8.2.11 Star Test

- The visual examination of the image of a point source is one of the most basic and important tests that can be performed.
  - Interpretation of the image is to a large degree a matter of experience.
  - Should be a dynamic process where the observer probes through focus and across the field.
  - Magnifying power should be such that the smallest significant detail subtends 10 to 15 minutes of arc at the eye.
  - The numerical aperture of the viewing optics must be large enough to collect the entire cone of light from the optics under test.

Airy Disk

- **For perfect optics the image of a point source as seen at best focus is called the Airy disk.**
  - The diameter of the central core is equal to 2.44λf#, where f# is the f/number of the converging light beam.
  - In the visible, the diameter of the central core is approximately equal to the f# in microns.
  - The central core contains approximately 84% of the total amount of light, while the total amount of light contained within the first, second, and third rings is approximately 91%, 94%, and 95%, respectively.
  - If the microscope is moved back and forth along the axis, the image will be seen to go in and out of focus. A perfect image will appear totally symmetrical on opposite sides of focus

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Spot Size – Spherical Aberration

$$\Delta W = \Delta W_s \rho^4$$

$$\varepsilon_y = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h} 4 \Delta W_s = -8\left(\frac{R}{2h}\right)\Delta W_s$$

Therefore,

$$2\varepsilon_y = 16 f^{\#} \Delta W_{sph}$$

The minimum blur diameter is ¼ of this or

$$\text{Minimum Blur Diameter } = 4 f^{\#} \Delta W_{sph}$$

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Spot Size – Astigmatism

$$\Delta W = \Delta W_a y^2$$

$$\varepsilon_y = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h} 2y\Delta W_a = -4\left(\frac{R}{2h}\right)\Delta W_a$$

Therefore,

$$2\varepsilon_y = 8 f^{\#} \Delta W_{ast}$$

Blur diameter at circle of least confusion is ½ of this or

$$\text{Minimum Blur Diameter } = 4 f^{\#} \Delta W_{ast}$$

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Spot Size – Coma

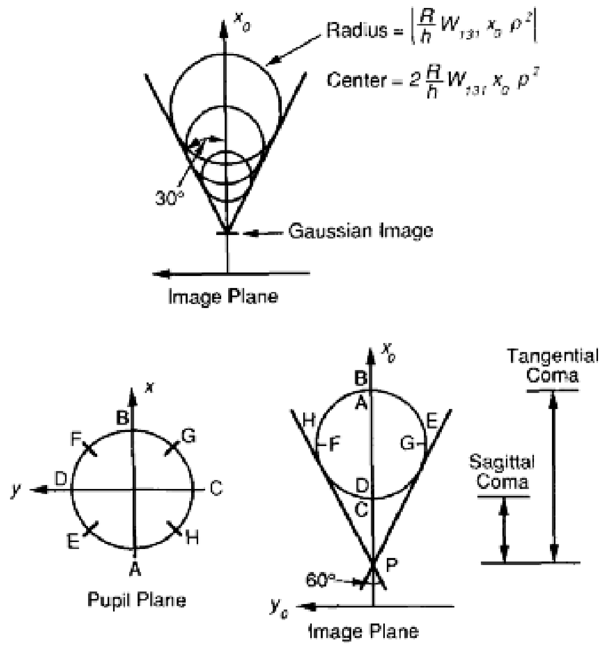

FIG. 26. Transverse ray aberration for coma.

$$\Delta W = \Delta W_c \rho^3 Cos[\phi] = \Delta W_c \left(x^2 + y^2\right) y$$

$$\varepsilon_y = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h}\Delta W_c\left(x^2 + 3y^2\right) = -\frac{R}{h}\Delta W_c\left(\rho^2 + 2y^2\right)$$

$$\text{At } y^2 = 1$$

$$\varepsilon_y = -2\frac{R}{2h}\Delta W_c(3) = -6 f^{\#} \Delta W_{coma}$$

The sagittal coma is 1/3 of this or

$$\text{Diameter of Blur Circle } = 4 f^{\#} \Delta W_{coma}$$

Therefore, the minimum spot diameter for third-order spherical, the width of the coma image (2/3 the tangential coma), and the diameter of the blur for astigmatism that falls halfway between the sagittal and tangential focus are all given by

$$d = 4 f^{\#} \Delta W$$

Ratio of Geometrical Blur to the Airy Disk Diameter

It is of interest to look at the ratio of geometrical blur to the Airy disk diameter.

$$\frac{Geometrical\ Blur\ Diameter}{Airy\ Disk\ Diameter} = \frac{4f\#\Delta W}{2.44\lambda f\#} = 1.64\left(\frac{\Delta W}{\lambda}\right)$$

That is, the ratio of the geometrical blur diameter to the Airy disk diameter is approximately equal to 1.64 times the amount of aberration in units of waves.

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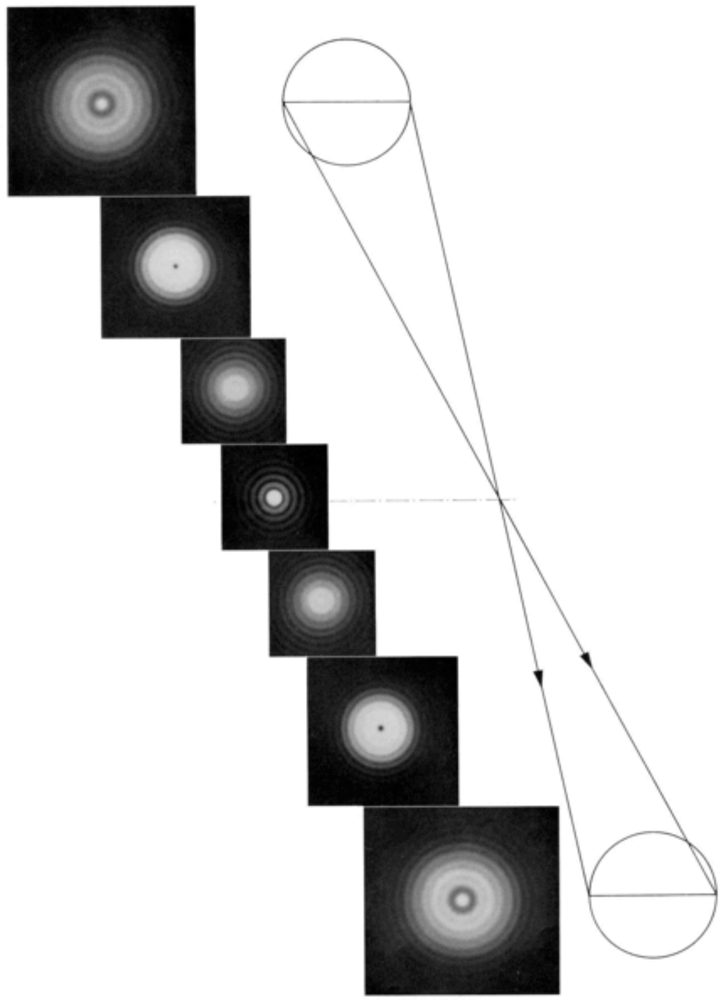
Diffraction by a circular aperture as a function of defocus for no aberration



Ref: “Atlas of Optical Phenomena” by Cagnet, Francon, and Thrierr.

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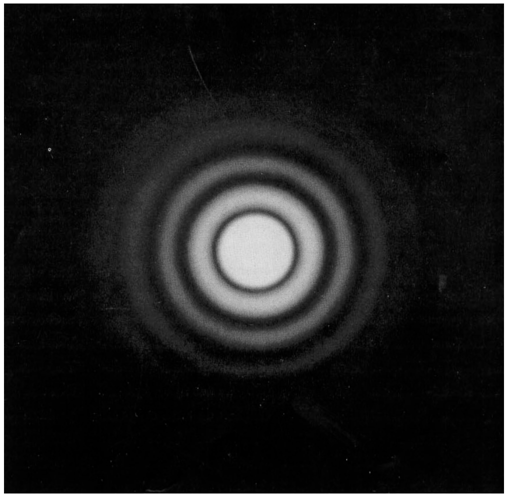
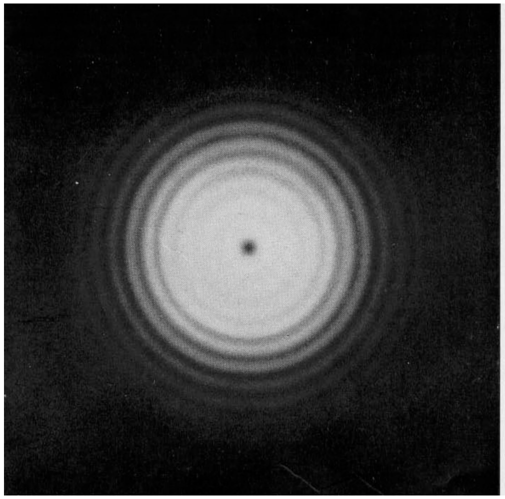
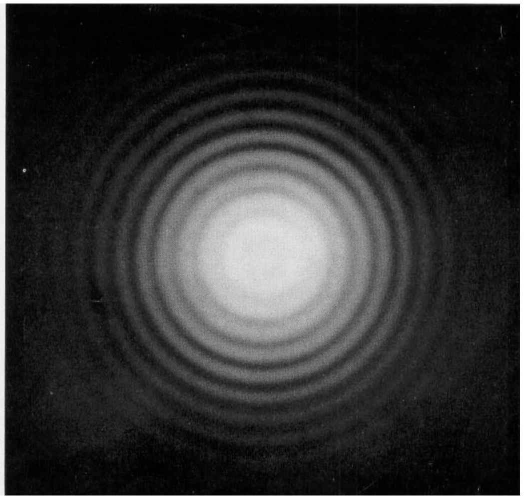
Diffraction by a circular aperture in the presence of defocus



Airy Disk

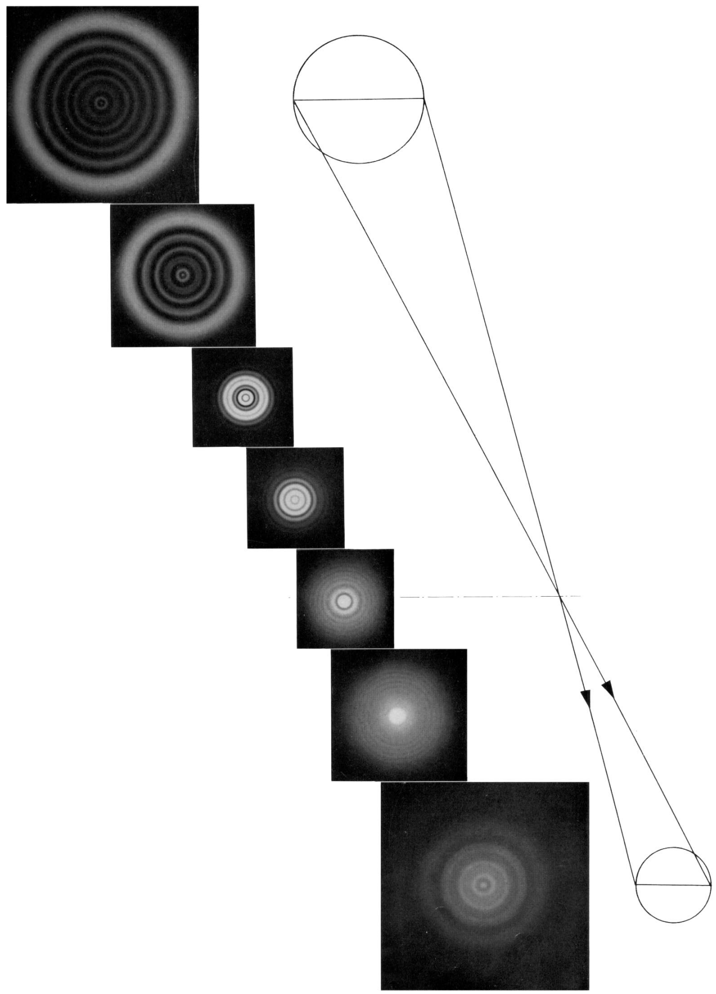
1 wave defocus





Less than 1 wave defocus

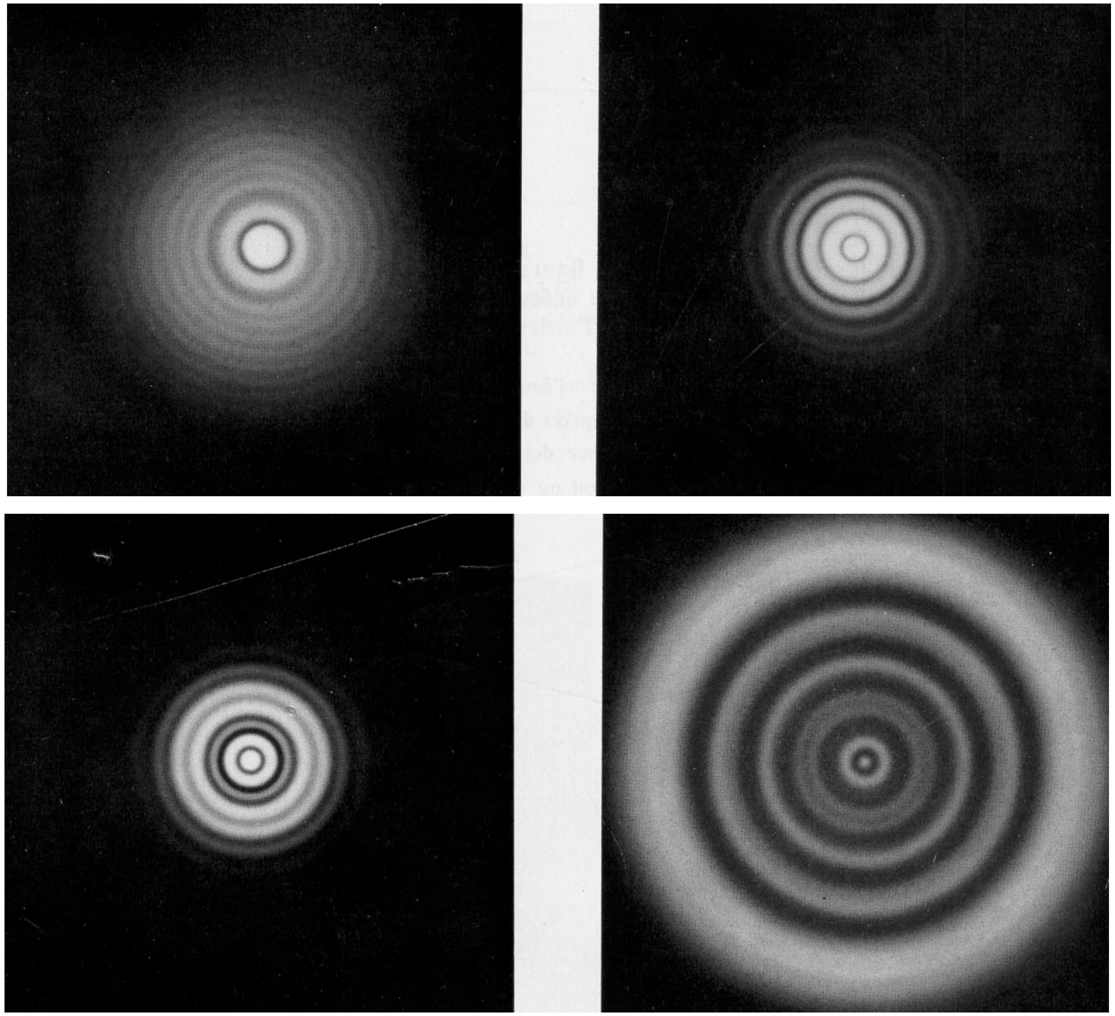
Diffraction by a circular aperture as a function of defocus for third-order spherical aberration



Diffraction by a circular aperture in the presence of third-order spherical aberration

Paraxial focus

Small distance inside paraxial focus



Moderate distance from marginal focus

Immediate neighborhood of marginal focus

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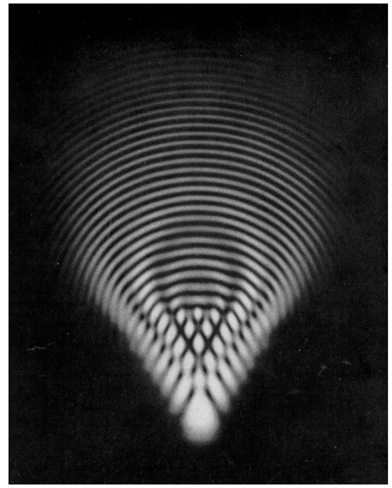
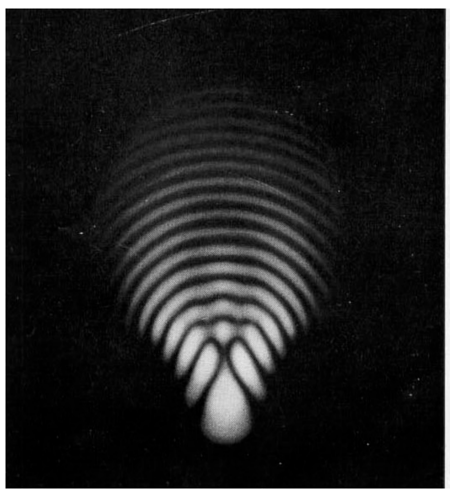
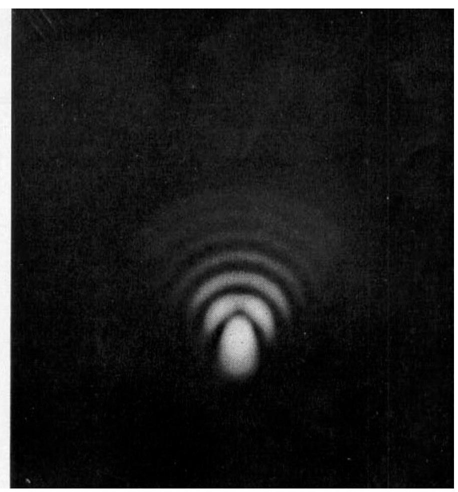
Diffraction by a circular aperture in the presence of third-order coma



