometer adjustment changed between the photographing of the three interferograms?

b) Obtain a plot (across one diameter) of the departure of the parabolic mirror relative to the sphere centered at the vertex center of curvature.

c) What is the focal length of the parabolic mirror?



Solution

a)

Photo on right

Scatterplate at paraxial focus

Center photo

Scatterplate slightly inside 0.7 zone (scatterplate moved further from mirror)

Photo on left

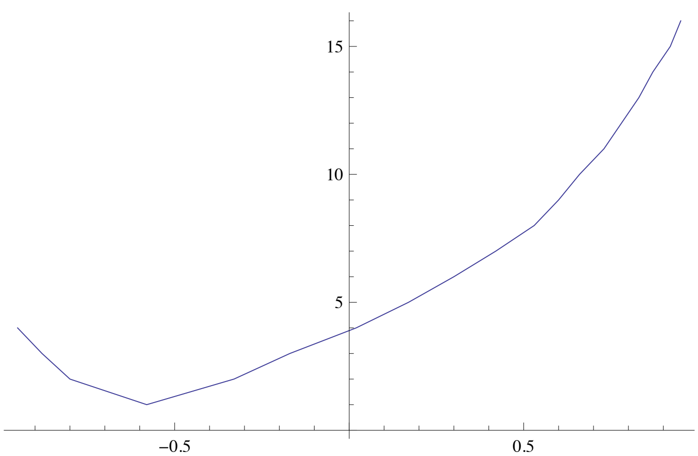
Scatterplate at marginal focus (scatterplate moved still further from mirror)

b)

The fringe position along a line through the center of the interferogram, perpendicular to the fringes in the center of the interferogram, is measured and the following Mathematica output gives the normalized x position for the various fringe orders.

```
opd = {{-.95, 4}, {-.88, 3}, {-.8, 2}, {-.58, 1}, {-.33, 2}, {-.17, 3},
   {.02, 4}, {.17, 5}, {.3, 6}, {.42, 7}, {.53, 8}, {.6, 9}, {.66, 10},
   {.73, 11}, {.78, 12}, {.83, 13}, {.87, 14}, {.92, 15}, {.95, 16}};
```

```
ListPlot[opd, Joined → True, Background → White]
```



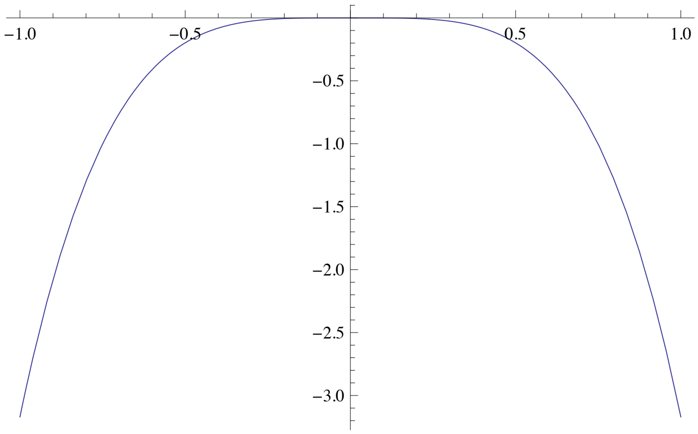
Next we will fit the data.

```
Fit[opd, {1, x, x², x³, x⁴}, x]
```

$3.89969 + 6.64994\,x + 1.05624\,x^2 - 0.357201\,x^3 + 6.34283\,x^4$

We need to divide this result for the wavefront by 2 to get the surface. We will now plot the departure of the parabolic mirror relative to the sphere centered at the vertex center of curvature. We know that the sag of a parabola is less than the sag of a sphere having a radius of curvature equal to the vertex radius of curvature of the parabola.

$$\text{Plot}\left[\frac{1}{2}(-6.34`)\,x^4,\ \{x,-1,1\},\ \text{PlotRange}\to\text{All},\ \text{Background}\to\text{White}\right]$$



c)

$$\text{sag} = -\frac{s^4}{64\,f^3}$$

$$\text{sag} = -3.17\ \text{wave}\ \frac{633\times10^{-9}\ \text{m}}{\text{wave}}$$

$$-2.00661\times10^{-6}\ \text{m}$$

$$f = \left(\frac{-s^4}{64\ \text{sag}}\right)^{1/3}\ /.\ s\to0.15\ \text{m} = 1.58\ \text{m}$$

CS-6

I am using a quasi-monochromatic spatially-incoherent circular source with a scatterplate interferometer. A 20 cm diameter, 200 cm radius of curvature spherical mirror is being tested. The lens used to image the source on the mirror has a 4 cm focal length. The interferometer is adjusted so as to produce fringes having a spacing of 2 cm when projected back onto the mirror being tested. How large can the diameter of the source be before a lack of spatial coherence causes the fringe contrast to go to zero? How does the result depend upon wavelength? State any simplifying assumptions you make.

Solution

For a circular incoherent source the coherence function, or fringe contrast, goes as

$$\text{contrast} = \frac{2\,\text{Bessel}[1,\ \pi\,x\,\alpha/\lambda]}{\pi\,x\,\alpha/\lambda}$$

α is the angular subtense of the source. Let the source diameter be d. The lens used to image the source on the mirror is assumed to be close to the scatterplate, and the radius of curvature of the mirror is large compared to the focal length of the lens, so the source is approximately the focal length of the lens away from the scatterplate.

$$\alpha \approx \frac{d}{f} = \frac{d}{4\text{ cm}}$$

x is the distance between the two point sources forming the interference fringes. For two beam interference

$$x\,\theta = \lambda$$

$$\theta = \frac{2\text{ cm}}{200\text{ cm}} = 10^{-2}$$

Therefore,

$$x = 100\,\lambda$$

For zero fringe visibility

$$\frac{\pi\,x\,\alpha}{\lambda} = 1.22\,\pi$$

$$\alpha = \frac{1.22\,\lambda}{x} = \frac{1.22\,\lambda}{100\,\lambda} = \frac{d}{4\text{ cm}}$$

$$d = \frac{4\,(1.22)}{100}\text{ cm} = 0.488\text{ mm},\ \text{independent of }\lambda$$

The size of the image of the source on the mirror is 0.488 mm $\frac{200}{4}$= 2.44 cm, which is approximately the fringe spacing. If the source were a square source the size of the image would be exactly equal to the fringe spacing.

