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College of Optical Sciences

5.0 Direct Phase Measurement Interferometry

5.0 Direct-Phase Measurement Interferometry

- 5.1 Introduction
- 5.2 Zero-Crossing Technique
- 5.3 Phase-Lock Interferometry
- 5.4 Up-Down Counters
- 5.5 Phase-Stepping and Phase-Shifting Interferometry
- 5.6 Phase-Shifting Non-Destructive Testing
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- 5.8 Vertical Scanning (Coherence Probe) Techniques

5.1 Introduction

- For the last several years there has been much interest in electronic or digital techniques for measuring the phase distribution across an interference fringe pattern.

- Principal reasons for this interest
  - High phase measurement accuracy
  - Rapid measurement
  - Fast and convenient way of getting the interference fringe data into a computer so the fringe data can be properly analyzed.

5.2 Zero-Crossing Technique

- In the zero-crossing technique a clock starts when the reference signal passes through zero, and stops when the test signal passes through zero.
- The ratio of the time the clock runs to the period of the signal gives the phase difference between the two signals.
- In practice, the sinusoidal signals are greatly amplified to yield a square wave to improve the zero-crossing detection.
- The phase measurement is performed modulo 2π.

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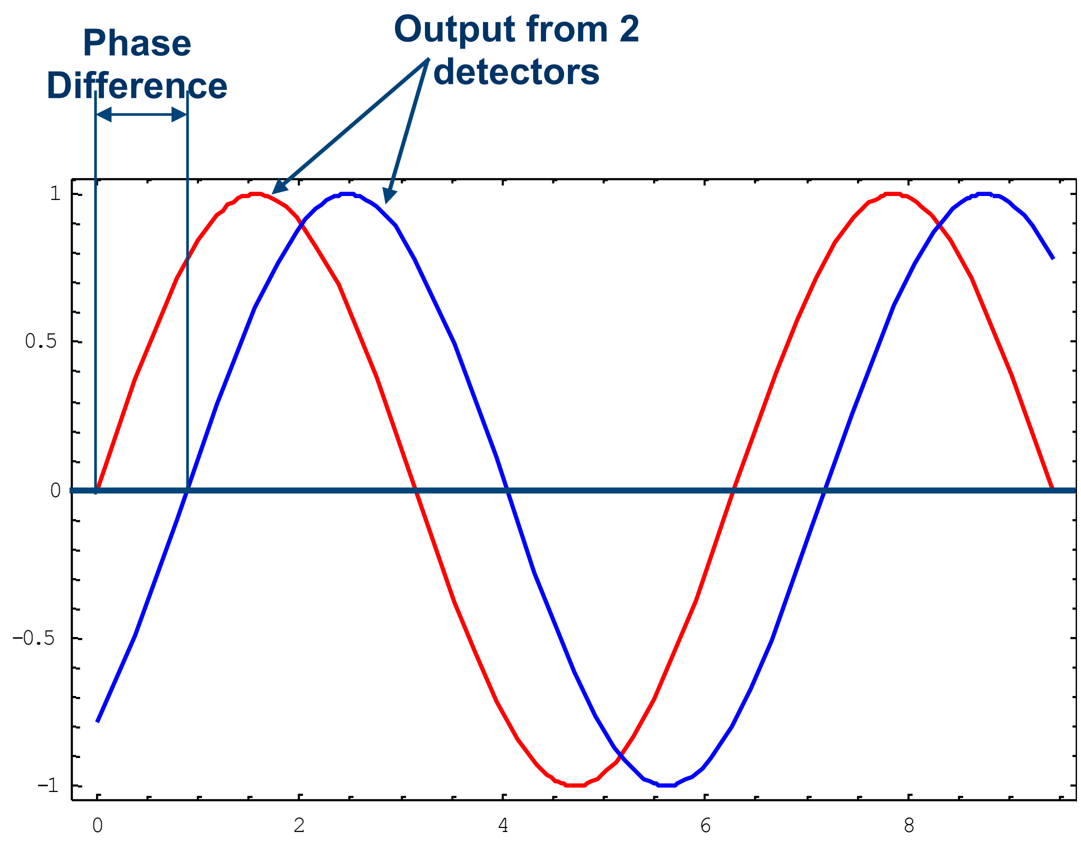
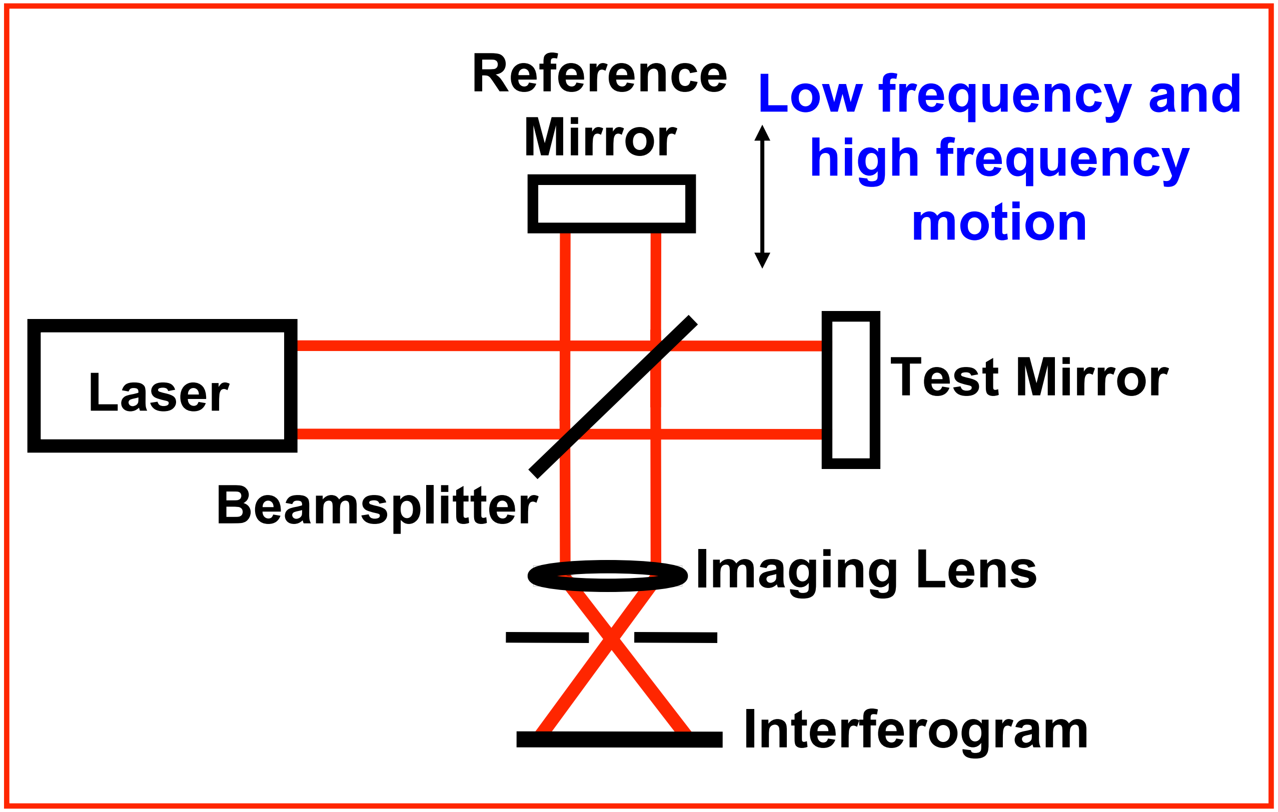
Zero Crossing



5.3 Phase Lock Interferometry



Phase-Lock Technique

High frequency dither $\delta[t] = aSin[\omega t]$

The detected signal can be written as

$$I[x,y,t] = I_1[x,y] + I_2[x,y] + 2\sqrt{I_1[x,y]I_2[x,y]}Cos[\phi[x,y] + aSin[\omega t]]$$

Expanding the cosine yields

$$I[x,y,t] = I_1[x,y] + I_2[x,y] + 2\sqrt{I_1[x,y]I_2[x,y]}$$
$$\left(Cos[aSin[\omega t]]Cos[\phi[x,y]] - Sin[aSin[\omega t]]Sin[\phi[x,y]]\right)$$

This can be written as

$$I[x,y,t] = I_1[x,y] + I_2[x,y] +$$
$$2\sqrt{I_1[x,y]I_2[x,y]}\begin{pmatrix} Cos[\phi[x,y]]\left(J_0[a] + 2J_2[a]Cos[2\omega t] + \cdots\right) \\ -Sin[\phi[x,y]]\left(2J_1[a]Sin[\omega t] + 2J_3[a]Sin[3\omega t] + \cdots\right) \end{pmatrix}$$

Use low-frequency phase shifter to make $\phi[x,y] = n\pi$, then $Sin[\phi[x,y]]=0$ and amplitude of signal at fundamental dither frequency =0.

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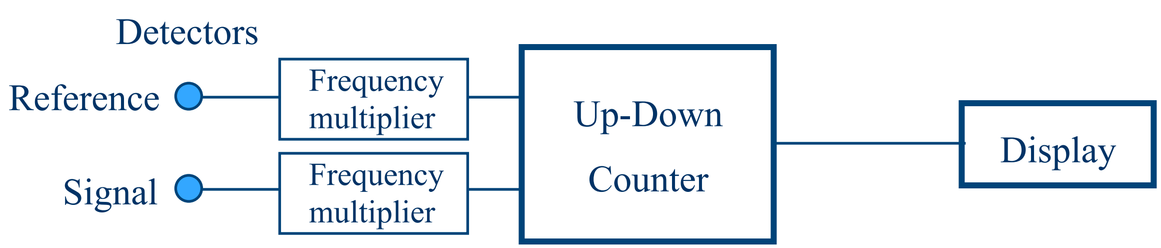
5.4 Up-Down Counters

- Most phase measurement techniques have the disadvantage that they measure the phase modulo 2π.
- Up-down counters technique does not have this disadvantage.
- Disadvantage that a signal loss at any time during the measurement will disrupt the phase measurement.
- Technique measures only changes in phase, so if the goal is to measure the phase distribution across a pupil, the detector must be scanned; a detector array cannot be used.

Up-Down Counters

To get sub-wavelength accuracy



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Basic idea of the use of up-down counters for phase measurement

- The output of the light detector is connected to the up terminal of an up-down counter.
- A reference signal having the same frequency as the difference between the two interfering light beams is connected to the down terminal of the up-down counter. Reference signal could be derived for example from a stationary detector observing the interference.

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Basic idea of the use of up-down counters for phase measurement

- When the sinusoidal signal goes positive, the up-down counter changes by one count. If both the reference and test signal see the same frequency, the output of the up-down counter will be zero.

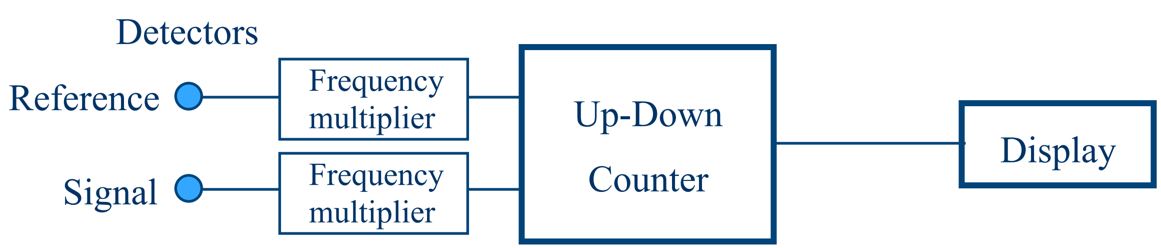
- If the test signal frequency increases, which would result when the test detector scans through fringes, the up-down counter will give an output signal equal to the number of fringes the test detector scans through.

Obtaining Sub-Fringe Accuracy

- Frequency multipliers are placed before the up-down counter.
- If a frequency multiplication of N is used, then 1/N fringe-measurement capability is obtained.
- Generally a phase-lock loop with a divide-by-N counter in the feedback loop is used as the frequency multiplier.

Up-Down Counters

To get sub-wavelength accuracy



5.5 Phase-Stepping and Phase-Shifting Interferometry

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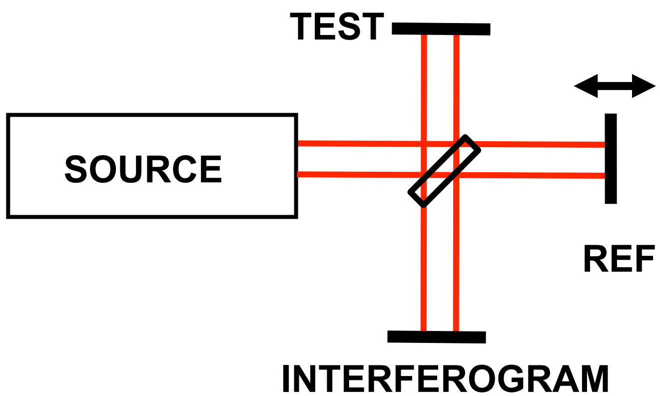
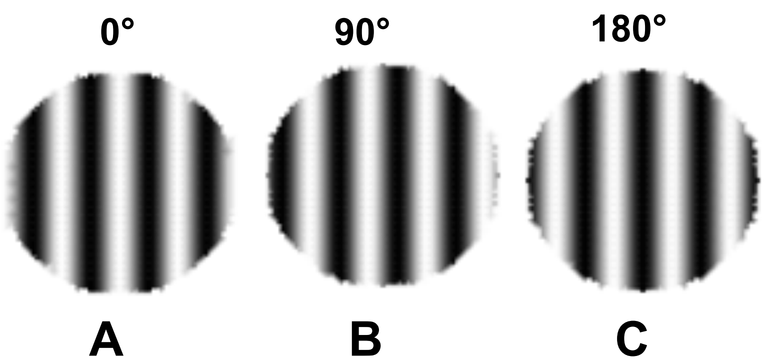
5.5.1 Phase-Stepping and Phase-Shifting Interferometry - Introduction

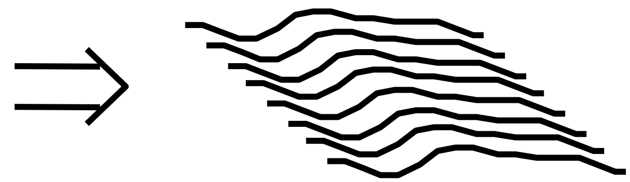


1) MODULATE PHASE

2) RECORD MIN 3 FRAMES

3) CALCULATE OPD





$$OPD = \frac{\lambda}{2\pi} \, TAN^{-1}\left[\frac{C - B}{A - B}\right]$$

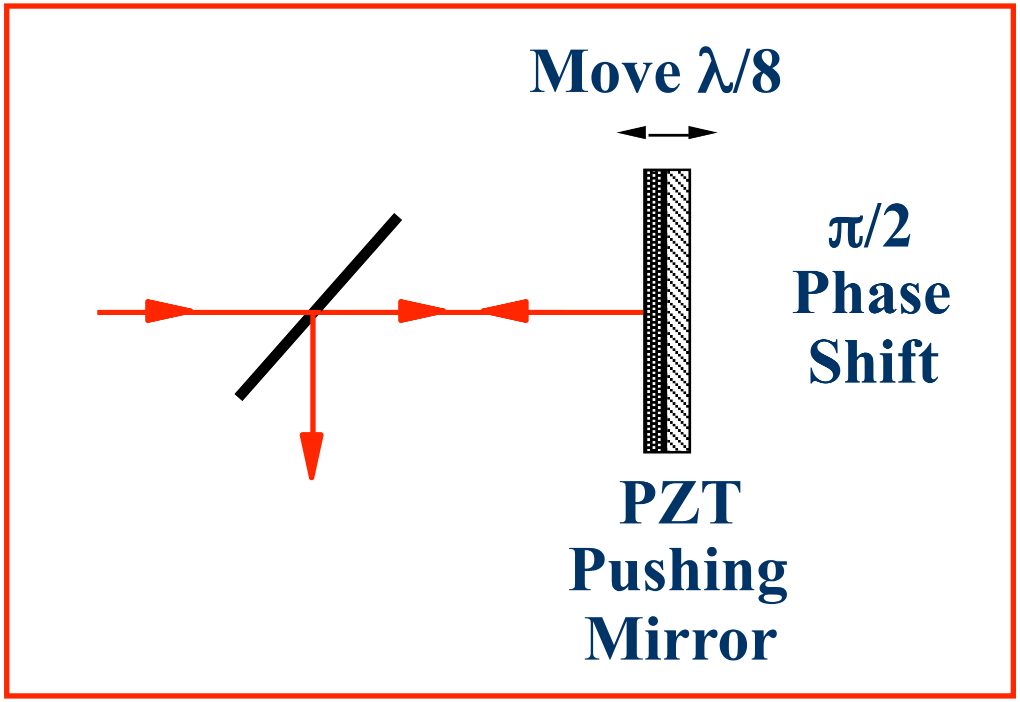
Advantages of Phase-Shifting Interferometry