6 λ

2.5 λ





1 λ

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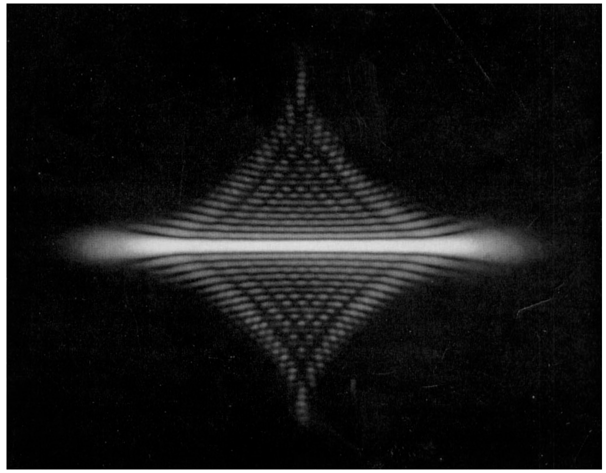
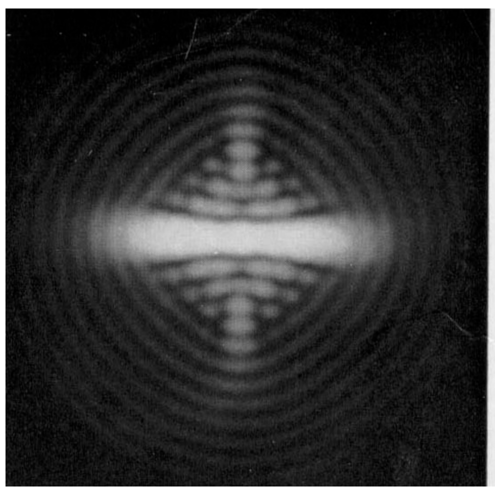
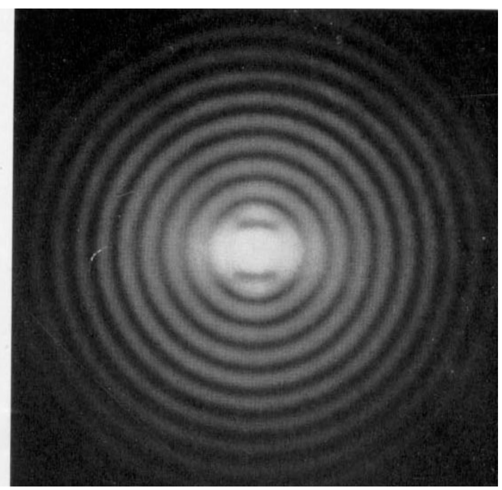
Diffraction by a circular aperture in the presence of astigmatism



7 λ

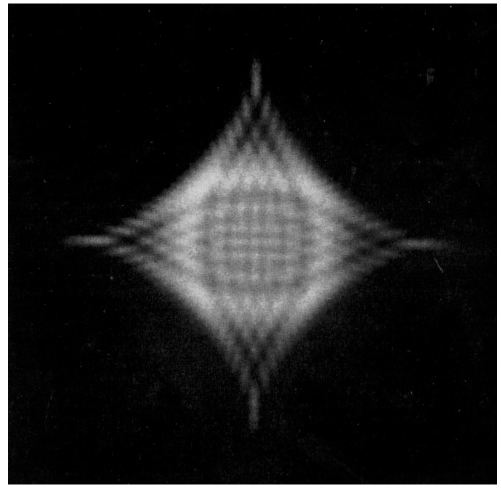
1.5 λ





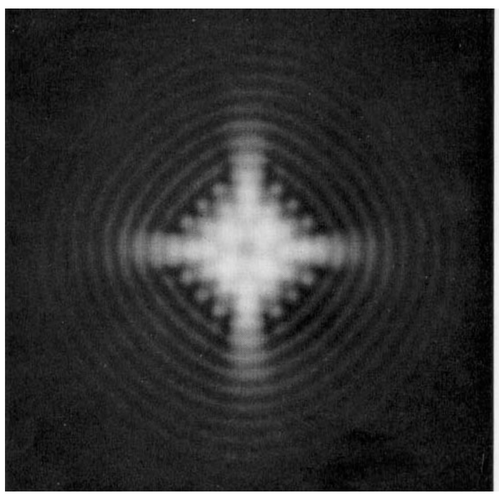
0.23 λ

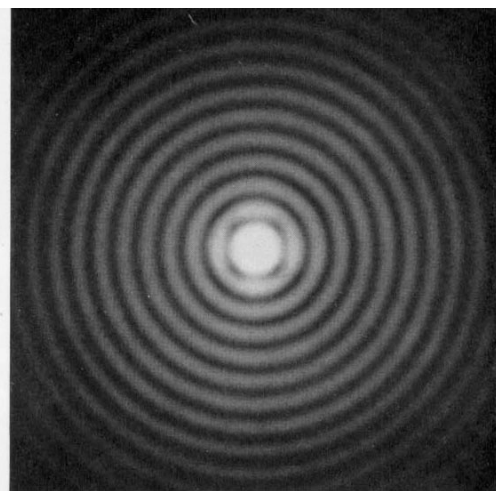
Diffraction by a circular aperture - astigmatism in the neighborhood of the circle of least confusion



7 λ

1.6 λ





0.23 λ

Star Test - Detecting Chromatic Aberration

- In a perfectly apochromatic system a symmetrical "white" image is obtained for all focal positions.
- If chromatic aberration is present the image color is a function of focal position. In moving away from the lens through the paraxial focal plane, a sequence of images is observed.
  - Well away from focus, a white flare is observed.
  - As the blue focus is reached, the color balance is seen to change as blue light appears to be removed from the flare and is concentrated in a core.
  - Farther away from the lens a similar color effect is observed as the foci for green and red are reached.
  - For overcorrected color, the colors appear in the opposite order.

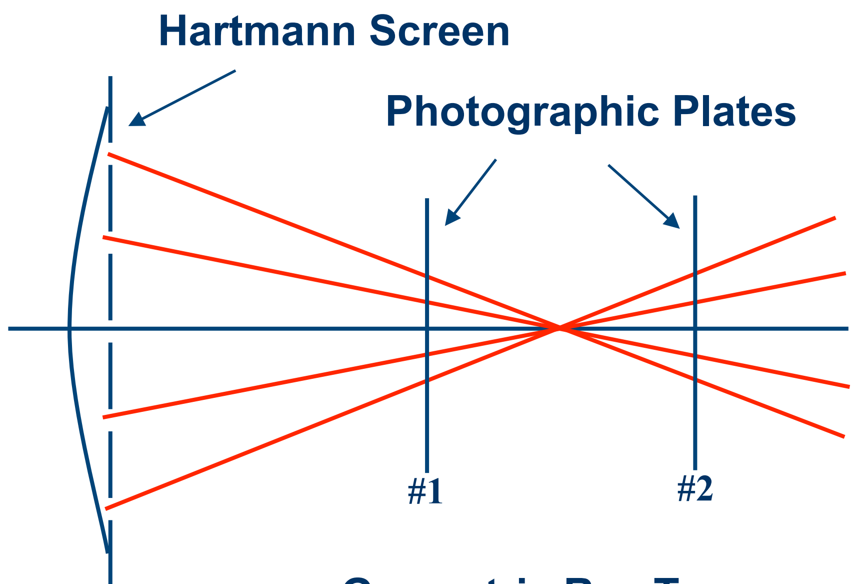
Lateral Chromatic Aberration

- The chromatic errors in an off-axis image are most spectacular in visual testing.
- The lateral separation of the images in red and blue light gives directly the amount of lateral chromatic aberration.
- If the red image is found to lie at a greater distance from the axis than the blue image, negative or undercorrected lateral color is present, while for overcorrected lateral color, the blue image is a greater distance from the axis than the red image.

8.2.12 Shack-Hartmann Test

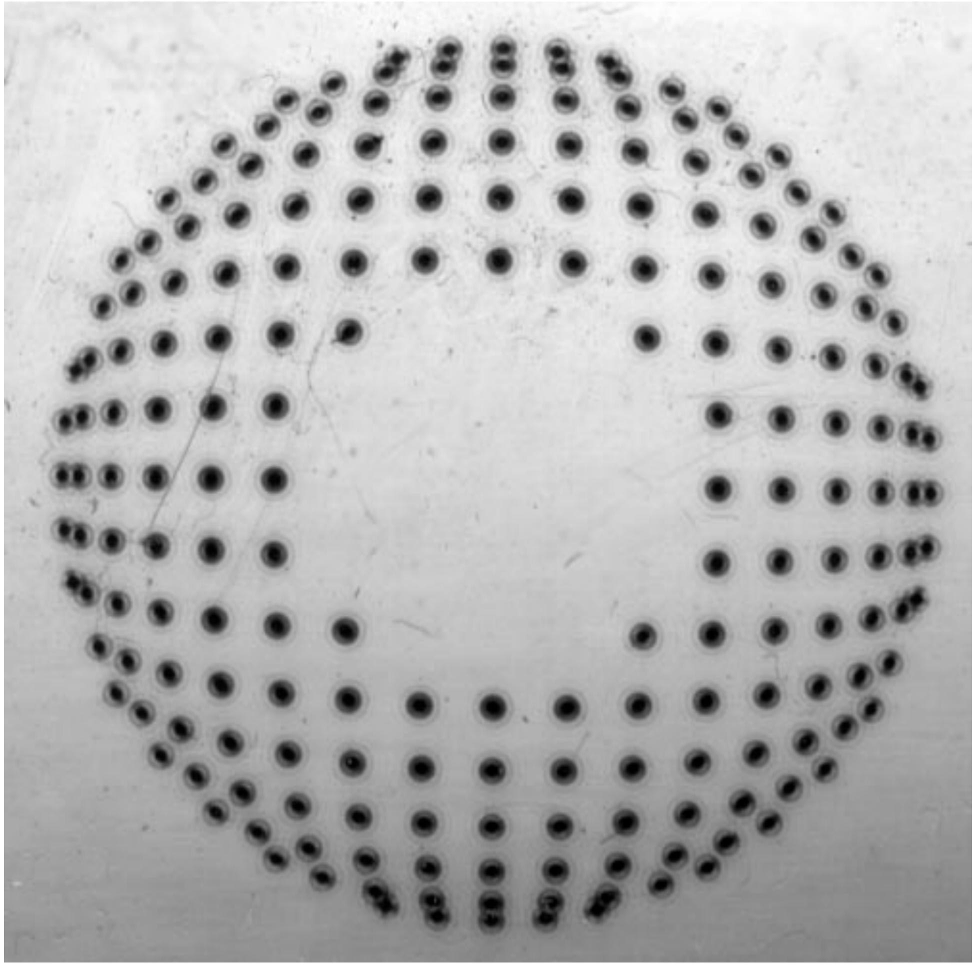
Geometrical ray trace that measures angular, transverse, or longitudinal aberrations from which numerical integration can be used to calculate the wavefront aberration.

Classical Hartmann Test



Geometric Ray Trace

Hartmann Test of Parabola Outside Position



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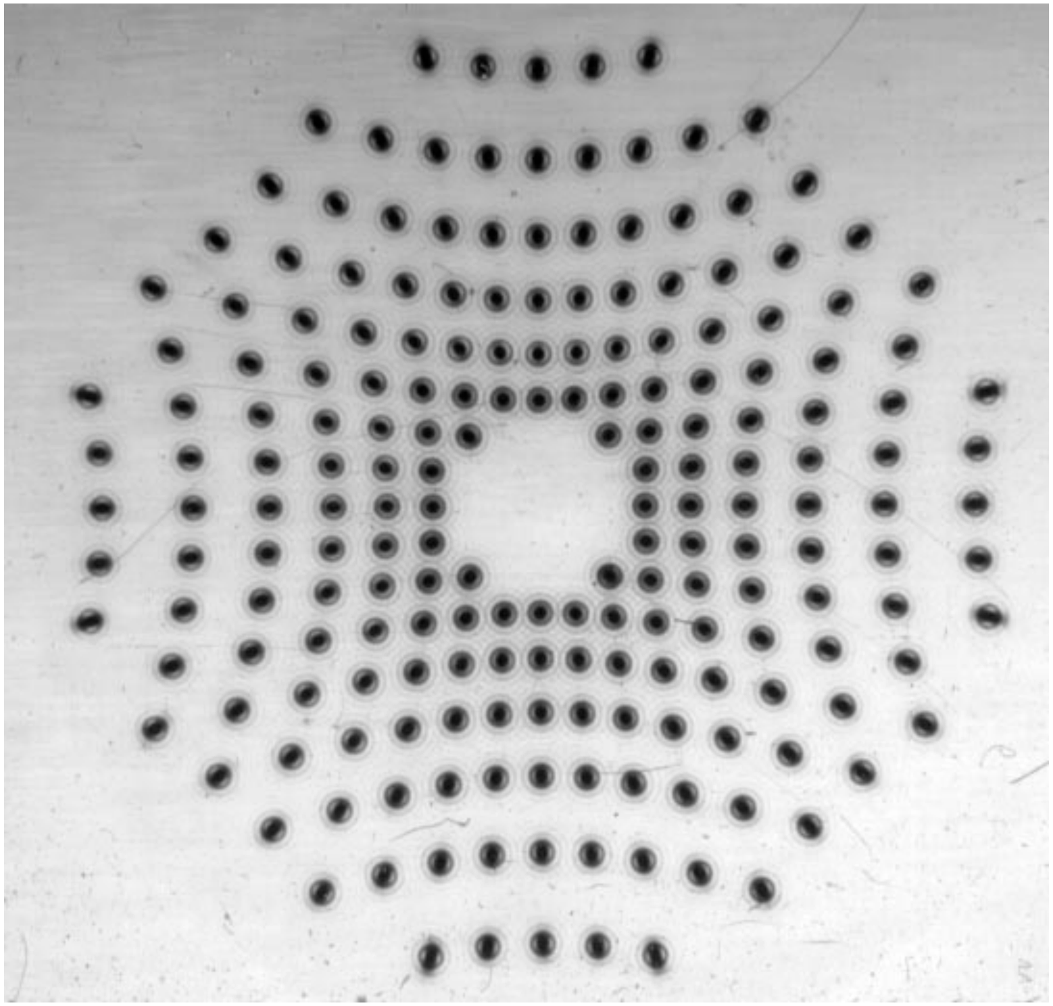
Hartmann Test of Parabola Inside Position



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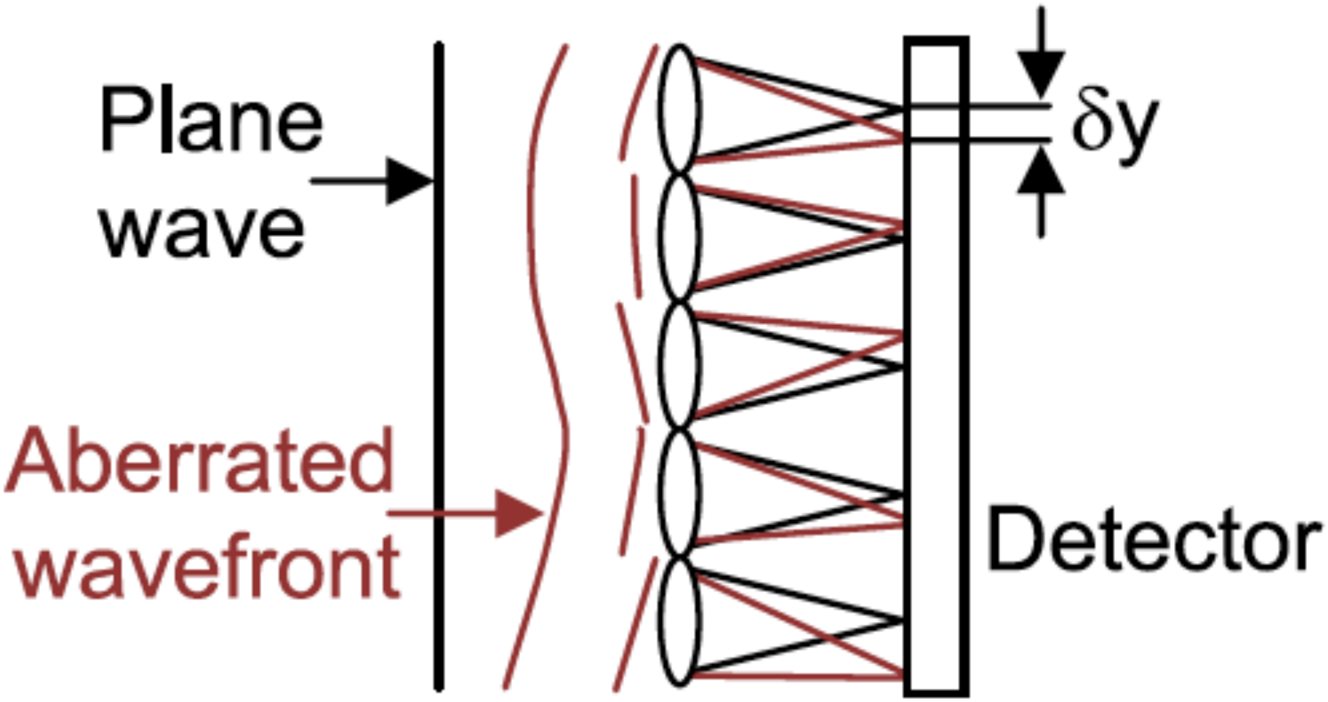
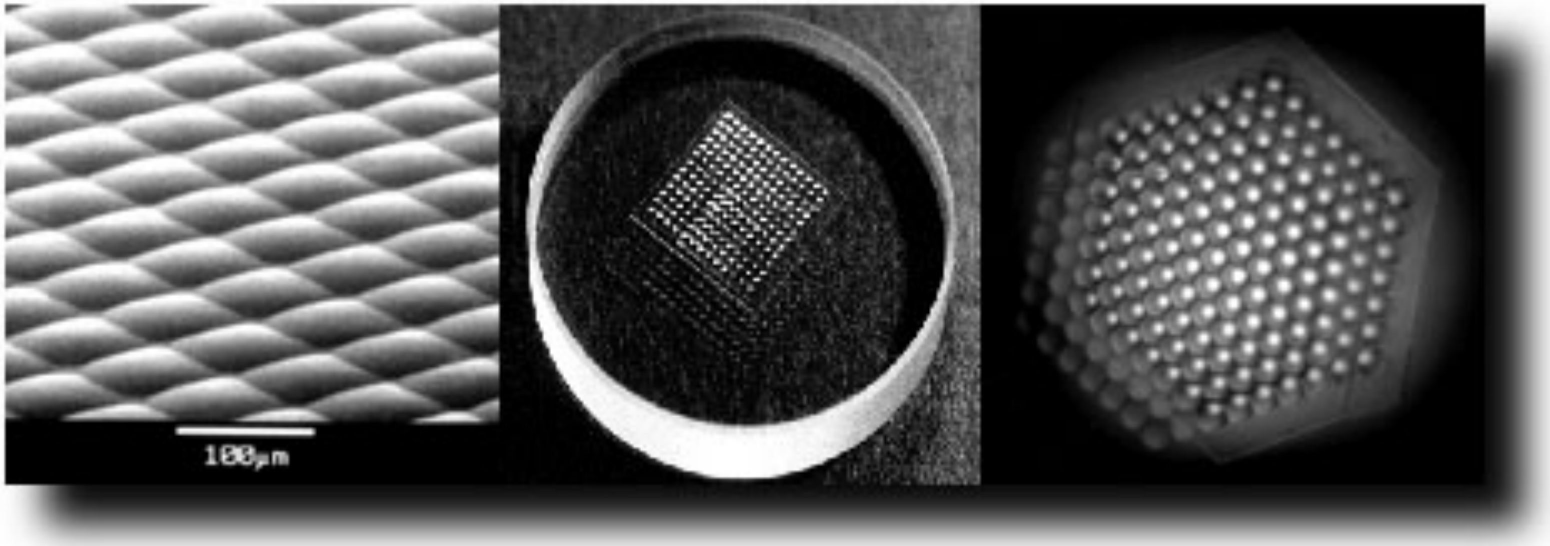
Shack-Hartmann Lenslet Array



Shack-Hartmann Lenslets



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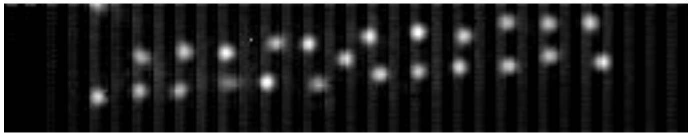
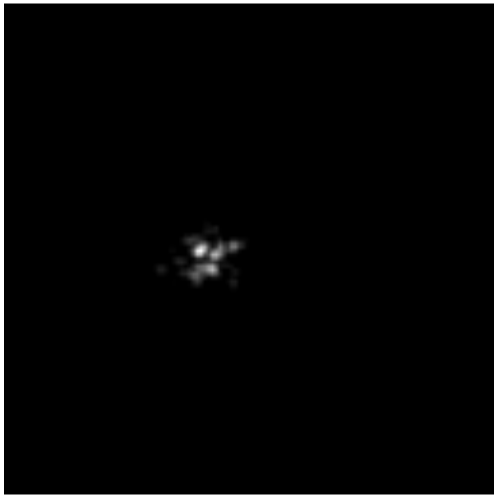
Shack-Hartmann Movie



Movie showing results obtained using Shack-Hartmann test to measure atmospheric turbulence.

