Some Further Aspects of Scatter-Fringe Interferometry

A. H. Shoemaker and M. V. R. K. Murty

It is shown that an interferometer using one scatter plate and a plane mirror is well suited for testing long focus optical systems. Stability is automatically achieved in this system because of its +1 magnification. Two methods of introducing straight fringes into the field in order to examine a nearly perfect optical system, and a method of making scatter plates to give high-contrast fringe patterns are described.

Introduction

The phenomena associated with scattering as a method of beam splitting were first described by Newton.[1] These phenomena were explained on the basis of wave theory first briefly by Young [2] and then more completely by Herschel[3] and Stokes.[4] More recently Bauchwitz and Shoenberg[5] and Shoenberg[6] have discussed some further aspects of multiple-beam scatter fringes. Although the phenomena associated with scattering as a means of beam splitting are thoroughly explained in the literature, this is not the case concerning the best conditions under which scattering can be used for testing optical systems. Burch,[7,8] Dyson,[9] and Martin[10] explain the principles and some of the advantages and disadvantages of scatter-fringe interferometers but go no further; Scott[11] has published a photograph taken with a scatter-fringe interferometer, but he has not given any particulars concerning the instrument. This paper describes a simple method of testing using scattering as a means of beam splitting.

Interferometer

The general method suggested by Burch is shown in Fig. 1. The interfering wavefronts are formed by the light which is scattered only once by the scatter plates. The light which is not scattered by either scatter plate (the direct–direct light) gives rise to a bright spot in the center of the aperture; it can be eliminated by a properly placed stop in the system. The light which is scattered by both scatter plates (the scattered–scattered light) simply adds to the background level and decreases the contrast in the fringe pattern. Since the magnification of this system is −1 it is extremely sensitive to vibration. In practice, the light source, the focusing lens, and the two scatter plates are all rigidly mounted on a three-dimensional motion to facilitate investigation of the region near the paraxial image.

In order to stabilize this system it is necessary to replace only the second scatter plate by a plane mirror, thus reimaging the scatter plate point for point back on itself. This is illustrated in the inset of Fig. 1. Since the magnification is +1 this system is extremely insensitive to vibration and therefore well suited for testing long focus systems. Dyson[12] has applied this method of stabilization to several other common-path interferometers. The effect of the plane mirror is to rotate the wavefronts through 180° in azimuth such that light striking the aperture of the system under test at $P(x,y)$ on the first pass strikes the aperture at $P(-x, -y)$ after reflection from the plane mirror. Thus the general expression for the optical path difference (OPD) δ between the reference and test wavefronts (primary aberrations only) due to the first pass is

$$\delta_1 = B(x^2 + y^2)^2 + F(x^2 + y^2)y + C(x^2 + y^2) + gx + hy + a(x^2 + y^2), \quad (1)$$

and due to the second is

$$\delta_2 = B(x^2 + y^2)^2 - F(x^2 + y^2)y + C(x^2 - y^2) - gx - hy + a(x^2 + y^2), \quad (2)$$

where B is the spherical aberration coefficient, F is the coma coefficient, C is the astigmatism coefficient, h,g are lateral displacement coefficients, and a is the longitudinal defocusing coefficient. Then the total OPD for this case is

$$\text{OPD} = 2B(x^2 + y^2)^2 + 2C(x^2 - y^2) + 2a(x^2 + y^2). \quad (3)$$

From Eq. (3) it is seen that this system is twice as sensitive to the even-order aberrations of the optical system

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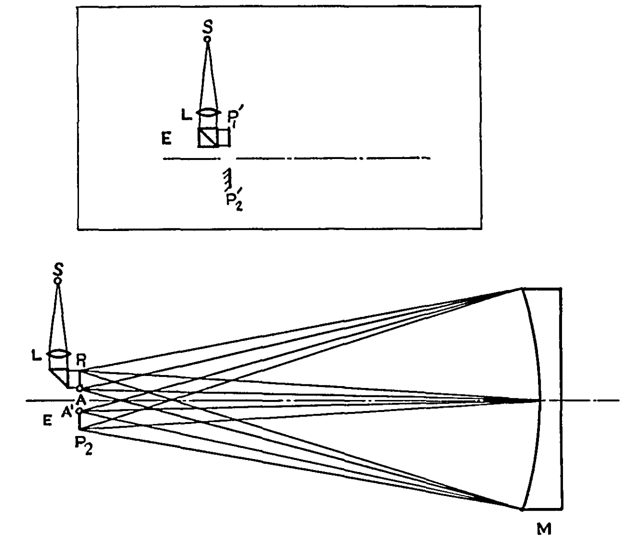