- High measurement accuracy (>1/1000 fringe, fringe following only 1/10 fringe)
- Rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil
- Phase obtained at fixed grid of points
- Easy to use with large solid-state detector arrays

5.5.2 Phase Shifters

- 5.5.2.1 Moving Mirror
- 5.5.2.2 Diffraction Grating
- 5.5.2.3 Bragg Cell
- 5.5.2.4 Polarization Phase Shifters
  - 5.5.2.4.1 Rotating Half-Wave Plate
  - 5.5.2.4.2 Rotating Polarizer in Circularly Polarized Beam
- 5.5.2.5 Zeeman Laser
- 5.5.2.6 Frequency Shifting Source

5.5.2.1 Phase-Shifting - Moving Mirror



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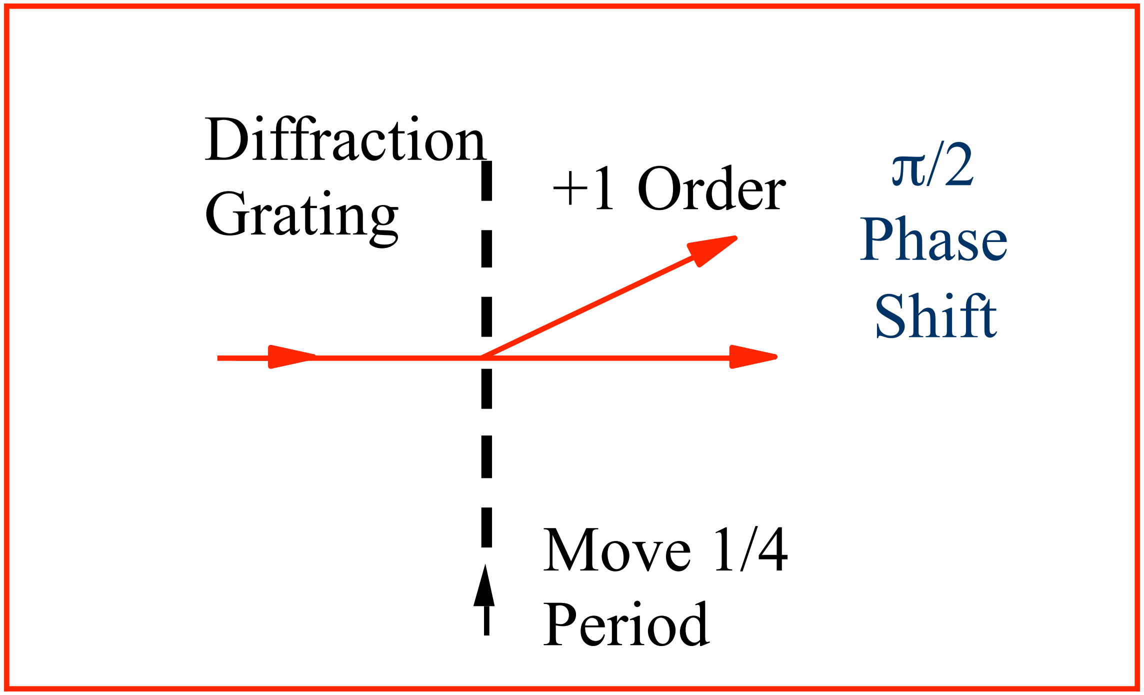
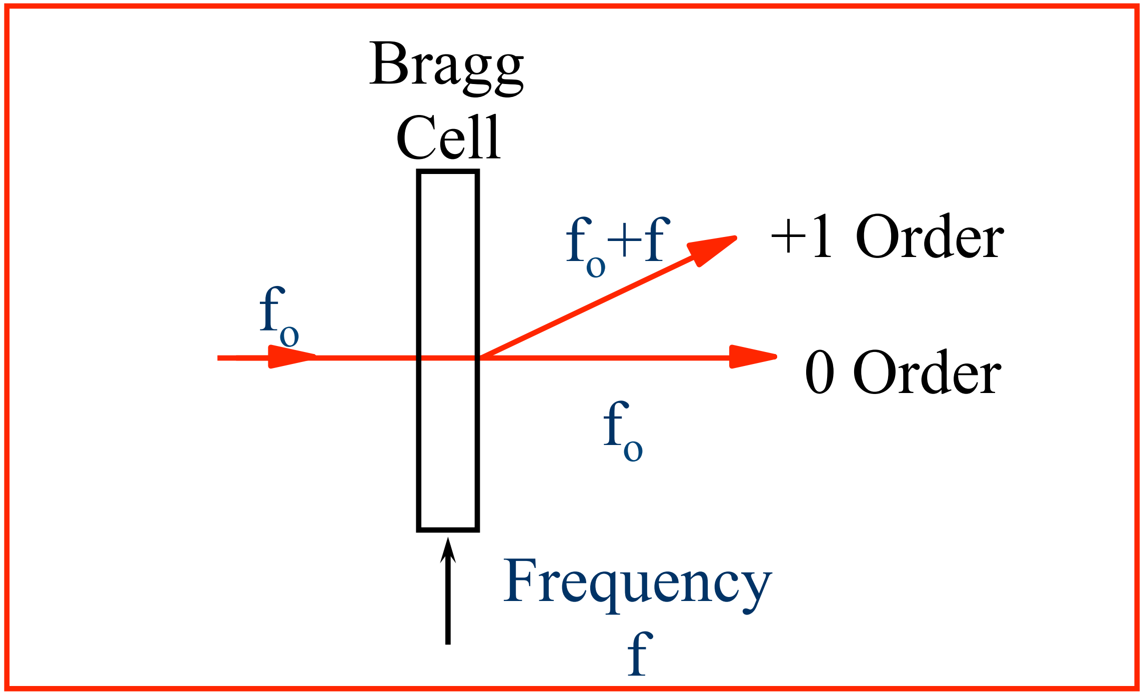
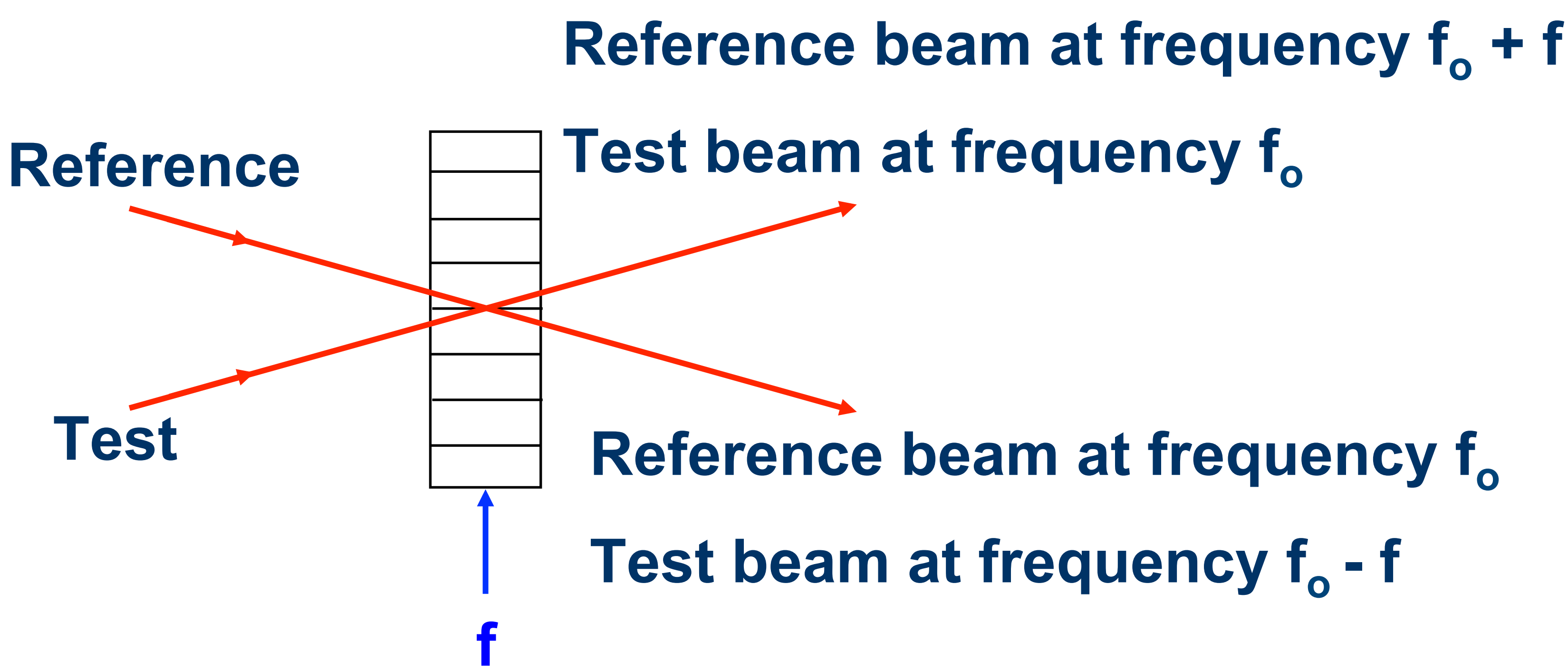
5.5.2.2 Phase Shifting - Diffraction Grating



5.5.2.3 Phase Shifting - Bragg Cell



Bragg Cell


Reference beam at frequency $f_o + f$
Test beam at frequency f_o
Reference
Test
Reference beam at frequency f_o
Test beam at frequency $f_o - f$
f

5.5.2.4 Polarization Phase Shifters

- There are polarization techniques for phase-shifting that introduce a phase-shift that depends little on the wavelength of the light. These phase-shifters are often called geometric phase shifters. In these notes we will discuss two geometric phase shifters.
  - Rotating half-wave plate
  - Rotating polarizer

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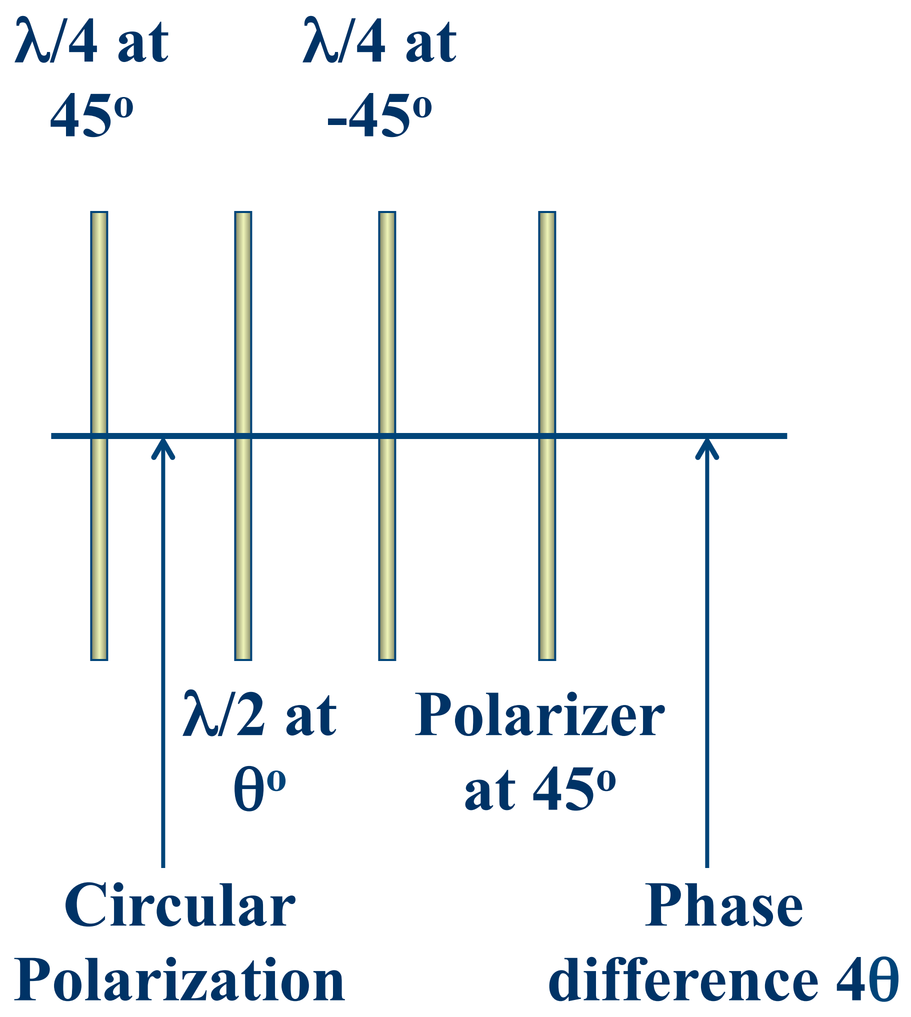
5.5.2.4.1 Phase Shifting - Rotating Half-Wave Plate

Input: Reference and test beams have orthogonal linear polarization at 0º and 90º.



Phase Shifting - Rotating Half-Wave Plate

$$input = \begin{pmatrix} ae^{i\phi} \\ b \end{pmatrix}$$

$$output = lpp45 \cdot rrot\left[\frac{\pi}{2}, \frac{-\pi}{4}\right] \cdot rrot[\pi, \theta] \cdot rrot\left[\frac{\pi}{2}, \frac{\pi}{4}\right] \cdot input$$

$$\text{intensity} = (output)(Conjugate[output])$$

$$= \frac{1}{2}\left(a^2 + b^2 - 2abCos[4\theta - \phi]\right)$$

As half-wave plate is rotated angle θ the phase difference between test and reference beams changes by 4θ.

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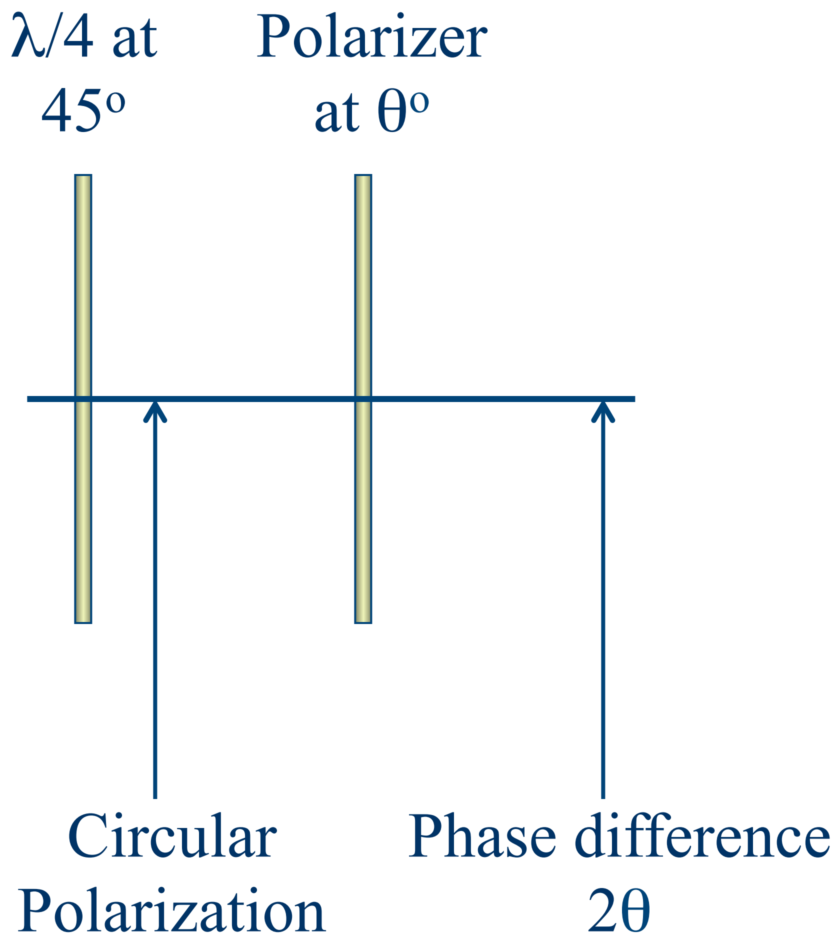
5.5.2.4.2 Rotating Polarizer in Circularly Polarized Beam

Input: Reference and test beams have orthogonal linear polarization at 0^o and 90^o.



