



THE UNIVERSITY
OF ARIZONA.



College of Optical Sciences

5.7 Multiple Wavelength and White Light Phase-Shifting Interferometry



Introduction

- One of the largest advantages of optical interferometry is also one of its largest disadvantages. Due to the short wavelength of the light, the sensitivity of a single wavelength interferometric measurement is very high, but also due to the short wavelength of the light the dynamic range is limited unless additional information is available.
- In a single wavelength interferometric measurement the phase of the interference pattern, repeats itself at distance intervals equal to the wavelength. Thus, an optical path difference of δ gives the same interferometric measurement as $(n \lambda + \delta)$, where n is an integer.



Two Solutions to Problem – Solution I

- If a white light source is used in an interferometer the best contrast interference fringes are obtained only when the two paths in the interferometer are equal.
- Thus, if an interferometer is made such the path length of the sample arm of the interferometer is varied, distances or height variations across the sample can be determined by looking at the mirror or sample positions for which the fringe contrast is a maximum.
- In this measurement there are no height ambiguities and since in a properly adjusted interferometer the sample is in focus when the maximum fringe contrast is obtained, there are no focus errors in the measurement of surface microstructure.
- This solution will be discussed in Section 5.8.



Two Solutions to Problem – Solution II

- A second solution is to perform the measurement at more than one wavelength and compare the measurement results for the different wavelengths to determine the true distance.
- By combining two-wavelength interferometry with phase-shifting techniques very powerful commercial interference microscopes have been made.
- We will discuss three ways of performing two-wavelength interferometry.



Technique 1 - Calculated Distances Equal

Let ϕ be a fractional fringe and n and Δ be integers. Then

$$OPD = (n + \phi)\lambda$$

and

$$n\lambda_1 + \phi[\lambda_1]\lambda_1 = (n + \Delta)\lambda_2 + \phi[\lambda_2]\lambda_2$$

$$n = \frac{-\Delta\lambda_2 + \lambda_1\phi[\lambda_1] - \lambda_2\phi[\lambda_2]}{\lambda_2 - \lambda_1}$$

We must find Δ such that n is an integer, to within some tolerance.

Accuracy is lowest of the three techniques discussed because any small errors in $\phi[\lambda_1]$ and $\phi[\lambda_2]$ are magnified in the calculation of n .



Technique 2 - Fractional Fringes

- The second technique described is called exact fractions first described by Michelson and Benoit in 1895.
- A length is measured by measuring the excess fractional fringes for two or more wavelengths. It is assumed that the length is known to within a few wavelengths. For our example assume it is known that the length lies between 1209 and 1215 wavelengths for red light.
- Let the excess fraction in red light be known to be 0.35. Then we know the actual length is 1209.35, 1210.35, ..., 1214.35. From this information the number of wavelengths of green light is calculated.
- A comparison of the calculated number of fractional fringes for green light with the measured number shows which of the calculated lengths is correct.
- Additional wavelengths can be used to increase the accuracy and reduce the chances of error.

Technique 3 - Equivalent Wavelength Interferometry



If we take the difference between the phases for the two wavelengths we have

$$\text{phase}\lambda_2 - \text{phase}\lambda_1 = 2\pi \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) OPD$$

It is convenient to write this as

$$\text{phase}\lambda_2 - \text{phase}\lambda_1 = \frac{2\pi}{\lambda_{eq}} OPD$$

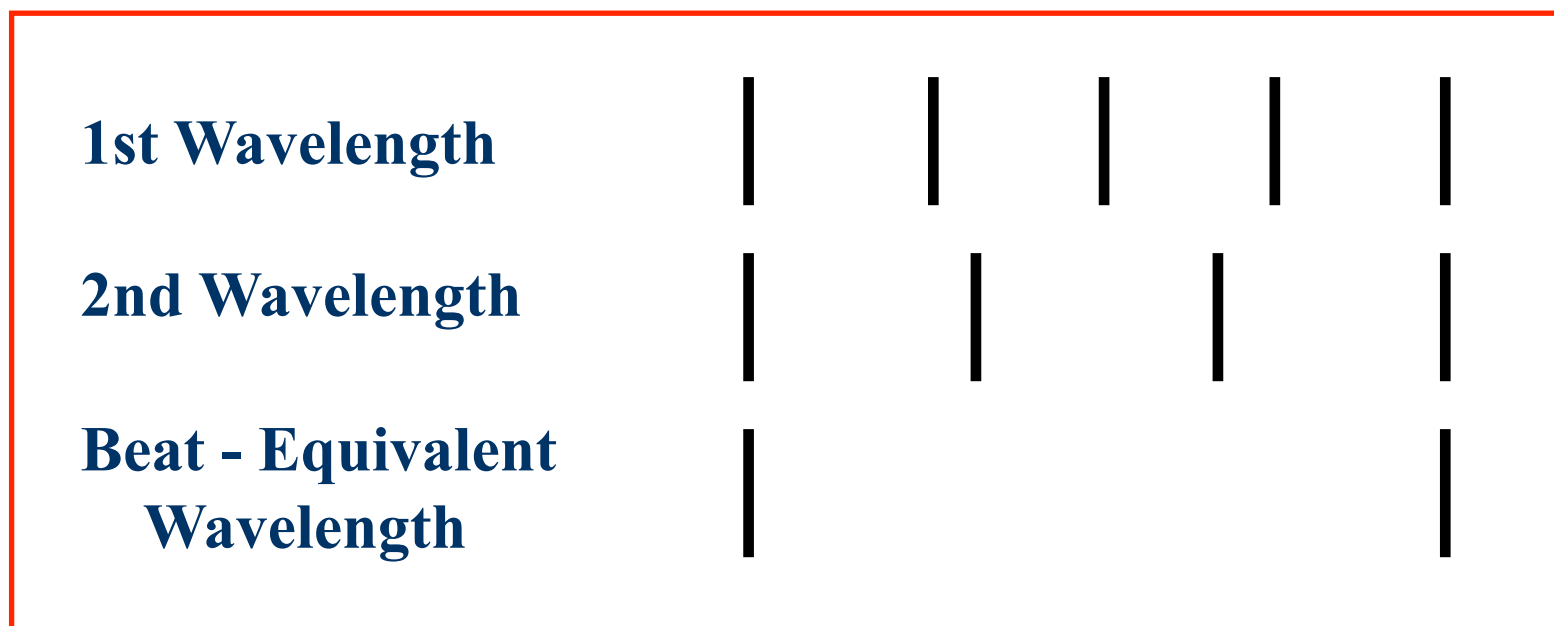
Where the equivalent wavelength is given by

