# Direct Phase Measurement

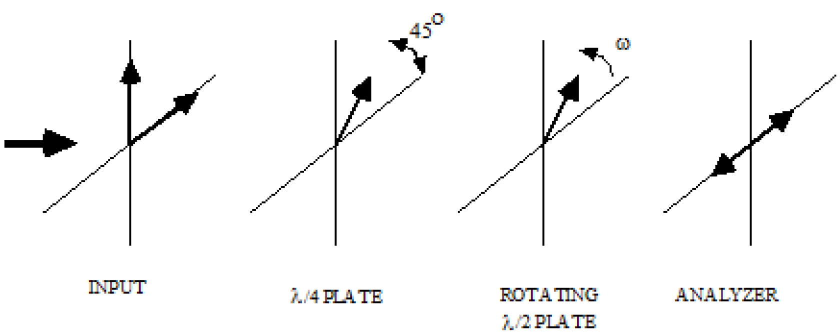
## DP-1

Phase shifting interferometry is used with a Twyman-Green interferometer to measure a spherical mirror. Both the reference beam and the beam incident upon the spherical mirror are collimated beams. The wavelength is 633 nm. Let the detector be a 1024 element linear detector array with a pixel spacing of 6 microns. Assuming unit magnification between the spherical mirror and the detector, what is the minimum radius of curvature spherical mirror that can be measured such that there are no $2\pi$ phase discontinuities in the measurements?

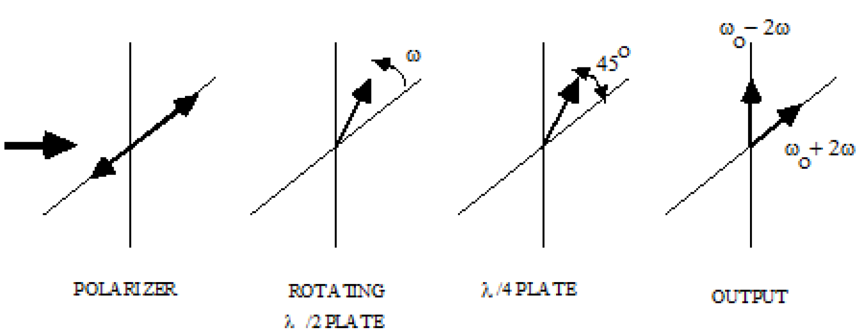
## DP-2

Use Jones calculus to show the following:

a) The output of an interferometer can be made to vary sinusoidally in time by using a stationary $\lambda/4$ plate at $45^{o}$, a rotating $\lambda/2$ plate at frequency $\omega$, and an analyzer. When the test and reference beams exit the interferometer, they are linearly polarized in orthogonal directions.

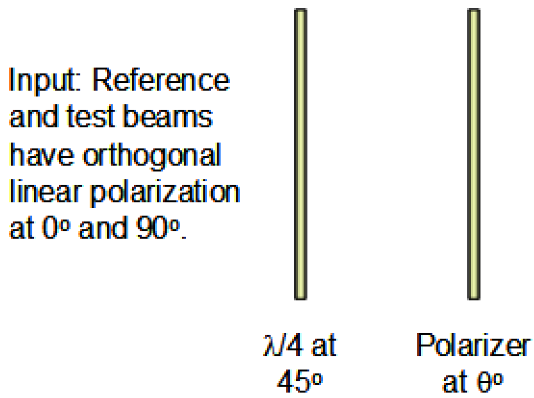



b) A $\lambda/2$ plate rotating at frequency $\omega$ followed by a stationary $\lambda/4$ plate with its axis at 45° with respect to the direction of the original polarization converts a linearly polarized beam into two linearly polarized beams of orthogonal polarization which differ in optical frequency by an amount $4\omega$.



# DP-3

The figure below shows a diagram of the phase shifter as it could be used in the output of an interferometer where the reference and test beams have orthogonal linear polarization. A quarter-wave plate converts one of the two interfering beams into right-handed circularly polarized beam and the second interfering beam into a left-handed circularly polarized beam. Show that as a polarizer is rotated an angle $\theta$ the phase difference between the test and reference beams changes by $2\theta$. The polarizer also makes it possible for the two beams to interfere.




# DP-4

In phase-shifting interferometry, the general expression for the intensity of an interferogram with a phase step $\delta$ is $i[x, y, \delta] = i_o[x, y]\,(1 + \gamma\,\mathrm{Cos}[\phi[x, y] + \delta[t]])$.
Rather than stepping the phase in discrete steps, we will integrate the intensity as the phase is linearly changed through a phase interval $\Delta$.

a) Determine $I_i$[x,y] by integrating over phase values of $\delta_i - \Delta/2 < \delta < \delta_i + \Delta/2$. Make sure to include a normalization factor which keeps the average integrated signal for each interferogram independent of $\Delta$.

b) What is the effect of integrating the interferogram intensity while the phase is changing?

c) Write out the equations for the three bucket technique with $\Delta = \pi/2$,and $\delta_i = \pi/4,\ 3\pi/4$,and $5\pi/4$.