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Phase Shifting - Rotating Polarizer

$$input = \begin{pmatrix} ae^{i\phi} \\ b \end{pmatrix}$$

$$output = rot[-\theta] \cdot hlp \cdot rot[\theta] \cdot rrot\left[\frac{\pi}{2}, \frac{\pi}{4}\right] \cdot input$$

$$\begin{aligned} \text{intensity} &= (output)(Conjugate[output]) \\ &= \frac{1}{2}\left(a^2 + b^2 + 2abSin[2\theta - \phi]\right) \end{aligned}$$

As polarizer is rotated an angle θ the phase difference between the test and reference beams changes by 2θ.

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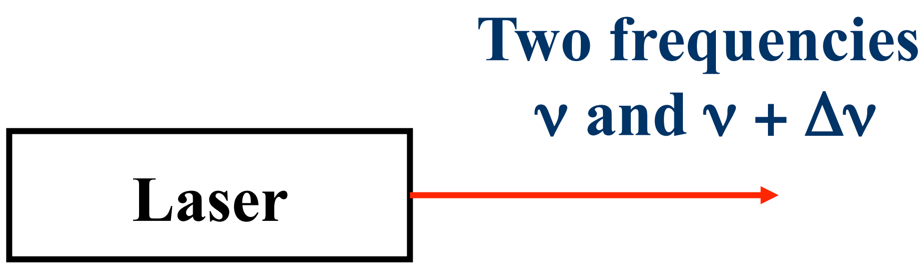
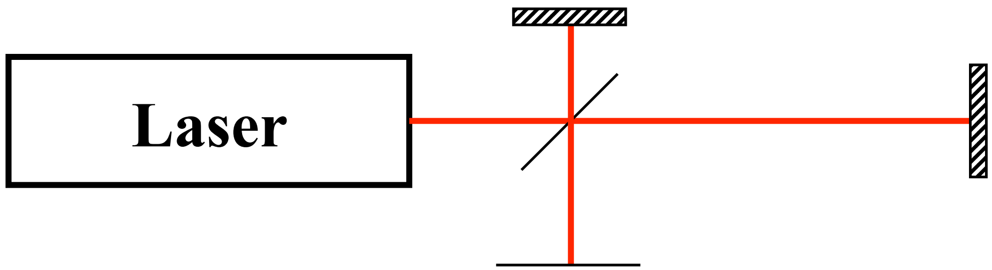
5.5.2.5 Zeeman laser



Two frequencies have orthogonal polarization

5.5.2.6 Frequency Shifting Source

Phase = $(2\pi/\lambda)$ (path difference) = $(2\pi/c)\,\nu$ (path difference)



Phase shift = $(2\pi/c)$ (frequency shift) (path difference)

5.5.3 Algorithms

Four-Step Method

phase shift

$$I(x,y) = I_{dc} + I_{ac}\cos[\phi(x,y) + \phi(t)]$$

measured object phase

$I_1(x,y) = I_{dc} + I_{ac}\cos[\phi(x,y)]$	$\phi(t) = 0 \quad (0°)$
$I_2(x,y) = I_{dc} - I_{ac}\sin[\phi(x,y)]$	$= \pi/2 \quad (90°)$
$I_3(x,y) = I_{dc} - I_{ac}\cos[\phi(x,y)]$	$= \pi \quad (180°)$
$I_4(x,y) = I_{dc} + I_{ac}\sin[\phi(x,y)]$	$= 3\pi/2 \quad (270°)$

$$\boxed{\mathrm{Tan}[\phi(x,y)] = \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}}$$

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Relationship between Phase and Height

$$\phi(x,y) = Tan^{-1}\left[\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right]$$

$$\text{Height Error}(\mathrm{x, y}) = \frac{\lambda}{4\pi}\phi(x,y)$$

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Phase-Measurement Algorithms

Three Measurements

$$\phi = ArcTan\left[\frac{I_3 - I_2}{I_1 - I_2}\right]$$

Four Measurements

$$\phi = ArcTan\left[\frac{I_4 - I_2}{I_1 - I_3}\right]$$

Schwider-Hariharan Five Measurements

$$\phi = ArcTan\left[\frac{2\left(I_4 - I_2\right)}{I_1 - 2I_3 + I_5}\right]$$

Carré Equation

$$\phi = ArcTan\left[\frac{\sqrt{\left[3\left(I_2 - I_3\right) - \left(I_1 - I_4\right)\right]\left[\left(I_2 - I_3\right) - \left(I_1 - I_4\right)\right]}}{\left(I_2 + I_3\right) - \left(I_1 + I_4\right)}\right]$$

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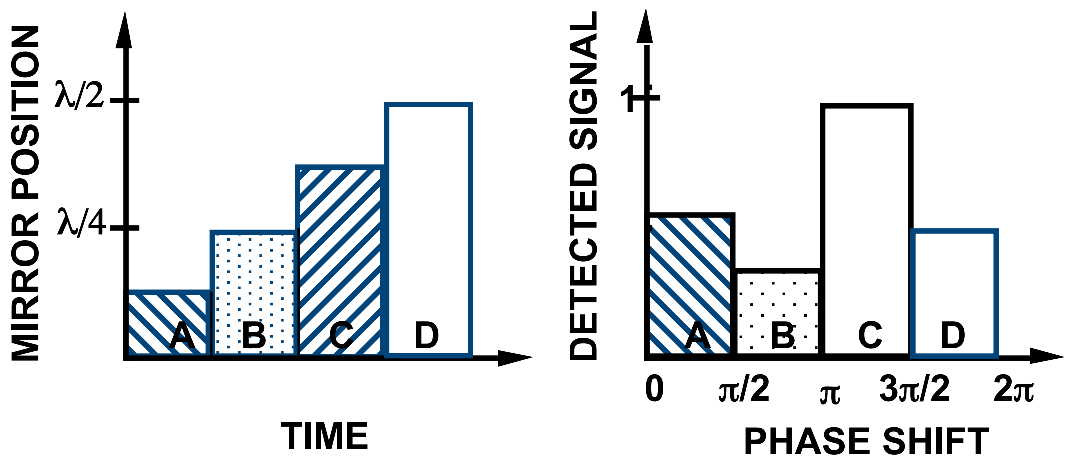
Phase-Stepping Phase Measurement



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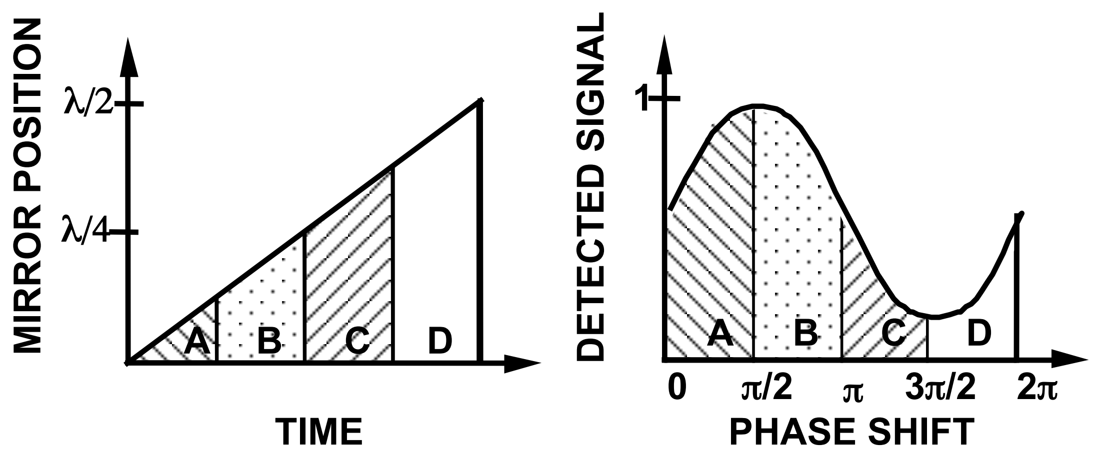
Integrated-Bucket Phase Measurement



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Integrating-Bucket and Phase-Stepping Interferometry

Measured irradiance given by

$$I_i = \frac{1}{\Delta} \int_{\alpha_i - \Delta/2}^{\alpha_i + \Delta/2} I_o \left\{1 + \gamma_o \cos\left[\phi + \alpha_i(t)\right]\right\} d\alpha(t)$$

$$= I_o \left\{1 + \gamma_o \operatorname{sinc}\left[\frac{\Delta}{2}\right] \cos\left[\phi + \alpha_i\right]\right\}$$

Integrating-Bucket $\Delta = \alpha$

Phase-Stepping $\Delta = 0$

Another Approach for Calculating Phase-Shifting Algorithms

If ϕ is the phase being measured, and δ is the phase shift, the irradiance can be written as

$$I = I_{avg}\left(1+\gamma Cos\left[\phi+\delta\right]\right)$$
$$= I_{avg} + I_{avg}\gamma Cos\left[\delta\right]Cos\left[\phi\right] - I_{avg}\gamma Sin\left[\delta\right]Sin\left[\phi\right]$$

Letting

$$a0 = I_{avg}, \quad a1 = I_{avg}\gamma Cos\left[\phi\right], \quad a2 = -I_{avg}\gamma Sin\left[\phi\right]$$

Then

$$I = a0 + a1 Cos\left[\delta\right] + a2 Sin\left[\delta\right]$$

It follows that

$$Tan\left[\phi\right] = -\frac{a2}{a1}, \qquad \gamma = \frac{\sqrt{a1^2 + a2^2}}{a0}$$

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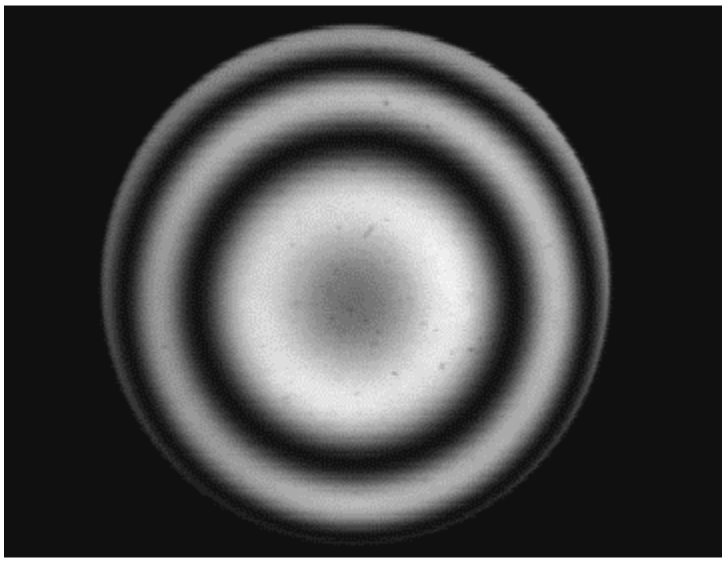
Least Squares Fitting

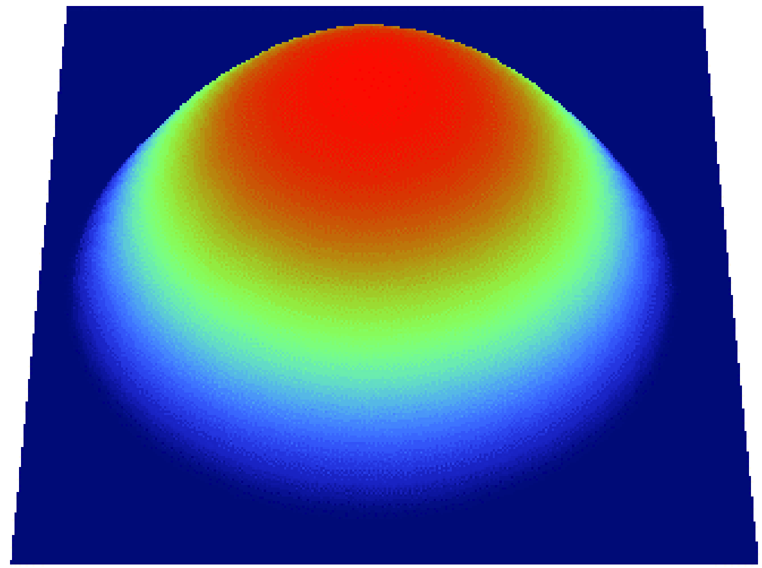
- The above approach shows measurements for at least three phase shifts are required.
- If measurements are performed for three phase shifts, it is possible to solve for a1 and a2, and the phase, ϕ, can be determined as a function of position across the pupil.
- If more than 3 phase shifts are used, a1 and a2 can be solved for using a least squares approach. That is, find the square of the difference between the measured irradiance and the irradiance predicted using the sinusoidal irradiance relationship given above. This error is minimized by differentiating with respect to each of the three unknowns and equating these results to zero. The simultaneous solution of these three equations produces the least square result.
- The least squares fitting approach is extremely powerful.

5.5.4 Phase-Unwrapping

Typical Fringes For Spherical Surfaces



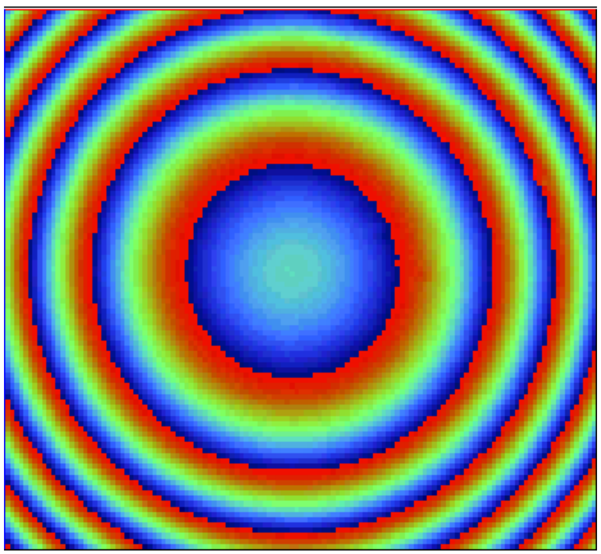
Fringes

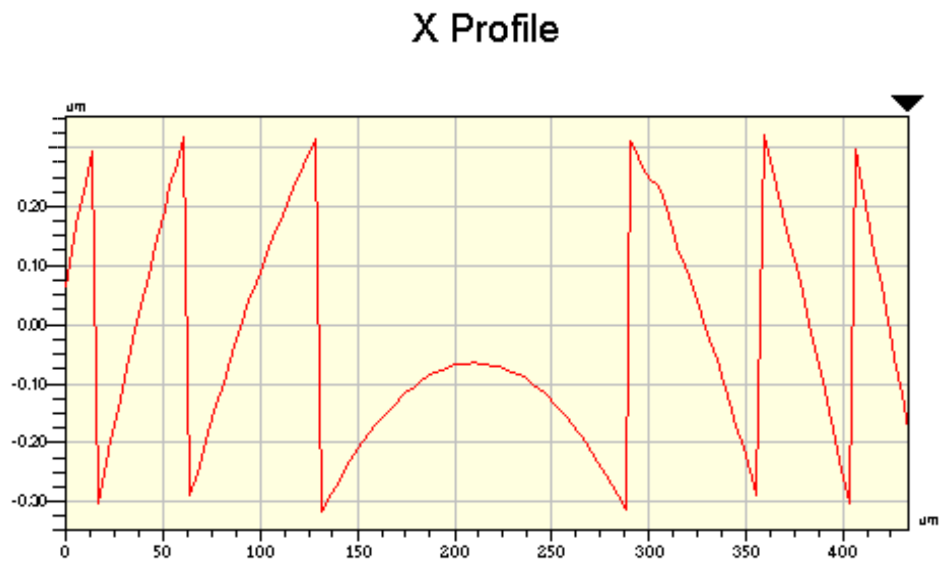


Phase map

Phase Ambiguities - Before Unwrapping

2 π Phase Steps





Removing Phase Ambiguities

- Arctan Mod 2π (Mod 1 wave)
- Require adjacent pixels less than π difference (1/2 wave OPD)