Another way to think about this problem is to consider the extended source to be made up of a collection of point sources. The zero order fringe for each point source will pass through the image of the point source. As the source becomes larger, the zero order fringes for the different point sources will spread. For simplicity, consider an extended source made up of only two equally bright point sources incoherent with respect to each other. The fringe contrast will go to zero when the dark fringe for one source falls at the location of the zero order fringe for the second source. Thus, for a uniform extended source we will have low visibility when the fringe spacing is equal to the source size. The exact value of the fringe contrast will depend upon the source distribution, as shown above.

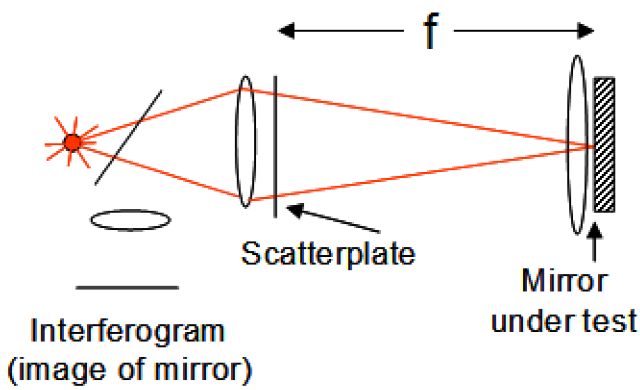
CS-7

a) Sketch a diagram for using a scatterplate interferometer to test a flat mirror 2 cm in diameter. What limits the maximum size of the scatterplate? What limits the minimum size?

b) Assume the flat mirror is flat to within 5 fringes, how large a spectral bandwidth can be used for the source? State any assumptions.

Solution

a)



The source should be imaged onto the flat mirror.
The scatterplate must be imaged back onto itself.

The maximum size is limited by off-axis aberrations.
The minimum size is limited by speckle size.

b)

$$\text{coherenceLength} \sim \frac{\lambda^2}{\Delta\lambda} \sim \pm 2.5\,\lambda$$

$$\Delta\lambda = \frac{500\ \text{nm}}{2.5} = 200\ \text{nm}$$

CS-8

a) I am using a scatterplate interferometer to test an uncoated parabolic mirror using a null configuration. Sketch a suitable experimental setup.

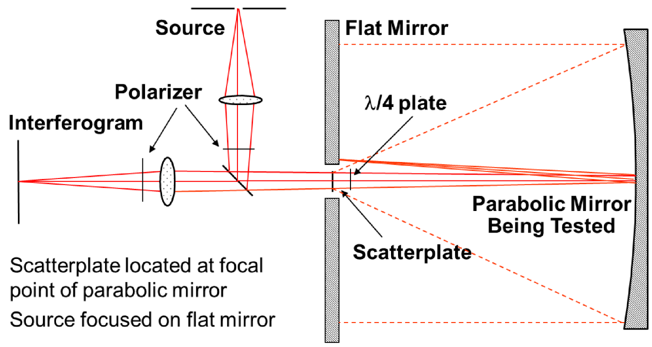
b) In the answer to part a you should be reflecting off the parabolic mirror twice. I find that in doing the experiment I have so much light reflected back from the scatterplate that my fringe contrast is very low. Describe a "polarization technique" that would at least partially correct this problem.

c) Give at least 2 tradeoffs on the optimum ratio of scattered to unscattered light for a scatterplate. How does your answer depend upon whether a coated or an uncoated mirror is being tested?

d) Does the "hotspot" for a scatterplate interferogram fall on a dark or a bright fringe? Explain.

Solution

a)



- Scatterplate located at focal point of parabolic mirror
- Source focused on flat mirror

b)

i) Use linearly polarized light
ii) Place a λ/4 plate between scatterplate and the mirror. Orientate the λ/4 plate at 45° so after passing thru the plate twice the direction of polarization is rotated 90°.
iii) Place a polarizer in front of the interferogram to block light which has not passed through the λ/4 plate twice. (Note: The scatterplates I have worked with do not depolarize the light much.)

c)

Too little scatter, too bright hot spot, too little light in interferogram. Too much scatter, scatter-scatter term is too bright and the fringe contrast is low. The answer does not depend upon whether the mirror is coated or not because both scattered and direct beams reflect off the mirror.

d)

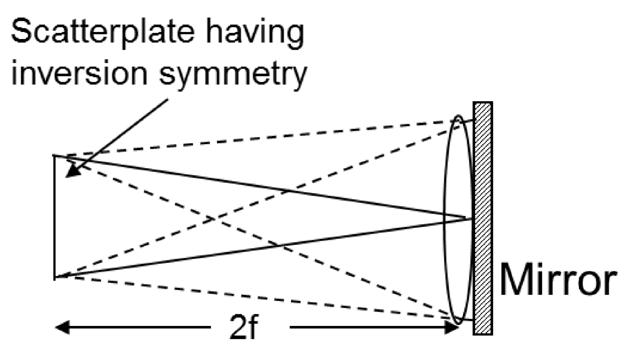
A bright fringe always passes through the "hot spot" because the OPD is zero.

CS-9

a) Describe how you would use a single scatterplate to test a lens at 1:1 conjugates.
b) If it is possible, devise a way to use two identical scatterplates that do not have inversion symmetry to test a lens at 1:1 conjugates.
In both a) and b) explain how tilt and defocus are introduced.

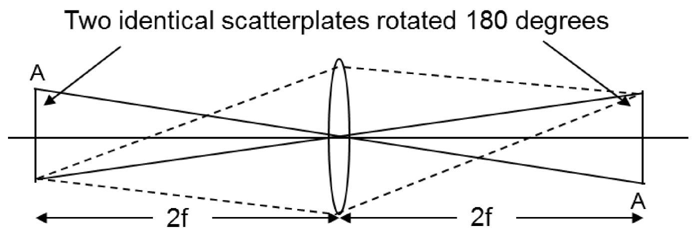
Solution

a)



Tilt is introduced by translating the scatterplate sideways. Defocus is introduced by longitudinal displacement of the scatterplate. The direct beam is focused at the center of the lens.

b)



Tilt is introduced by translating either scatterplate sideways. Defocus is introduced by longitudinal displacement of either scatterplate. The direct beam is focused at the center of the lens.