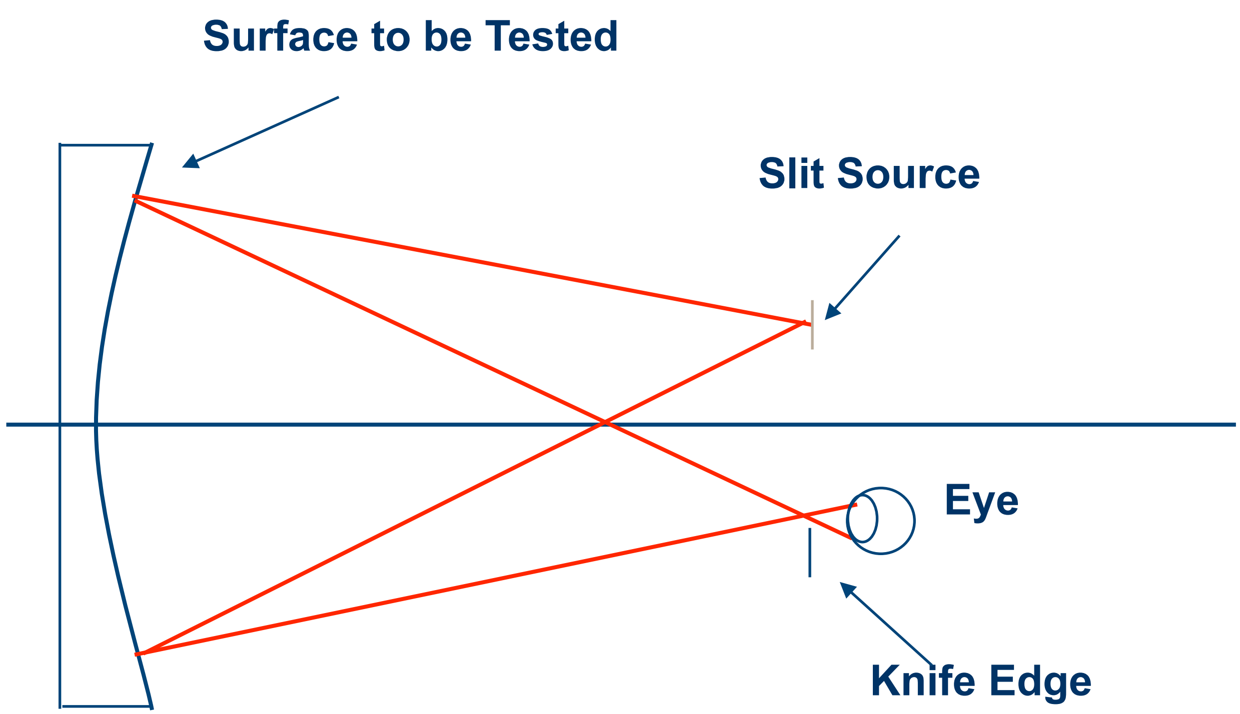


Movie showing stellar speckle image

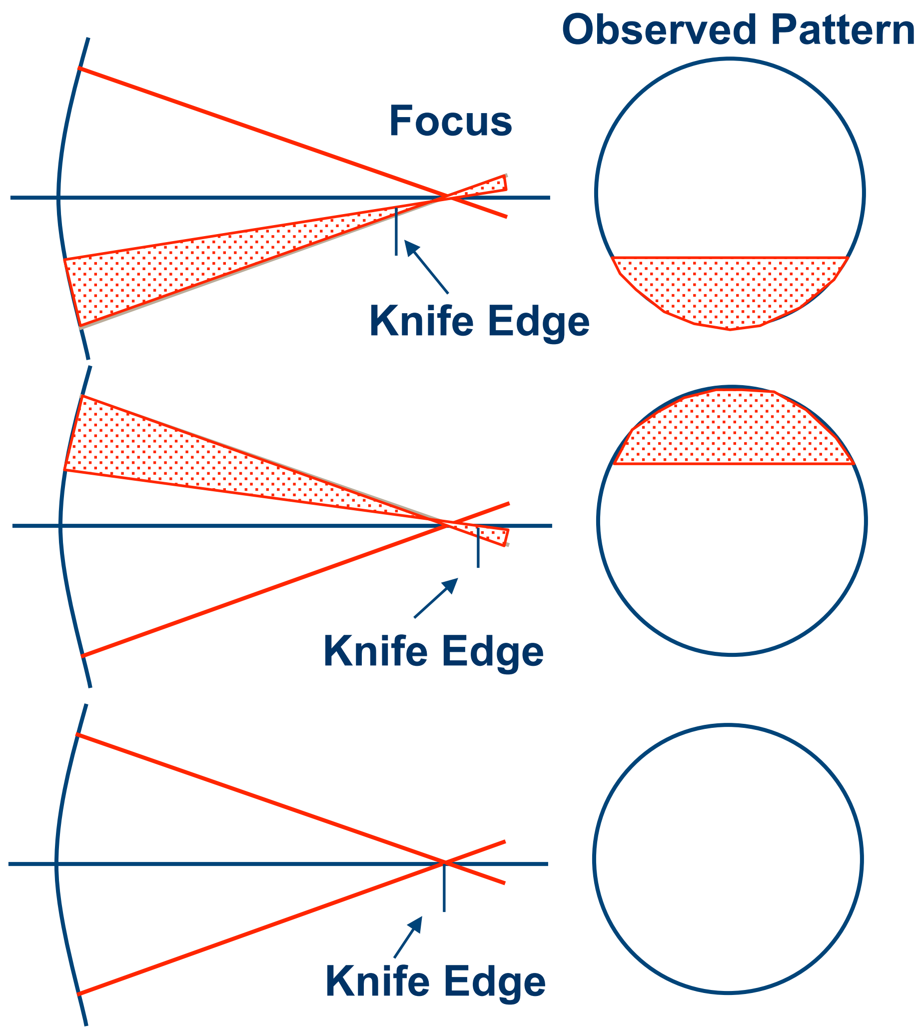
Shack Hartmann Test - Comments

- Air turbulence will average out as long as integration time is long compared to period of turbulence
- Holes in Hartmann screen large enough so diffraction does not limit measurement accuracy, but not so large surface errors are averaged out
- Test often used for adaptive optics

8.2.13 Foucault Knife-Edge Test



Ray Picture of Foucault Knife-Edge Test



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Shadows for Third-Order Spherical

$$\Delta W = W_{040}\left(x^2+y^2\right)^2 + \frac{\varepsilon_z h^2}{2R^2}\left(x^2+y^2\right)$$

Boundary of geometrical shadow is given by

$$d = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = \frac{-4RW_{040}y\left(x^2+y^2\right)}{h} - \frac{\varepsilon_z hy}{R}$$

If the knife edge is on the axis, d=0, and the solution is

$$y = 0 \qquad\qquad x^2+y^2 = -\frac{\varepsilon_z h^2}{4R^2 W_{040}}$$

One solution is straight line and second is circle of radius

$$\rho = \left(\frac{-\varepsilon_z h^2}{4R^2 W_{040}}\right)^{1/2}$$

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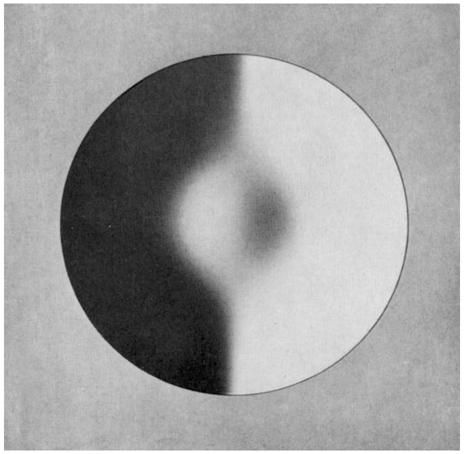
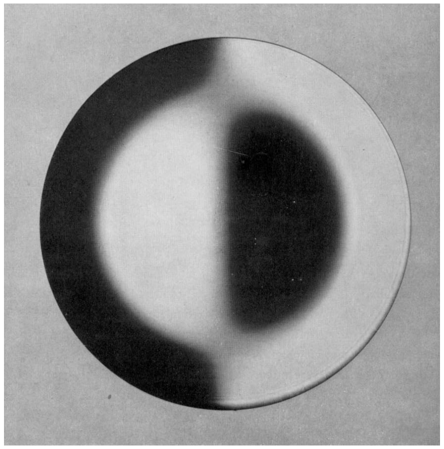
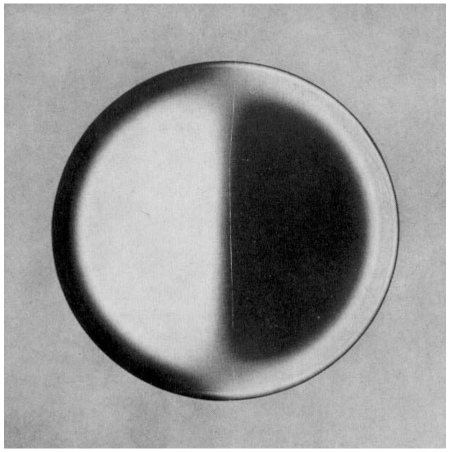
Third-Order Spherical
Knife-Edge on Optical Axis



Knife edge near paraxial focus



Knife edge partway between paraxial and marginal focus



Knife edge near marginal focus

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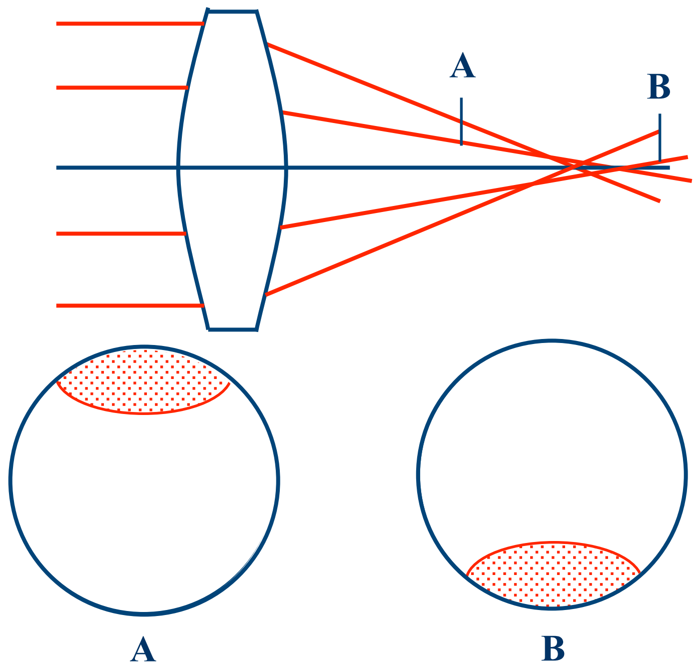
Knife-Edge Not on Optical Axis



Knife-edge ahead of marginal focus

Knife-edge behind of paraxial focus

Shadows for Third-Order Coma

$$\Delta W = W_{131} y_0 y \left(x^2 + y^2\right) + \frac{\varepsilon_z h^2}{2R^2}\left(x^2 + y^2\right)$$

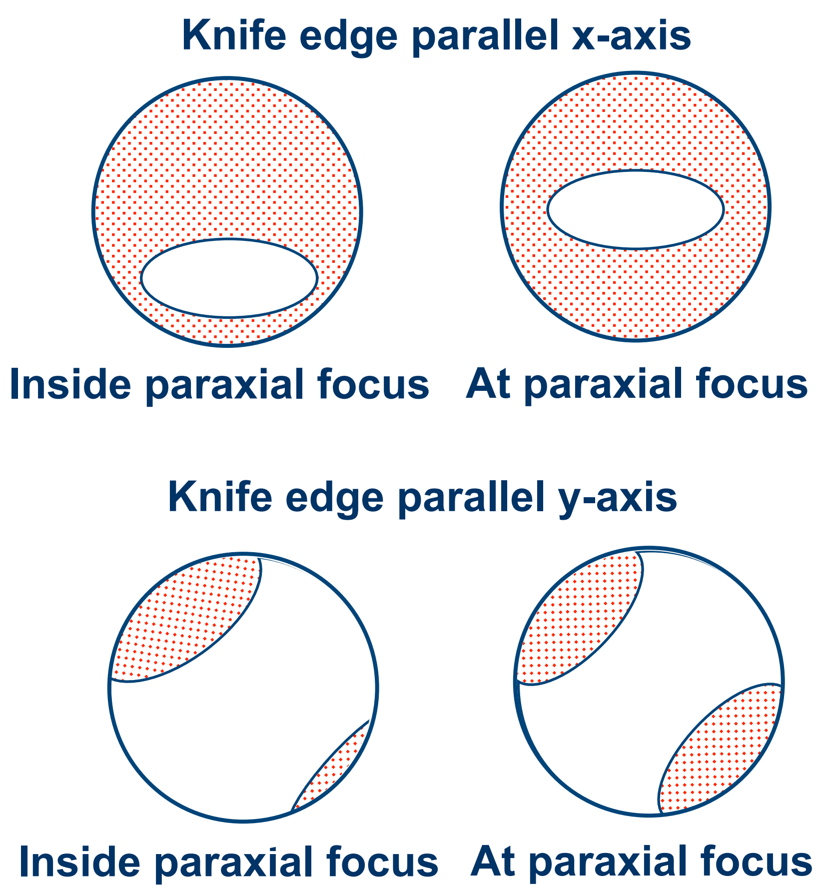
If the knife edge is parallel to the x-axis we get an ellipse

$$d = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\frac{R}{h} W_{131} y_0 \left(x^2 + 3y^2\right) - \frac{\varepsilon_z h y}{R}$$

If the knife edge is parallel to the y-axis we get a hyperbola

$$d = -\frac{R}{h}\frac{\partial \Delta W}{\partial x} = -\frac{2R}{h} W_{131} y_0 x y - \frac{\varepsilon_z h x}{R}$$

Knife-Edge Test Pattern Due to Coma



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Shadows for Third-Order Astigmatism

$$\Delta W = W_{222} y_0^2 y^2 + \frac{\varepsilon_z h^2}{2R^2}\left(x^2 + y^2\right)$$

If the KE is parallel to either the x-axis or the y-axis we get

$$d = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} = -\left(\frac{2R}{h} W_{222} y_0^2 + \frac{\varepsilon_z h}{R}\right) y \qquad \text{or} \qquad d = -\frac{R}{h}\frac{\partial \Delta W}{\partial x} = -\frac{\varepsilon_z h}{R} x$$

Which are straight lines, so the astigmatic wavefront would be indistinguishable from a spherical wavefront. Put KE at an angle α to the x-axis then

$$d = \varepsilon_y Cos[\alpha] - \varepsilon_x Sin[\alpha]$$

$$d = \frac{\varepsilon_z h}{R} x Sin[\alpha] - \left(\frac{2R}{h} W_{222} y_0^2 + \frac{\varepsilon_z h}{R}\right) y Cos[\alpha]$$

Angle of shadow changes as KE moved along axis

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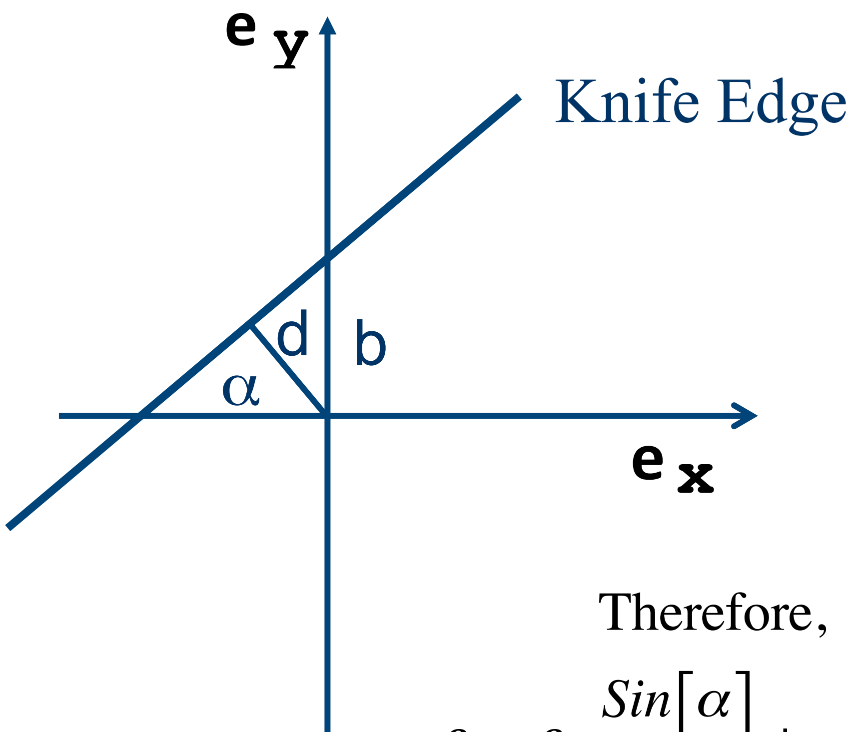
Knife Edge at Angle



Along the knife edge

$$\varepsilon_y = m\varepsilon_x + b$$

$$m = \frac{Sin[\alpha]}{Cos[\alpha]}$$

$$Cos[\alpha] = \frac{d}{b}$$

Therefore,

$$\varepsilon_y = \varepsilon_x \frac{Sin[\alpha]}{Cos[\alpha]} + \frac{d}{Cos[\alpha]}$$

and

$$d = \varepsilon_y Cos[\alpha] - \varepsilon_x Sin[\alpha]$$

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Foucault Knife Edge Test - Comments

- Advantage of test is simplicity
- Disadvantage is that it is sensitive to slopes, not wavefront, and measures slopes in a single direction with single orientation of KE.
- While it is possible to get numbers from the KE test, it is generally used as a qualitative test.
- An improvement would be a phase KE with transmits both sides with a phase difference between the two halves of 180°. The diffraction pattern is symmetric, and the boundary centers are easier to determine.