- Trace path
- When phase jumps by > π
  Add or subtract $N2\pi$

  Adjust so < π

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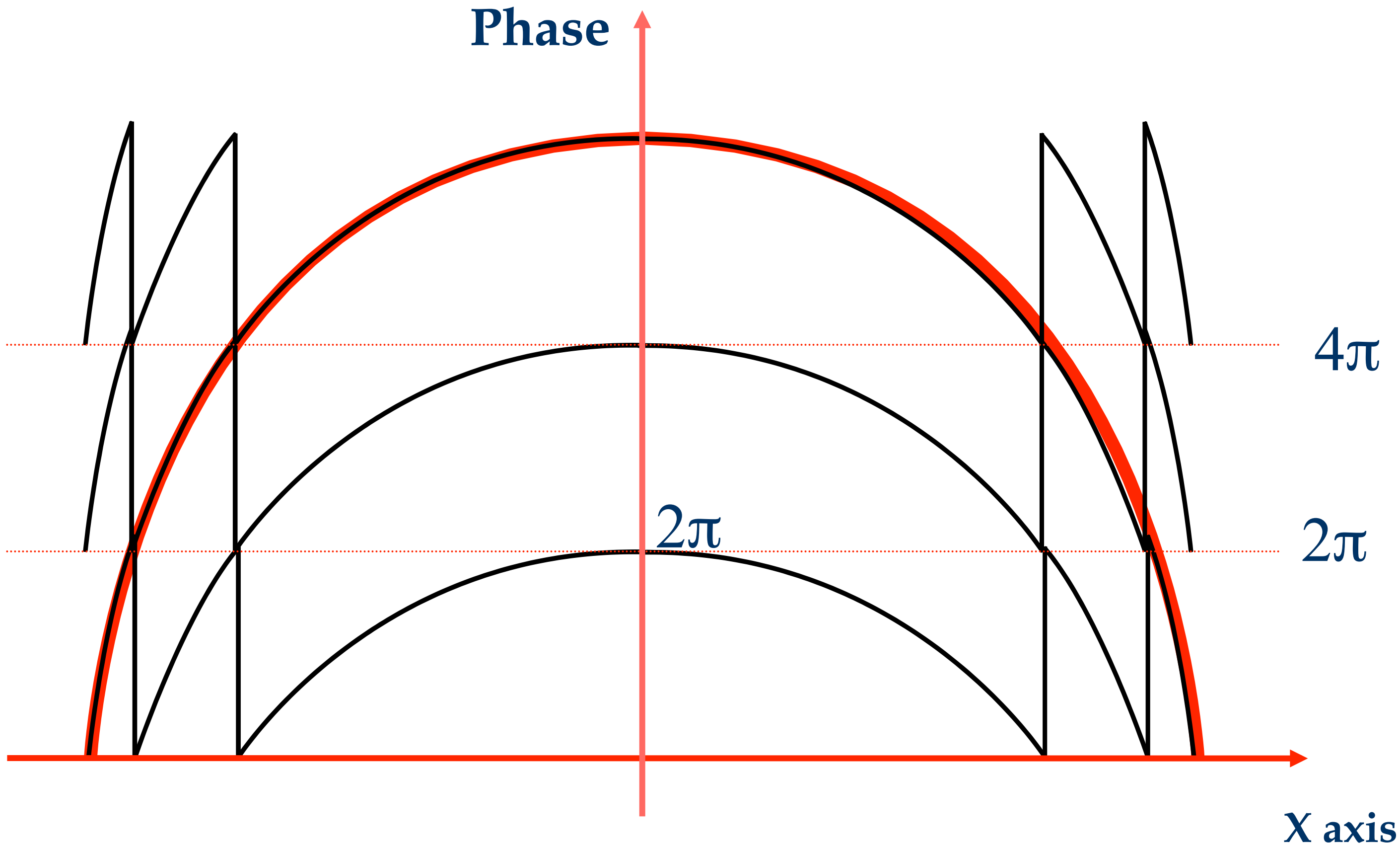
Phase Multiple Solutions



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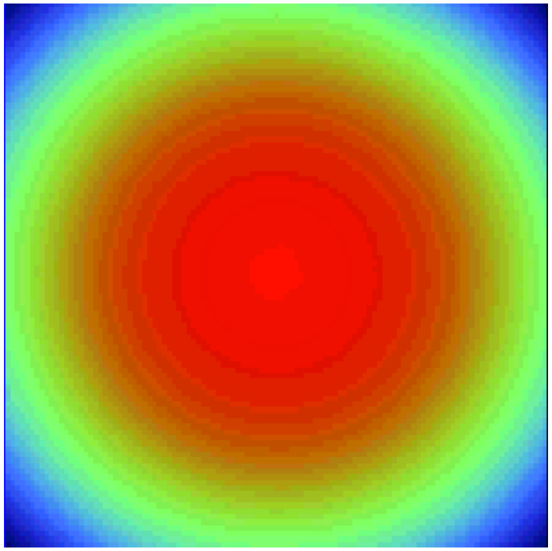
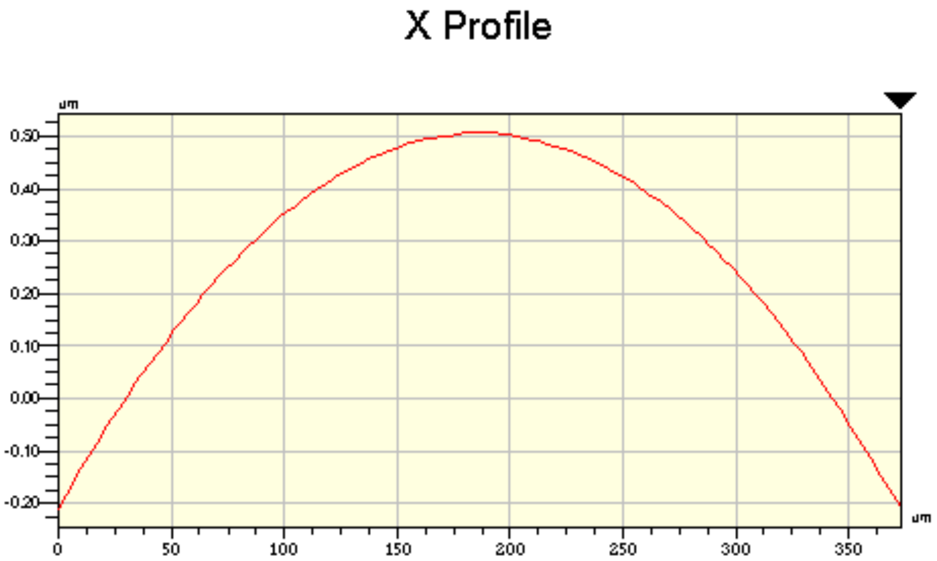
Phase Ambiguities – After Unwrapping

Phase Steps Removed





Phase Unwrapping Reference

Two-Dimensional Phase Unwrapping
Theory, Algorithms, and Software

Dennis C. Ghiglia and Mark D. Pritt

Wiley Interscience, 1998

5.5.5 Phase Shifter Calibration

- Phase shifter calibration is an important part of operating a phase-shifting interferometer.
- Several calibration techniques are available. The following is a commonly used procedure.
  - Take 5 frames of irradiance data where the phase shifts are -2α, -α, 0, α, 2α. It can be shown that α is given by

$$\alpha = ArcCos\left[\frac{1}{2}\frac{I_5 - I_1}{I_4 - I_2}\right]$$

  - Sign of numerator tells us whether α is too large or too small.
  - The algorithm has singularities and tilt fringes can be introduced into the interferogram and data points for which the numerator or denominator are smaller than a threshold are eliminated.
  - Often convenient to look at histogram of phase shifts. If the histogram is wider than expected there must be problems with the system such as too much vibration present.

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5.5.6 Errors

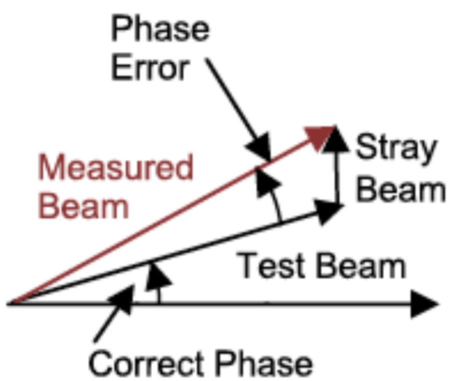
- 5.5.6.1 Error Due to Stray Reflections
- 5.5.6.2 Quantization Error
- 5.5.6.3 Detector Nonlinearity
- 5.5.6.4 Source Instabilities
- 5.5.6.5 Error Due to Incorrect Phase-Shift Between Data Frames

Error Sources

- The two most common sources of error are
  – Incorrect phase-shift between data frames. The incorrect phase-shift is often caused by vibration.
  – Stray reflections
- Less common errors
  – Quantization error
  – Detector nonlinearity
  – Source instabilities

5.5.6.1 Error Due to Stray Reflections

- A common problem in interferometers using lasers as a light source is extraneous interference fringes due to stray reflections.
- The easiest way of thinking about the effect of stray reflections is that the stray reflection adds to the test beam to give a new beam of some amplitude and phase.
- The difference between this resulting phase, and the phase of the test beam, gives the phase error.
- In well designed interferometers the stray light is minimal.
- Probably the best way of reducing or eliminating the error due to stray light is to use a short coherence length light source.



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5.5.6.2 Quantization Error

- Interferograms are analog signals that must be digitized to be processed by a computer. Typically 8 – 12 bits are used. If b is the number of bits and N is the number of steps in the algorithm, quantizing the signal will cause an rms phase error that goes as

$$\sigma = \frac{2}{2^b \sqrt{3N}}$$

- If the fringe modulation does not span the full dynamic range of quantization levels the effective number of bits is less than the quantization level.
- If the noise is greater than one bit, quantization error can be reduced by averaging data sets.

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Ref: Brophy, JOSA A, 7, 537 (1990)

5.5.6.3 Detector Nonlinearity

- Nonlinearity in a detector can cause phase errors in a measurement and care should be taken to adjust exposure so as not to operate near saturation or at extremely low signal levels.
- Most detectors are extremely linear over most of their dynamic range, so this is not usually a large source of error in PSI.

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5.5.6.4 Source Instabilities

- **If the source frequency changes and the paths are not matched, a phase shift will be introduced between the two interfering beams. If d is the path difference, c is the velocity of light, the phase difference introduced by a frequency change of $\Delta\nu$ is given by**

$$\Delta\phi = 2\pi \frac{d}{c} \Delta\nu$$

- **If N is the number of steps in the algorithm, irradiance fluctuations introduce a standard deviation in the measured phase of** $$\sigma = \frac{1}{SNR\sqrt{n}}$$

- **In the ideal situation the noise limitation is set by photon shot noise. If p is the number of detected photons, the standard deviation of the measured phase goes as**

$$\sigma = \frac{1}{\sqrt{p}}$$

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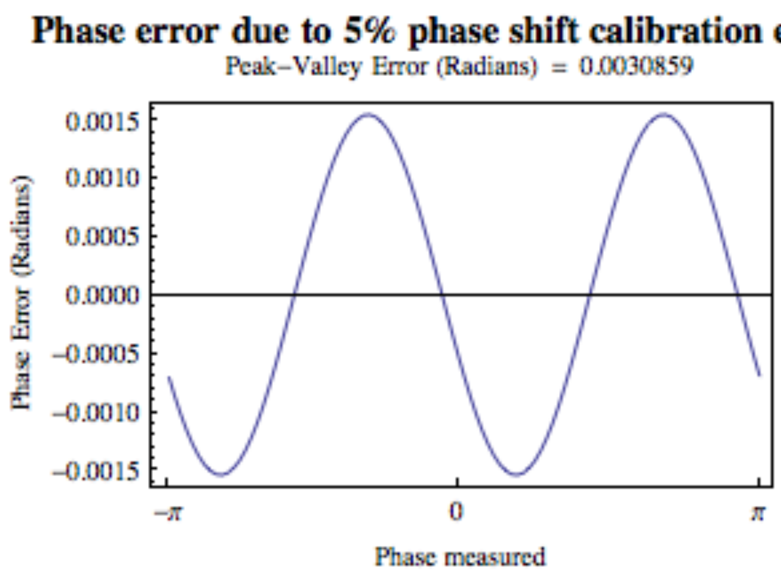
5.5.6.5 Error Due to Incorrect Phase-Shift Between Data Frames

- **The resulting error is a ripple in the measured phase that is twice the frequency of the original interference fringes.**
- **The magnitude of the ripple depends on the amount of error in the phase-shift and the algorithm used to calculate the phase.**



Results for Schwider-Hariharan 5-step algorithm

Reducing Double Frequency Error

- Since errors occur at twice the frequency of the interference fringes, it should be possible to perform the measurement twice with a 90^o offset in the phase shift and then average the two results having errors 180^o out of phase to nearly cancel the double frequency error.
- Do not have to actually perform the measurement twice, but as long as the phase step is 90^o all we had to do is to add one more frame of data and use frames 1 thru N-1 for the first calculation and frames 2 thru N for the second calculation and average the two results. The result gives greatly reduced error due to phase shifter calibration.

Better Approach for Reducing Double Frequency Error

A better approach is to average the numerators and denominators of the arctangent function. That is, if two data sets are taken with a 90° phase step between the two data sets the phase calculation can be of the form

$$Tan[\phi] = \frac{n_1 + n_2}{d_1 + d_2}$$

where n_i and d_i are the numerator and denominator for phase calculation algorithm for each data set. If the phase step is $\pi/2$, only one additional data frame is required. n_1 and d_1 are calculated from frames 1 thru N-1 and n_2 and d_2 are calculated from frames 2 thru N.

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Ref: Schwider et al, Digital Wavefront Measuring Interferometry: Some Systematic Error Sources," Appl. Opt., 22, 3421 (1983).

Derivation of Schwider-Hariharan Algorithm

4-step algorithm $Tan[\phi] = \frac{I_4 - I_2}{I_1 - I_3}$ or $Tan[\phi] = \frac{I_4 - I_2}{I_5 - I_3}$

Thus $Tan[\phi] = \frac{2(I_4 - I_2)}{I_1 - 2I_3 + I_5}$

- **Note that for 90-degree steps I_1 and I_5 are nominally identical and differ only because of the measurement errors.**
- **We could now add another data frame and repeat the procedure to obtain an even better 6-frame algorithm. Then of course we could add yet another frame and get an even better 7-frame algorithm.**
- **Going from 4 to 5 steps can reduce error by an order of magnitude, and by going from 4 to 7 steps can reduce the error by 4 orders of magnitude**

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5.5.7 Solving the Vibration Problem

- 5.5.7.1 2+1 Algorithm
- 5.5.7.2 Measure vibration and introduce vibration 180 degrees out of phase to cancel vibration
- 5.5.7.3 Spatial Synchronous and Fourier Methods
- 5.5.7.4 Spatial Carrier Technique
- 5.5.7.5 Simultaneous Phase-Measurement Interferometer
- 5.5.7.6 Single-Shot Holographic Polarization Dynamic Interferometer
- 5.5.7.7 Pixelated Polarizer Array Dynamic Interferometer

The Vibration Problem

- Probably the most serious impediment to wider use of PSI is its sensitivity to external vibrations.
- Vibrations cause incorrect phase shifts between data frames.
- Error depends upon frequency of vibration present as well as phase of vibration relative to the phase shifting.

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Best Way to Fix Vibration Problem

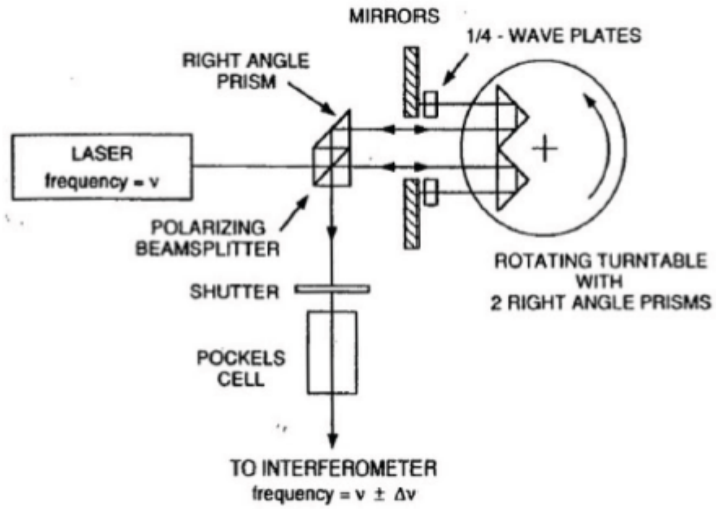
- Control environment
- Common-path interferometers
- Retrieve frames faster
- Measure vibration and introduce vibration 180 degrees out of phase to cancel vibration
- Single-Shot Direct Phase Measurement
  – Spatial Synchronous and Fourier Methods
  – Spatial Carrier
  – Single-Shot Holographic Polarization Dynamic Interferometer
  – Pixelated Polarizer Array Phase Sensor Dynamic Interferometer

5.5.7.1 2 + 1 Algorithm

- The 2 + 1 algorithm can be used to attack the problem of measurement errors introduced by vibration. Two interferograms having a 90 degree phase shift are rapidly collected and later a third interferogram is collected that is the average of two interferograms with a 180 degree phase shift.
- An interline transfer CCD can be used for rapidly obtaining the two interferograms having the 90^{o} phase shift. In an interline transfer CCD each photosite is accompanied by an adjacent storage pixel. The storage pixels are read out to produce the video signal while the active photosites are integrating the light for the next video field. After exposure, the charge collected in the active pixels is transferred in a microsecond to the now empty storage sites, and the next video field is collected.