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{\text{Abs}[\lambda_2 - \lambda_1]}$$



Two Wavelength Measurement

- Measure Beat Frequency
- Long Effective Wavelength





Two Wavelength Calculation

Equivalent Wavelength $\lambda_{eq} = \frac{\lambda_1 \lambda_2}{Abs[\lambda_2 - \lambda_1]}$

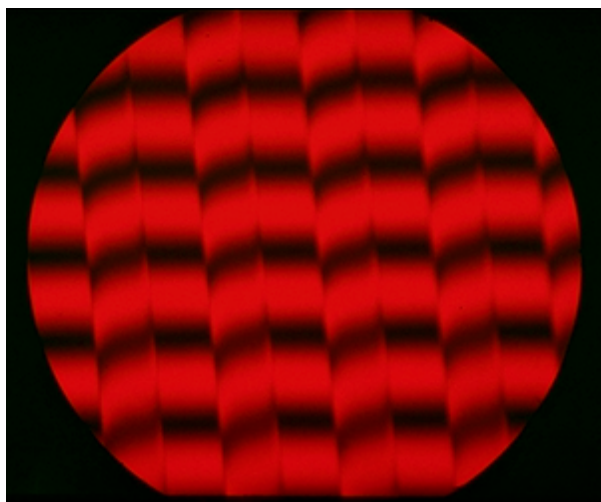
Equivalent Phase $\Delta\varphi_{eq} = \Delta\varphi_2 - \Delta\varphi_1$