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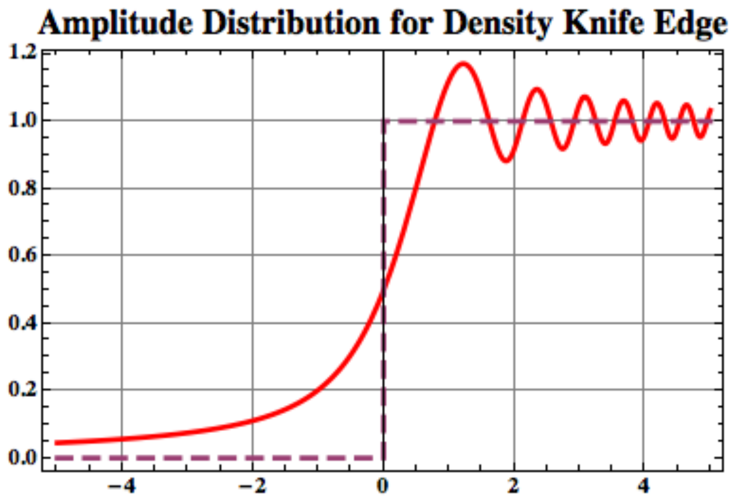
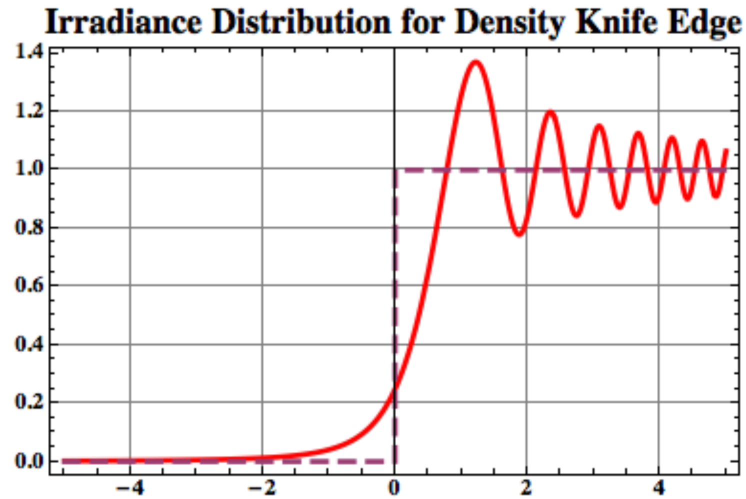
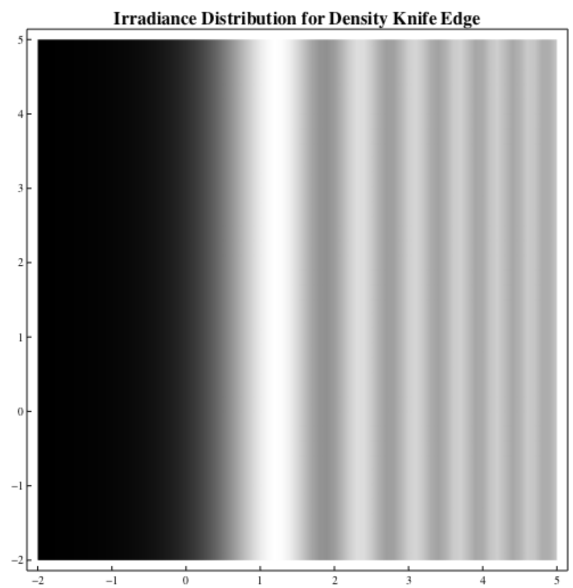
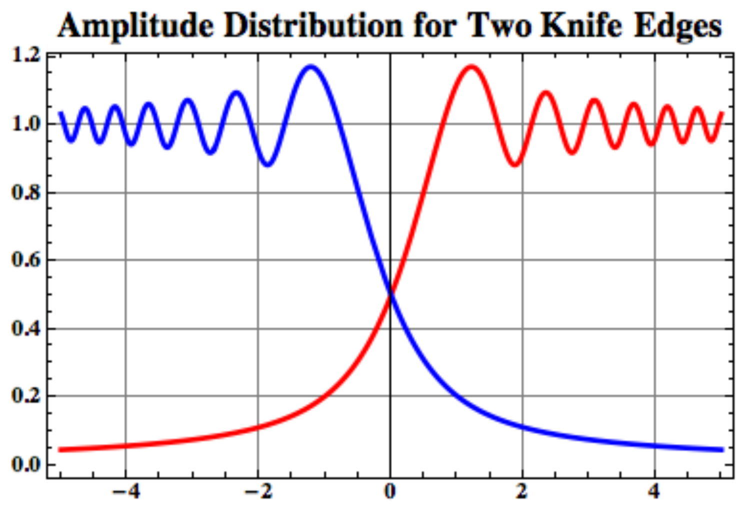
Density Knife Edge

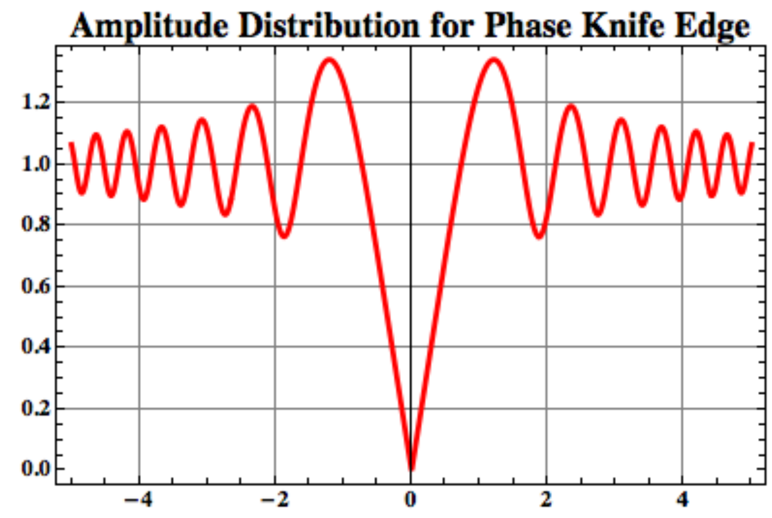


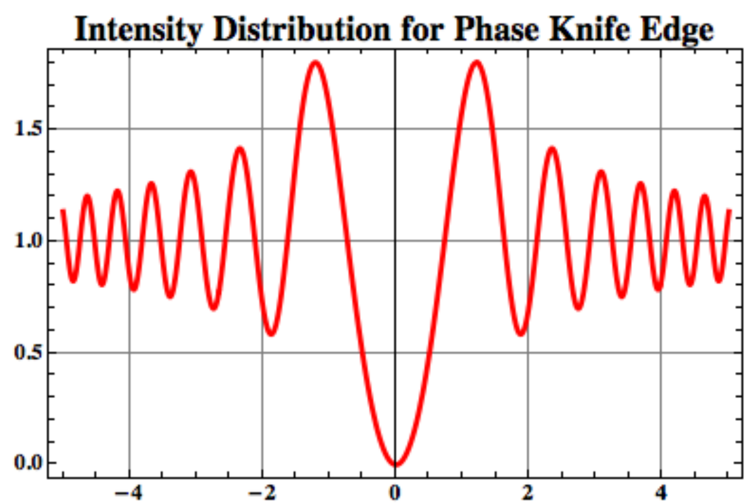


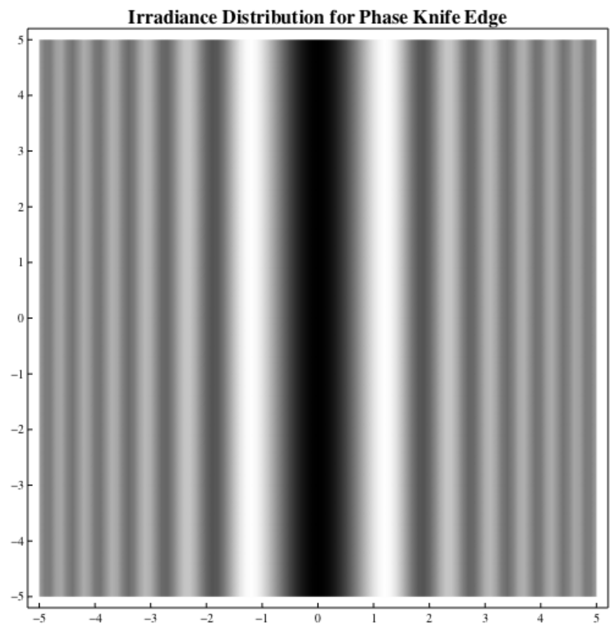


Phase Knife Edge









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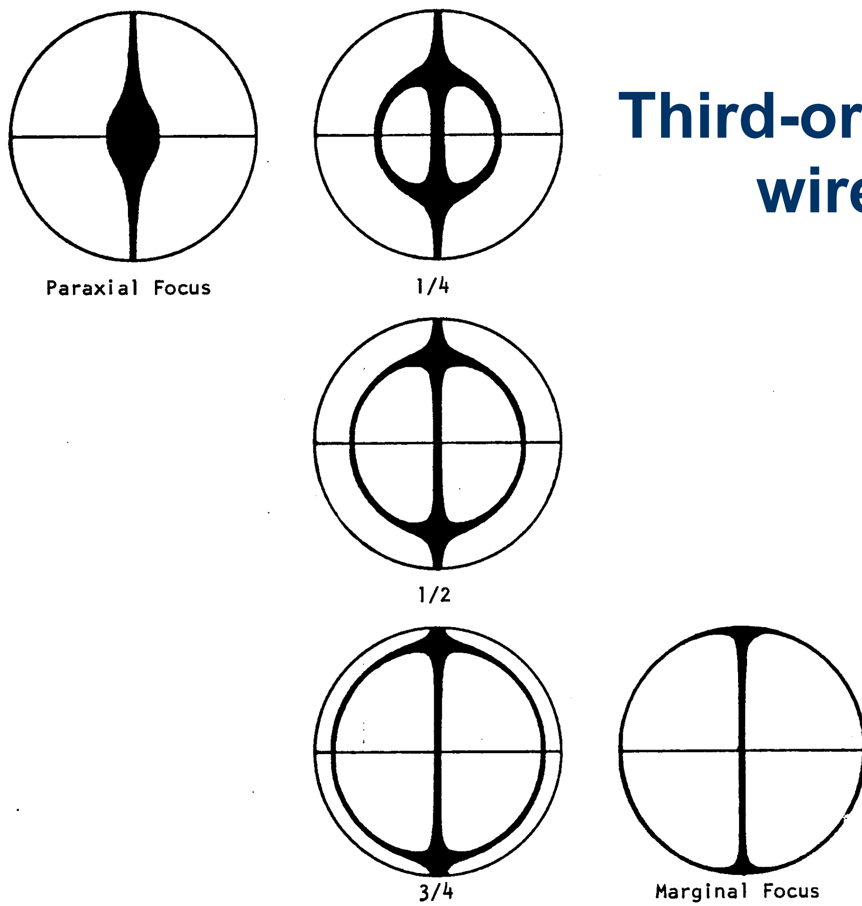
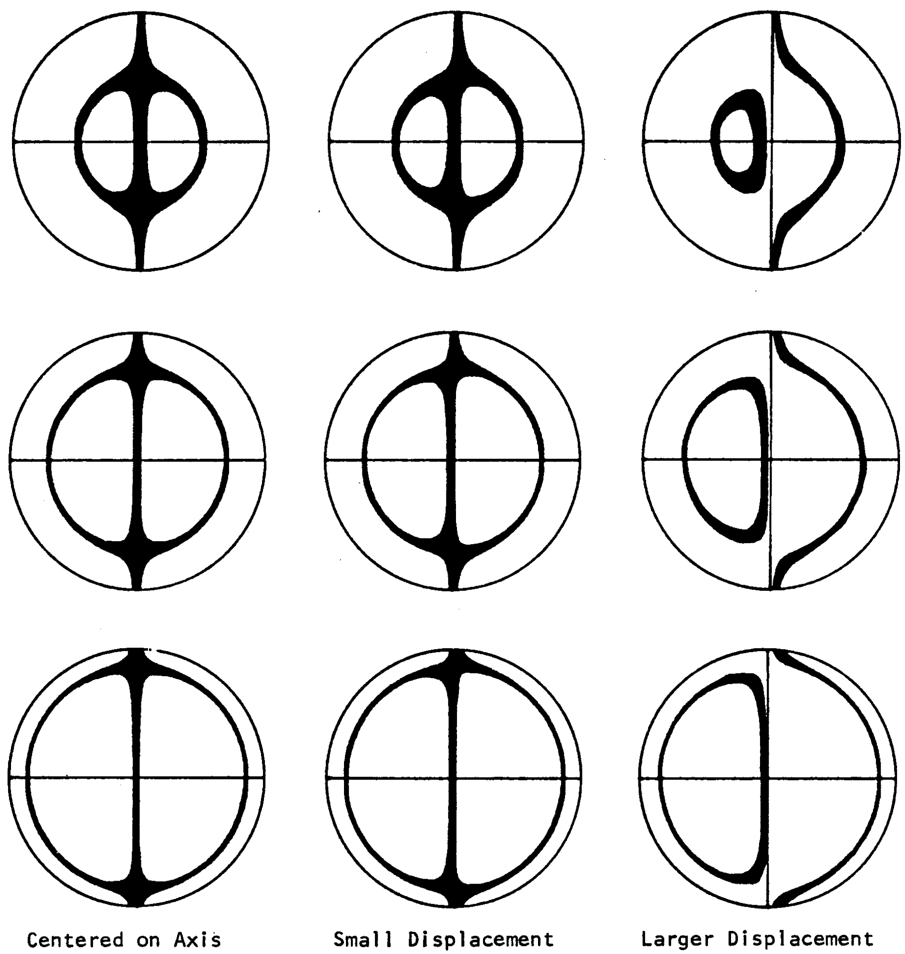
8.2.14 Wire Test

Same as knife edge test, except knife edge is replaced with a wire.



Third-order spherical wire on axis

Wire Test
Third-order spherical, wire off axis



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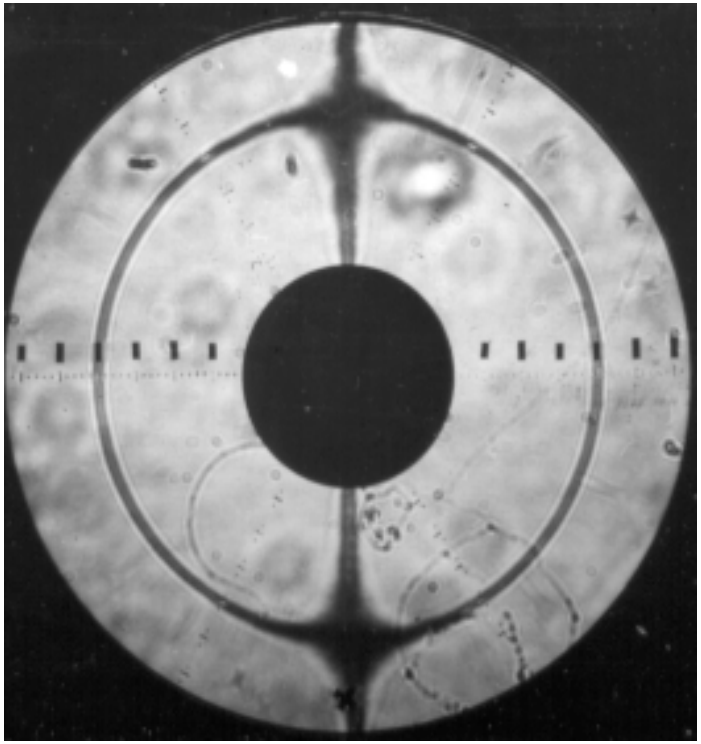
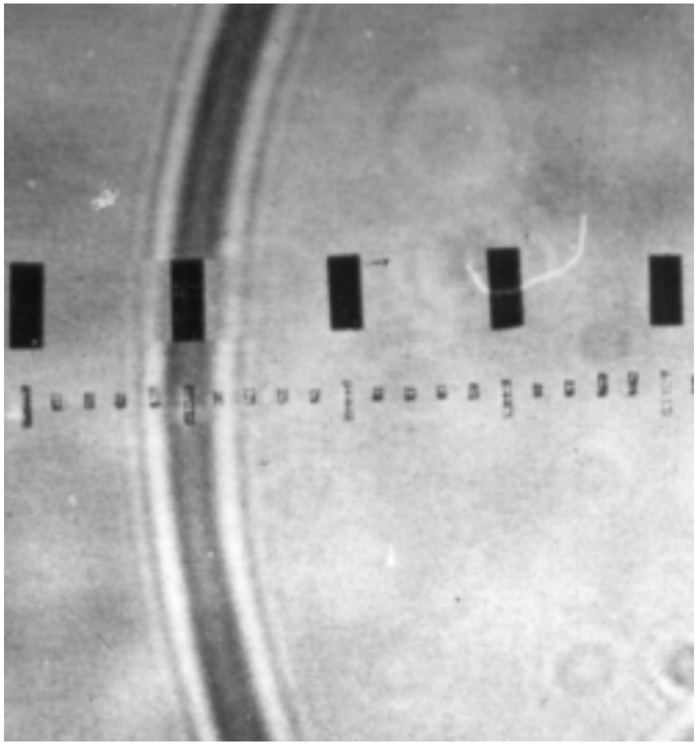
Wire Test



Wire test experimental results for parabolic mirror tested at center of curvature



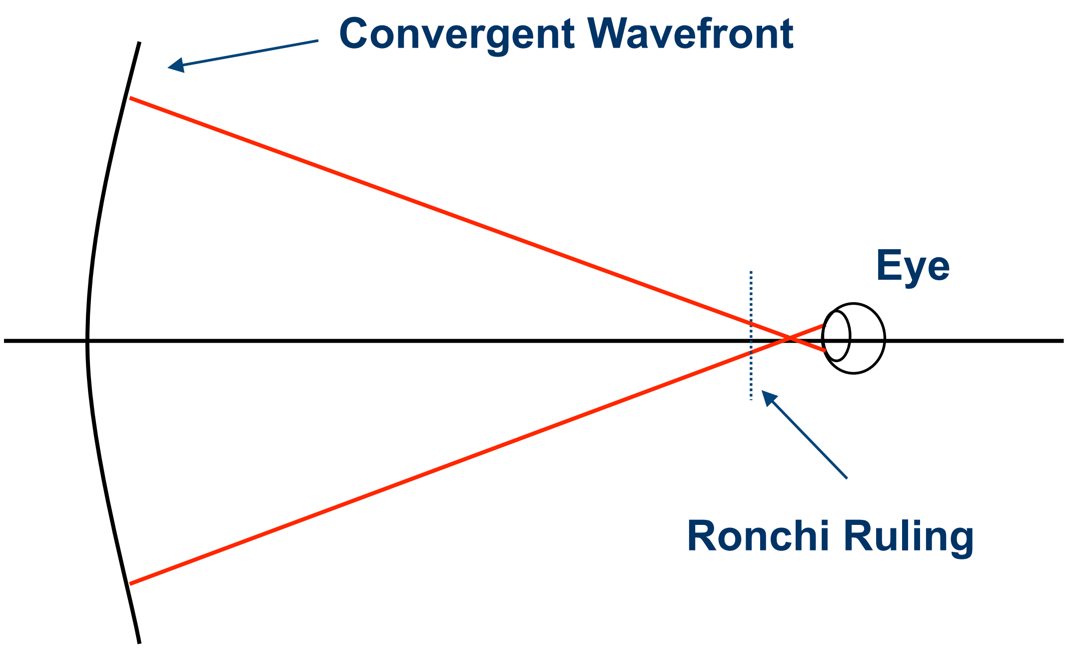
Close-up showing diffraction pattern

Wire Test - Comments

Wire test better than knife-edge test for quantitative measure, but not as good for qualitative

8.2.15 Ronchi Test

A low frequency grating is substituted for knife edge or wire. The test can be understood by considering the Ronchi ruling as equivalent to multiple wires.