Fig. 1. Schematic arrangement for testing a concave mirror using an interferometer with two identical scatterplates. M is the concave mirror under test; P_1 and P_2 are two identical scatter plates (P_2 has been rotated 180° in azimuth with respect to P_1 such that A and A' are identical scatterers); L is the lens used to focus source S on M; and E is the position of the eye or camera. The inset shows how +1 magnification is obtained. P_1' is the scatter plate and P_2' is the plane mirror.

being tested as the Twyman–Green interferometer[13] used for testing. It is also seen from Eq. (3) that the odd-order aberrations are cancelled out, and therefore there are two major disadvantages to this system. The first is that any coma in the system being tested will not be observed (this is not a disadvantage when testing mirrors). The second is that it is not possible to introduce a tilt between the reference and test wavefronts by a lateral displacement of the scatter plate and plane mirror in order to examine an almost perfectly spherical test wavefront in detail. This last disadvantage can be overcome by introducing a plane-parallel plate into the system.

Method of Obtaining Tilt between Reference and Test Wavefronts

Tilt between the reference and test wavefronts can be obtained by introducing a small tilt to a thin plane parallel plate which has been placed into one-half of the cone of light which makes up the two wavefronts as shown in Fig. 2. The effect of the tilted plate is to introduce a lateral displacement of the image of the scatter plate with respect to itself. Since the tilt angle is small and the wavefronts pass through the plate twice, the OPD introduced by the tilted plate is

$$\mathrm{OPD} = (2t\alpha/nr)(n-1)\,x, \tag{4}$$

where t is the thickness of the parallel plate, α is the tilt angle of the parallel plate, n is the index of refraction of the parallel plate, and r is the distance from the exit pupil of the system under test to the paraxial image. This corresponds to a family of straight fringes in the field. Any deviation from straightness is, of course, a measure of the aberration in the nearly perfect test wavefront. If it is desired to have say four fringes in the field for $\alpha = 1°$, f/number $= 3$, $n = 1.5$, and $\lambda = 0.5 \times 10^{-3}$ mm, t must be about 1.0 mm. Since the parallel plate is in a converging beam of light, a certain amount of spherical aberration (along with much smaller amounts of coma and astigmatism) will be introduced, and this limits the use of the parallel plate for introducing straight fringes. For example, using the conditions mentioned above, for systems faster than about $f/3.0$ the amount of OPD (due to spherical aberration) introduced by the parallel plate becomes greater than $\frac{1}{10}\lambda$, and another method of introducing straight fringes should be used if the accuracy of the test is to be better than $\frac{1}{10}\lambda$. In Fig. 3 are photographs of the fringe patterns for a nearly perfect optical system and for a system with a small amount of spherical aberration.

Effect of the Separation of the Scatter Plate and Plane Mirror on the Aberration in the Wavefront

In applying Eq. (3) directly to the aberrations of the system under test, it is assumed that the scatter plate and plane mirror have no lateral separation: an impossibility. The effect of this separation is to introduce astigmatism into the wavefront. Using Coddington's equations it is easy to determine that the OPD introduced due to a separation d of the scatter plate and plane mirror is

$$|\mathrm{OPD}| = d^2/8r(f^\#)^2, \tag{5}$$

where r is the distance from exit pupil of the system under test to the paraxial image and $f^\# = F$/number of the system under test. Rearranging Eq. (5), the permissible separation is

$$d \leq 2f^\#(2r|\mathrm{OPD}|)^{\frac{1}{2}} \tag{6}$$

For example, if it is desired to test to an accuracy of $\frac{1}{10}\lambda$ for f/number $= 3$ and $r = 1000$ mm, $d \leq 1.9$ mm.