**No height ambiguities as long as height difference
between adjacent detector pixels < equivalent
wavelength / 4**

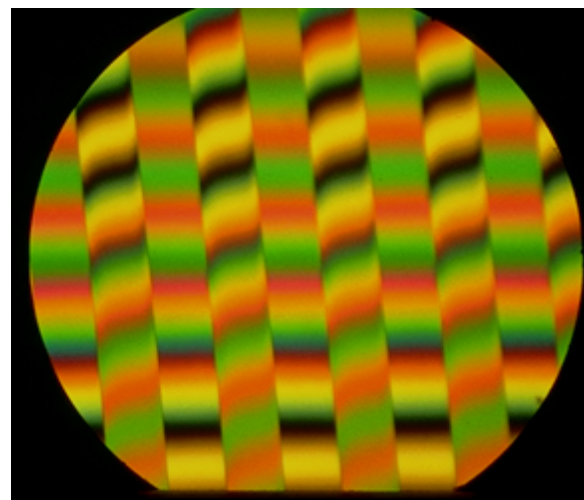


Interferograms of Diffraction Grating

Quasi-Monochromatic Light



White Light



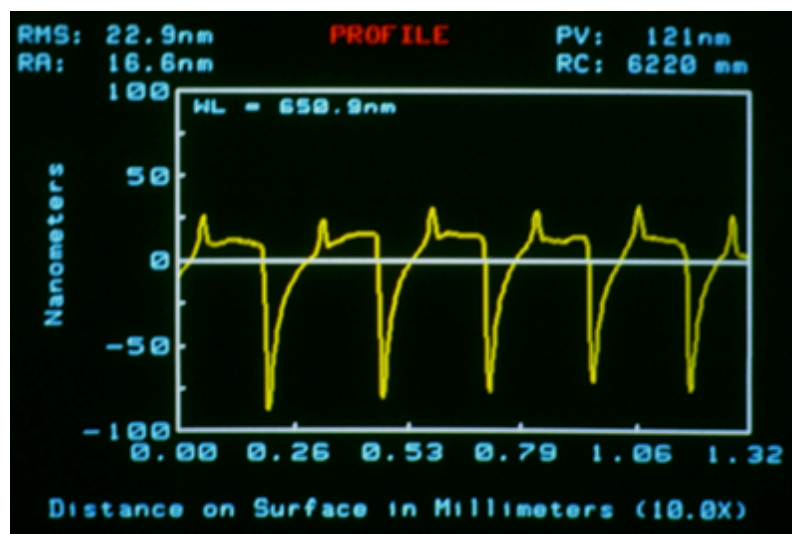
Profile



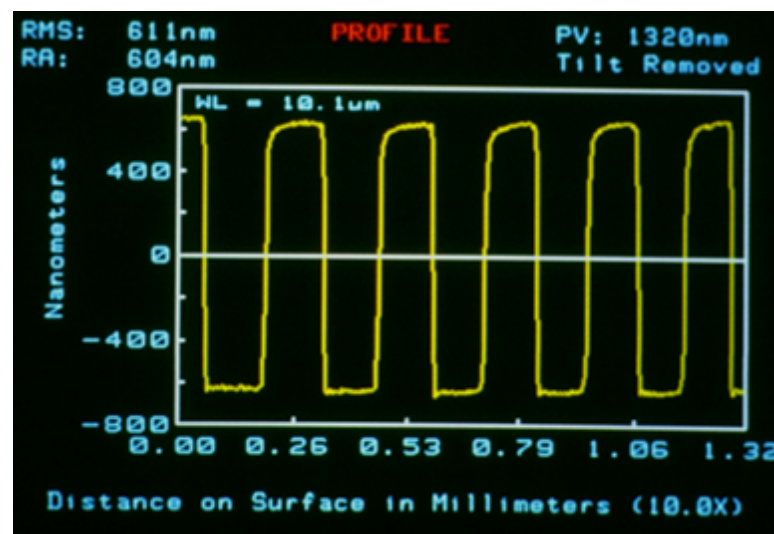


Diffraction Grating Measurement

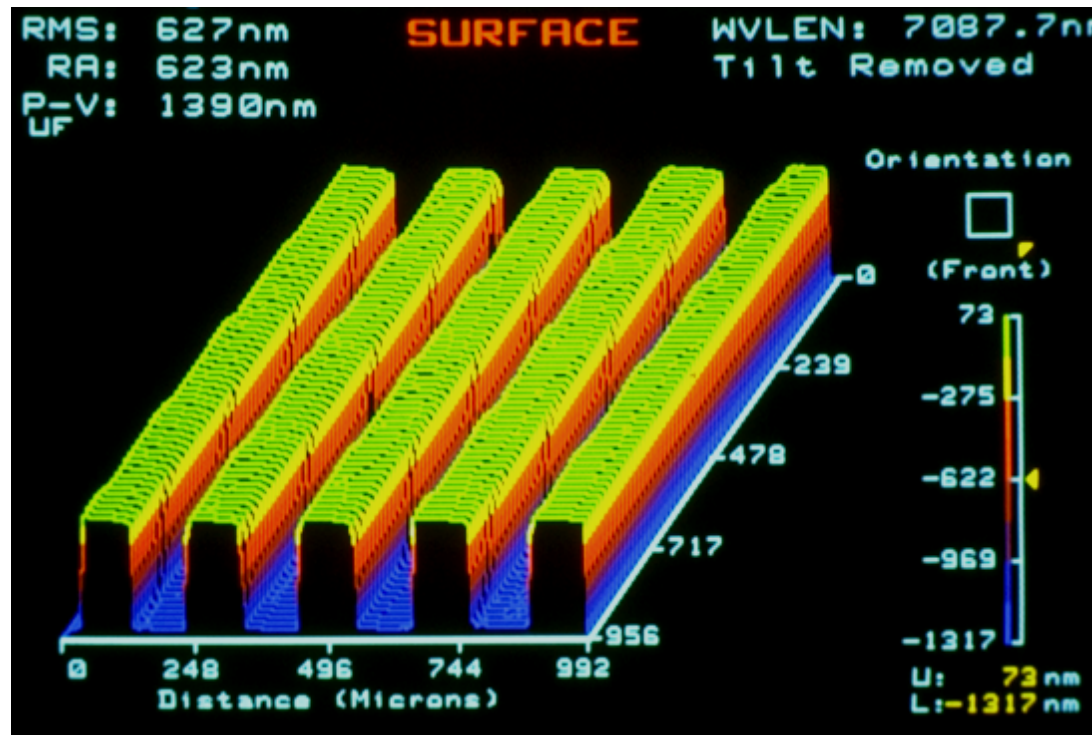
Single wavelength (650 nm)



Equivalent wavelength (10.1 microns)

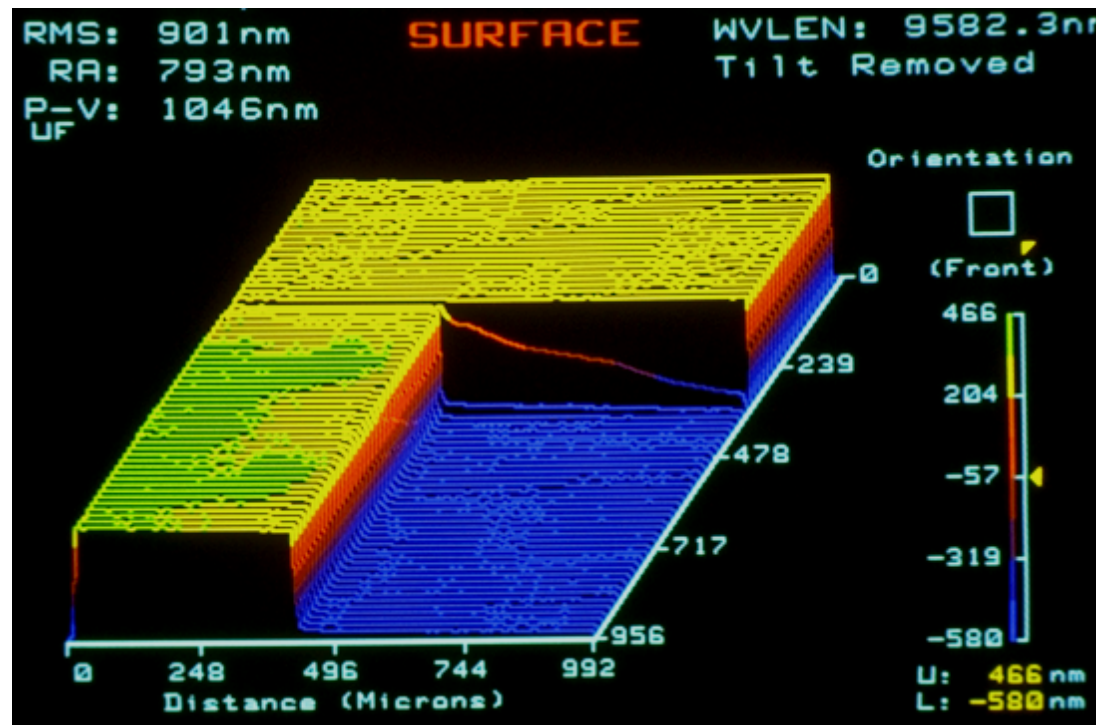


3-D Two-Wavelength Measurement (Equivalent Wavelength, 7 microns)