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Ronchi Test of Perfect Lens

$$md = \varepsilon_y Cos[\alpha] - \varepsilon_x Sin[\alpha] = -\frac{R}{h}\frac{\partial \Delta W}{\partial y} Cos[\alpha] + \frac{R}{h}\frac{\partial \Delta W}{\partial x} Sin[\alpha]$$

$$\text{If } \alpha = 90^o, \quad md = \frac{R}{h}\frac{\partial \Delta W}{\partial x} = -\frac{\varepsilon_z h}{R} x$$

Ruling near focus

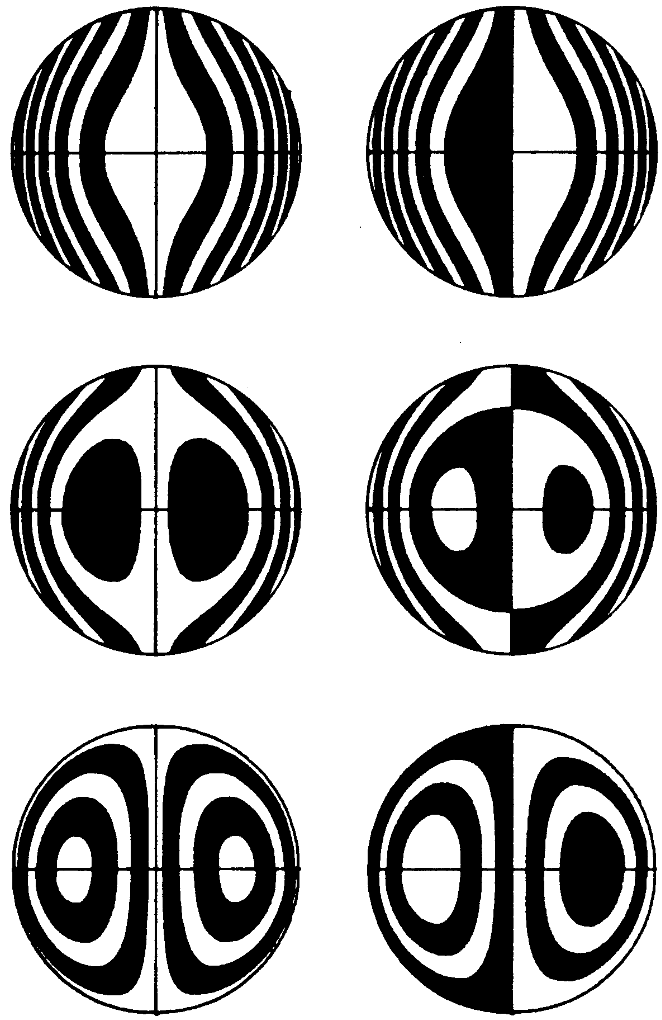
Ruling away from focus

Ronchi Test – Third-Order Spherical

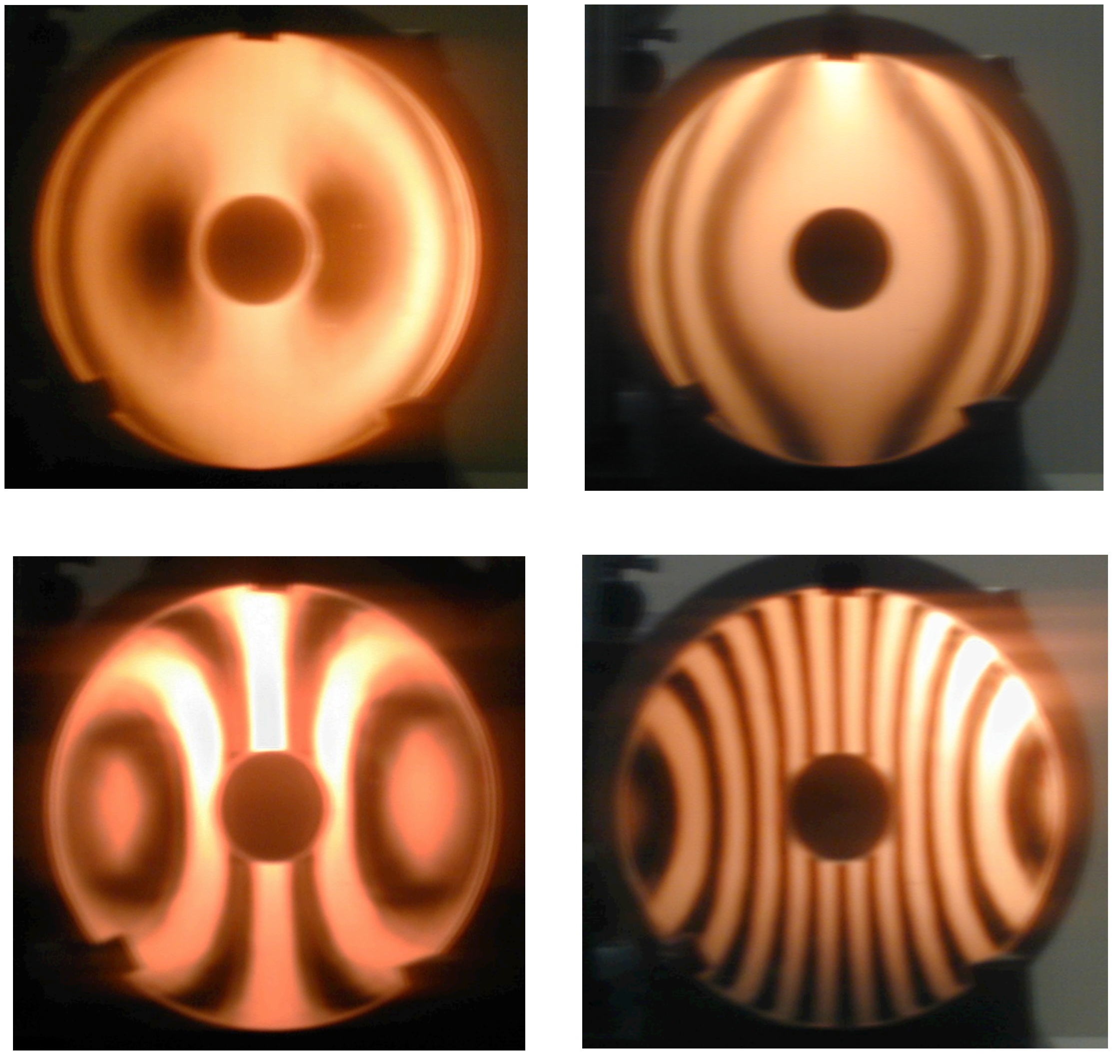
Paraxial
Focus

Mid
Focus

Marginal
Focus



Ronchigrams From the Lab



Margy Green, 2002

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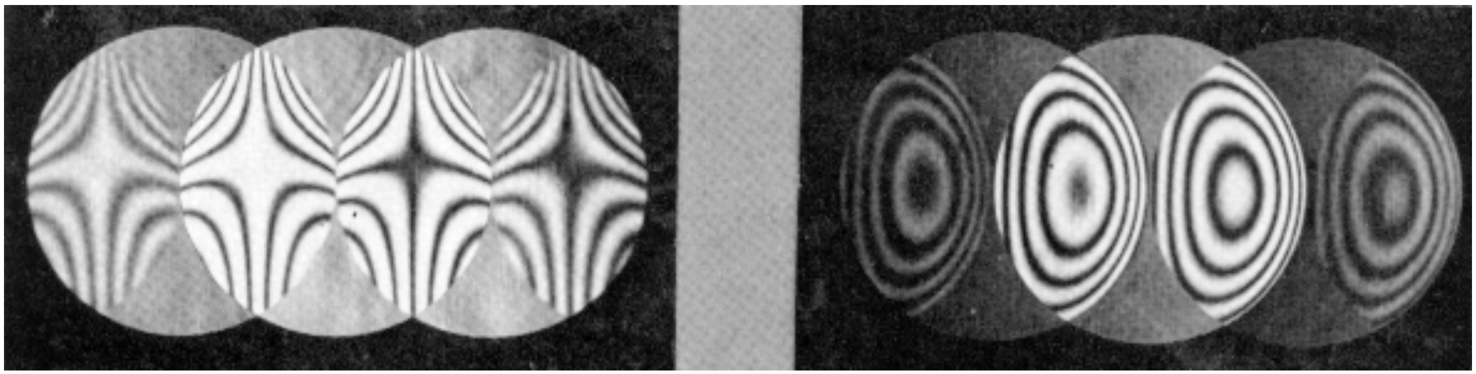
Ronchi Test Patterns for Third-Order Coma



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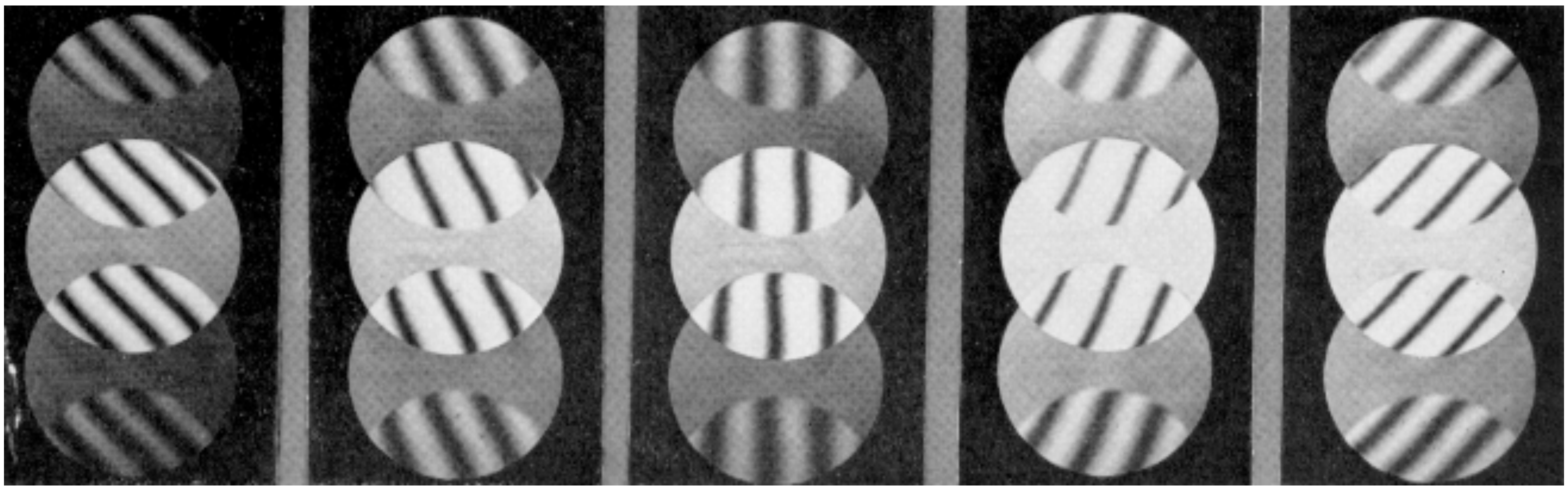
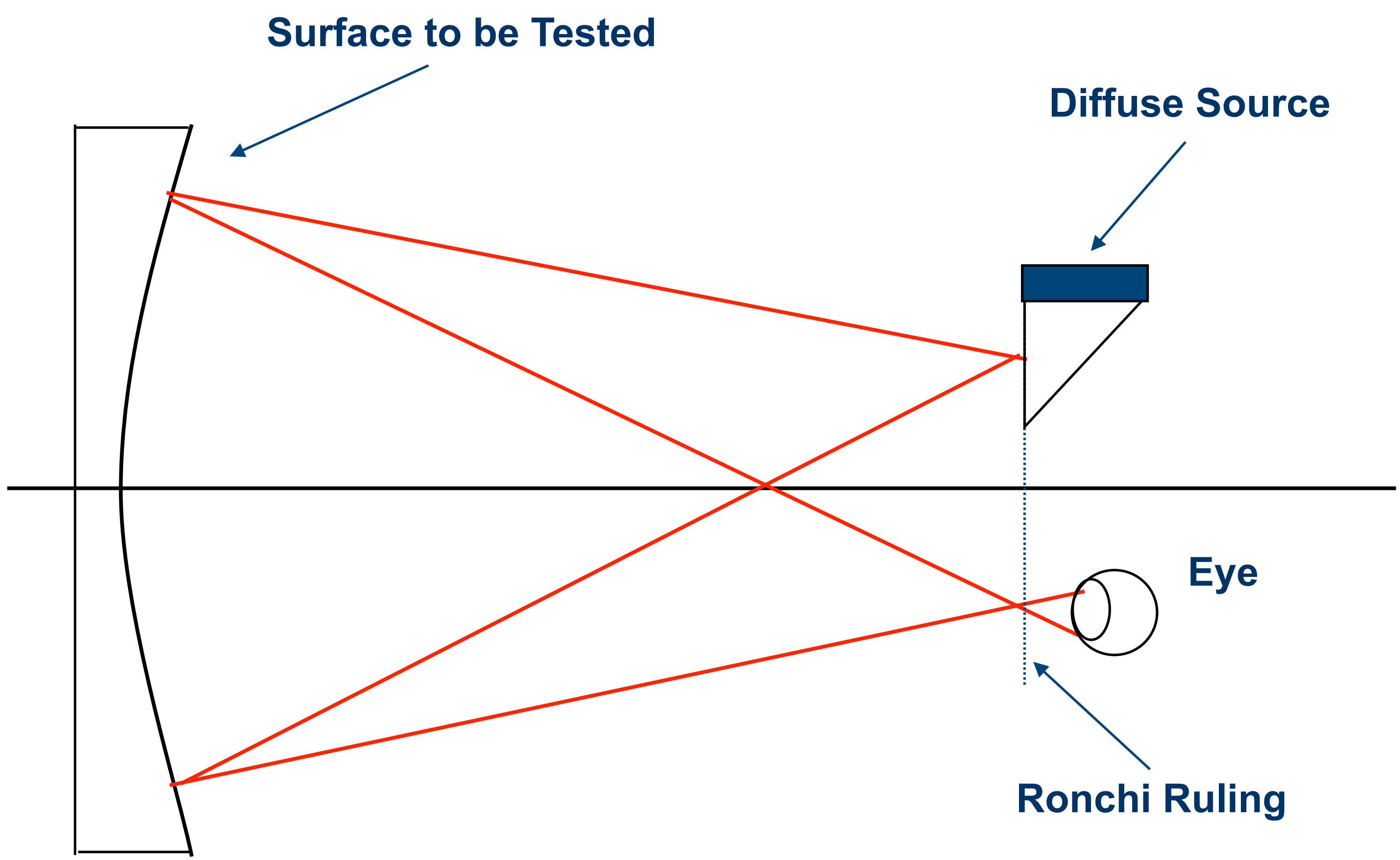
Ronchi Test Patterns for Astigmatism



Ronchi Test Using Extended Diffuse Source



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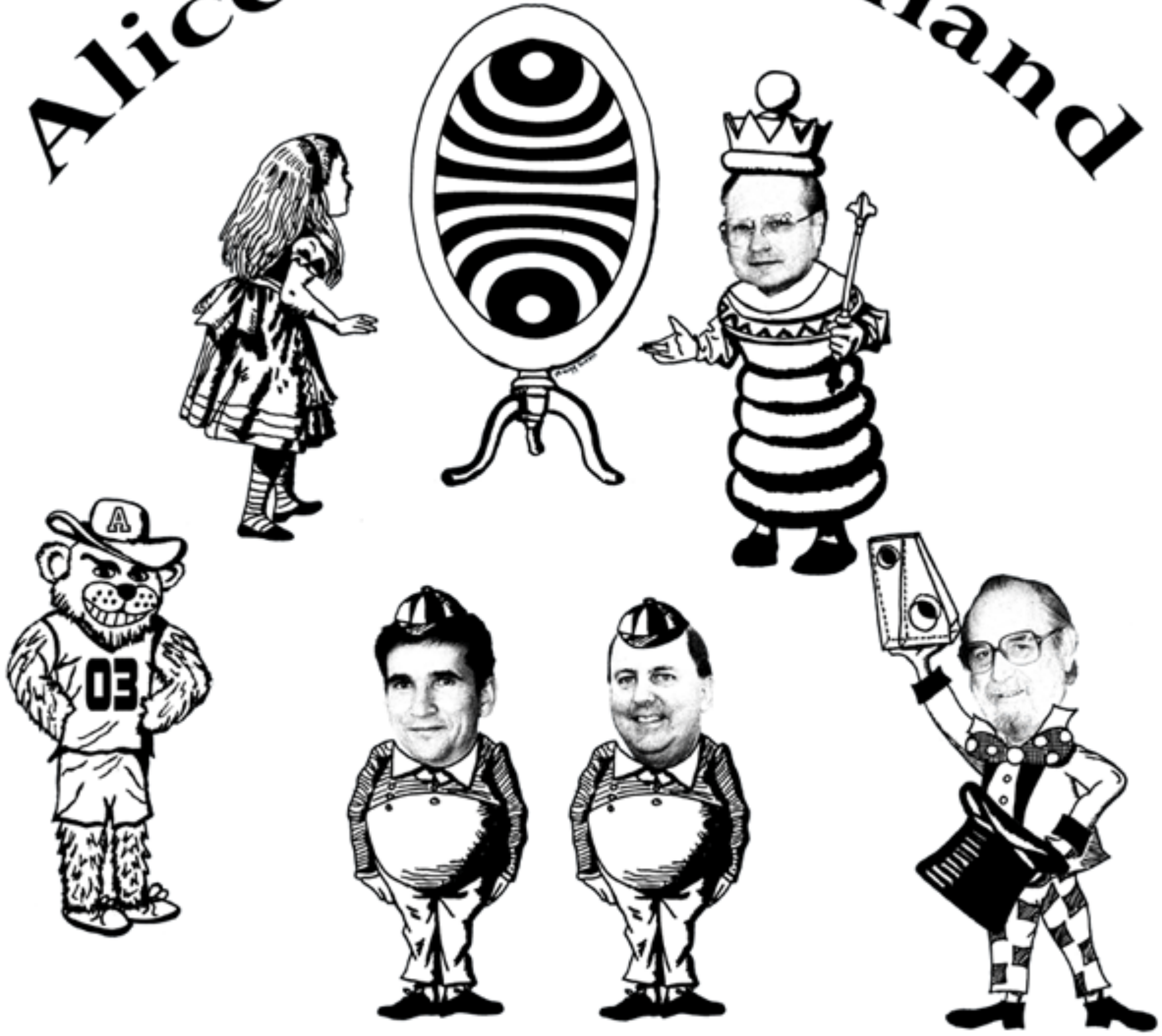
Ronchi Test - Comments