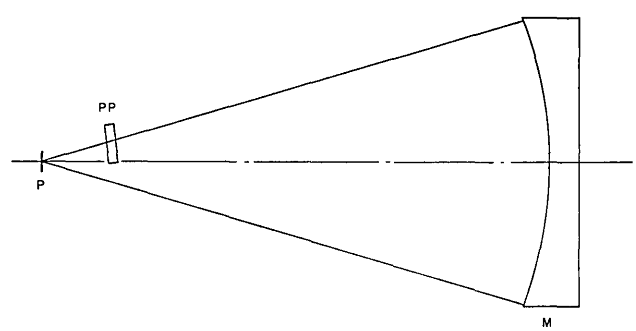
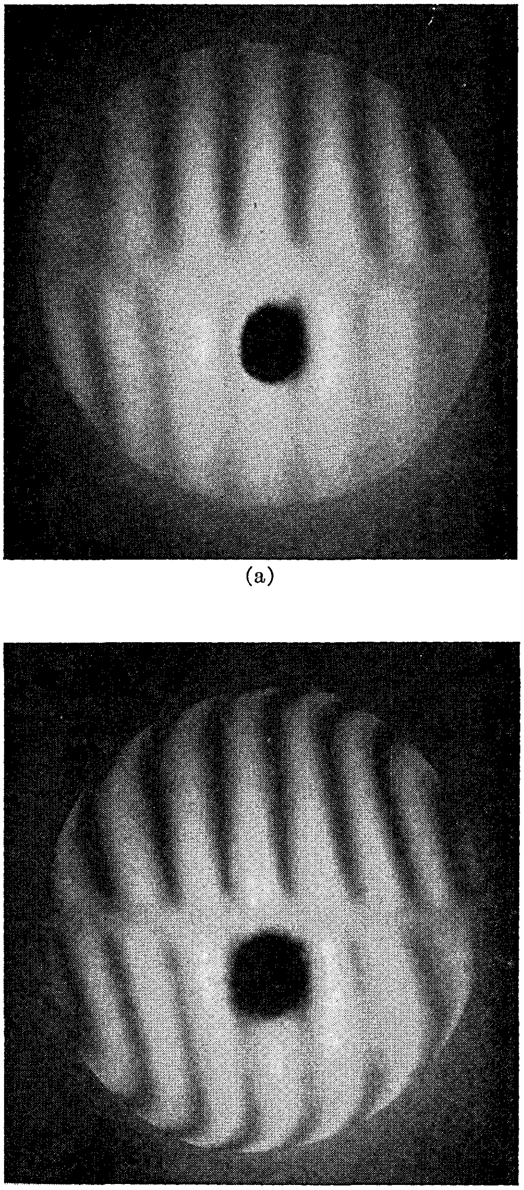


Fig. 2. Diagram illustrating method of introducing a plane-parallel plate into the system to obtain tilt between reference and test wavefronts. M is the concave mirror under test; P is the position of the scatter plate and plane mirror: and PP is the plane-parallel plate.

$$\mathrm{OPD} = 2B(x^2 + y^2)^2 - 4Cy^2. \tag{7}$$

Since $4C \gg 2B$, this represents a family of straight fringes slightly modified by the spherical aberration term. The disadvantage in this case is that the spacing between fringes is not constant but decreases as $|y|$ increases with a broad fringe in the center of the field. This means that the center of the aperture cannot be examined in detail. It should be noted that in this case the fringes correspond to the tangential astigmatic focus. In Fig. 4 are photographs of the fringe patterns