Two-Wavelength Measurement of Step

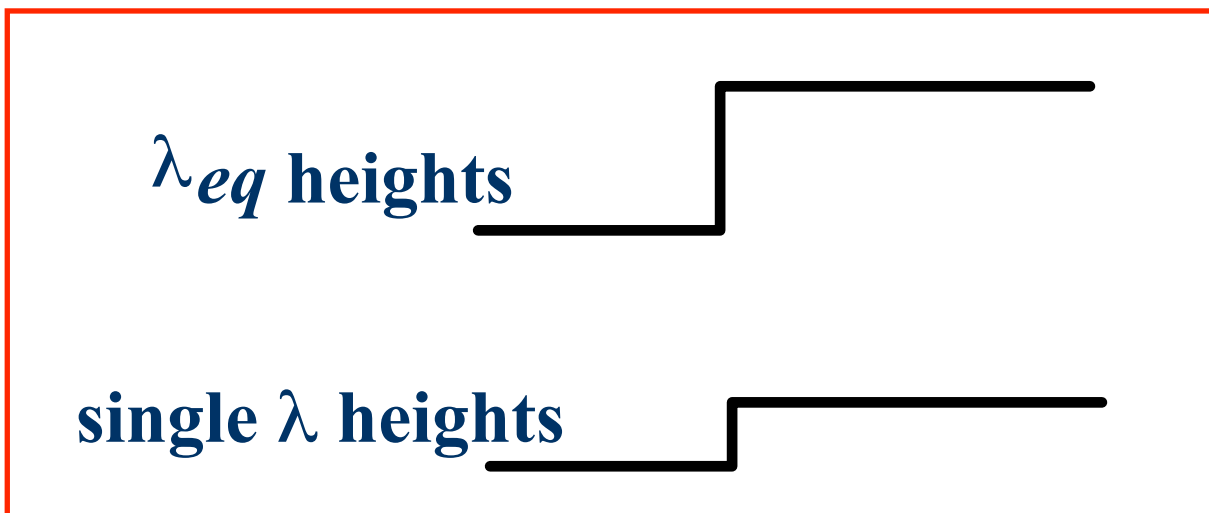




Wavelength Correction

Compare

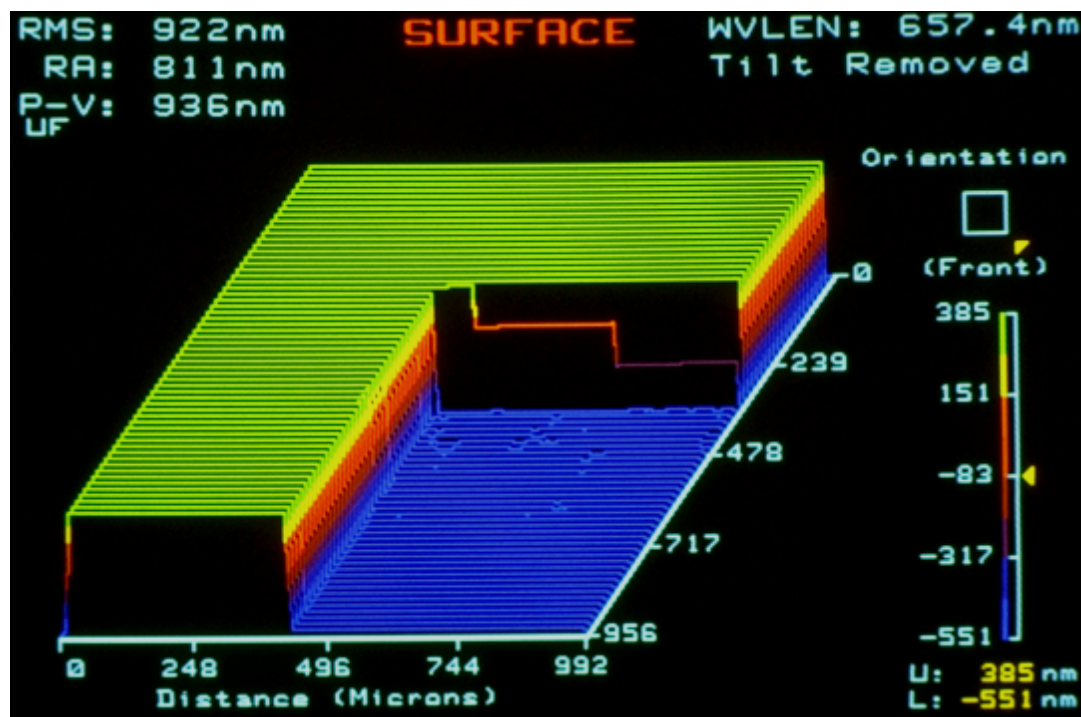
- Heights calculated using equivalent wavelength
- Heights calculated using single wavelength



Add $N \times \lambda/2$ to heights calculated using single wavelength so difference $< \lambda/4$



Wavelength Correction Measurement of Step



Two-Wavelength Phase-Shifting Interferometry – Subtract Phases



2 Wavelengths can be used to extend dynamic range.

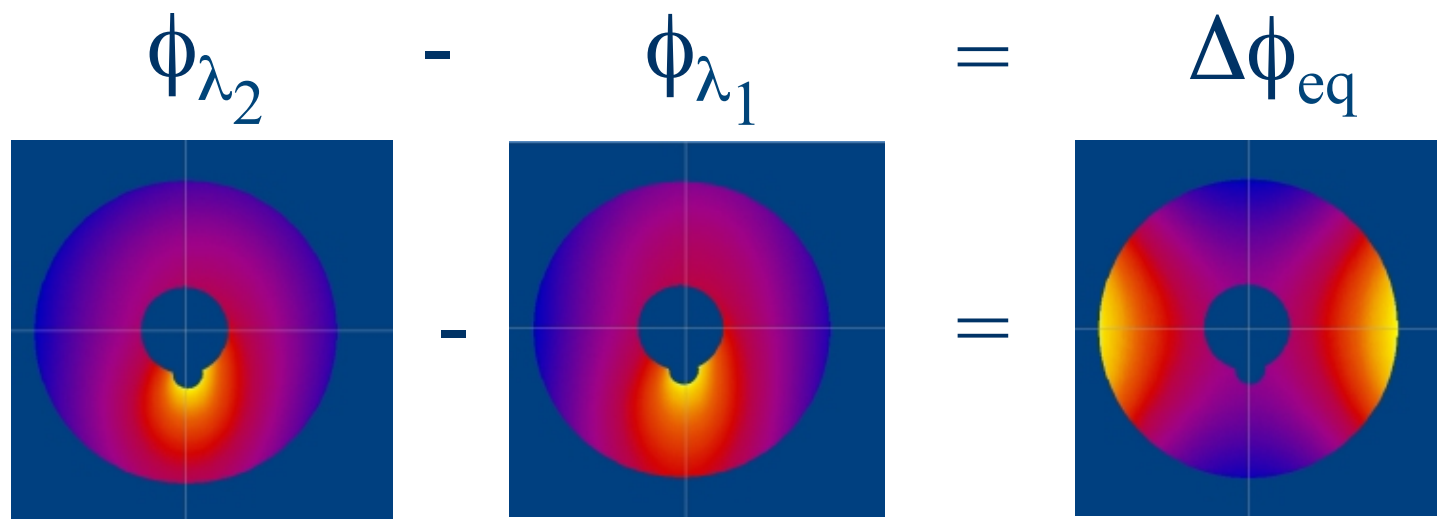
A measurement is made at each wavelength and the phases are subtracted.