- The advantages are that the test is simple and will work with a white light source
- Disadvantage is that it does not give the wavefront directly, and for a single Ronchi ruling orientation slope in only one direction is obtained
- The diffraction effects are very troublesome and limit the accuracy of the test

Alice in Ronchiland



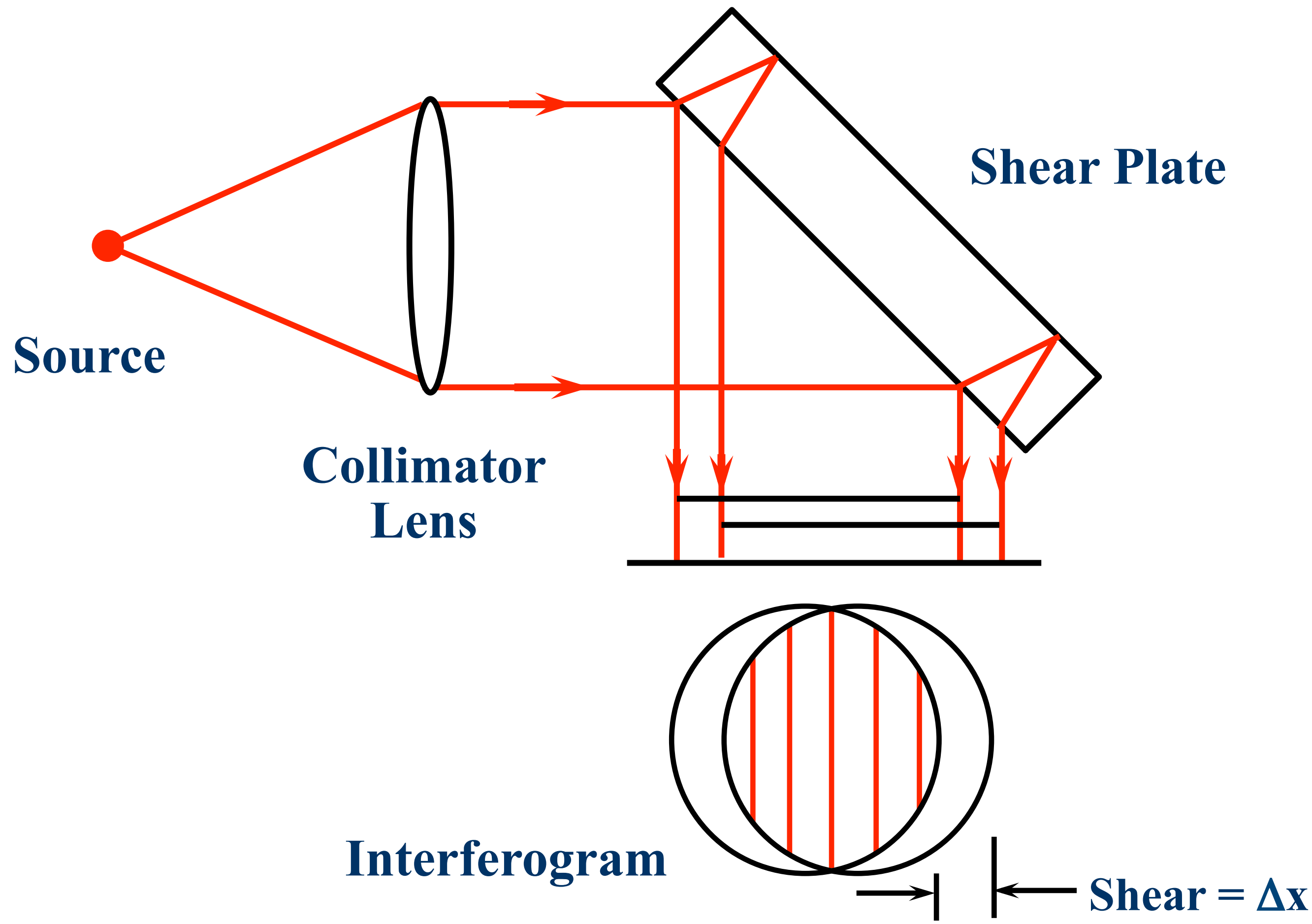
Optical Sciences Center

Tucson, AZ

**Created by
Margy Green, 2003**

8.2.16 Lateral Shear Interferometry

Measures wavefront slope



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Lateral Shear Fringes

$\Delta W(x,y)$ is wavefront being measured

Bright fringe obtained when

$$\Delta W(x+\Delta x, y) - \Delta W(x,y) = m\lambda$$

$$\left(\frac{\partial \Delta W(x,y)}{\partial x}_{\substack{\text{Average over} \\ \text{shear distance}}} \right)(\Delta x) = m\lambda$$

Measures average value of slope over shear distance

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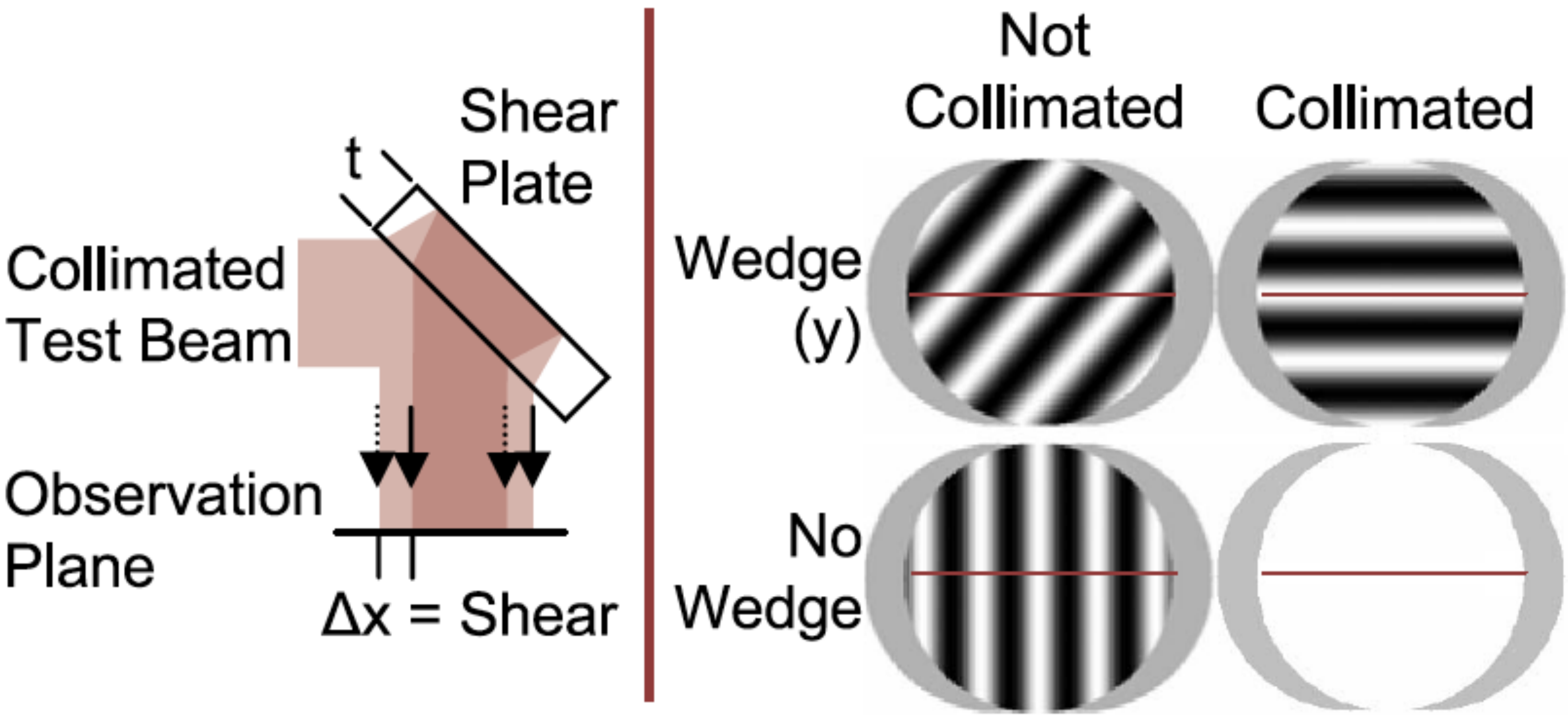
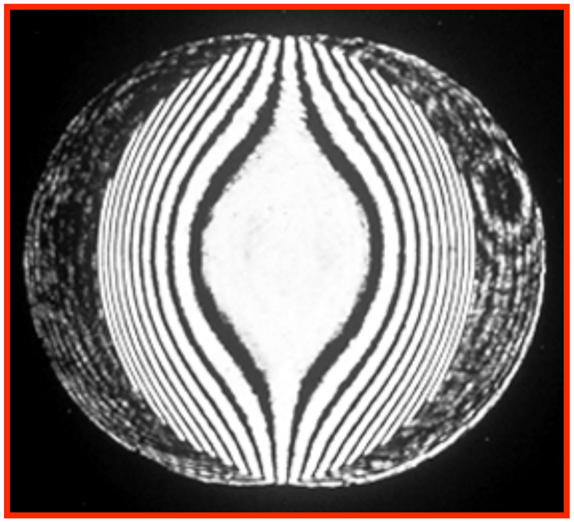
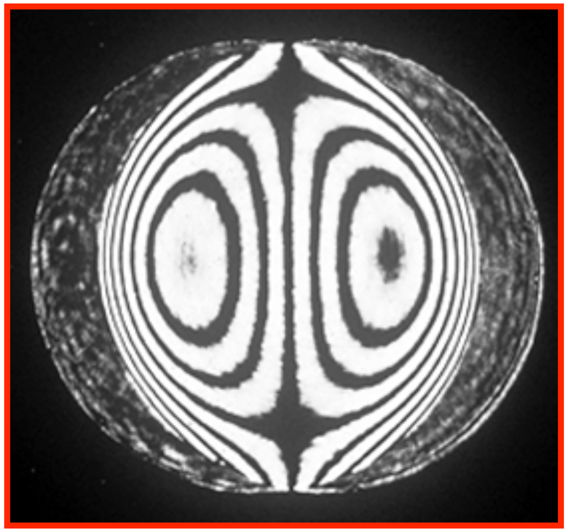
Collimation Measurement



Typical Lateral Shear Interferograms





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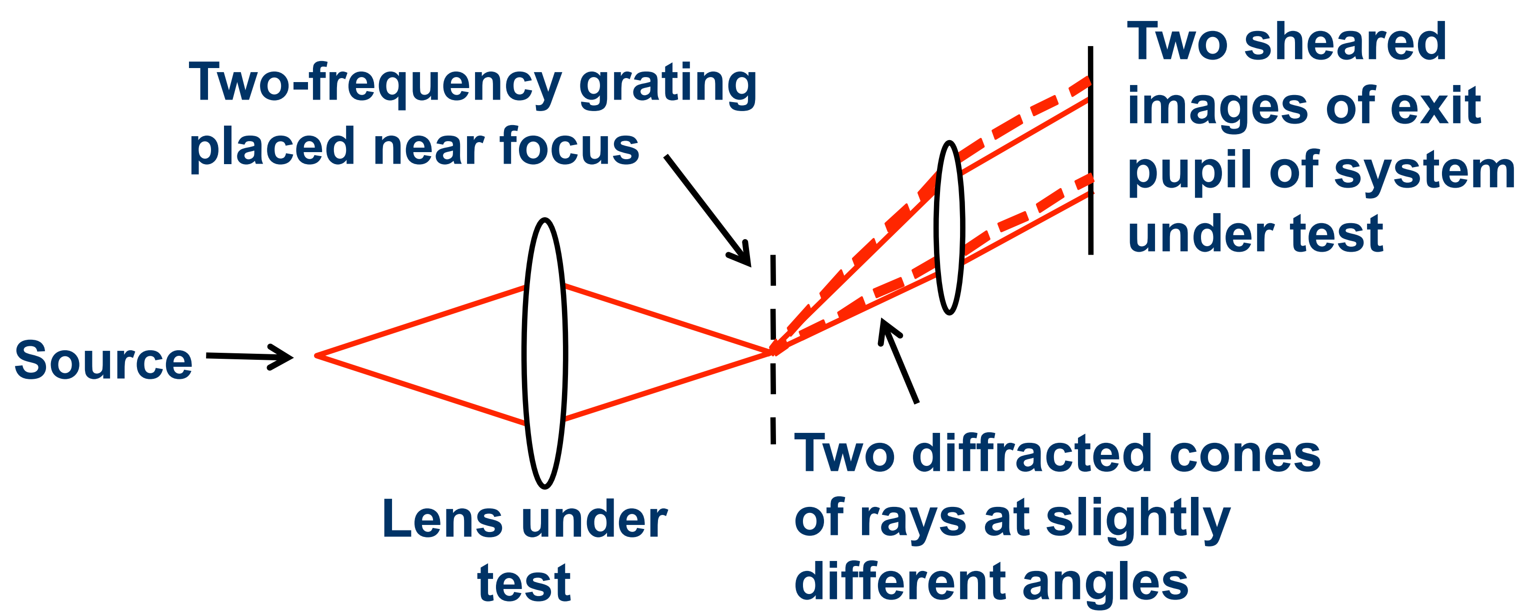
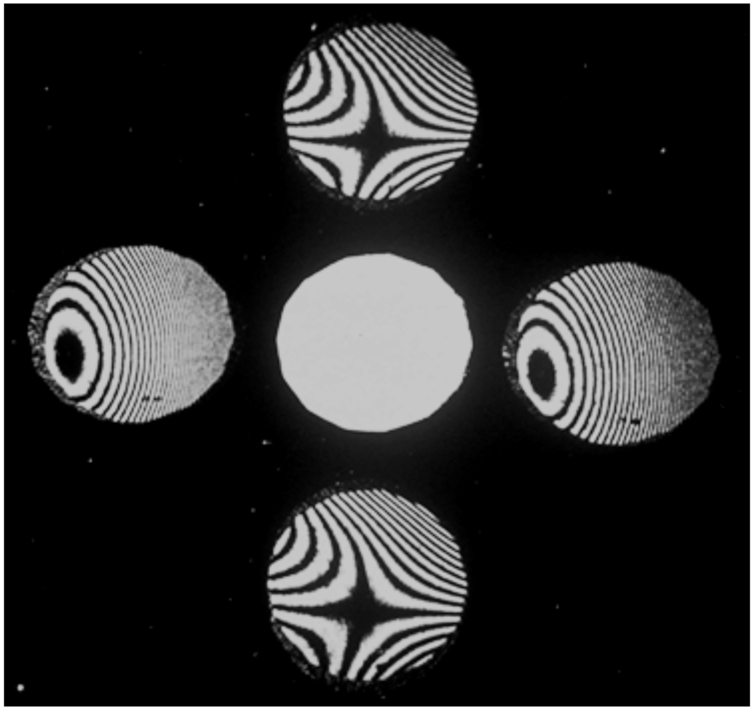
Lateral Shear Interferometer

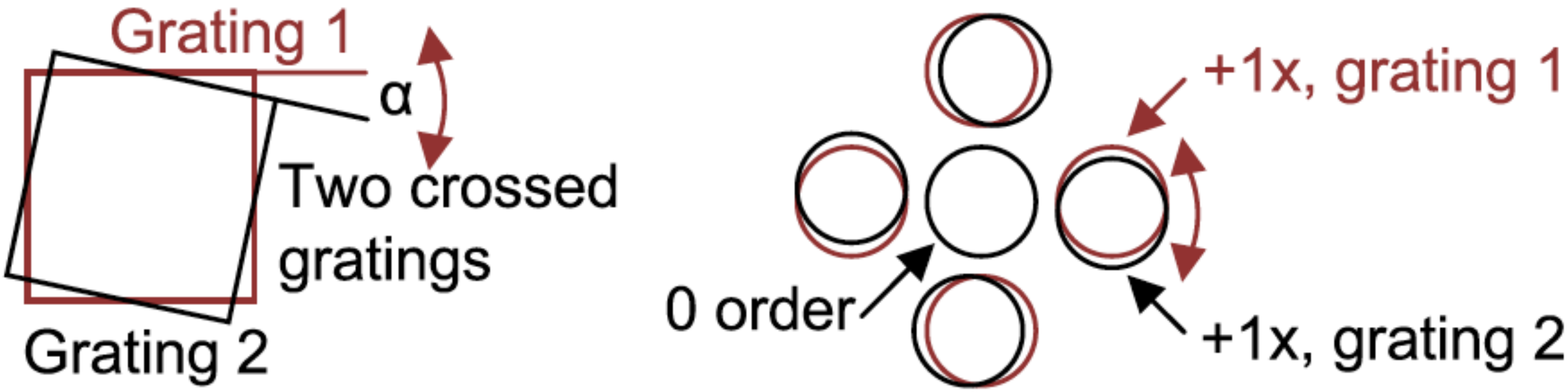


Measures slope of wavefront, not wavefront shape.