CS-10

a) What is known about the fringe that passes through the "hot spot" in a scatterplate interferogram?
b) Give at least 2 tradeoffs on the optimum ratio of scattered to unscattered light for a scatterplate.

Solution

a)

The fringe that passes through the "hot spot" is a zero order fringe.

b)

If we have too much light scattered the scattered-scattered beam will be too bright and the fringe contrast will be low. If we have too little light scattered the "hot spot" will be too bright and the interferogram will be dim.

CS-11

A scatterplate interferometer is used to test a concave spherical mirror having a 9 mm diameter and 75 mm radius of curvature.

a) The interferometer is adjusted to give a 7 bright vertical tilt fringes across the interferogram with a bright fringe at both the left and right edges of the interferogram for a wavelength of 600 nm. In units of microns, how far was the scatterplate translated from the null fringe position to obtain these 7 tilt fringes?

b) The interferometer is adjusted to give 7 bright circular defocus fringes across the interferogram with a bright fringe at both the center of the interferogram and at the edge of the interferogram for a wavelength of 600 nm. In units of microns, how far was the scatterplate translated from the null fringe position to obtain these 7 circular defocus fringes?

c) In part a) above, what is the fringe order number for the fringe passing through the hot spot if the wavelength is changed to 700 nm?

Solution

a)

For Young's two pinholes interference

$$\Delta \frac{x}{L} = \lambda$$

where Δ is the distance between the two pinholes, x is the fringe spacing, and L is the distance from the two pinholes to the fringes. We need to find Δ.

$$\Delta = \frac{\lambda L}{x} = \frac{.6\ \mu m\ (75\ mm)}{1.5\ mm} = 30\ \mu m$$

The scatterplate needs to be translated half of this or 15 μm

b)

Let Δ be the distance the scatterplate is moved, r be the radius of curvature of the mirror, and ρ be the radius. The OPD at the edge of the wavefront is 6 λ.

$$\text{opd} = \text{Together}\left[\frac{\rho^2}{2\ (r - \Delta)} - \frac{\rho^2}{2\ (r + \Delta)}\right]$$

$$\frac{\Delta \rho^2}{(r - \Delta)\ (r + \Delta)}$$

Since Δ is small

$$6\ \lambda = \frac{\Delta \rho^2}{r^2}$$

$$\Delta = \frac{6\,\lambda\,r^2}{\rho^2} = \frac{6\,(.6\,\mu m)\,(75\,mm)^2}{(4.5\,mm)^2} = 1000\,\mu m = 1\,mm$$

c)

Zero.

CS-12

A Smartt PDI is used to test a lens. How will the interferogram change if

a) The PDI plate is translated laterally?

b) The PDI plate is translated longitudinally?

Solution

a)

Tilt fringes will be introduced. The fringe contrast will also change because the amount of light incident upon the pinhole will change.

b)

Defocus fringes will change. There will be some change in the fringe contrast because the amount of light incident upon the pinhole will change. The fringe contrast change will not be as much as when the PDI plate is shifted laterally.

CS-13

I am using a Smartt point diffraction interferometer to test a lens that is essentially aberration free.

a) If I place the pinhole in the Smartt interferometer at the center of the 4th bright ring in the Airy disk, how many fringes do I see across the image of the lens I am testing? Be sure to list any assumptions you make.

b) If the diameter of the pinhole of part a) is equal to the Airy diameter/2.44, what should the density of the mask around the pinhole be to give maximum fringe contrast? I want only an approximate answer. You do not need to go through complicated numerical integration or diffraction calculations.

c) If the density calculated in part b) is off by 1 (i.e. transmittance off by an order of magnitude), how much will the fringe visibility suffer?

Solution

a)

From Table I of Chapter 1, "Basic Wavefront Aberration Theory for Optical Metrology" in Volume XI of Applied Optics and Optical Engineering we find that the distance from the center of the Airy disk to the 4th bright fringe is 4.72 λ f#. If D is the lens diameter, R is the distance from the lens to the image, and m is the order number at the edge of the pupil, then

$$4.72 \frac{\lambda R}{D} \left(\frac{D/2}{R}\right) = m \lambda$$

Thus m is given by

$$\text{Solve}\left[4.72 \frac{\lambda R}{D} \left(\frac{D/2}{R}\right) == m \lambda, m\right]$$

$$\{\{m \to 2.36\}\}$$

The number of fringes is 2(2)+1=5.

I have assumed the Smartt interferometer does not introduce a constant phase between two interfering wavefronts. Since the order number at the edge of the pupil is 2.36, a constant phase term could move one of the fringes out of the pupil without bringing another fringe in, so the total could be 4 instead of 5 fringes.

b)

From the same reference given above, we find that the radius of the 4th dark ring is 4.24 $\lambda f^{\#}$, while the radius of the 5th dark ring is 5.24 $\lambda f^{\#}$. The reference also says that approximately 1% of the total light is contained within the 4th dark ring. The diameter of the pinhole is $\lambda f^{\#}$. To within the accuracy needed, the fraction of the total light that passes through the pinhole is

$$(\text{\% of energy within 4 th bright fringe}) \left(\frac{\text{area of pinhole}}{\text{area of 4 th bright ring}}\right) =$$

$$0.01 \left(\pi \left(\lambda f^{\#}\right)^2 / 4\right) / \left(\pi \left(\left(5.24 \lambda f^{\#}\right)^2 - \left(4.24 \lambda f^{\#}\right)^2\right)\right) = 2.6 \times 10^{-4}$$

The area of the central portion of the diffraction pattern for the pinhole will be approximately $(2.44)^2$ times the area of the lens. Thus, only about 10^{-4} of the total energy will be in the reference beam over the image of the lens. Therefore, the density of the mask should be approximately 4.

c)

Let I_T = intensity of the test beam

Let I_R = intensity of the reference beam

For perfect coherence

$$\text{FringeVisibility} = \frac{2\sqrt{I_T T_R}}{I_T + T_R} = V$$

$$V = \frac{2\sqrt{10}}{10+1} = 0.57, \text{ which is not bad.}$$

A Smartt PDI can be modified to simultaneously give three interferograms with relative phase differences between adjacent interferograms of 90°. One way to accomplish this is to add a sinusoidal amplitude transmission grating in the plane containing the pinhole and partially transmitting filter. Once these three interferograms are detected, the phase of the wavefront can be calculated using the standard three-step equations given in the class notes.

a) What is the total amplitude transmittance of the interferometer in terms of the pinhole translation, and grating spacing?

b) To create interferograms in the ±1 diffraction orders which have phase shifts of ±90° relative to the zero order, where should the pinhole be placed with respect to the grating period?