Ref: Wizinowich, P. L., "Phase-Shifting Interferometry in the Presence of Vibration: A New Algorithm and System," Appl. Opt., 29, 3271 (1990).

Implementation of 2 + 1 Algorithm



- Two orthogonally polarized light beams are produced having two sets of interference fringes 90^o out of phase. A Pockel cell is used to select which set of fringes is present on the detector. The third exposure is made with two sets of fringes 180 degrees out of step present.
- The 2 + 1 algorithm has found limited use because the small number of data frames makes it susceptible to errors resulting from phase-shifter nonlinearity and calibration.

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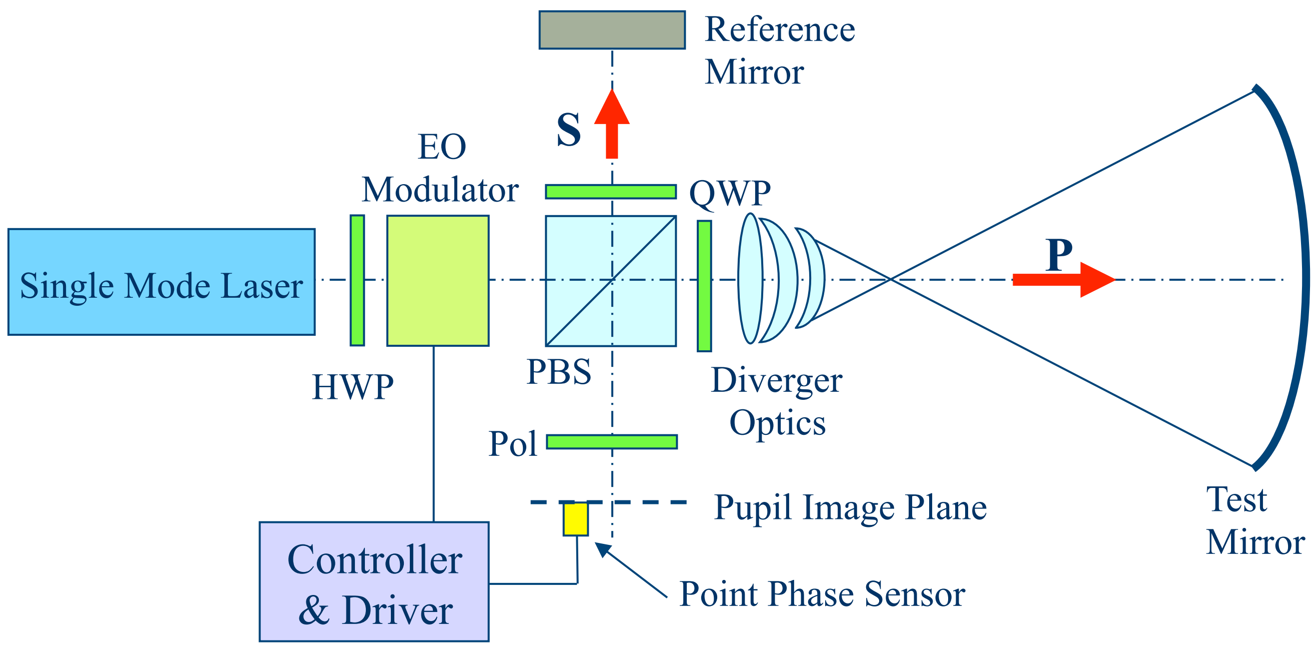
5.5.7.2 Measure vibration and introduce vibration 180 degrees out of phase to cancel vibration

- Use polarization Twyman-Green configuration
- EOM changes relative phase between 'S' & 'P' components
  – Can be very fast: 200 kHz - 1 GHz response



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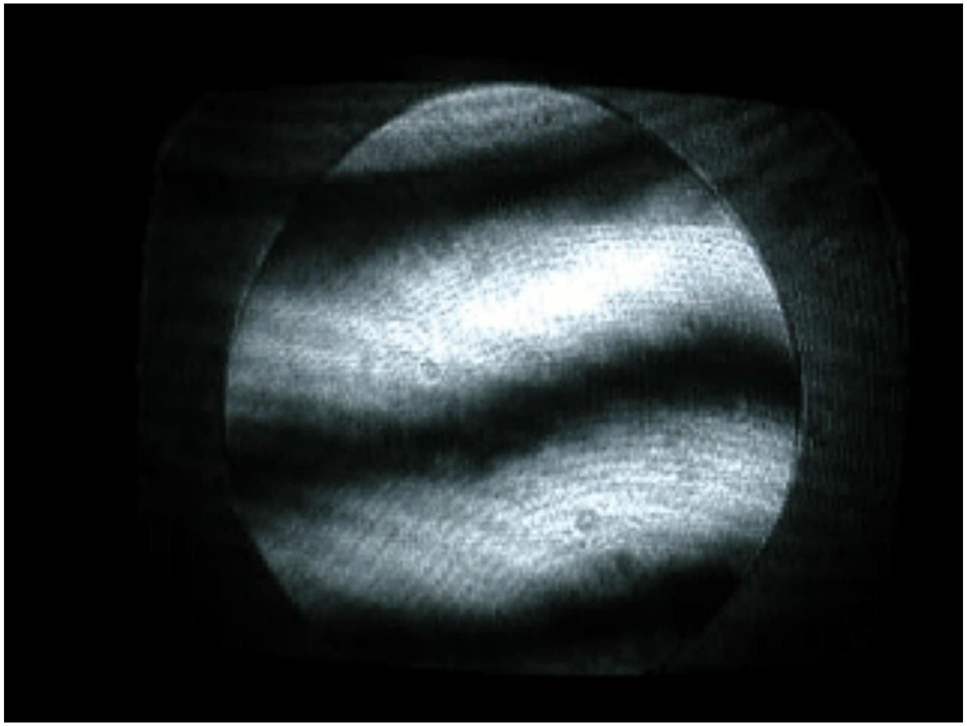
Results



Conclusions - Active Vibration Cancellation Interferometer

System works amazingly well, but it is rather complicated and expensive.

5.5.7.3 Spatial Synchronous and Fourier Methods

Both techniques use a single interferogram having a large amount of tilt

Can write the interference signal as

$$irradiance[x,y] = i_{avg}\left(1+\gamma Cos\left[\phi[x,y]+2\pi fx\right]\right)$$

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Spatial Synchronous

The interference signal is compared to reference sinusoidal and cosinusoidal signals

$$r\cos[x,y] = Cos[2\pi fx]$$

$$r\sin[x,y] = Sin[2\pi fx]$$

Multiplying the reference signal times the irradiance signal gives sum and difference signals

$$irradiance[x,y]\, r\cos[x,y] =$$

$$\frac{1}{2}\left(2i_{avg}Cos[2\pi fx] + i_{avg}\gamma Cos[\phi[x,y]] + i_{avg}\gamma Cos[\phi[x,y] + 4\pi fx]\right)$$

$$irradiance[x,y]\, r\sin[x,y] =$$

$$\frac{1}{2}\left(2i_{avg}Sin[2\pi fx] - i_{avg}\gamma Sin[\phi[x,y]] + i_{avg}\gamma Sin[\phi[x,y] + 4\pi fx]\right)$$

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Spatial Synchronous – Calculating the Phase

The low frequency second term in the two signals can be written as

$$s1 = \frac{i_{avg}}{2}\gamma Cos\left[\phi\left[x,y\right]\right]$$

$$s2 = -\frac{i_{avg}}{2}\gamma Sin\left[\phi\left[x,y\right]\right]$$

$$Tan\left[\phi\left[x,y\right]\right] = -\frac{s2}{s1}$$

The only effect of having the frequency of the reference signals slightly different from the average frequency of the interference signal is to introduce tilt into the calculated phase distribution.

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Fourier Method

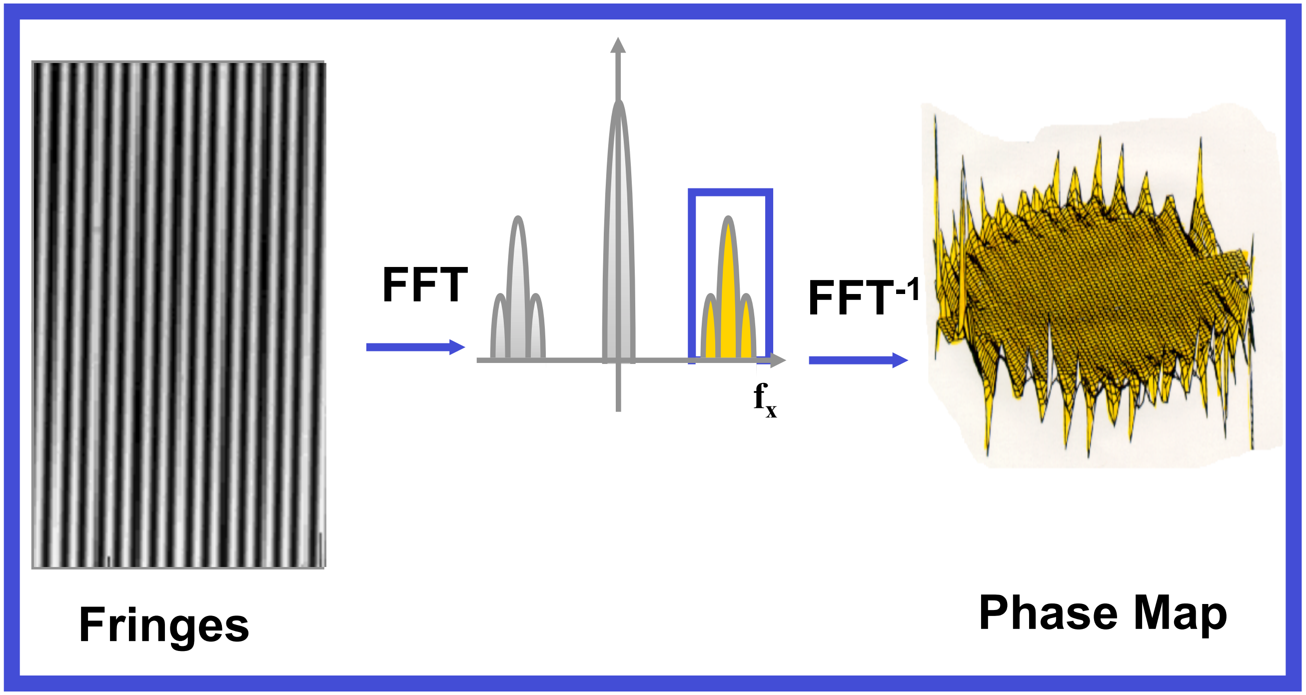
- The interference signal is Fourier transformed, spatially filtered, and the inverse Fourier transform of the filtered signal is performed to yield the wavefront.
- The Fourier analysis method is essentially identical to the spatial synchronous method.

$$irradiance[x,y] = i_{avg}\left(1+\gamma Cos\left[\phi[x,y]+2\pi fx\right]\right)$$

This can written as

$$irradiance[x,y] = i_{avg}\left(1+\frac{1}{2}\gamma\left(e^{i(\phi[x,y]+2\pi fx)}+e^{-i(\phi[x,y]+2\pi fx)}\right)\right)$$

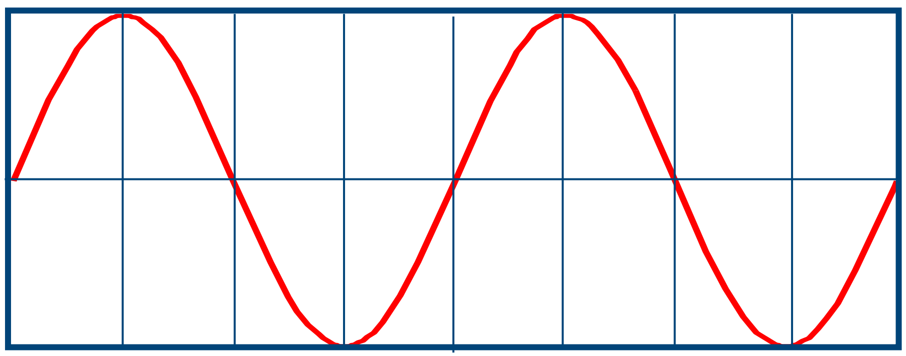
Fourier Transform and Spatially Filtered



Since a spatially limited system is not band limited, the orders are never completely separated and the resulting wavefront will always have some ringing at the edges. The requirement for large tilt always limits the accuracy of the measurement.

5.5.7.4 Spatial Carrier Technique

Phase shifting algorithms applied to consecutive pixels thus requires calibrated tilt



4 pixels per fringe for 90 degree phase shift

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Creating the Carrier Frequency

- Introduce tilt in reference beam
  - Aberrations introduced due to beam transmitting through interferometer off-axis
- Wollaston prism in output beam
  - Requires reference and test beams having orthogonal polarization
- Pixelated array in front of detector
  - Special array must be fabricated

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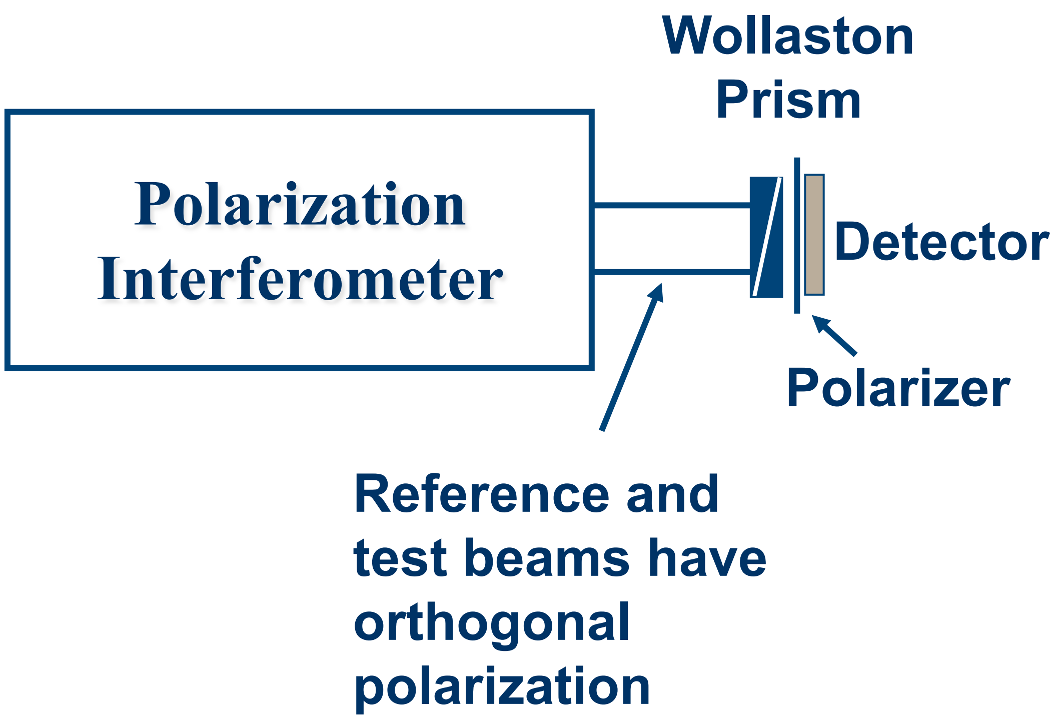
Use of Wollaston Prism to Produce Carrier Fringes



Two Examples of Spatial Carrier Interferometers

- 193 nm wavelength interferometer for testing DUV Lithographic Optics
- High Speed, 525 to 1400 frames per second interferometer