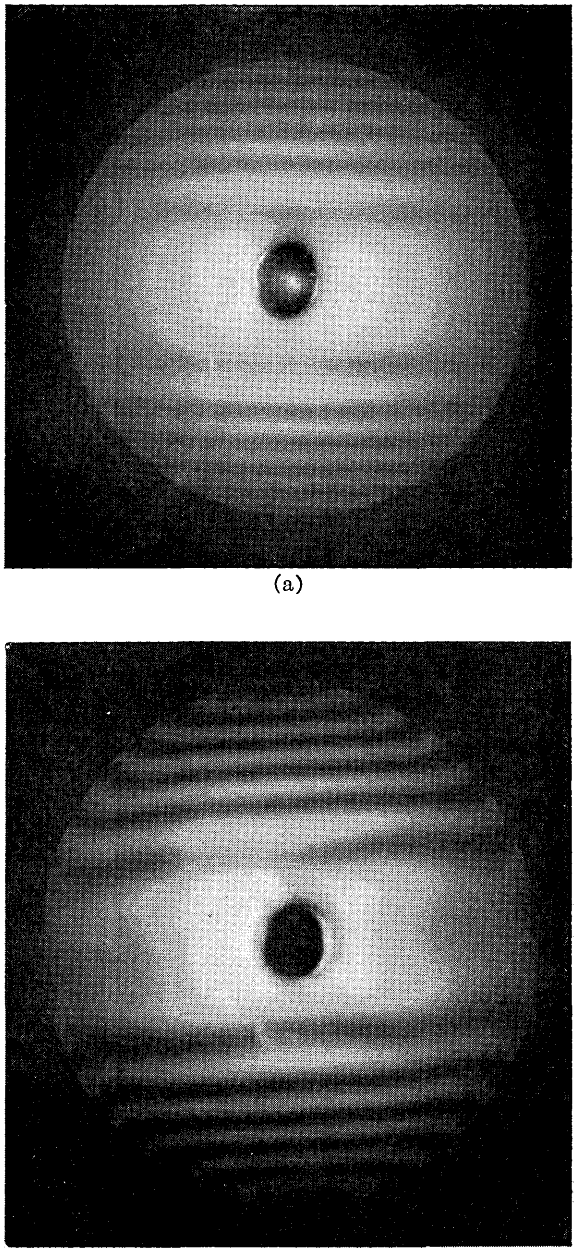
(a)

(b)

Fig. 3. Photographs showing the fringe patterns for (a) a nearly perfect optical system, and (b) an optical system with a small amount of spherical aberration when the plane-parallel plate is used to obtain straight fringes.

Separation as small as this is not strictly necessary since the amount of astigmatism introduced is easily calculated using Eq. (6) and can be taken into account in evaluating the fringe pattern.

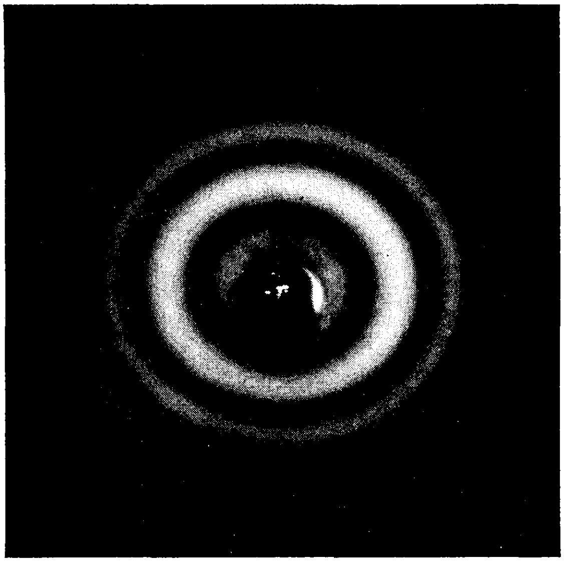
Equation (5) suggests another method for obtaining straight fringes in the field to examine an almost perfect optical system. For an almost perfect optical system B and C in Eq. (3) are both small. Then, by separating the scatter plate and plane mirror such that OPD in Eq. (5) is large with respect to $2C$ in Eq. (3) and adjusting the longitudinal defocusing of the system such that $a = -C$, Eq. (3) reduces to