Solution

a)

Let $t_g[x, y]$ be the transmission function due to the grating.

$$t[x, y] = t_g + (1 - t_g)\ \mathrm{cyl}\left[\frac{\sqrt{(x - x_o)^2 + (y - y_o)^2}}{d}\right]$$

$$t_g[x, y] = \frac{1}{2}\left(1 + \beta\,\mathrm{Cos}\left[2\pi\nu\left(x - x_g\right)\right]\right)$$

d = diameter of pinhole

$\frac{\beta^2}{4}$ = diffraction efficiency of the grating

ν = spatial frequency

x_g = relative position of pinhole with respect to peak of one grating period

a)

The pinhole should be placed 1/4 of the grating period from the peak of the grating to yield ± 90° phase shifts in the ± 1 diffraction orders.

CS-15

Question

A Smartt point diffraction interferometer is used to test an f/7.0 lens at a wavelength of 500 nm.

a) Sketch the setup.

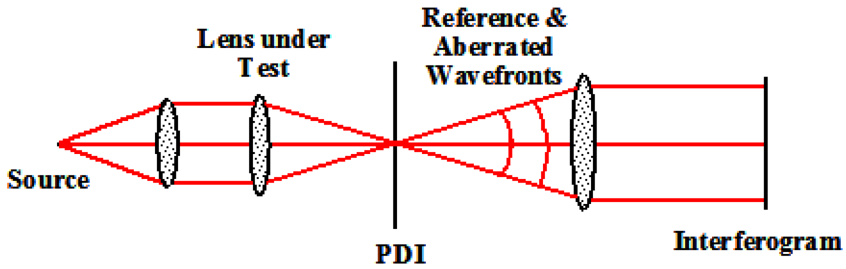
b) What is the approximate maximum-size pinhole that would be acceptable?

c) What is the effect of having too large a pinhole?

d) Sketch the interferogram obtained testing a lens having 4 fringes of third-order spherical aberration and 6 fringes of tilt across the diameter. The lens is tested at paraxial focus.

Solution

a)



b)

AiryDiskDiameter = 2.44 λ f # = 2.44 (0.5 μm) 7 = 8.54 μm

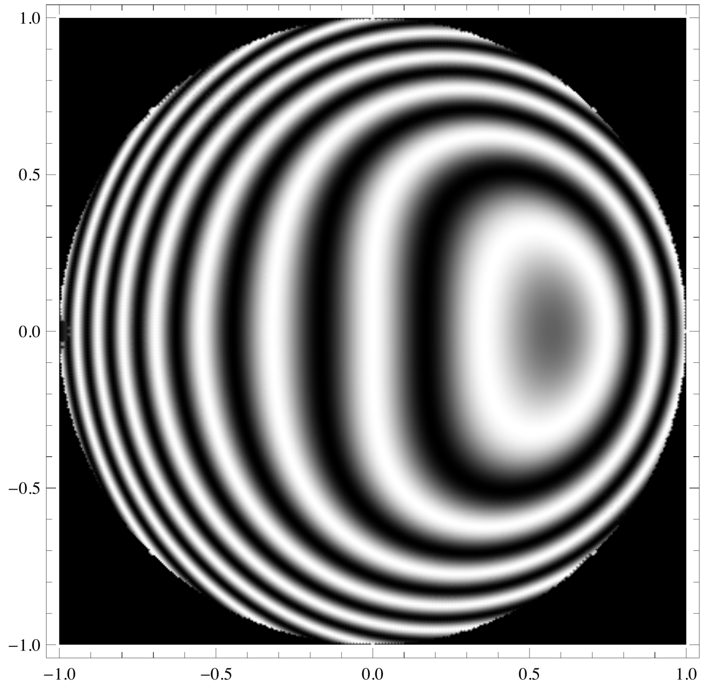
Approximately one-half the Airy disk diameter, or about 4.27 microns.

c)

The reference beam will have aberrations in it. The reference wavefront is given by the convolution of the Fourier transform of the pinhole with the test beam.

d)

```
DensityPlot[If[x^2 + y^2 ≤ 1, 0.5` (1 + Cos[2 π (4 (x^2 + y^2)^2 - 3 x)]), 0], {x, -1, 1},
 {y, -1, 1}, PlotPoints → 100, Mesh → False, ColorFunction -> GrayLevel]
```

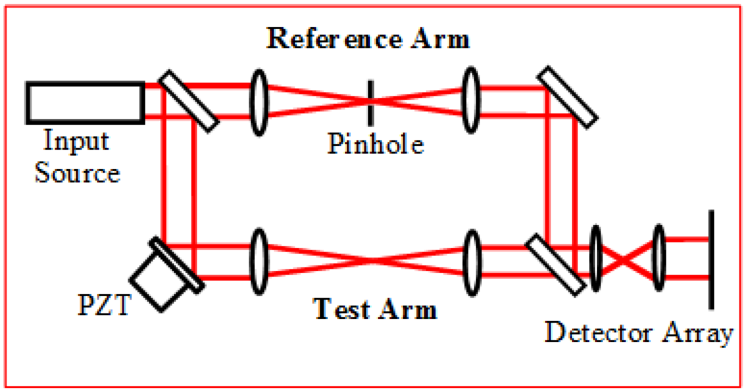


CS-16

Design a self-referencing interferometer to test the wavefront quality of a laser beam. The wavelength is approximately 500 nm, the average power is 10 watts, the beam is 2 cm in diameter, the beam is pulsed at a rate of 1 MHz and the wavefront should be measured to an accuracy of approximately 1/100 wave rms. (Warning - there are at least 7 key points I am looking for.)

Solution

There are a lot of possible answers.
One possibility is a Mach-Zehnder interferometer with a spatial filter in one arm.



To get 1/100 wave rms accuracy probably need to use phase shifting.
Need to average over enough pulses so small error is introduced.
Have to worry about power level and integration time so we use all possible bits.
Need to worry about vibration.
Need to worry about quality of components in interferometer.
If we use a spatial filter to clean up the reference beam the size of spatial filter should not be larger than one-half of the Airy disk diameter.
If we match paths we do not have to worry about coherence length problems. May need to put same focusing lenses in test arm so dispersion is same for both arms.
There may be polarization problems.