$$\Delta\varphi_{eq} = \Delta\varphi_2 - \Delta\varphi_1$$

$$\Delta opd = \frac{\Delta\varphi_{eq}}{2\pi} \lambda_{eq} \qquad \lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}$$



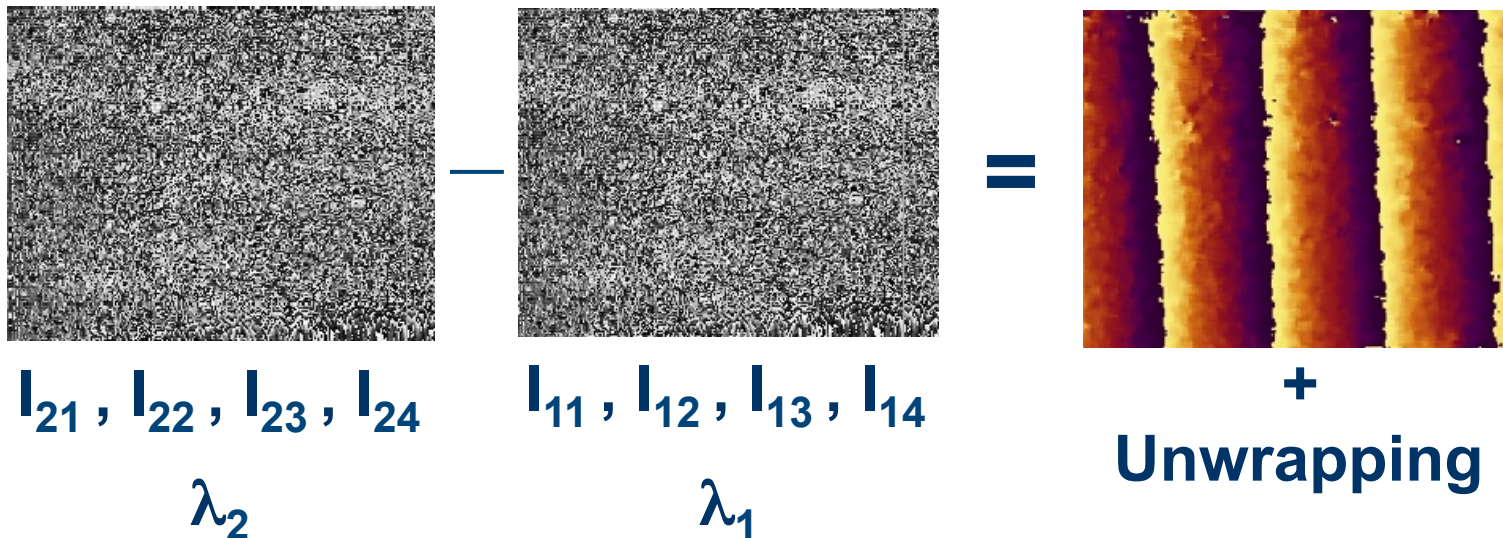
Subtract Phases



Two-Wavelength Phase-Shifting Interferometry – Subtract Intensities