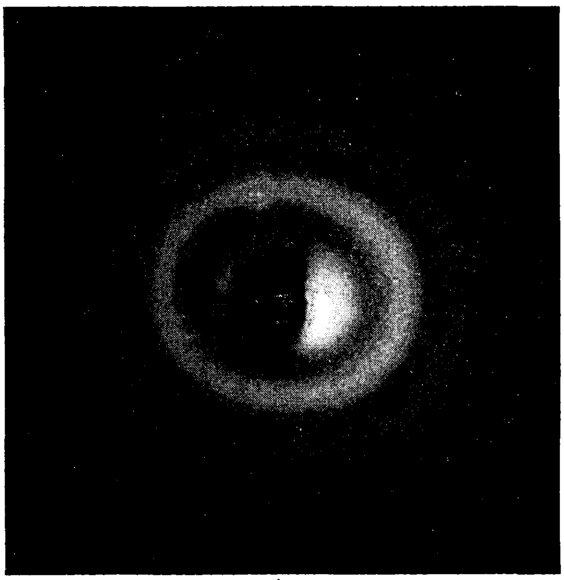
(a)

(b)

Fig. 4. Photographs showing the fringe patterns for (a) a nearly perfect optical system, and (b) an optical system with a small amount of spherical aberration when a separation of the scatter plate and plane mirror is used to obtain straight fringes.



(a)



(b)

Fig. 5. Contrast photographs obtained using the scatter plates made with the (a) 12-μ and (b) 60-mμ grinding powders.

obtained for a nearly perfect optical system and for a system with a small amount of spherical aberration.

Contrast of the Fringe Patterns

The two parameters which most affect the contrast of the fringe patterns are the source size and the scattering properties of the scatter plate (it is assumed that stray light in the vicinity of the interferometer has been minimized). Ideally the source should be a mathematical point in order to obtain the highest possible contrast. However, for a mathematical point the intensity of the source would be vanishingly small and in practice sufficiently high contrast can be obtained if the size of the source is such that it does not contribute more than $\frac{1}{4}$ λ of OPD to the wavefront. This means that the size of the image of the source on the paraxial region of the optical system under test should be less than $\frac{1}{4}$ of the width of the white light fringe.

If, for simplicity, we assume that all the pits of a given size scatter light into thin-walled cones of the same half-angle and that the scattering coefficient is constant with scattering angle and pit size, the expression for the contrast of the fringe pattern reduces to

$$V = 2(1 - k)/(2 - k), \tag{8}$$

where $k(\theta)$ is the scattering coefficient and V denotes contrast. From Eq. (8) it may be noted that, for $V = 1$, $k = 0$, i.e., the maximum contrast occurs when the intensity of the fringe pattern is zero. On the other hand, when $k = 1$, $V = 0$, and the intensity in the fringe pattern is a maximum. Then, for some value of k between 0 and 1 there is a compromise for the contrast and the intensity in the fringe pattern. In general, the assumptions used to derive Eq. (8) are not true, and k is some complex function of scattering angle and pit size. In order to determine a method for making scatter plates which give high contrast fringe patterns of reasonable intensity, an investigation of the contrast for scatter plates with various degrees of roughness was carried out. Each scatter plate was made by grinding one side of a small glass plate with a grinding powder with a certain maximum particle size and then polishing it to give a smooth surface with pits in it. For the present investigation, only scatter plates made in the abovementioned manner are considered because they are easy to make, they are permanent, and as will be seen they give adequate contrast. The scatter plates 30 were made with grinding powders with 12 μ, 16 μ, 23 μ, μ, and 60 μ particle sizes. Photographs of the fringe patterns obtained with each scatter plate were taken, and a Hilger and Watts Model L50 recording microphotometer was used to determine the maximum and minimum intensities in each case. The contrast in the fringe patterns went from 0.29 for the scatter plate made with 60-μ particles to 0.70 for the scatter plate made with the 12-μ particles. The pit density on the 12-μ scatter plate was about 8000 pit/mm^2 and the pit sizes ranged up to 12 μ as shown in Table I. In Fig. 5 are the contrast photographs for the 12-μ and the 60-μ scatter plates.