If we were to use a lateral shear interferometer we must have two interferograms with shear in different directions.

CS-17

I am using the star test to evaluate an optical system. How does the minimum blur diameter due to third-order spherical aberration compare to

a) the blur diameter due to astigmatism at the circle of least confusion?

b) the width of blur due to third-order coma which is twice the sagittal coma?

Assume equal amounts of third-order spherical, coma, and astigmatism as the maximum wavefront aberration.

Solution

Spherical aberration

$$\Delta W = \Delta W_s \, \rho^4$$

$$\epsilon_y = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h} 4 \, \Delta W_s = -8\left(\frac{R}{2\,h}\right)\Delta W_s$$

Therefore,

$$2\,\epsilon_y = 16\, f^{\#}\, \Delta W_{sph}$$

The minimum blur diameter is $\frac{1}{4}$ of this or

$$\text{MinimumBlurDiameter} = 4\, f^{\#}\, \Delta W_{sph}$$

a)

Astigmatism

$$\Delta W = \Delta W_a \, y^2$$

$$\epsilon_y = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h} 2\, y\, \Delta W_a = -4\left(\frac{R}{2\,h}\right)\Delta W_a$$

$$2\,\epsilon_y = 8\, f^{\#}\, \Delta W_{ast}$$

Blur diameter at circle of least confusion is one-half of this or

$$\text{BlurDiameterAtCircleOfLeastConfusion} = 4\, f^{\#}\, \Delta W_{ast}$$

b)

Coma

$$\Delta W = \Delta W_c \, \rho^3 \, \text{Cos}[\phi] = \Delta W_c \left(x^2 + y^2\right) y$$

$$\epsilon_y = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h}\Delta W_c \left(x^2 + 3\, y^2\right) = -\frac{R}{h}\Delta W_c \left(\rho^2 + 2\, y^2\right)$$

At $y^2 = 1$

$$\epsilon_y = -2 \frac{R}{2\,h} \triangle W_c\ (3) = -6\, f^{\#} \triangle W_{coma}$$

The sagittal coma is 1/3 of this and the diameter of the blur circle is 2/3 of this or

$$\text{DiameterOfBlurCircle} = 4\, f^{\#} \triangle W_{coma}$$

From Volume 11 of Applied Optics and Optical Engineering page 23 we have

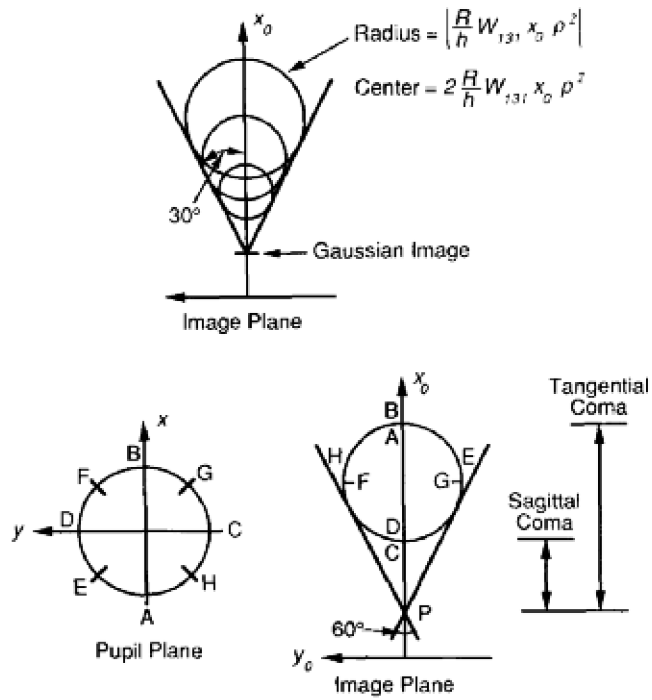


FIG. 26. Transverse ray aberration for coma.

Therefore, the minimum spot diameter for third-order spherical, the width of the coma image (2/3 the tangential coma), and the diameter of the blur for astigmatism that falls halfway between the sagittal and tangential focus are all given by

$$d = 4\, f^{\#} \triangle W$$

CS-18

An 8 1/2 inch diameter, 48 inch focal length parabolic mirror tested at "center of curvature" using a Hartmann test. The Hartmann screen placed at the mirror surface has holes spaced 1 inch apart on a square grid. The coordinates of the Hartmann spots at paraxial focus are given below.

x coordinate of hole in Hartmann screen	x coordinate of Hartmann spot
-4 inch	-9.944 x 10-3 inch
-3	-4.930 x 10-3
-2	-1.268 x 10-3
-1	-2.085 x 10-4

0	0
1	1.085 x 10-4
2	9.681 x 10-4
3	3.930 x 10-3
4	1.194 x 10-2

Give the surface departure of the mirror from the desired parabolic mirror along the line tested. Is it practical to perform the Hartmann test at paraxial focus? Explain.

Solution

For a parabola

$$\triangle z = \frac{-s^4}{64\, f^3}$$

$$\text{slope} = \alpha = \frac{-s^3}{16\, f^3}$$

Let δ be the perfect displacement of a spot at paraxial focus $\delta = 2\,(-2\,f)\,\alpha$.

$$\delta = \frac{s^3}{4\, f^2};$$

The focal length is given by

f = 48;

The data was taken at the following points across the mirror.

s = {-4, -3, -2, -1, 0, 1, 2, 3, 4};

The measured error in the position of the spots is

$$\delta\text{measured} = \{-9.944,\ -4.930,\ -1.268,\ -0.2085,\ 0,\ 0.1085,\ 0.9681,\ 3.930,\ 11.94\} \times 10^{-3};$$

The calculated "ideal error" in the position of the spots is

$$\delta\text{calculated} = s^3\, \frac{1}{4\, f^2};$$

We will now calculate the surface slope error, αError

$$\alpha\text{Error} = \frac{\delta\text{measured} - \delta\text{calculated}}{-4\, f};$$

We will now calculate the surface height error. We will integrate the slope from the center out.