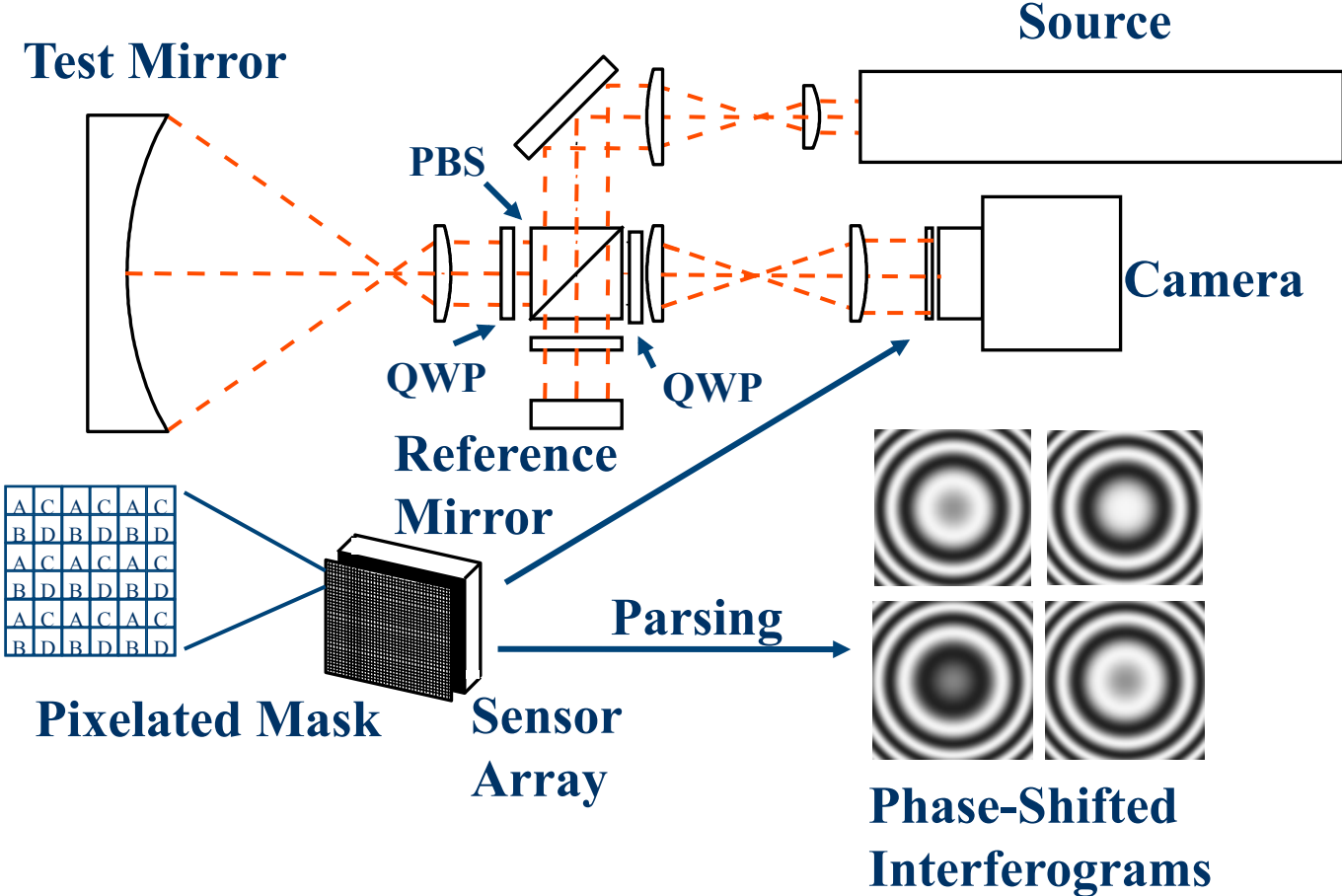


Stetson - 8 frame phase-difference

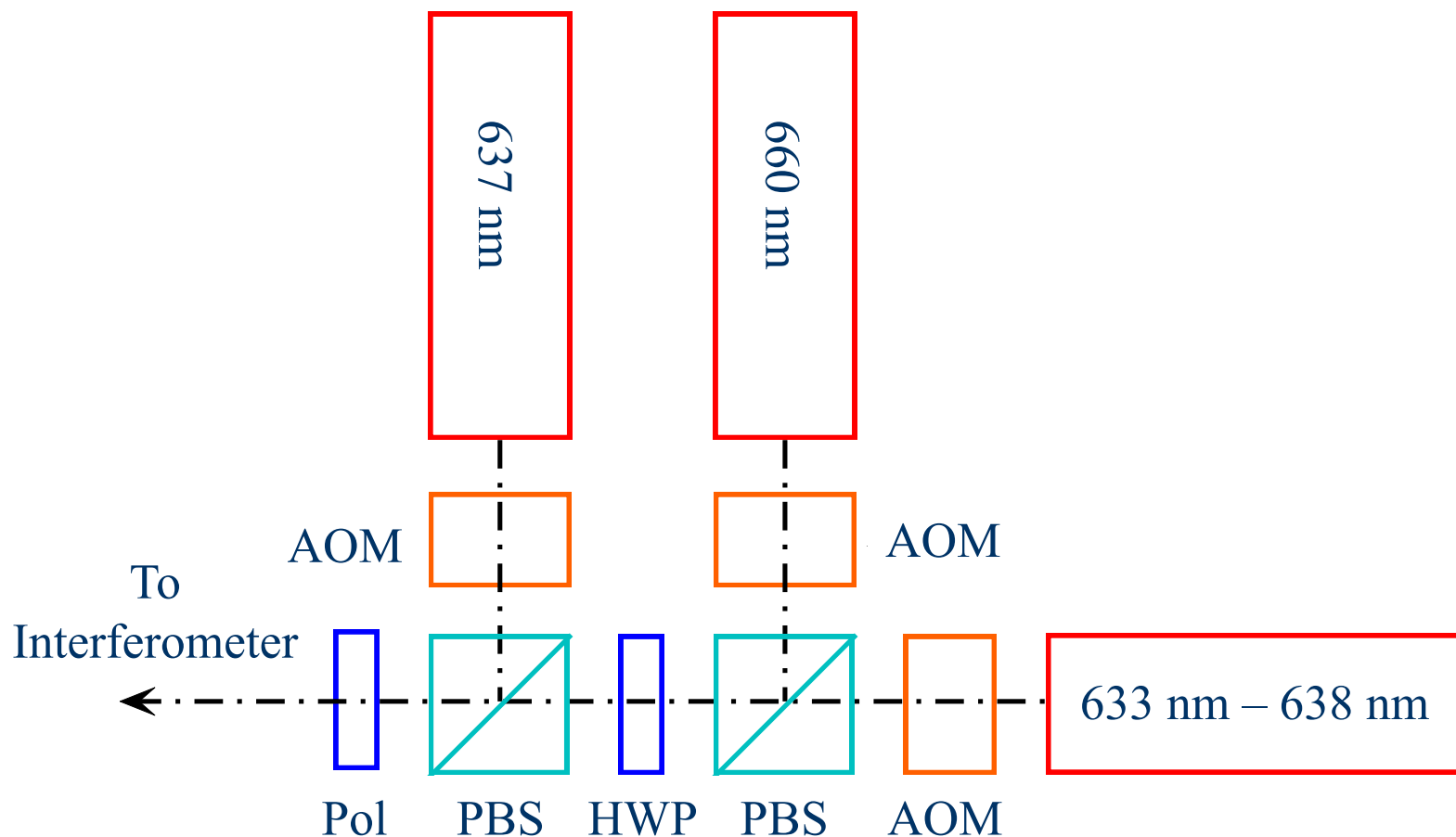


$$\Delta OPD = \frac{\lambda_{eq}}{2\pi} \text{ArcTan} \left(\frac{[I_{23} - I_{21}][I_{14} - I_{12}] + [I_{11} - I_{13}][I_{24} - I_{22}]}{[I_{11} - I_{13}][I_{21} - I_{23}] + [I_{14} - I_{12}][I_{24} - I_{22}]} \right)$$