Summary

In the foregoing it has been shown that an interferometer using one scatter plate and a plane mirror is well

Table I. Distribution of Pit Sizes on Scatter Plates Made Using 12-μ Grinding Powder

Pit size	Relative population
12 μ	5%
10 μ	5%
8 μ	10%
5 μ	30%
3 μ or less	50%

suited for testing long focus optical systems. The first requirement of an instrument used to test long focus systems is that it should be insensitive to vibration. This is achieved automatically in the present system because of its +1 magnification. Also the +1 magnification makes it extremely easy to align the system for testing. In order to test nearly perfect optical systems, it must be possible to introduce straight fringes into the field so that small deviations of the test wavefront from a sphere can be observed. In the present system this is achieved in two ways. One method is to introduce a small tilt to a plane parallel plate which has been placed into one-half of the cone of light making up the reference and test wavefronts, thus introducing a tilt between the reference and test wavefronts. The other method is to separate the scatter plate and plane mirror a certain amount to introduce a given amount of astigmatism, which when viewed at either the sagittal or tangential focus gives straight fringes. The disadvantage with this method is that the fringe spacing is not constant (as it is in the first method); it decreases with increase in aperture, having a broad fringe in the center of the field. However, for very fast systems the amount of spherical aberration introduced by the plane-parallel plate in the first method is excessive and the second method is preferred. Since the test wavefront passes twice through the optical system being tested, the sensitivity of this system is twice that of the Twyman–Green interferometer for test mirrors. The major disadvantage with this method is that coma cannot be observed. However, this is not a disadvantage when testing mirrors. Finally, it has been shown that a contrast in the fringe pattern of 0.7 can be realized, if scatter plates are made by grinding one side of a glass plate with a grinding powder whose maximum particle size is 12 μ and polishing to give a smooth surface with pits in it.

References

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9. J. Dyson, Appendix B "Interferometers," in J. B. Strong, *Concepts of Classical Optics* (W. H. Freeman and Company, San Francisco, Calif., 1958), p. 377.
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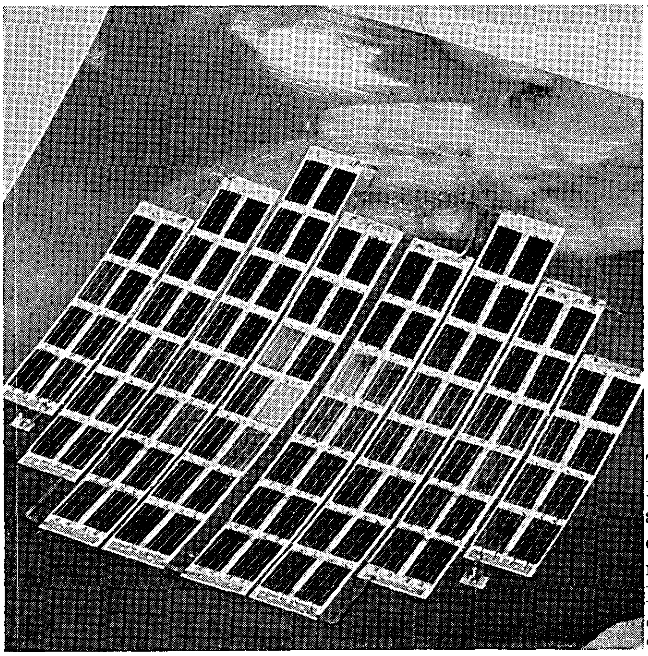
13. See, for example, C. Candler, *Modern Interferometers* (Hilger and Watts, London, 1951).



These film-mounted solar cells built by Electro-Optical Systems Inc., a subsidiary of Xerox Corporation, are only 0.1-mm thick and 60 mg in weight each compared to conventional silicon cell thicknesses of about 0.5 mm and weights in excess of 170 mg. Designed by Stephen Kaye and Perry Rolik of the EOS Semiconductor Department, the units were fabricated from diffused single cyrstal silicon and have silver titanium contacts. The panel shown is one of two polyimide plastic films containing 100 such cells produced by EOS under contract to G. T. Scheldhal & Company for electric power application aboard high altitude meteorological balloons. Efficiency per cell is 9% (at air mass one) compared to normal efficiencies (at same atmospheric density) of $12^1/_2$% to 13%. It is believed contact modification would improve cell performance and thus allow their possible use in deep space electric power systems. Their extreme light weight and flexibility, for example, could help make possible ultimate fabrication of spacecraft photovoltaic arrays as large as football fields.