$$\Delta\text{zPlusS} = \text{Table}\left[\frac{1}{2}\sum_{i=6}^{n} (\alpha\text{Error}[[i-1]] + \alpha\text{Error}[[i]])\ (s[[i]] - s[[i-1]]),\ \{n, 6, 9\}\right];$$

$$\Delta\text{zMinusS} = \text{Reverse}\left[\text{Table}\left[\frac{1}{2}\sum_{i=6}^{n} (\alpha\text{Error}[[11-i]] + \alpha\text{Error}[[10-i]])\ (s[[10-i]] - s[[11-i]]),\ \{n, 6, 9\}\right]\right];$$

```
Δz = Join[ΔzMinusS, {0}, ΔzPlusS];

TableForm[Partition[{s, αError, Δz}, 3],
 TableHeadings -> {None, {"s", "α Error", "Height Error"}}]
```

s	α Error	Height Error
-4	0.0000156227	-0.0000208335
-3	0.0000104183	-7.81299×10^{-6}
-2	2.08304×10^{-6}	-1.56232×10^{-6}
-1	5.20797×10^{-7}	-2.60399×10^{-7}
0	0	0
1	3.6169×10^{-11}	1.80845×10^{-11}
2	-5.21065×10^{-7}	-2.60496×10^{-7}
3	-5.20996×10^{-6}	-3.12601×10^{-6}
4	-0.0000260185	-0.0000187402

We would have trouble performing the test at paraxial focus because due to diffraction, the spots would not separate.

CS-19

State at least 3 important criteria for determining the spacing and size of the holes used in a classical Hartmann test.

Solution

1) Aberration variation over hole < $\lambda/4$.
2) Geometrical image and diffraction image for hole approximately the same size.
3) Diffraction images for different holes should not overlap.
4) Holes close enough together to sufficiently sample the aberration.

CS-20

An 8 inch diameter. 24-inch focal length parabolic mirror is tested using a Foucault test. In the test the light source moves with the knife-edge. The mirror is found to be rotationally symmetric, however a surface error probably exists. The table gives the measured zone focus, relative to the paraxial center of curvature, as a function of zone radius.

a) How significant is the last decimal place in the data?

b) Determine the departure, in wavelengths (λ = 0.6328 μm), of the mirror surface relative to the desired parabolic surface.

ρ(in)	ϵ_z
0.5	0.001
1.0	0.008
1.5	0.015
2.0	0.030
2.5	0.045
3.0	0.087

3.5	0.130
4.0	0.182

Solution

Since both the light source and the knife edge move together we are finding where the normal to the surface cuts the axis.

For a parabola the perfect position of zonal focus relative to paraxial focus is given by

$$\delta z = \frac{s^2}{4\,f};$$

The shift in the retro reflection position due to a surface error, Δz, is given by

$$\Delta\epsilon_z = -\frac{R^2}{s}\frac{\partial\Delta z}{\partial s} = -\frac{(2\,f)^2}{s}\frac{\partial\Delta z}{\partial s};$$

Then the slope error is given by

$$\frac{\partial\Delta z}{\partial s} = -\frac{s}{(2\,f)^2}\Delta\epsilon_z;$$

a)

Can the shadow position be measured to sufficient accuracy to make the last decimal place in the data sufficient?

$$\partial(\delta z) = \frac{2\,s}{4\,f}\partial s;$$

At mid zone $s = 2$ in.

$$\partial(\delta z) = \frac{4}{4\,(24)}\partial s = 0.04\,\partial s;$$

∂s can perhaps be measured to 0.1 inch, so $\partial(\delta z)$ ~ 0.004 inch. Hence, two decimal places in the data would be more reasonable than 3.

b)

The data was taken at the following points across the mirror.

```
s = {0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4};
```

The zonal focus positions are

```
ϵz = {0, 0.001, 0.008, 0.015, 0.030, 0.045, 0.087, 0.130, 0.182};
```

The ideal positions would be given by

$$\delta z = \frac{s^2}{4\,f};\ f = 24;$$

The slope error is given by

$$\alpha\text{Error} = -\frac{s}{(2\,f)^2}\,(\epsilon_z - \delta z);$$

$$\Delta z = \text{Table}\left[\text{If}\left[n == 1,\, 0,\, \frac{1}{2}\sum_{i=2}^{n}(\alpha\text{Error}[[i-1]] + \alpha\text{Error}[[i]])\,(s[[i]] - s[[i-1]])\right],\, \{n, 1, \text{Length}[s]\}\right];$$

$$\text{TableForm}\left[\text{Partition}\left[\left\{s, \Delta z, 0.01\,\text{Round}\left[\Delta z\,\frac{25.40\times 10^3}{0.6328}\,100\right]\right\}, 3\right],\ \text{TableHeadings} \to \{\text{None}, \{\text{"s"}, \text{"}\Delta\text{w (inches)"}, \text{"}\Delta\text{w (waves)"}\}\}\right]$$

s	Δw (inches)	Δw (waves)
0	0	0
0.5	8.70316×10^{-8}	0
1	4.36288×10^{-7}	0.02
1.5	2.0718×10^{-6}	0.08
2	5.97692×10^{-6}	0.24
2.5	0.0000139624	0.56
3	0.0000216132	0.87
3.5	0.0000229006	0.92
4	0.0000153356	0.62

CS-21

You are given the problem of analyzing the output of an AC radial Ronchi ruling interferometer. The spatial frequency of the grating is given by ν = (100 lines/mm)(25 mm/r), where r is the distance from the center of the grating. The grating is rotating at a rate of 1 revolution per second. We will consider only the interference between the 0 order and the +1 order and the 0 and the -1 order.

a) Why can we limit our discussion to only interference between 0 and +1 and 0 and -1 orders?

b) What is the modulation frequency of the signal? How does the frequency depend upon spectral wavelength, and r, the distance between the center of the grating and the position of the focused spot?

c) What is the angular shear as a function of r?

d) Show that the output signal can be written as

$$i_s = C_1 + C_2\left(\text{Cos}\left[\Delta\,\phi^{i}[x,y] + \frac{\Delta^3}{6}\,\phi^{iii}[x,y] + \frac{\Delta^5}{120}\,\phi^{v}[x,y] - \omega\,t\right]\right)\left(\text{Cos}\left[\frac{\Delta^2}{2}\,\phi^{ii}[x,y] + \frac{\Delta^4}{24}\,\phi^{iv}[x,y] + \cdots\right]\right)$$

where C_1 and C_2 are constants, Δ is the shear, ω is the modulation frequency, and $\phi(x,y)$ is the phase distribution we are measuring.