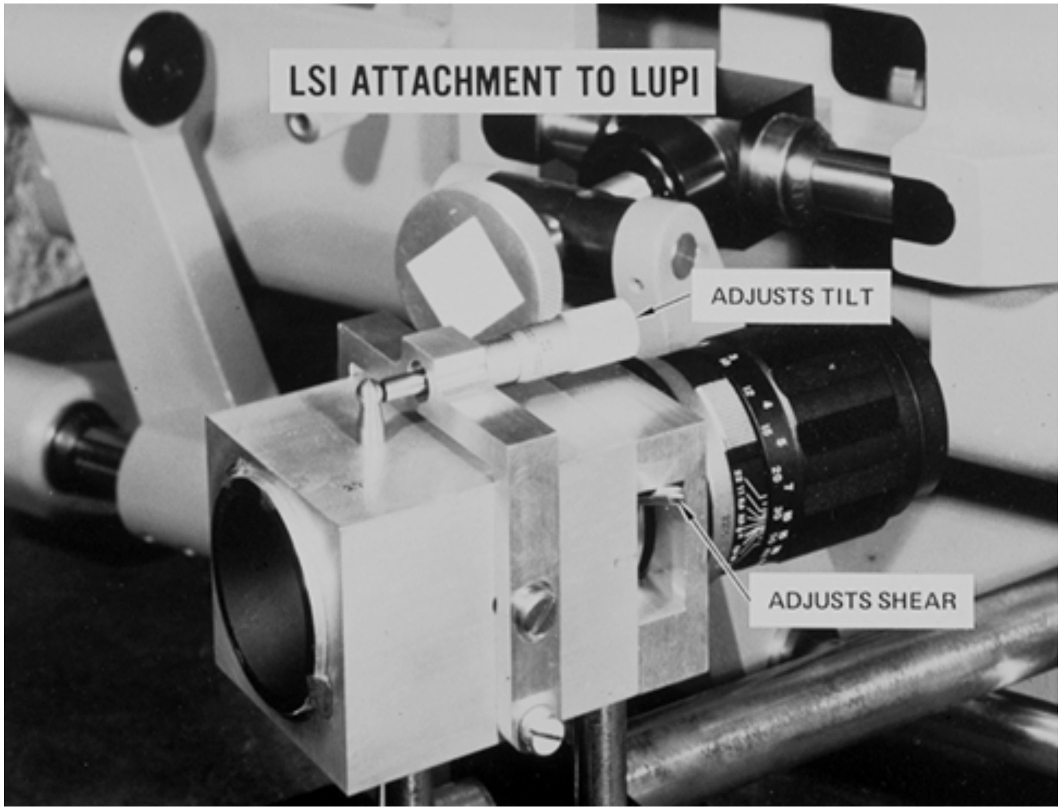
Interferogram Obtained using Grating Lateral Shear Interferometer



Rotating Grating LSI (Variable Shear)



Rotating Grating LSI



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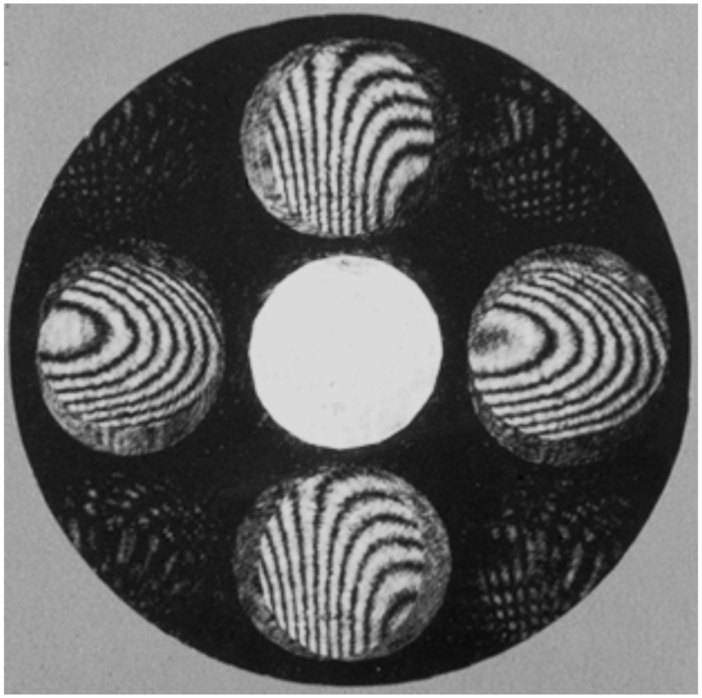
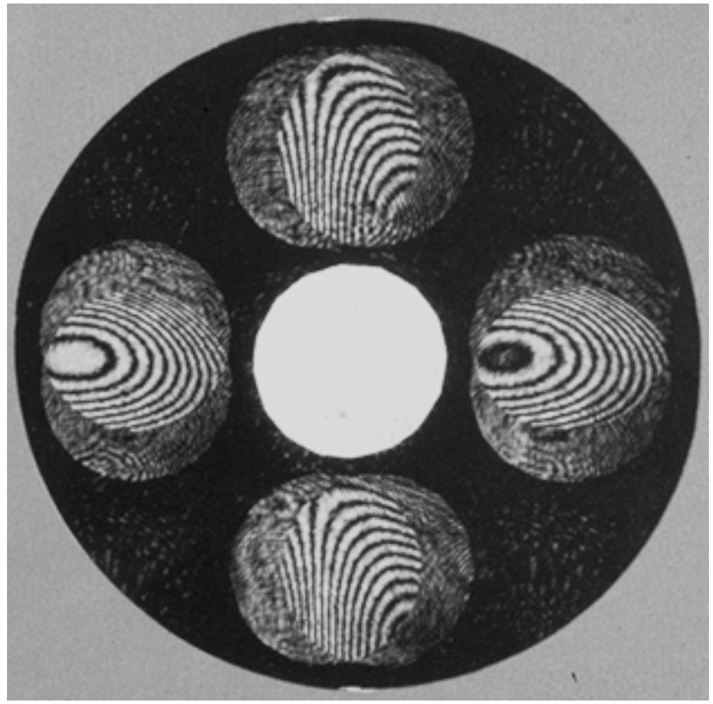
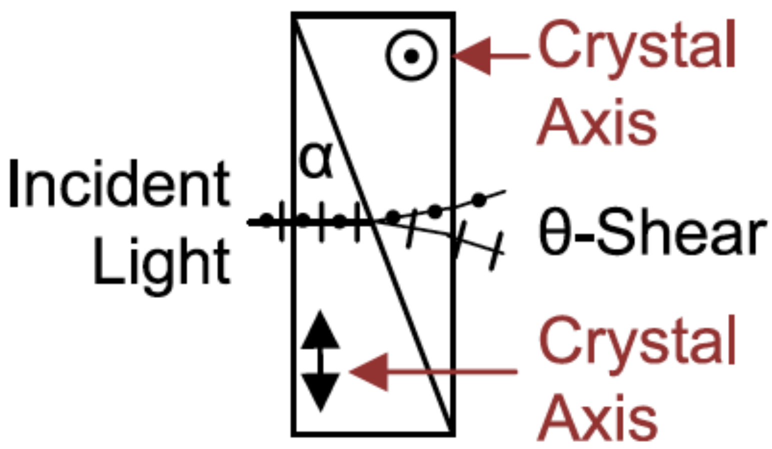
Shearing Interferograms (Different Shear)

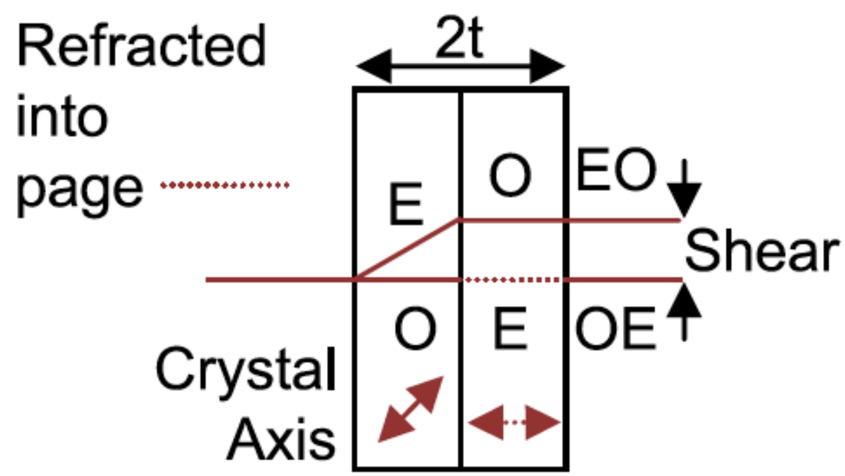




Polarization Interferometers

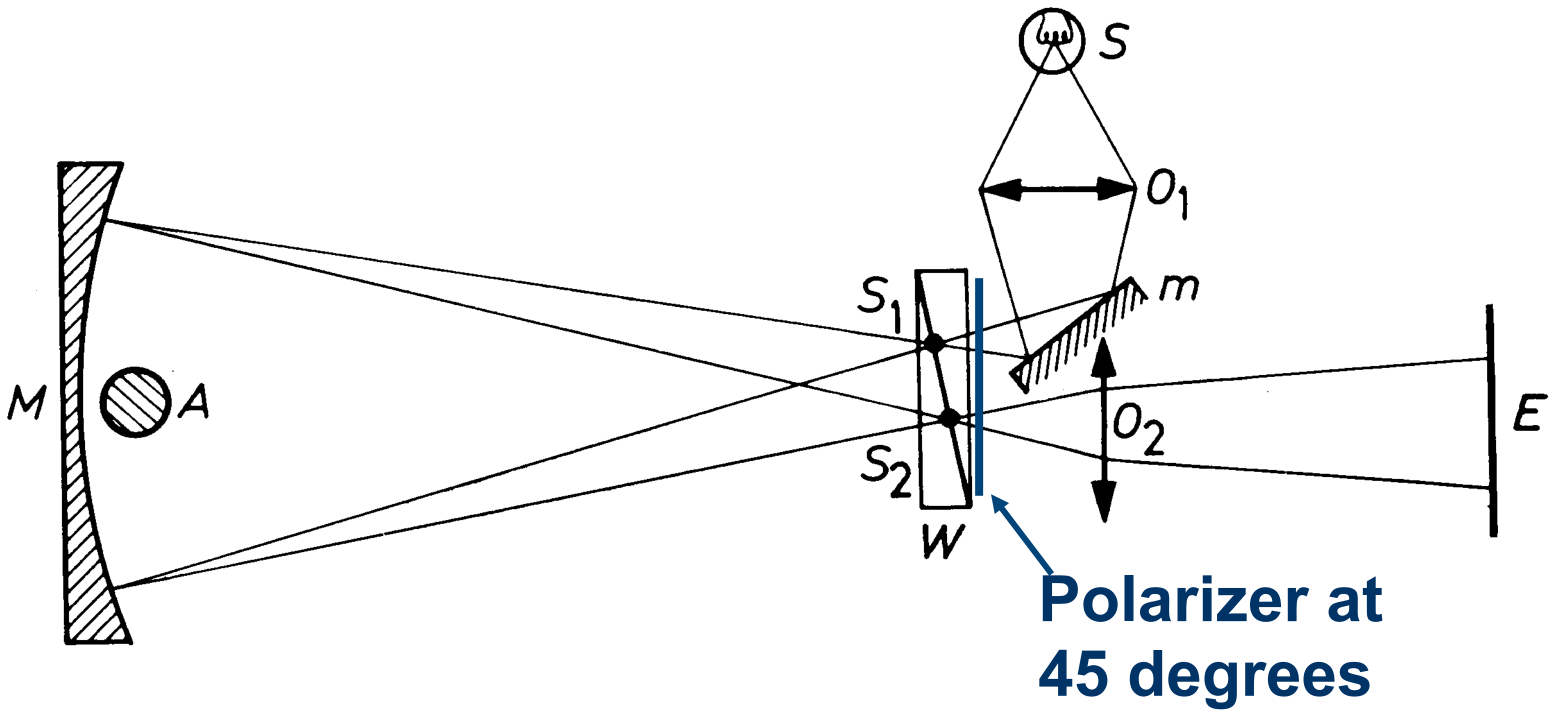


Wollaston Prism

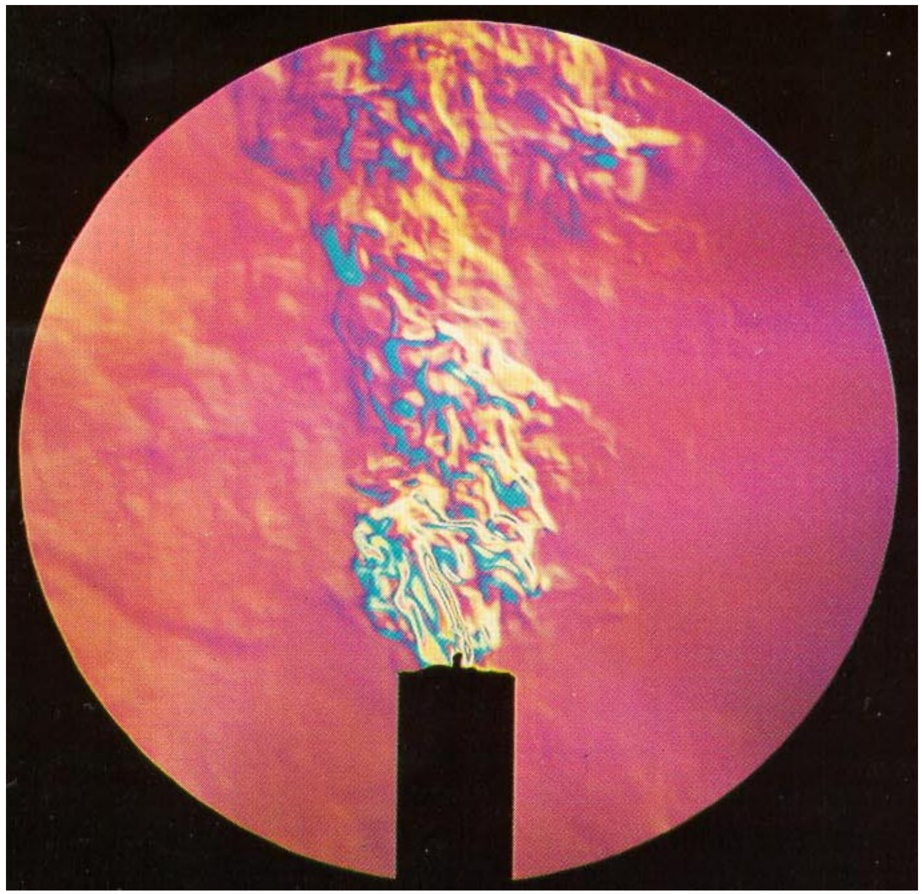


Savart Plate

Polarization Lateral Shear Interferometer



Convection currents in vicinity of candle flame observed with polarization interferometer



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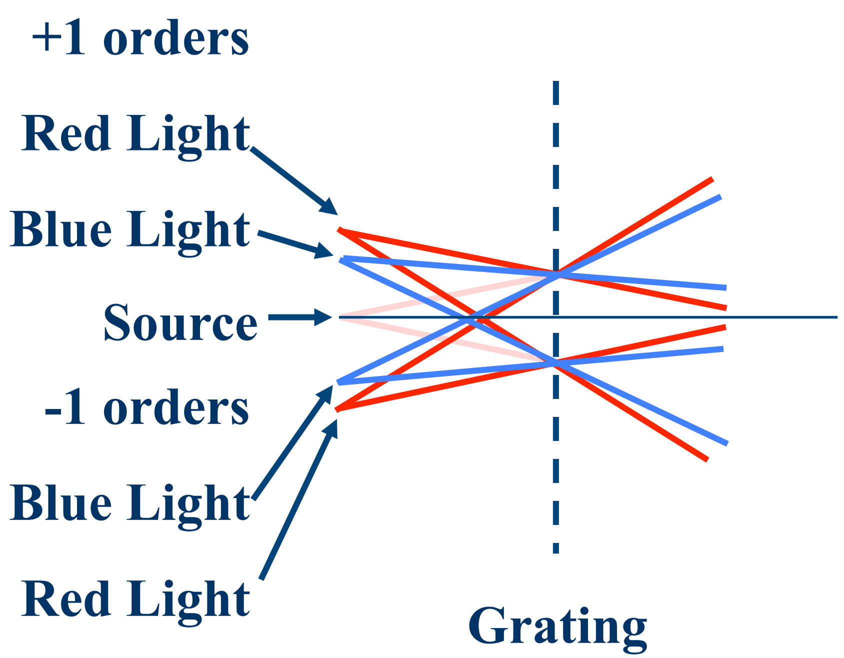
Convection currents in vicinity of candle flame observed with polarization interferometer

Defects of glass plate observed with polarization interferometer

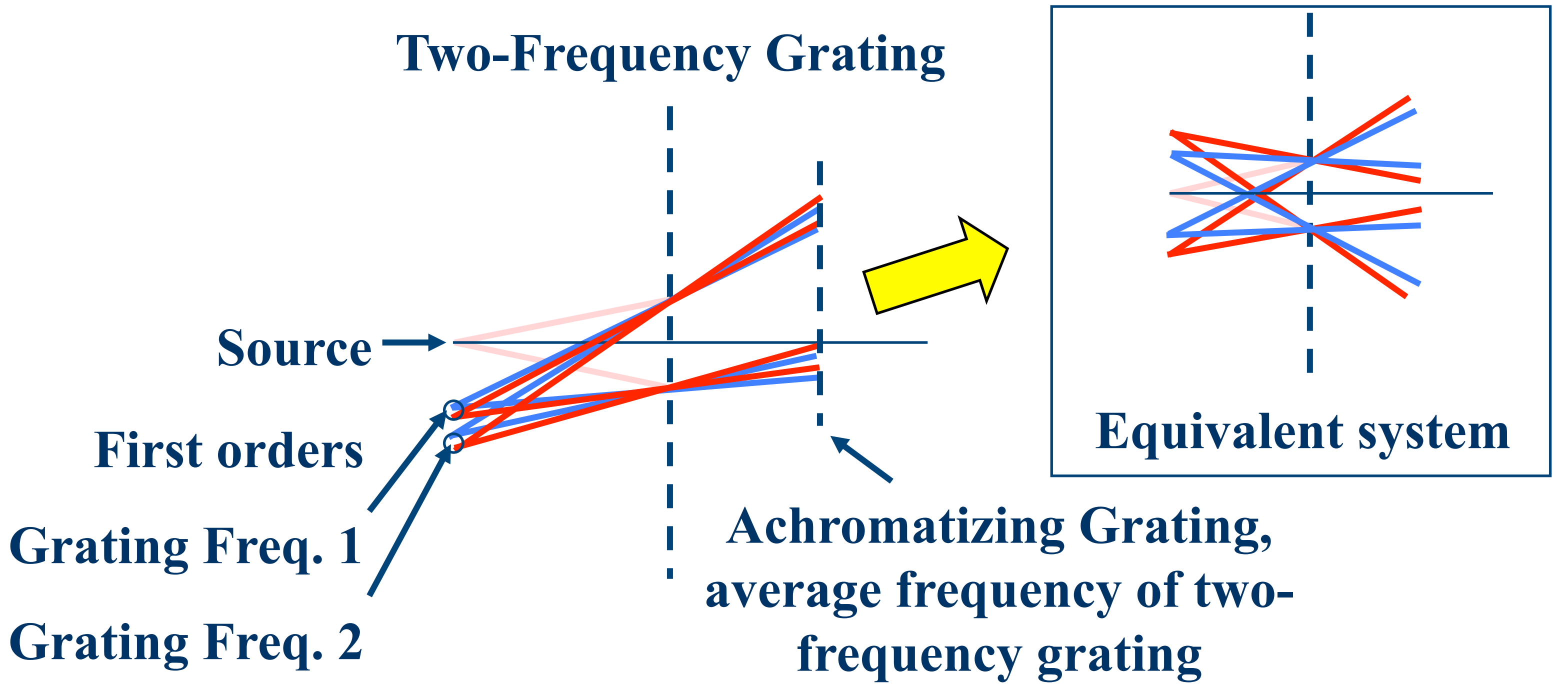
White Light Grating Interferometer



Separation between +1 and –1 orders is proportional to the wavelength. Therefore, fringe spacing same for all wavelengths.