Testing DUV Lithographic Optics

193 nm wavelength, 50mm Diameter Fizeau Interferometer

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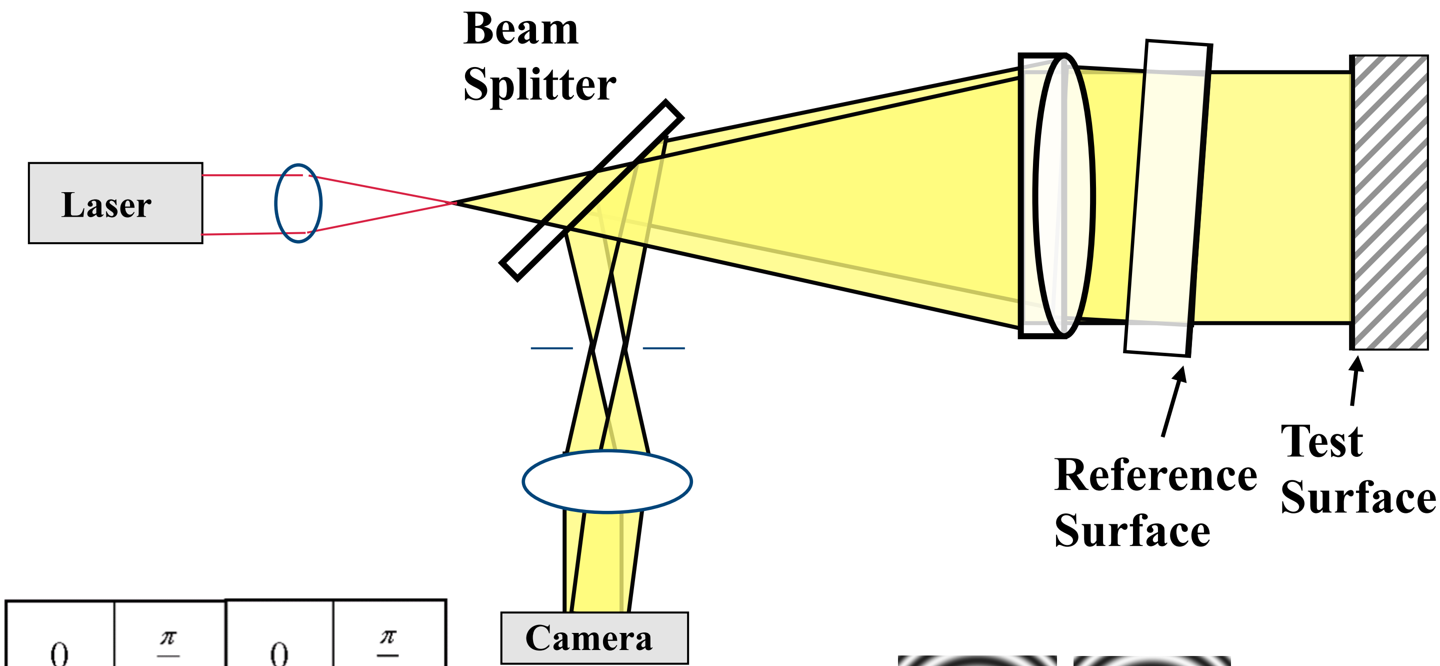
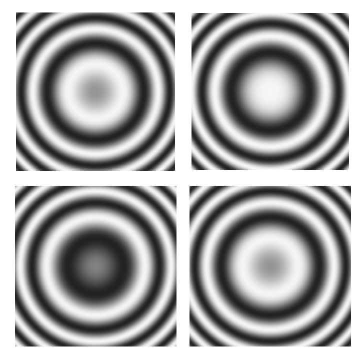
Single Frame Dynamic Mode



0	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$-\frac{\pi}{2}$	1 π	2 $-\frac{\pi}{2}$	3 π
0	4 $\frac{\pi}{2}$	5 0	6 $\frac{\pi}{2}$
$-\frac{\pi}{2}$	7 π	8 $-\frac{\pi}{2}$	9 π

Parsing of Phase-Shifted Pixels



Calculation of phase using 3 x 3 element array

0	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$-\frac{\pi}{2}$	1 π	2 $-\frac{\pi}{2}$	3 π
0	4 $\frac{\pi}{2}$	5 0	6 $\frac{\pi}{2}$
$-\frac{\pi}{2}$	7 π	8 $-\frac{\pi}{2}$	9 π

$$Tan[\theta_5] = \frac{2(I_2 + I_8 - I_4 - I_6)}{-I_1 - I_3 + 4I_5 - I_7 - I_9}$$

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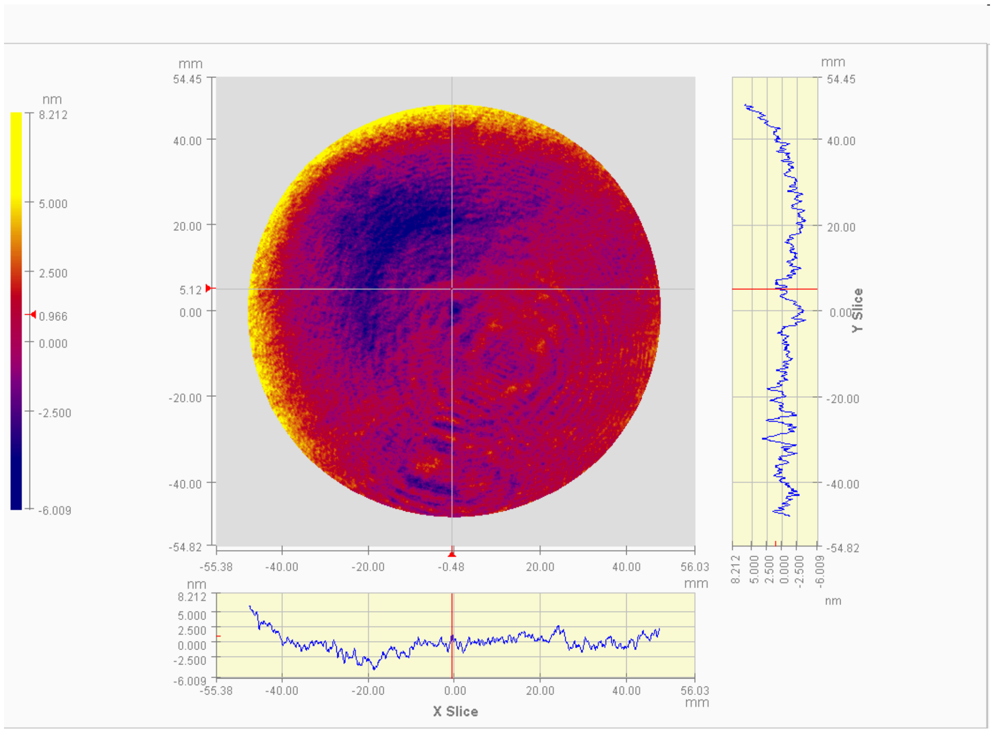
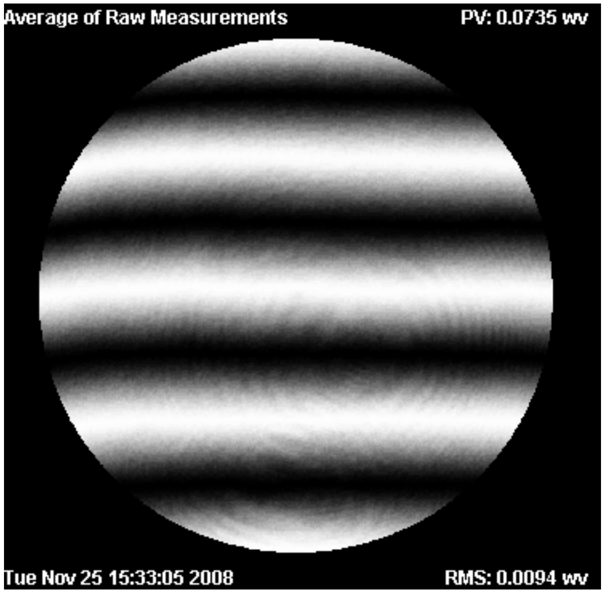
Measured Performance

- Uncalibrated accuracy = 1.8nm rms
- RMS repeatability = 0.07nm

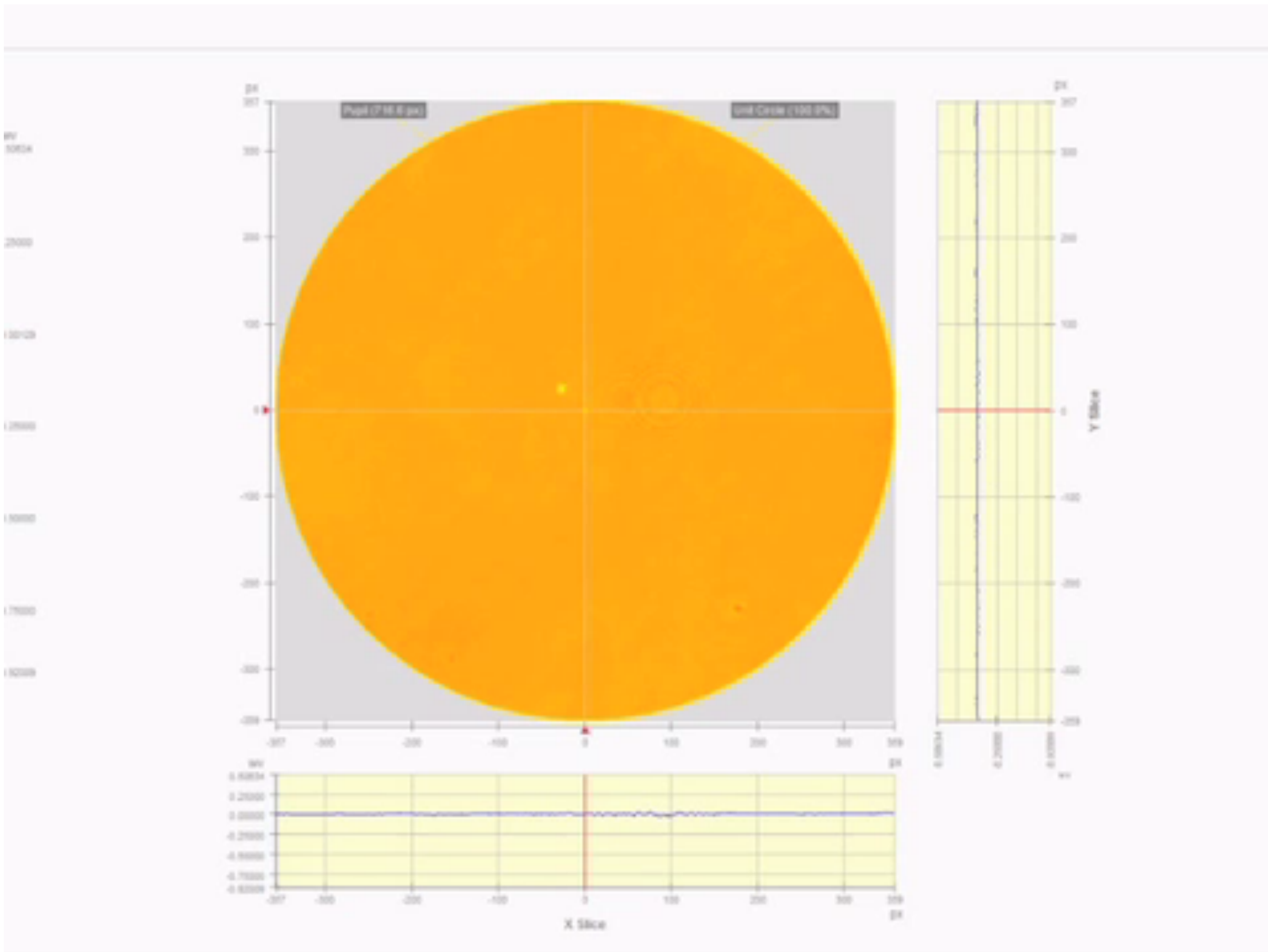




High Speed Interferometer

- Twyman Green Spatial Carrier
- CMOS Camera
- 880 x 880, 525 frames/second
- 720 x 720, 1000 frames/second
- 550 x 550, 1400 frames/second

Air Stream – 525 Frames/Second



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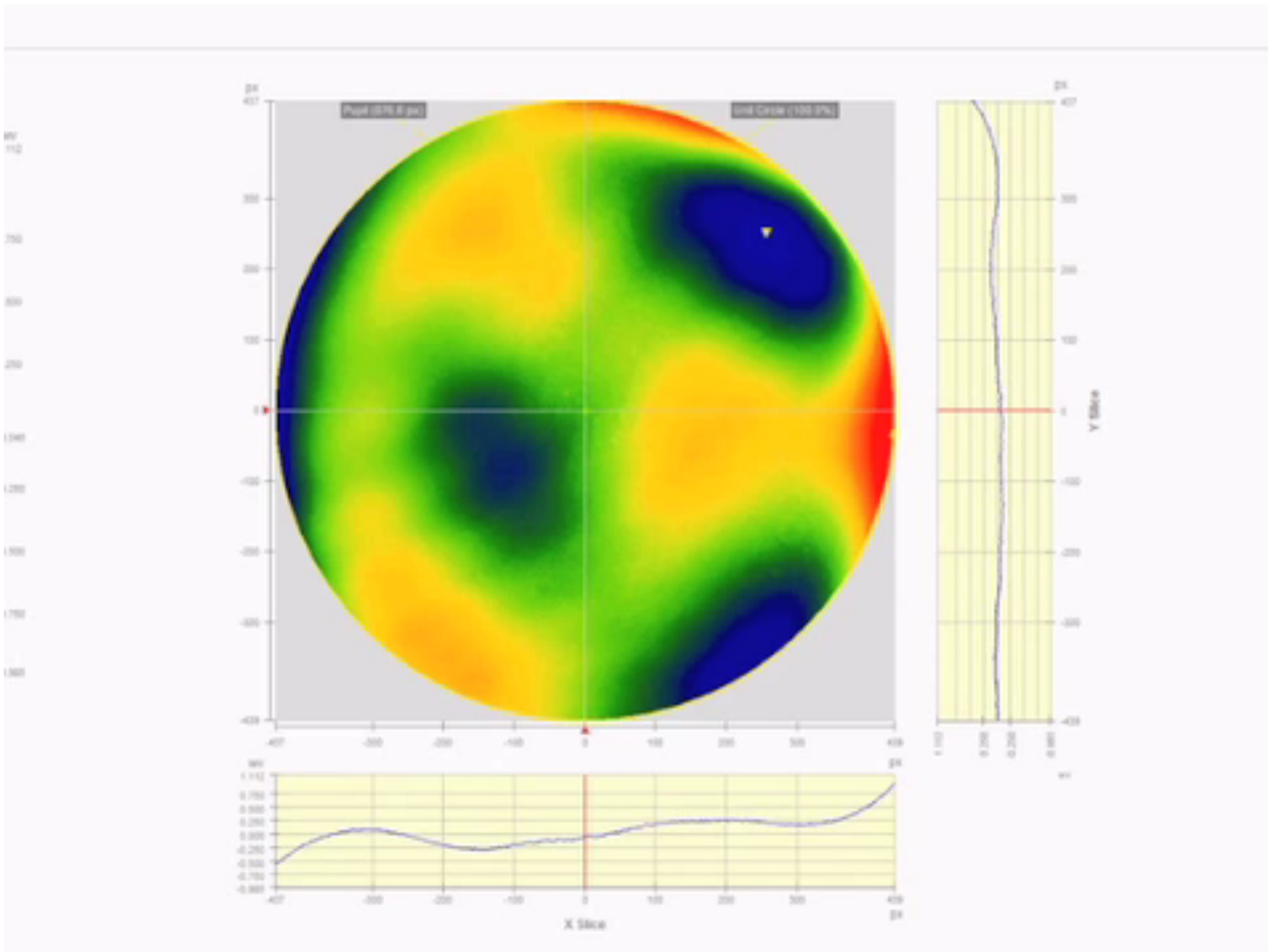
Water Surface Fringes, 525 Frames/Second