Meetings Calendar *continued from page 59*

23–30 Internatl. Conf. on Luminescence, Budapest *G. Szigeti, Hungarian Academy of Sciences, P.O.B. Ujpest 1, no. 76, Budapest, Hungary*

29–Sept. 2 Symp. on Solar-Terrestrial Physics, Belgrade, *Dejan Bajic, ETAN/URSI, P.O.B. 356, Belgrade, Yugoslavia*

29–Sept. 2 Symp. on Applications of Photogrammetry for Surveying the Earth's Surface, Prague *Ing. L. Skiladal, Internatl. Soc. for Photogrammetry, Hybernska, Prague 1, Czechoslovakia*

31–Sept. 7 12th Gen. Assem. IUPAP, Berkeley, Calif.

September

? Internatl. Conf. on Magnetic Resonance and Relaxation, Ljubljana *R. Blinc, Nuklearni Institut Jozef Stefan, Ljubljana, Yugoslavia*

5–9 Internatl. Conf. on Semiconductor Physics, Tokyo *G. M. Hatoyama, Sony Research Lab., 174 Fujitsukacho, Hodogayaku, Yokohama, Japan*

5–9 2nd Internatl. Biophysics Congr., Vienna *A. Locker, Vienna Acad. of Medicine, Alserstr. 4, Vienna IV, Austria*

5–10 Internatl. Conf. on Magnetic Resonance and Relaxation, Ljubljana *Sec. XIV Colloque Ampere, Nuklearni Institute Jozef Stefan, Ljubljana, Yugoslavia*

6–9 17th Symp. on Molecular Structure and Spectroscopy, Columbus *H. H. Nielsen, Dept. of Physics and Astronomy, Ohio State U., Columbus, Ohio*

12–16 6th Internatl. Conf. on Microwave and Optical Generation and Amplification, Univ. of Cambridge *IEE, Savoy Pl., London, WC2, U.K.*

12–17 Internatl. Commission on Glass, Tokyo and Kyoto *M. Imaoka, Ceramic Assoc. of Japan, 7, Shiba-Nishikubo-Sakuragawa-cho, Minato-ku, Tokyo, Japan*

14–22 Symp. on Some Problems of Photointerpretation, Strasbourg *A. Bauer, 47 av. du Marechal Fayolle, Paris 16, France*

October

1–9 1st Internatl. Congr. for Photography and Film in Industry and Technology, Cologne *Kongressbüro, Deutsche Gesselschaft für Photographie, Neumarkt 49, 5 Koln, Germany*

3–7 1st Internatl. IMR Symp., Gaithersburg *R. G. Bates, Institute for Materials Research NBS, Washington, D.C. 20234*

9–15 IAF Congr., Madrid *P. Contensou, ONERA, Chatillonsous-Bagneux, Seine, France*

13–14 Canadian Symp. on Communications, Montreal *G. W. Farnell, McGill U., 805 Sherbrooke St. W., Montreal, Canada*

17–21 Internatl. Conf. on Electron Nuclear Hyperfine Interactions in Spectroscopy, Wellington, N.Z. *S. F. Duncan, Royal Soc., of N.Z., c/o Chem. Div., DSIR, Private Bag, Petone, Wellington, N.Z.*

17–22 Physicists Conf., Munich *K. H. Riewe, German Physical Soc., Heraeusstr. 12–14, 645 Hanau/Main, Germany*

19–21 **Optical Society of America, 51st Ann. Mtg., Jack Tar Hotel, San Francisco, Calif.** *M. E. Warga, OSA, 1155 16th St., N.W., Washington, D.C. 20036*

24–27 ISA Ann. Instrument-Automation Conf. & Exhibit, Coliseum, New York City *D. R. Stearn, ISA, Penn Sheraton Hotel, Pittsburgh 19, Pa.*

26–29 SPSE Colloq. on the Photographic Interaction Between Radiation and Matter, Marriott Motor Hotel, Wash., D.C. *SPSE, Main P.O. Box 1609, Washington, D.C.*

November

8–9 Soc. of Glass Technology, Jubilee Mtg., Sheffield *D. Hawksworth, Soc. of Glass Technology, Thornton, 20 Hallam Gate Road, Sheffield 10, U. K.*