Solution

a)

Since for a Ronchi grating the even orders are missing, the only two interference patterns which oscillate at the fundamental frequency ω will be the 0, +1 and the 0, -1 orders.

b)

$$\text{modulation\_frequency} = \omega = 2\pi \left(100\,\frac{\text{lines}}{\text{mm}}\right)\left(2\pi\left(25\,\frac{\text{mm}}{\text{sec}}\right)\right) = 2\pi\,(15\,708)\ \text{Hz}$$

Note that this is independent of λ and r.

c)

The angular shear between the 0 and either the +1 or -1 orders is given by

$$\text{angular\_shear} = \lambda\,\nu = \lambda\left(\frac{100}{\text{mm}}\right)\left(\frac{25\ \text{mm}}{r}\right)$$

d)

Over the region of overlap of 0, +1, and -1 orders the amplitudes of the signal can be written as

$$A_s = a\,e^{i\,\phi[x,y]} + b\left(e^{i\,\phi[x-\Delta,y]}\,e^{i\,\omega\,t} + e^{i\,\phi[x+\Delta,y]}\,e^{-i\,\omega\,t}\right)$$

The resulting intensity containing terms of frequency ω or lower is given by

$$i_s = a' + b'\,(\text{Cos}[\phi[x,y] - \phi[x-\Delta,y] - \omega\,t] + \text{Cos}[\phi[x,y] - \phi[x+\Delta,y] + \omega\,t])$$

By the binomial expansion

$$\phi[x \pm \Delta, y] =$$

$$\phi[x,y] \pm \Delta\,\phi^{i}[x,y] + \frac{\Delta^2}{2}\,\phi^{ii}[x,y] \pm \frac{\Delta^3}{6}\,\phi^{iii}[x,y] + \frac{\Delta^4}{24}\,\phi^{iv}[x,y] \pm \frac{\Delta^5}{120}\,\phi^{v}[x,y] + \cdots$$

Therefore,

$$i_s = a' + b'\left(\text{Cos}\left[\Delta\,\phi^{i}[x,y] - \frac{\Delta^2}{2}\,\phi^{ii}[x,y] + \frac{\Delta^3}{6}\,\phi^{iii}[x,y] + \cdots - \omega\,t\right] + \right.$$
$$\left.\text{Cos}\left[\Delta\,\phi^{i}[x,y] + \frac{\Delta^2}{2}\,\phi^{ii}[x,y] + \frac{\Delta^3}{6}\,\phi^{iii}[x,y] + \cdots - \omega\,t\right]\right)$$

$$i_s = C_1 + C_2\left(\text{Cos}\left[\Delta\,\phi^{i}[x,y] + \frac{\Delta^3}{6}\,\phi^{iii}[x,y] + \frac{\Delta^5}{120}\,\phi^{v}[x,y] - \omega\,t\right]\right)$$
$$\left(\text{Cos}\left[\frac{\Delta^2}{2}\,\phi^{ii}[x,y] + \frac{\Delta^4}{24}\,\phi^{iv}[x,y] + \cdots\right]\right)$$

CS-22

The grating used in a Ronchi test can be either a density (amplitude) grating that has a 50% duty cycle (equal width transparent and opaque lines) or a phase grating where the entire grating is 100% transmitting, but every other line has a phase retardation of ϕ with respect to the adjacent lines. The 0 phase and the ϕ phase lines have equal width. Assume the spatial frequency of the grating is sufficient that at most only two orders overlap.

a) Show that for the phase grating the interference of the 0, +1 orders give a pattern that is complimentary to that of the 0, -1 orders. That is, a dark fringe in one pattern corresponds to a bright fringe in the other pattern. (See for example, Figure 9.19 of Malacara.) Is the situation the same for the density grating?

b) Comment upon conservation of energy for the use of phase and density (amplitude) gratings. That is, if I get a single black fringe in the interference pattern for a density or a phase grating, where has the energy gone?

Solution

a)

For a phase grating the amplitude transmission, T_A can be written as

$$T_A = J_0[a] + J_1[a]\, e^{i x \nu}\, e^{i k x \mathrm{Sin}[\theta]} + J_{-1}[a]\, e^{-i x \nu}\, e^{-i k x \mathrm{Sin}[\theta]}$$

a = amplitude of sinusoidal phase variation

$J_{-1}[a] = -J_1[a]$

θ = diffraction angle

$$k = \frac{2\pi}{\lambda}$$

$$\nu = \frac{2\pi}{d},$$ where d = period of grating

x = displacement of grating from reference position, or equivalently the position at which a given ray passes through the grating relative to the reference ray

The irradiance for the interference of the 0 and the +1 order is given by

$$I = |J_0[a]|^2 + |J_1[a]|^2 + 2\, J_0[a]\, J_1[a]\, \mathrm{Cos}[x\nu + k x \mathrm{Sin}[\theta]]$$

The irradiance for the interference of the 0 and the -1 order is given by

$$I = |J_0[a]|^2 + |J_1[a]|^2 - 2\, J_0[a]\, J_1[a]\, \mathrm{Cos}[x\nu + k x \mathrm{Sin}[\theta]]$$

Therefore the two patterns are complementary for a phase grating.

For a density grating

$$T_A = a + b e^{i x \nu}\, e^{i k x \mathrm{Sin}[\theta]} + b e^{-i x \nu}\, e^{-i k x \mathrm{Sin}[\theta]}$$

The irradiance for the interference of the 0 and the +1 order is given by

$$I = a^2 + b^2 + 2\, a\, b\, \mathrm{Cos}[x\nu + k x \mathrm{Sin}[\theta]]$$

The irradiance for the interference of the 0 and the -1 order is given by

$$I = a^2 + b^2 + 2\, a\, b\, \mathrm{Cos}[x\nu + k x \mathrm{Sin}[\theta]]$$

Therefore the two patterns are not complementary for a phase grating.

b)

Energy conservation

Phase grating

With a perfect phase grating the grating absorbs no light and the light is distributed amongst the different orders (i.e. complementary patterns).

Density grating
The grating absorbs energy, so amount of energy in fringe patterns plus amount absorbed by the grating is equal to the incident energy.