Midpoint between sources independent of wavelength, so fringe position independent of wavelength

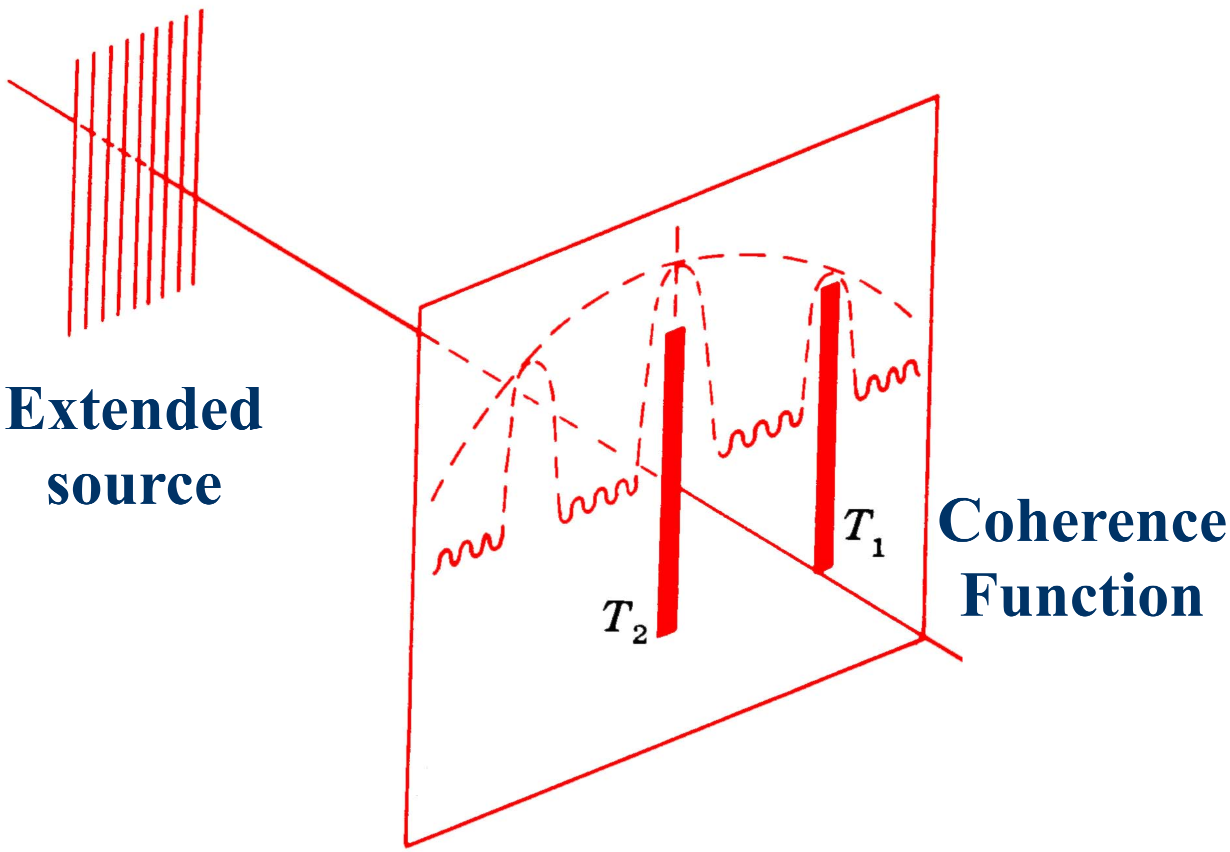
Two-Frequency White Light Grating Interferometer



Separation between the first orders of the two gratings is proportional to the wavelength. Therefore, fringe spacing same for all wavelengths.

Achromatizing grating must be added to make midpoint between sources independent of wavelength, so fringe position independent of wavelength.

White Light Extended Source Lateral Shearing Interferometer



Periodic source, then periodic coherence function. Period of coherence function proportional to wavelength. Therefore, shear should be proportional to wavelength.

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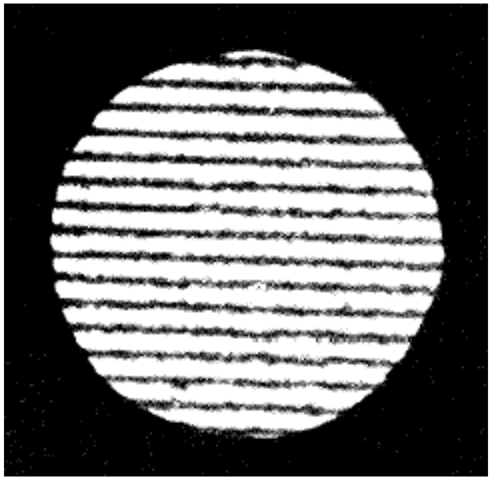
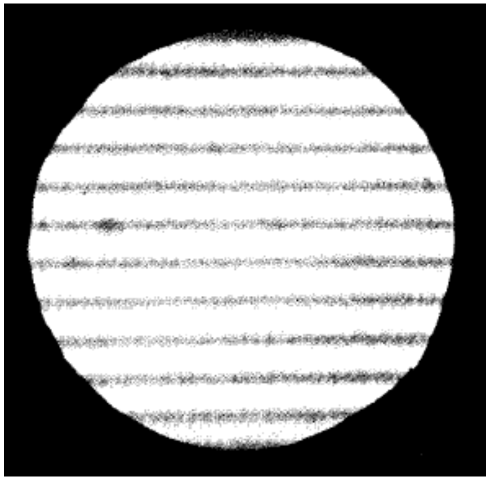
White Light Interferograms



Shearing interferogram obtained using tungsten arc source.



Shearing interferogram obtained using 60 watt incandescent bulb with Ronchi ruling in front of bulb.

One-Dimensional Analysis of Lateral Shearing Interferograms

- It is often sufficient to obtain the wavefront profile for a single scan across an interferogram.
- If the shear is sufficiently small a lateral shear interferogram gives the derivative of the wavefront in the direction of shear. For small shears the wavefront difference function can be fit to a polynomial and this polynomial can be integrated to obtain the wavefront.
- As the shear becomes larger it is no longer valid to assume the wavefront difference function is equal to the derivative.

One-Dimensional Analysis of Lateral Shearing Interferograms – Larger Shear

- This approach for analyzing LSIs that is valid for both large and small lateral shear
  - Least-squares fit the wavefront difference function to a polynomial and then set this polynomial equal to the finite difference wavefront difference function.
  - Solve for the polynomial coefficients describing the wavefront in terms of the polynomial coefficients describing the wavefront difference function.

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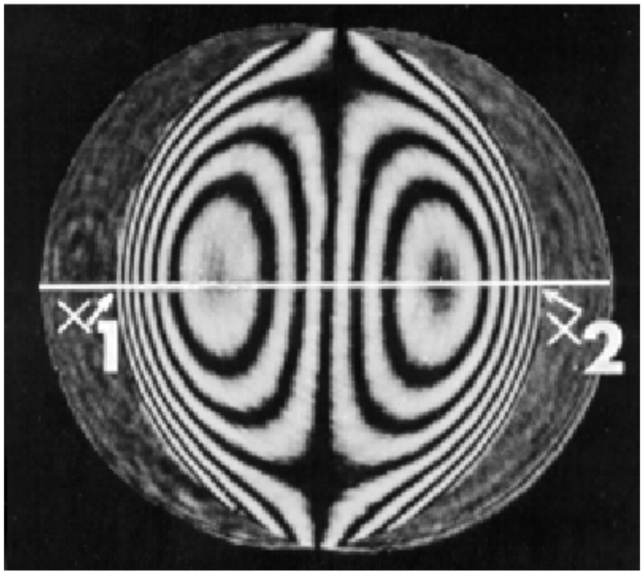
Determine Wavefront from Wavefront Difference Function



Wavefront difference function from interferogram

$$wdf = \sum_{n=0}^{nMax-1} b[n]x^n$$

Wavefront can be written as

$$\Delta w = \sum_{n=1}^{nMax} a[n]x^n$$

Wavefront difference function can be written as

$$\text{v} = \sum_{n=1}^{nMax} a[n]\left(\left(x+\frac{\Delta}{2}\right)^n - \left(x-\frac{\Delta}{2}\right)^n\right)$$

Solve for a's in term of the b's

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Solve a's in terms of b's

Spherical $a[4] \quad \frac{b[3]}{4\Delta}$

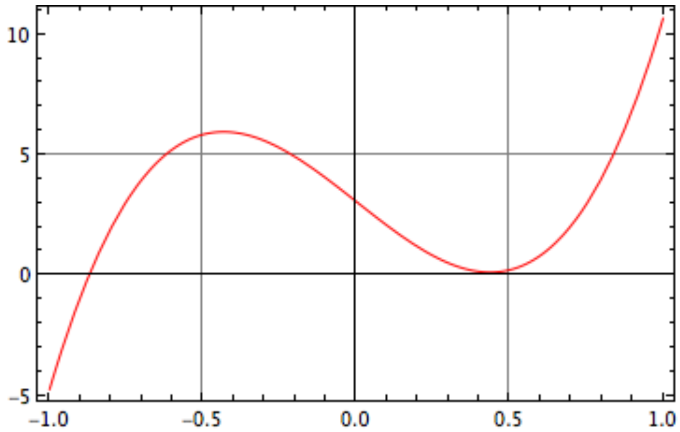
Coma $a[4] \quad \frac{b[2]}{3\Delta}$

Defocus $a[4] \quad \frac{b[1]}{2\Delta} - \frac{1}{8}\Delta b[3]$

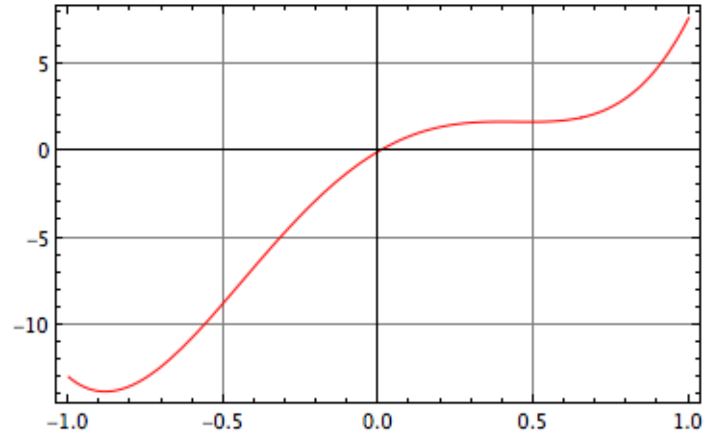
Tilt $a[4] \quad \frac{b[0]}{\Delta} - \frac{1}{12}\Delta b[2]$

Simple integration would have given only the first term in each expression above. Using the fact that a lateral shearing interferometer involves a finite-difference, rather than a derivative, makes it possible to obtain better results when the shear is not extremely small.

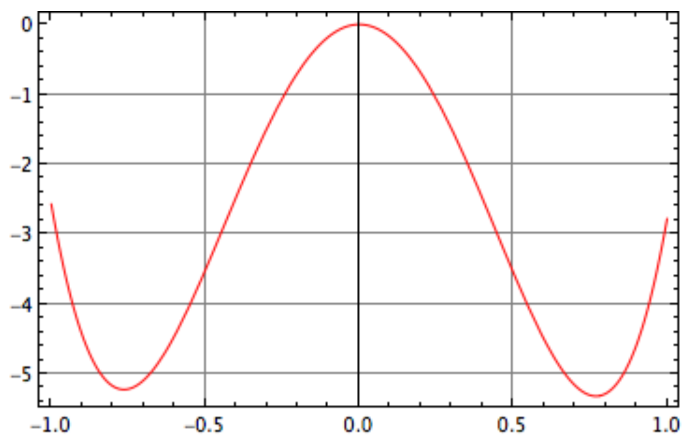
Results



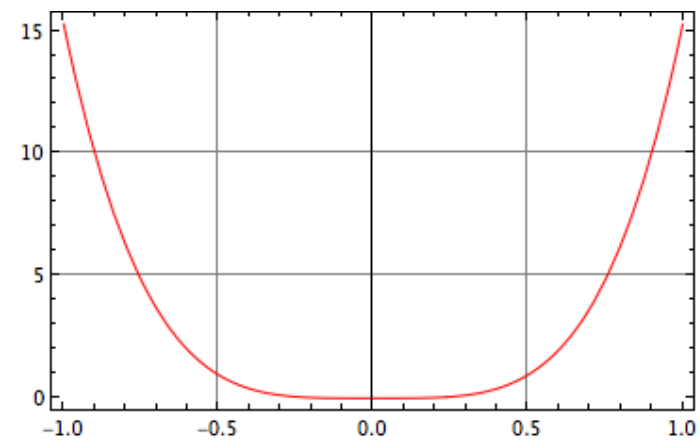
Wavefront Difference Function



Wavefront



Wavefront minus Tilt



4th Order Portion

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Shack Approach for Analyzing LSIs

- Wavefront difference function can be thought of as convolving the wavefront with an odd-impulse pair separated the shear distance.
- The Fourier transform of a convolution is the product of the two Fourier transforms.
- Divide the Fourier transform of the WDF by the FT of the two delta functions (2 i Sin()).
- Do an inverse transform to get the wavefront.

Ref: Ronald Gruenzel, JOSA, 66, No. 12, 1341 (1976).

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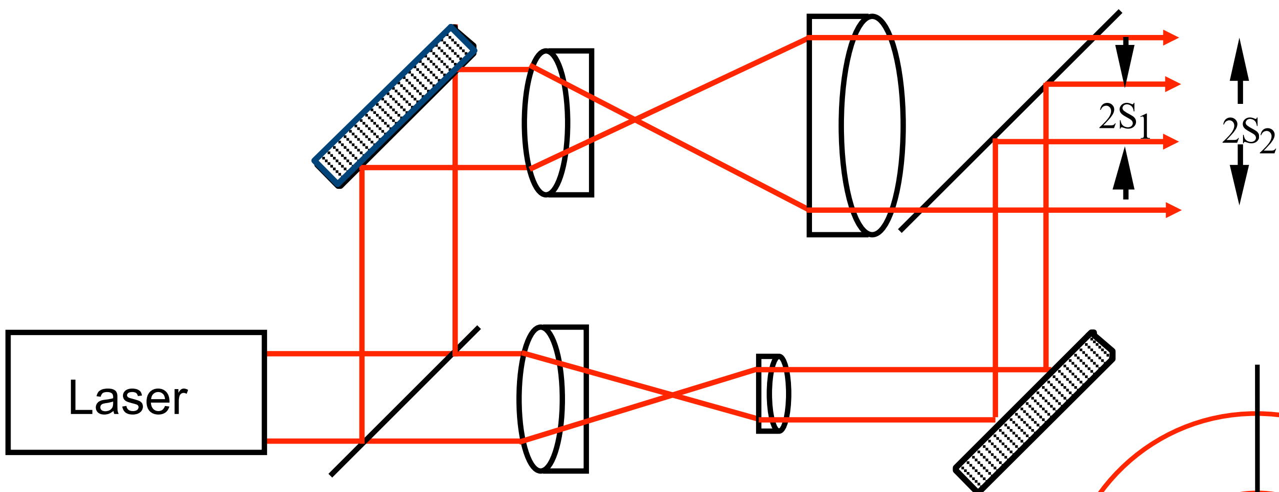
Lateral Shear Interferometer - Comments

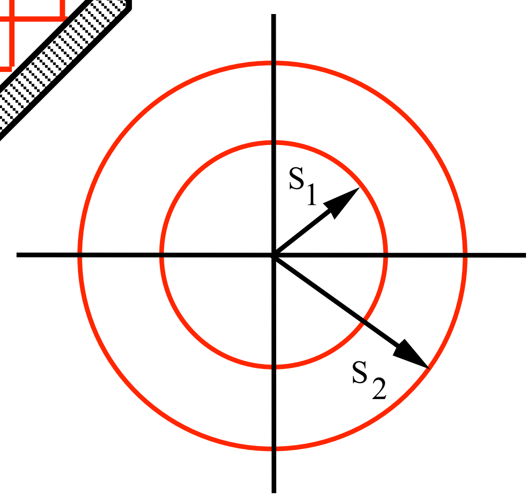
- The advantages are that the test is simple
- Disadvantage is that it does not give the wavefront directly, and for a single direction of shear slope in only one direction is obtained

8.2.17 Radial Shear Interferometry

Wavefront is interfered with expanded version of itself



$$\text{Radial Shear} = R = \frac{S_1}{S_2}$$



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Analysis of Radial Shear Interferograms

Wavefront being measured

$$\Delta W(\rho,\theta) = W_{020}\rho^2 + W_{040}\rho^4 + W_{131}\rho^3\cos\theta + W_{222}\rho^2\cos^2\theta$$

Expanded beam can be written

$$\Delta W(R\rho,\theta) = W_{020}(R\rho)^2 + W_{040}(R\rho)^4 + W_{131}(R\rho)^3\cos\theta + W_{222}(R\rho)^2\cos^2\theta$$

Hence, a bright fringe is obtained whenever

$$\Delta W(\rho,\theta) - \Delta W(R\rho,\theta) = W_{020}\rho^2(1-R^2) + W_{040}\rho^4(1-R^4) + W_{131}\rho^3(1-R^3)\cos\theta + W_{222}\rho^2(1-R^2)\cos^2\theta$$

Same as Twyman - Green if divide each coefficient by $(1 - R^n)$

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Radial Shear Interferometer - Comments

- Variable Sensitivity Test
  - Large shear - results same as for Twyman-Green
  - Small shear - Low sensitivity test

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