# DP-5

The 4 step technique described by Carré is used to measure phase. If between each readout of the intensity the phase of the reference beam is changed by an amount $2\alpha$ the 4 irradiance measurements are given by

$$A[x,y] = I_1 + I_2\,\mathrm{Cos}[\phi[x,y] - 3\alpha]$$
$$B[x,y] = I_1 + I_2\,\mathrm{Cos}[\phi[x,y] - \alpha]$$
$$C[x,y] = I_1 + I_2\,\mathrm{Cos}[\phi[x,y] + \alpha]$$
$$D[x,y] = I_1 + I_2\,\mathrm{Cos}[\phi[x,y] + 3\alpha]$$

Show that tan $\alpha$ and tan $\phi$ are given by

$$\mathrm{Tan}[\alpha] = \sqrt{\frac{3\,(B[x,y] - C[x,y]) - (A[x,y] - D[x,y])}{(B[x,y] - C[x,y]) + (A[x,y] - D[x,y])}}$$

$$\mathrm{Tan}[\phi[x,y]] = \left(\frac{(A[x,y] - D[x,y]) + (B[x,y] - C[x,y])}{(B[x,y] + C[x,y]) - (A[x,y] + D[x,y])}\right)\mathrm{Tan}[\alpha]$$

How would you determine $\phi$(x,y) modulo $2\pi$?

## DP-6

The three integrating bucket technique described in class is used to measure the phase distribution across an interferogram. The interferometer is a Twyman-Green with a helium neon laser as the light source. The detector used contains 100 X 100 detector elements. The detector is read out at a rate of $10^6$ detector elements per second. In the following assume that the detector can be read out continuously with no dead time.

a) Let the phase shifter be a rotating 1/4 wave plate in the reference arm of the interferometer. How many revolutions per second must the 1/4 wave plate rotate?

b) Let the phase shifter be a moving mirror mounted on a piezoelectric transducer. How fast must the mirror be moving during the taking of the data? Give velocity in units of microns per second.

## DP-7

a) Give 3 main sources of error for each conventional interferometry and phase-shifting interferometry.

b) Why are at least 3 separate measurements required in phase-shifting interferometry?

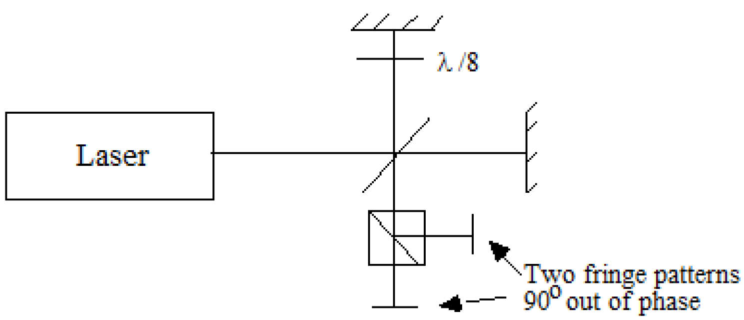
## DP-8

a) Give three advantages of phase-shifting interferometry over simply measuring fringe centers.

b) Phase stepping interferometry is used with a Twyman-Green interferometer to measure a nearly flat mirror. Let the detector be a 1024 element linear detector array. In units of waves/radius, what is the maximum wavefront slope that can be measured such that there are no $2\pi$ phase discontinuities in the measurements?

## DP-9

I have a Twyman-Green interferometer whose light source is a linearly polarized laser. I am told that if I place a l/8 wave plate in one arm of the interferometer, and a polarization beamsplitter in the output beam I obtain two fringe patterns $90^o$ out of phase. Is this right or wrong? Why might I want to obtain two interference patterns $90^o$ out of phase?




## DP-10

I am using phase shifting interferometry to test an optical system. Unfortunately, I am getting a stray reflection off the diverger lens used in the interferometer.

a) Let the stray reflection have 1/25 the intensity of the test beam, the phase of the test beam be $60^{\text{o}}$, and the phase of the stray reflection be $90^{o}$. How much phase error will the stray reflection introduce into the measurement of the test beam?

b) What phase difference between test beam and stray reflection will introduce the maximum error into the phase measurement?

## DP-11

a) Show that if $\alpha$ is the phase step between consecutive frames in a phase shifting interferometer, and $i_0, i_1, i_2, i_3$, and $i_4$ are the intensities of the five measured frames, the phase step $\alpha$ can be obtained from the expression

$$\frac{i_4 - i_0}{i_3 - i_1} = 2\,\mathrm{Cos}[\alpha]$$

b) I am using a direct phase measurement interferometer to test a coated spherical mirror. The diverger lens reflects 1% of the incident light. Give an estimate as to how large an error in measurement of the phase can be introduced by this extraneous reflected light. Other than putting an AR coating on the lens, how would you reduce the error?