Water Surface, 525 Frames/Second



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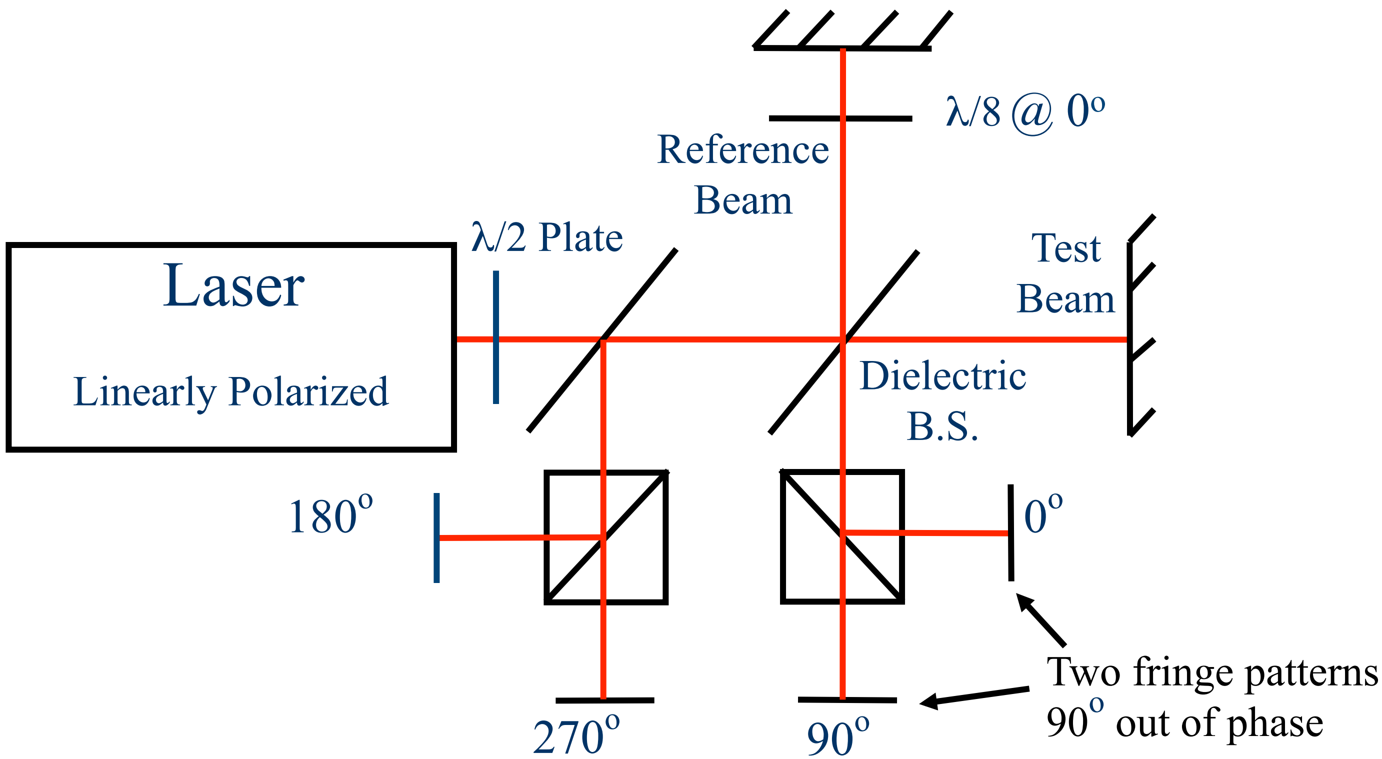
5.5.7.5 Simultaneous Phase-Measurement Interferometer



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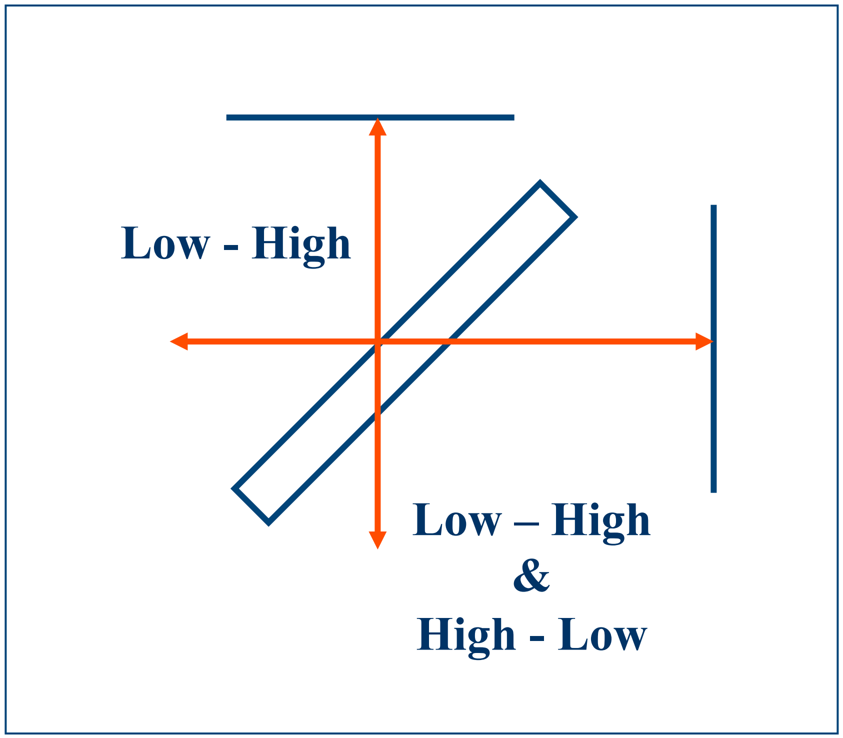
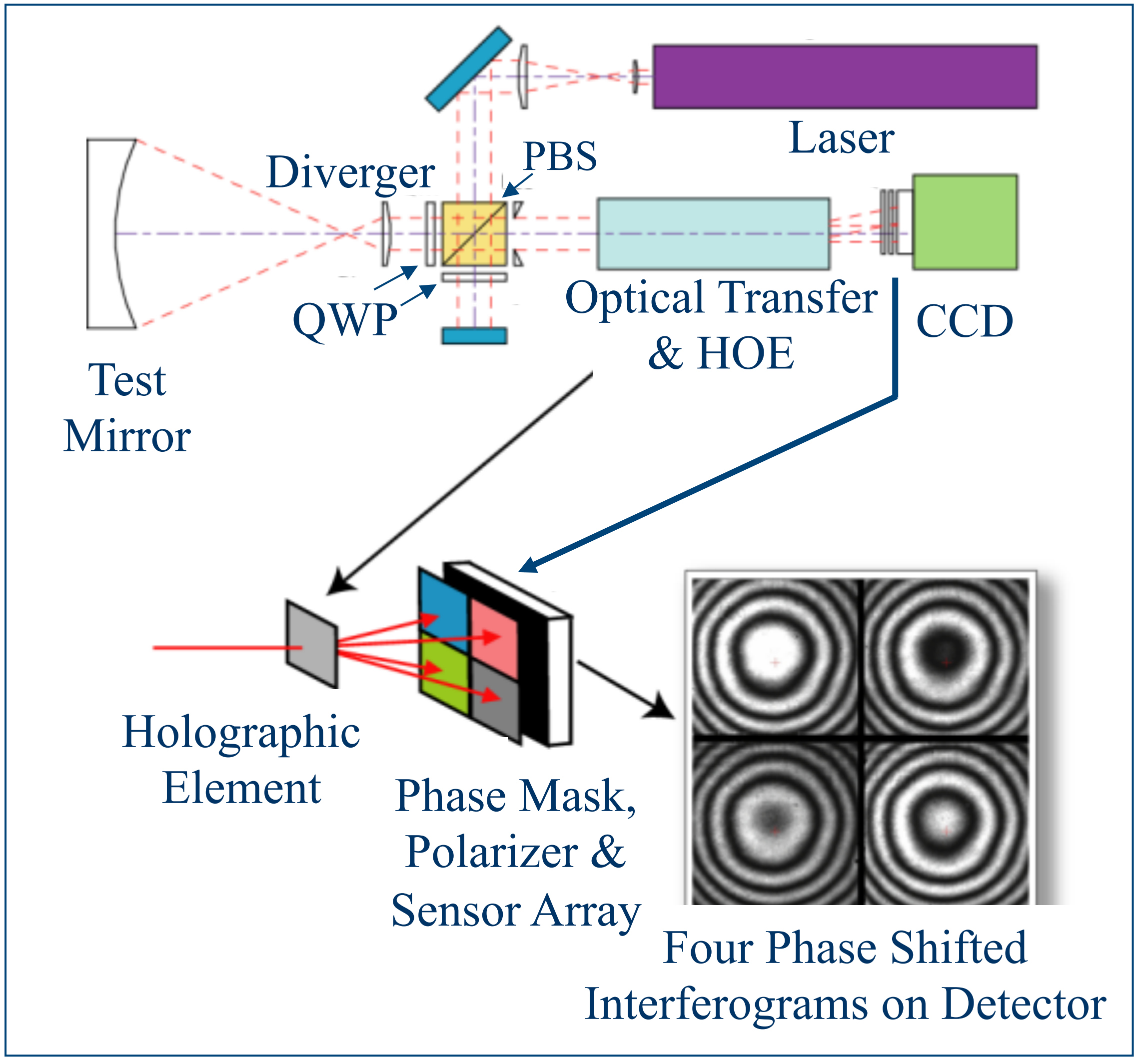
Dielectric beamsplitter and phase shift upon reflection for test and reference beams



5.5.7.6 Single-Shot Holographic Polarization Dynamic Interferometer

- **Twyman-Green**
  - Two beams have orthogonal polarization
- **4 Images formed**
  - Holographic element
- **Single Camera**
  - 1024 x 1024
  - 2048 x 2048
- **Polarization used to produce 90-deg phase shifts**



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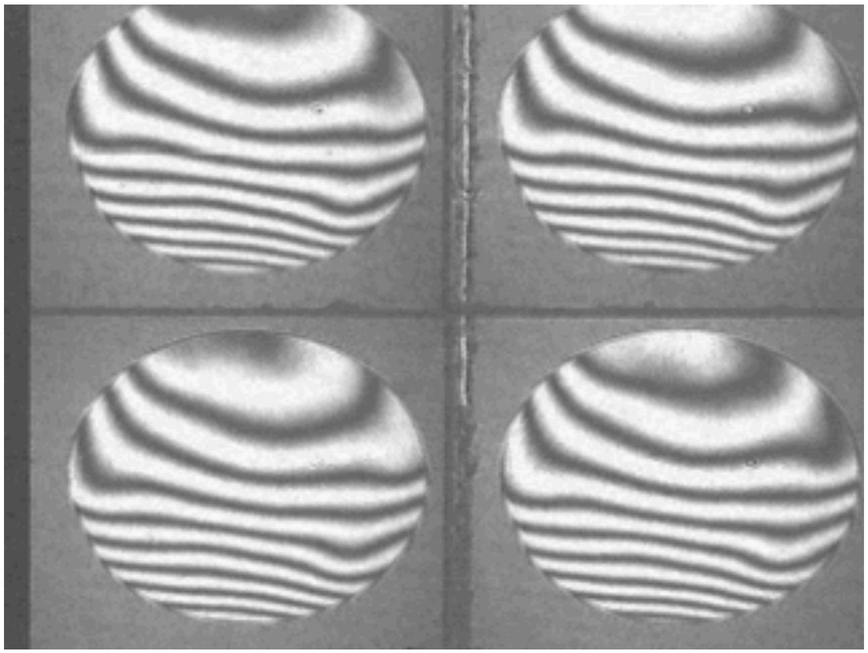
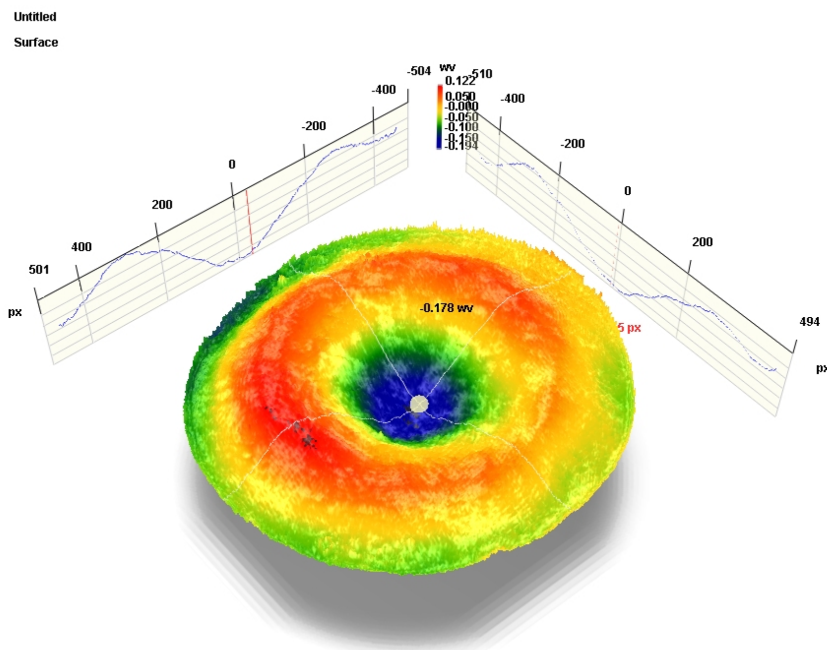
Dynamic Interferometry

Fringes Vibrating



Phase relationship is fixed

Dynamic interferometry enables measurements in the presence of vibration



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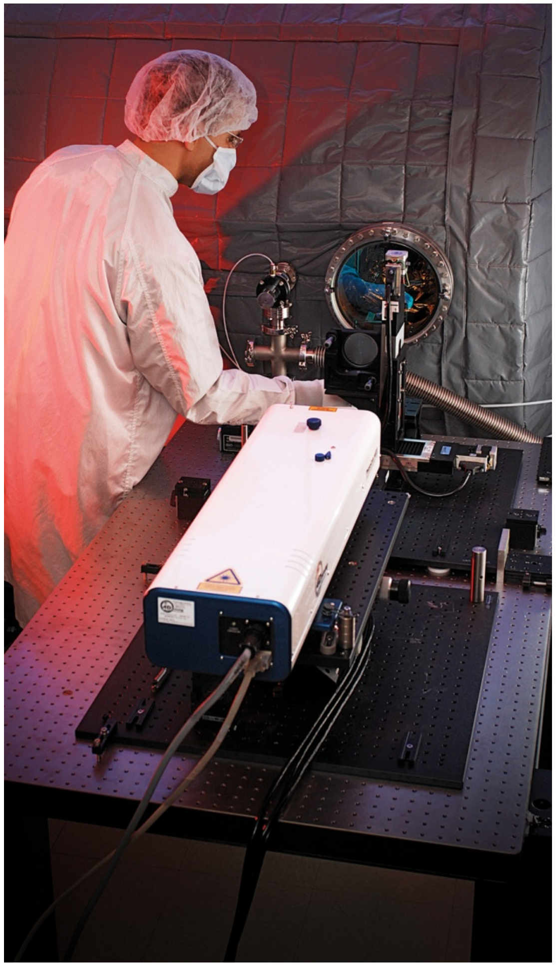
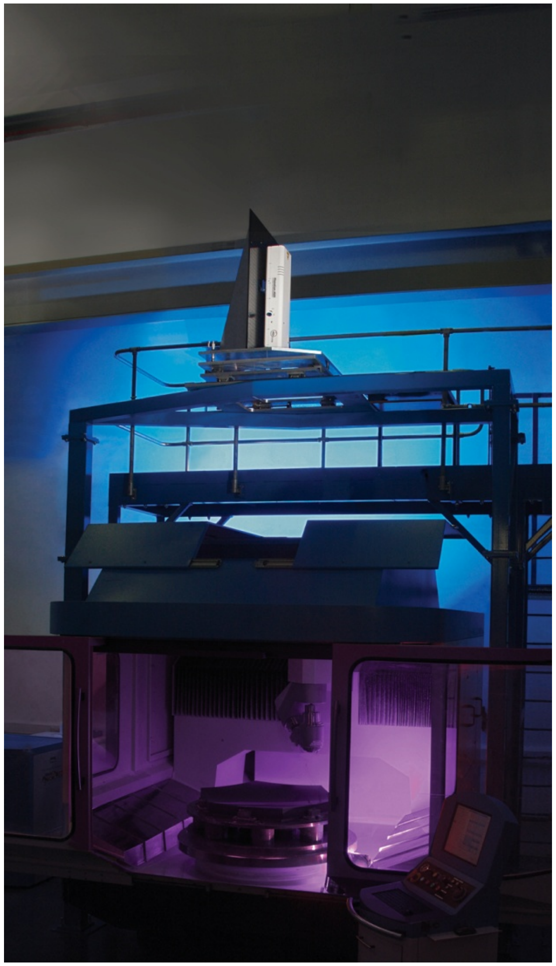
Testing of Large Optics



Testing in Environmental Chamber
(Courtesy Ball Aerospace)



Testing on Polishing Machine
(Courtesy OpTIC Technium)