CS-23

How are the number of fringes in a lateral shear interferogram related to the number of fringes in a Twyman-Green interferogram? Let S = shear distance / pupil semi-diameter.

a) By looking at the aberrations $A(x^2 + y^2)^2$ and $B(x^2 + y^2)$, and having the shear in the x direction, determine a rule valid for small shears which gives the ratio of the number of fringes in a lateral shear interferogram as a function of S and the maximum power of x in the aberration formula.

b) For the two aberrations in part a), give the results for a large amount of shear as well as a small amount of shear.

Solution

a)

With small shear we can take the derivative.

Defocus

$\delta w = 2\, B\, x\, \delta x$

Ratio of the number of fringes (y = 0)

$$\frac{LSI}{TG} = \frac{2\, B\, x\, \delta x}{B\, x^2} = 2\, \frac{\delta x}{x} = 2\, S$$

Spherical

$\delta w = 4\, A\, \left(x^2 + y^2\right)\, x\, \delta x$

Ratio of the number of fringes (y = 0)

$$\frac{LSI}{TG} = \frac{4\, A\, \left(x^2\right)\, x\, \delta x}{A\, \left(x^2\right)^2} = 4\, \frac{\delta x}{x} = 4\, S$$

Therefore, for small shears

$$\#\, \text{fringes}\, \frac{LSI}{TG} = (\text{max power of } x)\, S$$

b)

For large shears we must take the finite difference.

Defocus

$$\Delta w = B\left(x + \frac{\Delta x}{2}\right)^2 - B\left(x - \frac{\Delta x}{2}\right)^2 = 2\,B\,x\,\Delta x$$

Ratio of the number of fringes (y = 0)

$$\frac{LSI}{TG} = \frac{2\,B\,x\,\Delta x}{B\,x^2} = 2\,\frac{\Delta x}{x} = 2\,S$$

Spherical

$$\Delta w = A\left(\left(x + \frac{\Delta x}{2}\right)^2 + y^2\right)^2 - A\left(\left(x - \frac{\Delta x}{2}\right)^2 + y^2\right)^2 = 4\,A\,x\left(x^2 + y^2\right)\Delta x + A\,x\,(\Delta x)^3$$

Ratio of the number of fringes (y = 0)

$$\frac{LSI}{TG} = \frac{4\,A\,x\left(x^2\right)\Delta x + A\,x\,(\Delta x)^3}{A\left(x^2\right)^2} = 4\,\frac{\Delta x}{x} + \frac{(\Delta x)^3}{x^3} = 4\,S + S^3$$

CS-24

A wedged plate lateral shear interferometer is used to check for collimation. The plate has an index of 1.5 and a wedge of 5 arc-seconds. The plate is oriented so the direction of the wedge is perpendicular to the direction of shear. The shear is set at 6 mm. A 50 mm diameter beam is being checked. Let the wavelength be 0.6328 microns.

a) How many bright fringes are there across the beam for perfect collimation?

b) If the beam is defocused such that at the edge of the pupil the OPD due to defocus is W, give the equation describing the shape and orientation of the bright fringes as a function of W.

c) If we claim we can measure fringe orientation to an accuracy of 1°, how accurately can we measure defocus, i.e., how small of a defocus, W, can we detect?

Solution

a)

For a bright fringe 2 n α y = m λ, where α is the wedge angle.
Assuming a bright fringe at one edge, Δm, the number of bright fringes is given by

$$\Delta m = \text{IntegerPart}\left[\left(2\ (1.5)\ (5\ \text{sec})\ \left(50\times 10^{3}\ \mu m\right)\right) \Big/ \left(0.6328\ \mu m\right)\ \frac{5\times 10^{-6}}{1\ \text{sec}}\right] + 1 = 6$$

If we do not have a bright fringe at one edge we may have only 5 fringes.

b)

OPD due to wedge = 2 n α y.

$$\text{OPDDueToDefocusAndShear} = \frac{w}{(25\ \text{mm})^2}\left(\left(x + \frac{\Delta}{2}\right)^2 - \left(x - \frac{\Delta}{2}\right)^2\right) = \frac{2\,w\,x\,\Delta}{625\ \text{mm}^2}$$

Therefore, if we neglect a constant phase between the two interfering beams we will obtain a bright fringe when

$$2\,n\,\alpha\,y + \frac{2\,w\,x\,\Delta}{625\ \text{mm}^2} = m\,\lambda$$

c)

$$\text{slopeOfFringes} = \frac{dy}{dx} = -\frac{w\,\Delta}{625\ \text{mm}^2\,n\,\alpha} = \frac{1}{60}$$

$$w = \frac{n\,\alpha\,625\ \text{mm}^2}{60\,\Delta} = \frac{1.5\ (5\ \text{sec})\ 625\ \text{mm}^2}{60\ (6\ \text{mm})}\ \frac{5\times10^{-6}}{1\ \text{sec}}\ \frac{\lambda}{.632\times10^{-3}\ \text{mm}} = 0.1\,\lambda$$

CS-25

A lateral shear interferometer is used to perform a single pass test of a lens having 4 waves of spherical aberration. The shear is equal to 10% of the lens diameter. The resulting interferogram looks like a Twyman-Green interferogram having A fringes of coma and B fringes of tilt. What are A and B?

Solution

$$\text{wdf} = 4\,\lambda\left(\left(\left(x+\frac{\Delta}{2}\right)^2+y^2\right)^2-\left(\left(x-\frac{\Delta}{2}\right)^2+y^2\right)^2\right);$$

```
Expand[Simplify[wdf /. Δ → 0.2]]
```

$$0.032\,x\,\lambda + 3.2\,x^3\,\lambda + 3.2\,x\,y^2\,\lambda$$

A = 3.2 λ; B = 0.032 λ

CS-26

What is the primary quantity the following instruments measure? (One word for each answer is sufficient.)

a) Shack cube interferometer
b) Hartmann Test
c) Shack-Hartmann test
d) Lateral shear interferometer
e) Radial shear interferometer
f) Wire test
g) Foucault test
h) Scatterplate interferometer
i) Nomarski Interferometer
j) Spherometer

Solution

a) Wavefront

b) Wavefront slope

c) Wavefront slope

d) Wavefront slope

e) Wavefront slope in radial direction

f) Wavefront slope

g) Wavefront slope

h) Wavefront

i) Wavefront slope

j) Sag