Pixelated Polarizer Array Phase Sensor Dynamic Interferometer Configuration

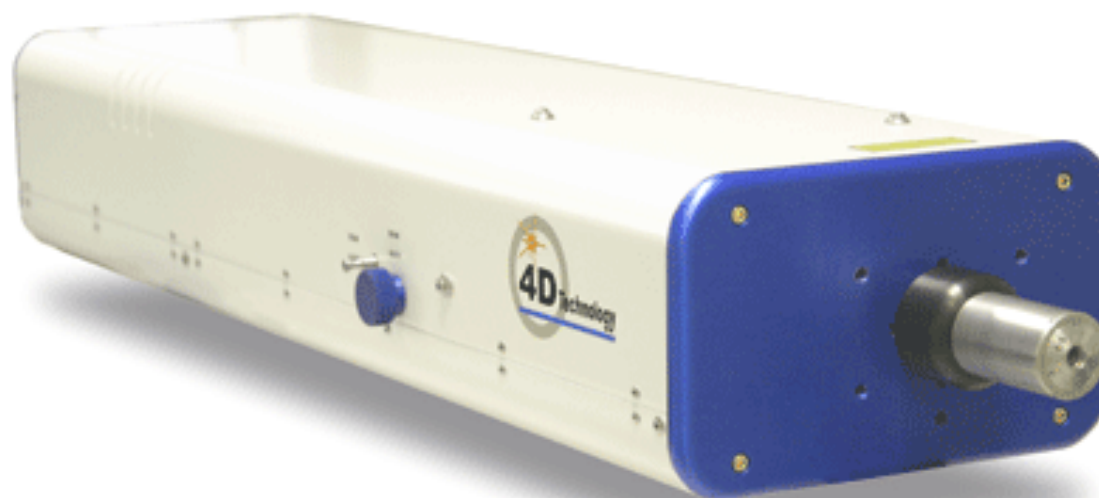


Polarization Facilitates Relatively Efficient Coupling of Sources



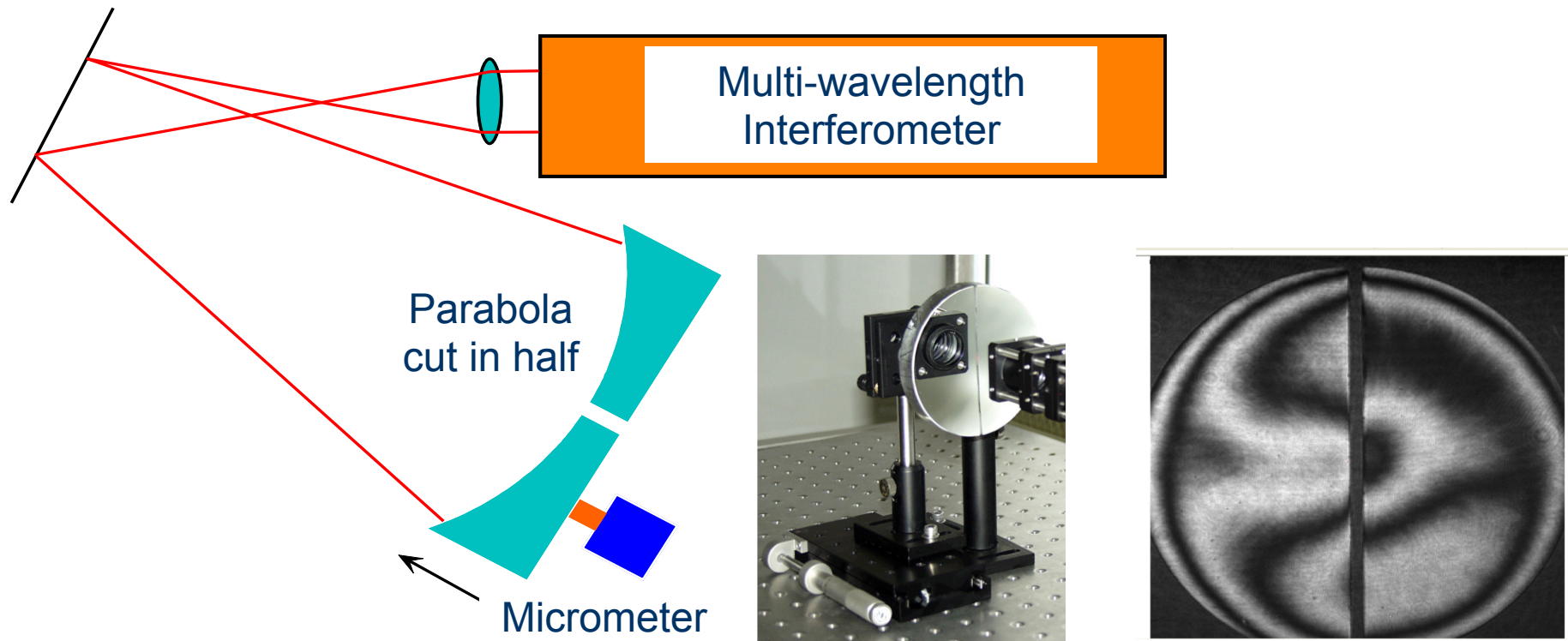


Multi-Wavelength PhaseCam



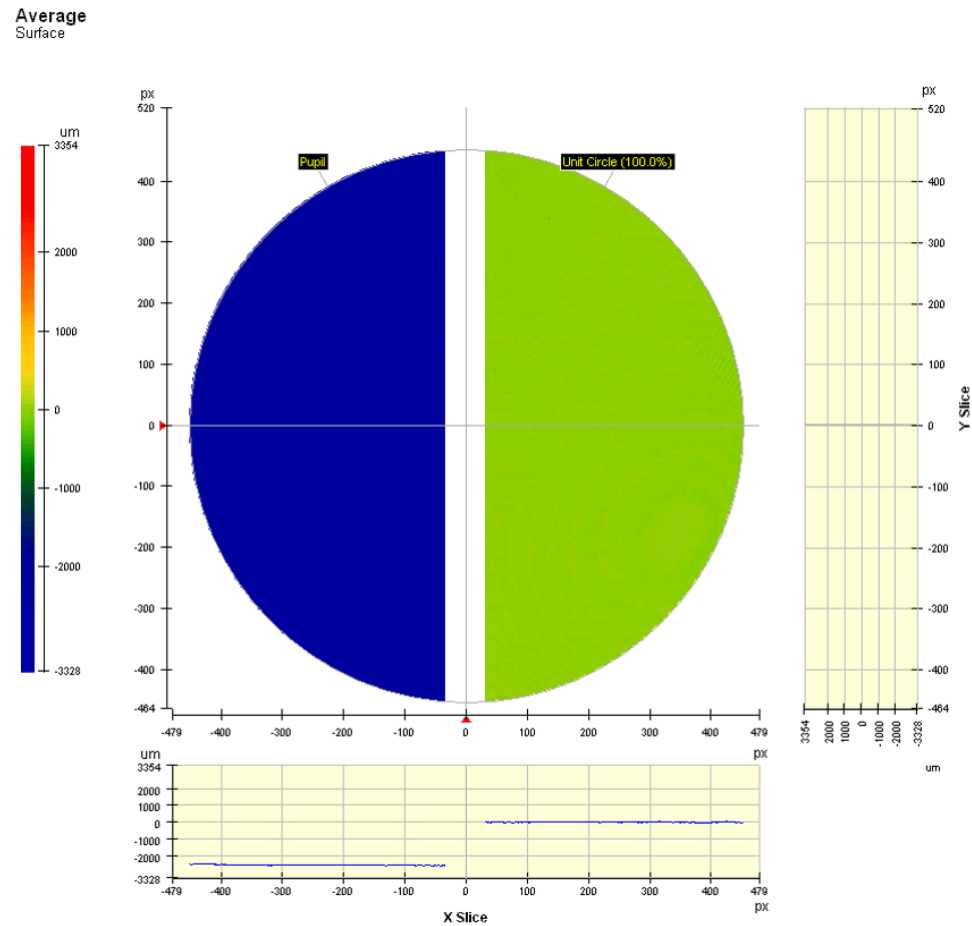
- Fundamental at 637 nm
- Acquisition Time $< 100\mu\text{s}$
- 3 Laser Sources, Two Fixed, One Tunable
- Synthetic Wavelengths from $18\ \mu\text{m}$ to 10 mm

Correlation of Step Height Measurement with Micrometer

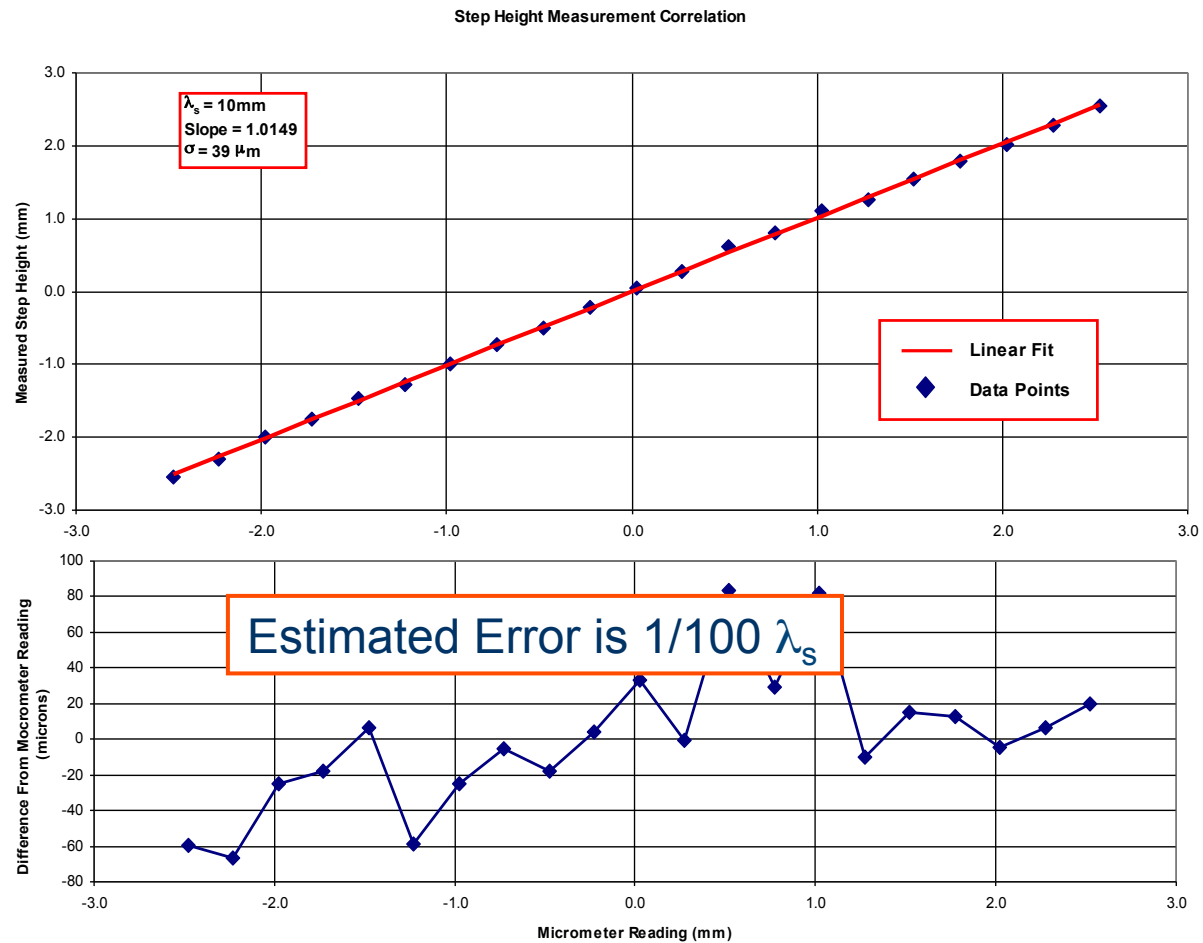


Measurements of two separated halves of a parabola were used to correlate step height measurements with a micrometer.

Correlation of Step Height Measurement with Micrometer



Correlation of Step Height Measurement with Micrometer





Conclusions

- **Several techniques exist for increasing the dynamic range of an interferometric test by using two or more wavelengths while still keeping the accuracy of a single wavelength.**
- **While these techniques have been known for some time, their usefulness depend upon the low noise, high accuracy, measurements made possible using modern electronics, computers, and software.**