## DP-12

In the three-step phase-shifting technique for measuring phase where the phase is shifted $\frac{\pi}{2}$ between the first and second measurement and another $\frac{\pi}{2}$ between the second and third measurement the common equation for $\phi$ is of the form

$$\phi = \mathrm{ArcTan}\left[\frac{i_1 + i_3 - 2\,i_2}{i_1 - i_3}\right];$$

a) Show that the above equation is correct.

b) Show that we can get a similar equation for $\phi$ if instead of measuring $\phi$ we measure $\phi \pm \frac{\pi}{4}$. (You tell me whether it is $\frac{+\pi}{4}$ or $\frac{-\pi}{4}$.) There is still a $\frac{\pi}{2}$ phase shift between consecutive intensity measurements.

c) How will this $\frac{\pi}{4}$ phase shift influence our measurement of the phase distribution across the exit pupil of the system under test?

## DP-13

Four-bucket phase-shifting interferometry is being used to test an optical system. Unfortunately, the detector has some linearity such that if the intensity incident upon the detector is I, the response of the detector goes as $I + \epsilon I^2$. If the phase being measured is $\phi$, how much error is introduced into the phase measurement?

## DP-14

In phase shifting interferometry the measured phase values at six consecutive data points are 0°, 150°, 300°, 90°, 240°, and 30°. We know that for the particular sample being measured the phase difference between consecutive data points must be less than 180°. What are the phase values after correcting for the $2\pi$ ambiguities arising from using the arc tangent function?

## DP-15

The zero crossing technique is used in a Twyman-Green interferometer to measure a mirror. The frequency difference between the two interfering beams is 1000 Hz.

a) Sketch the setup showing one possible method for producing the frequency difference.

b) If the time difference between the zero crossings of the output of two detectors is 0.0003 seconds, what is the corresponding surface height difference on the sample?

## DP-16

Phase-shifting interferometry is used with a Twyman-Green interferometer to measure a nearly spherical mirror that has a small aspheric height variation that goes as $A\,\rho^4$, where $0 \le \rho \le 1$. Let the detector be a 1024 x 1024 element CCD having a pixel spacing of 9 microns. The wavelength is 633 nm.

a) How large can A, the coefficient of the $\rho^4$ term, be such that there are no $2\,\pi$ phase discontinuities in the measurements if conventional phase-shifting techniques are used? You should adjust the interferometer to maximize A. State any assumptions being made.

b) Repeat part b) for the case where the detector is a 1024 x 1024 element CCD having a pixel spacing of 12 microns. State any assumptions being made.

## DP-17

A 20X Nomarski interference microscope contains a Wollaston prism of unknown material and prism angle. It is known that interference fringes having a spacing of 5 mm are obtained if the Wollaston prism is illuminated with a linearly polarized collimated HeNe laser beam and a properly oriented analyzer is placed after the prism. One method for obtaining the phase shift required for a phase-shifting interferometer is to translate the Wollaston prism. How far does the Wollaston prism need to be translated to obtain a $90^o$ phase shift between the two interfering beams?

## DP-18

A system using a Zeeman split laser producing two orthogonally polarized beams having a difference frequency of 100 MHz as the light source and up-down counters is used to measure the motion of one mirror in Twyman-Green interferometer. The wavelength is approximately 633 nm.

a) Sketch the system showing all important components.

b) If the electronics works well for frequencies between 80 and 120 MHz, how fast can we move the mirror in the Twyman-Green interferometer?

## DP-19

Briefly describe the spatial synchronous method of direct phase measurement. Be sure to give the major advantage and the major disadvantage compared to phase-shifting interferometry.

## DP-20

The Fourier Method is used for direct phase measurement interferometry for testing a mirror whose surface departs from a spherical surface by an amount $8\lambda\,\rho^4$, where $0 \le \rho \le 1$. Remember that in the Fourier method a single interferogram having a large amount of tilt is used and the irradiance falling on the detector array is Fourier transformed and filtered to obtain the wavefront. For simplicity we will assume the detector array consists of an array of point detectors.

a) What is the minimum amount of tilt in units of fringes/radius required to perform the test in order to separate the orders if we are allowed to introduce defocus?

b) What is the minimum number of point detectors needed in the two-dimensional array used to detect the interferogram?

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# DP-21

A concave spherical mirror is measured in a phase-shifting laser-based Fizeau interferometer made for the testing of flats. The diameter of the spherical mirror is 10 cm. A 1024 x 1024 element CCD array having a 5 micron pixel separation is used in the interferometer. The wavelength is 633 nm.

a) Assuming we want to test the entire mirror surface, what is the shortest radius of curvature mirror that we can measure? State any assumptions you are making.

b) Repeat part a) for an 8 micron pixel separation.