15–18 12th Ann. Conf. on Magnetism and Magnetic Materials, Washington, D.C. *AIP, 335 E. 45th St., New York, N.Y. 10017*

December

26–31 133rd Ann. Mtg. AAAS, Washington, D.C. *AAAS, 1515 Mass. Ave. N.W., Washington, D.C. 20005*

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January

25–28 Ann. Mtg. American Physical Soc., New York *AIP, 335 E. 45th St., New York, N.Y. 10017*

Spring

? **Optical Society of America Spring Mtg., Columbus, Ohio** *M. E. Warga, OSA, 1155 16th St. N.W., Washington, D.C. 20036*

March

21–24 Internatl. Mtg. IEEE, New York *Box A, Lenox Hill Stn., New York, N.Y. 10021*

April

5–7 Internatl. Nonlinear Magnetics Conf., Washington, D.C. *IEEE, 345 E. 47th St., New York, N.Y. 10017*

16–21 101st Semiann. Conf. SMPTE, New York *Michael Denson, 9 E. 41st St., New York, N.Y. 10017*

May

30–June 6 SPSE Ann. Conf., LaSalle Hotel, Chicago, Ill. *SPSE, Main P.O. Box 1609, Washington, D.C.*

15–19 SPSE Ann. Conf. on Photographic Science and Engineering, Sherman House, Chicago *SPSE, Main P.O. Box 1609, Washington, D.C.*

June

19–23 13th Internatl. Spectroscopy Colloq., Carleton U., Ottawa *C. S. Joyce, Pulp and Paper Research Inst. of Canada, 570 St. John's Rd., Pointe Claire, Que., Canada*

20–24 Conf. of Oil and Colour Chemists' Assn., Scarborough *E. B. Hamblin, Wax Chandlers' Hall, Gresham St., London, EC2, U.K.*

20–28 Internatl. Comm. on Illumination, Plenary Mtg., 16th Quadrennial, Wash. D.C. *L. Barbrow, U.S. Natl. Com. on Illumination, NBS, Washington, D.C. 20234*

25–30 ASTM 70th Ann. Mtg., Statler-Hilton Hotel, Boston, Mass.

August

22–31 13th Gen. Assem. Internatl. Astron. Union, Prague

September

10–16 Internatl. Conf. on Magnetism, Cambridge *L. F. Bates, Dept. of Physics, The University, Nottingham, U.K.*

11–14 ISA Ann. Instrument-Automation Conf. & Exhibit, McCormick Place, Chicago, Ill. *D. R. Stearn, ISA, Penn Sheraton Hotel, Pittsburgh 19, Pa.*

October

11–13 **Optical Society of America, 52nd Ann. Mtg., Sheraton-Cadillac, Detroit, Mich.** *M. E. Warga, OSA, 1155 16th St., N.W., Washington, D.C. 20036*

15–20 102nd Semiann. Conf. SMPTE, Chicago *Michael Denson, 9 E. 41st St., New York, N.Y. 10017*

26–28 SPSE Symp., on Unconventional Photographic Systems, Marriott Motor Hotel, Wash., D.C. *SPSE, Main P.O. Box 1609, Wash., D.C.*

1968

March or April

? **Optical Society of America Spring Mtg., Wash., D.C.** *M. E. Warga, OSA, 1155 16th St., N.W., Washington, D.C. 20036*

June

10–14 SPSE Ann. Conf., Sheraton-Boston, Boston, Mass. *SPSE, Main P.O. Box 1609, Wash., D.C.*

23–28 ASTM 71st Ann. Mtg. and 18th Materials Testing Exhibit, Statler-Hilton Hotel, San Francisco, Calif.

October

8–11 **Optical Society of America, 53rd Ann. Mtg., Pittsburgh Hilton Hotel, Pittsburgh, Pa.** *M. E. Warga, OSA, 1155 16th St., N.W., Washington, D.C. 20036*

28–31 ISA Ann. Instrument-Automation Conf. & Exhibit, Coliseum, New York, N.Y. *D. R. Stearn, ISA, Penn Sheraton Hotel, Pittsburgh 19, Pa.*

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