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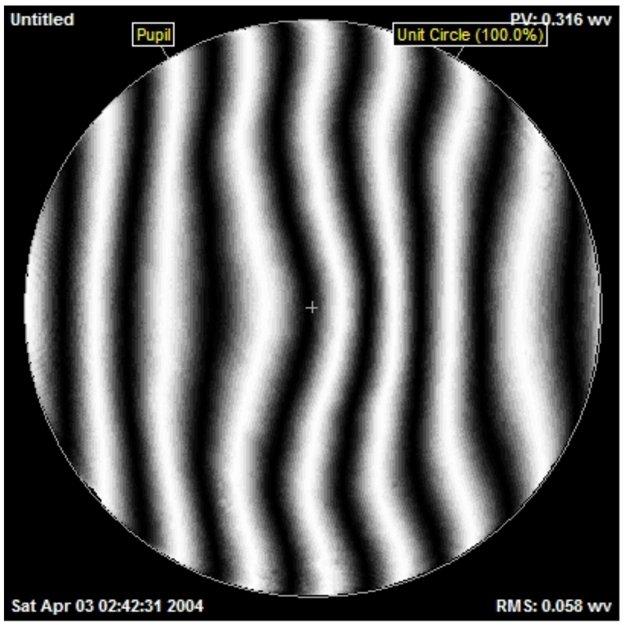
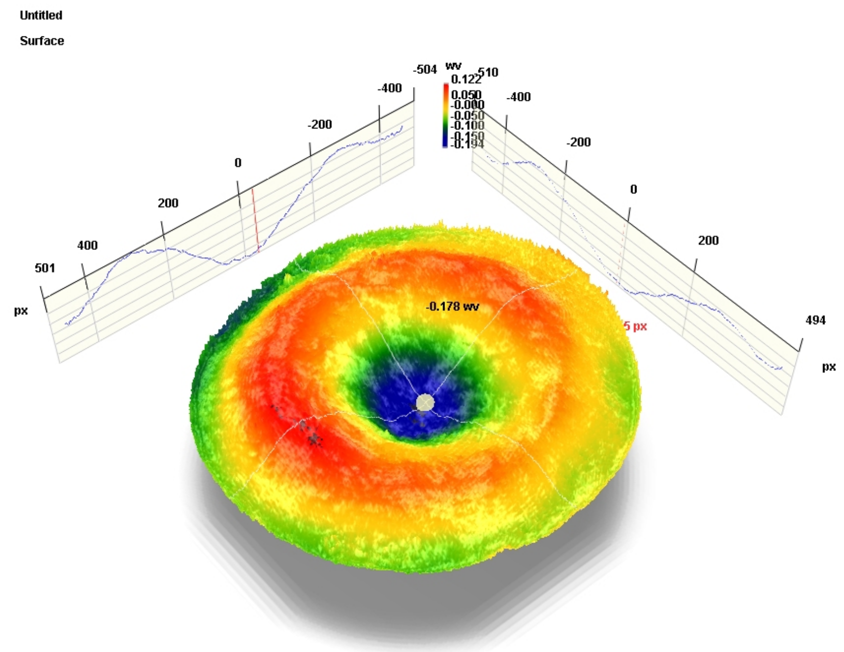
Measurement of 300 mm Diameter, 2 Meter ROC Mirror





Mirror and interferometer on separate tables!

Conclusions –Single-Shot Holographic Polarization Dynamic Interferometer

- Vibration insensitive, quantitative interferometer
- Surface figure measurement (nm resolution)
- Snap shot of surface height
- Acquisition of “phase movies”

Still not perfect

Not easy to use multiple wavelength or white light interferometry

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5.5.7.7 Pixelated Polarizer Array Dynamic Interferometer

- Compacted pixelated array placed in front of detector
- Single frame acquisition
  - High speed and high throughput
- Achromatic
  - Works from blue to NIR
- True Common Path
  - Can be used with white light

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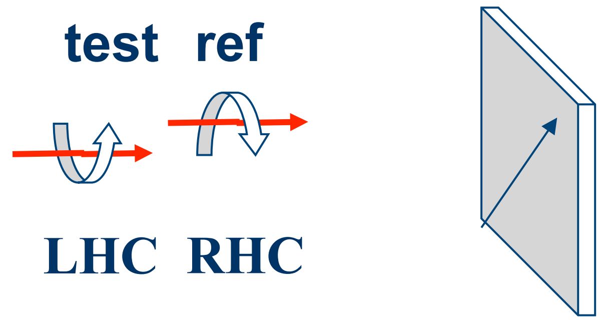
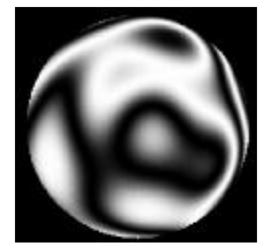
Use polarizer as phase shifter





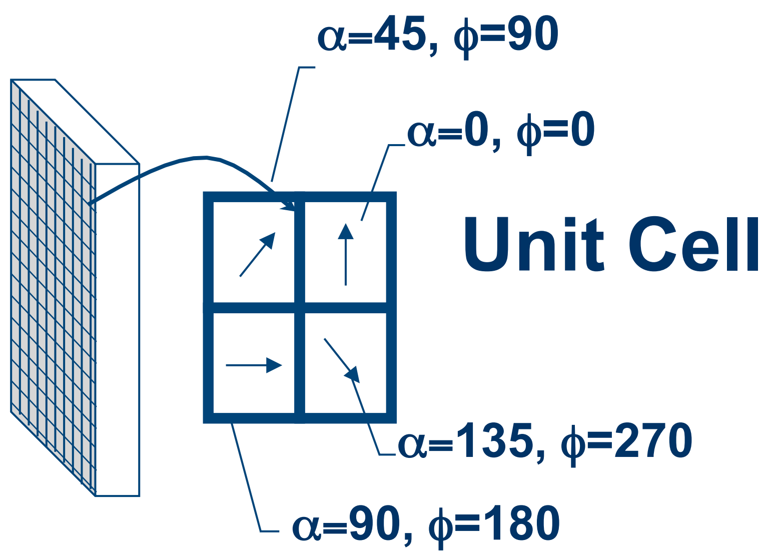
Circ. Pol. Beams ($\Delta\phi$) + linear polarizer $\longrightarrow$ cos ($\Delta\phi + 2\alpha$)

Phase-shift depends on polarizer angle

Reference: S. Suja Helen, M.P. Kothiyal, and R.S. Sirohi, "Achromatic phase-shifting by a rotating polarizer", Opt. Comm. 154, 249 (1998).

Array of Oriented Micropolarizers

Polarizer array
Matched to
detector array
pixels



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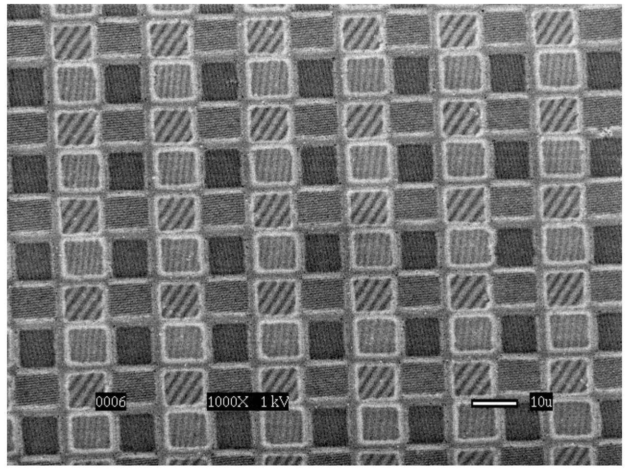
SEM of Patterned Polarizers



10 micron elements

Photolithography used to pattern polarizers

- Ultra-thin (0.1 - 0.2 microns)
- Wide acceptance angle (0 to 50 degrees)
- Wide chromatic range (UV to IR)

Array bonded directly to CCD

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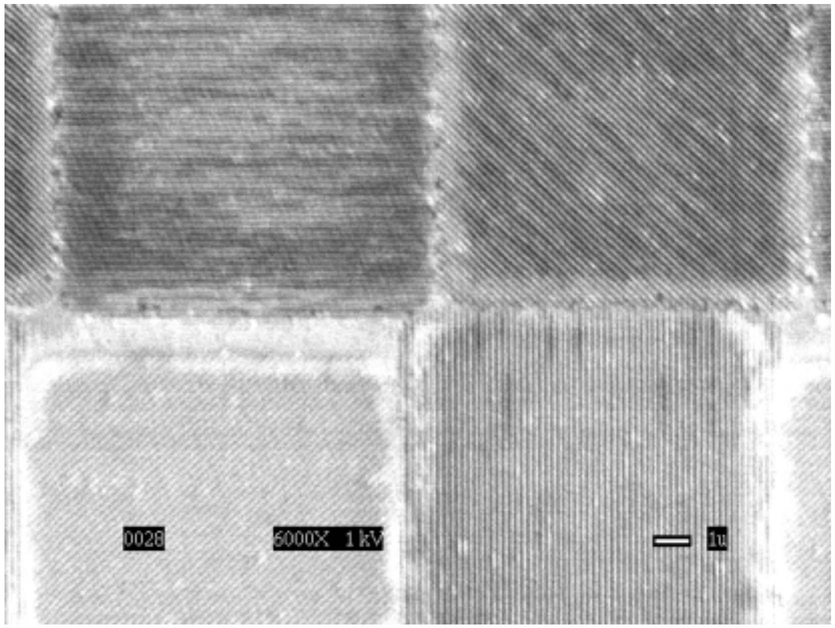
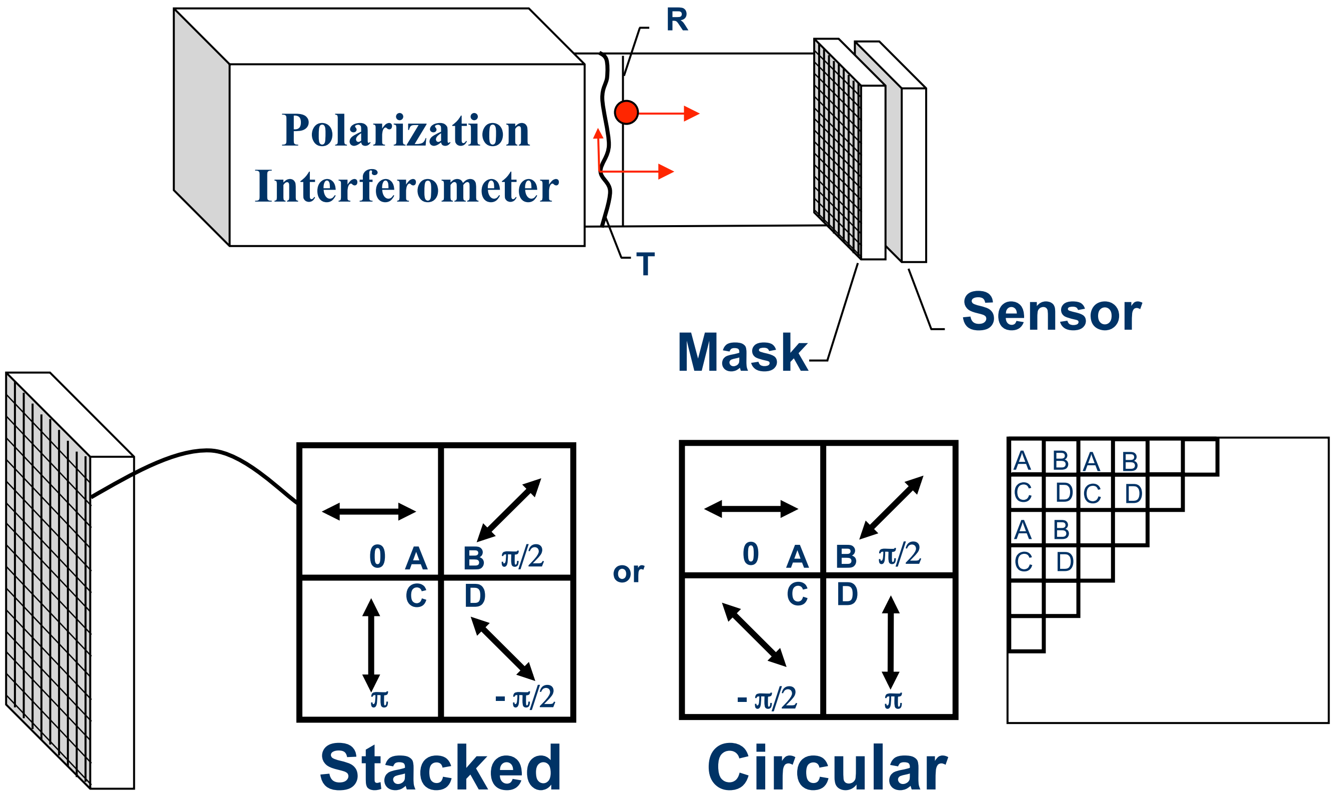
Electron micrograph of wire grid polarizers

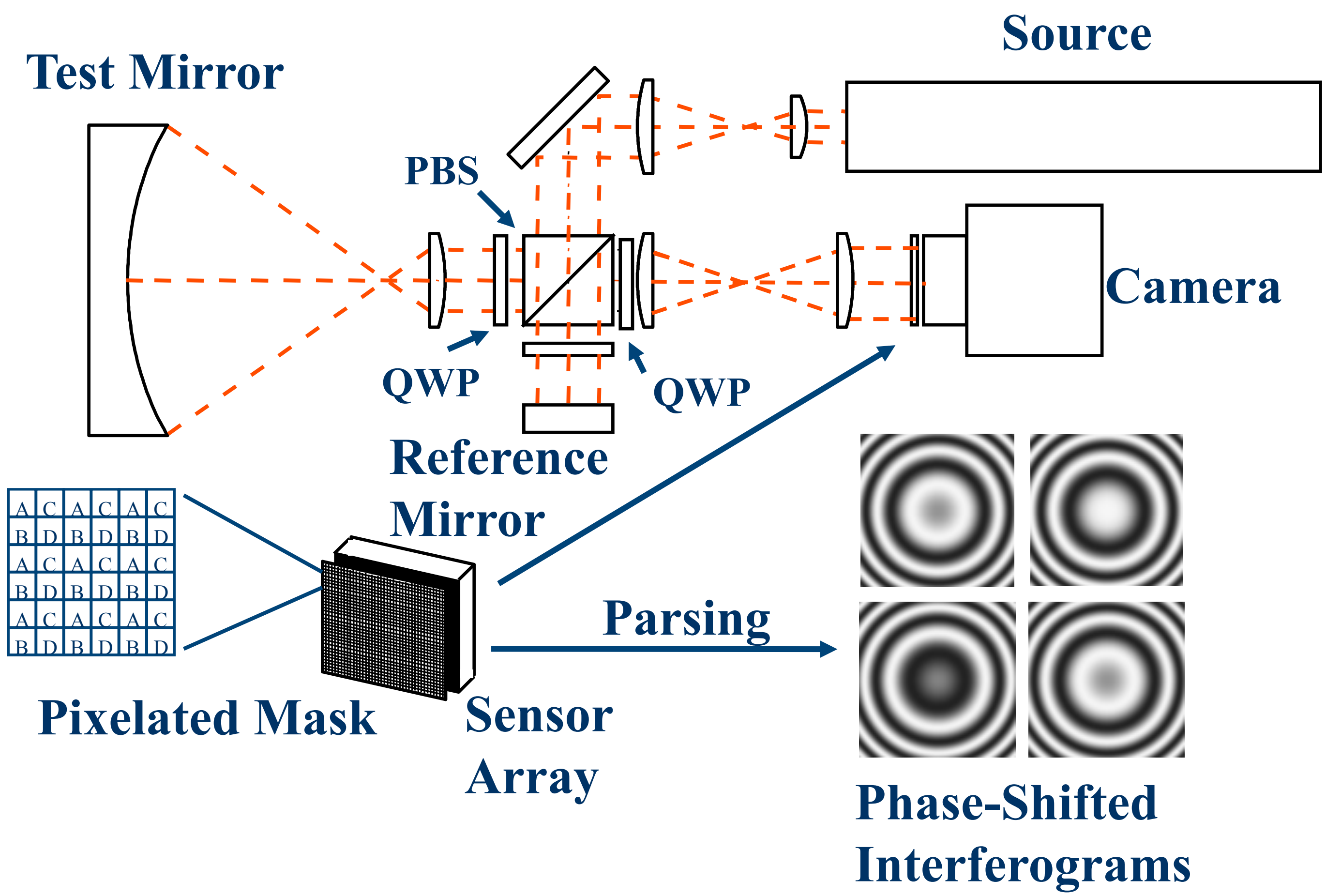


20 um

Array of phase-shift elements unique to each pixel



Pixelated Polarizer Array Phase Sensor Dynamic Interferometer Configuration



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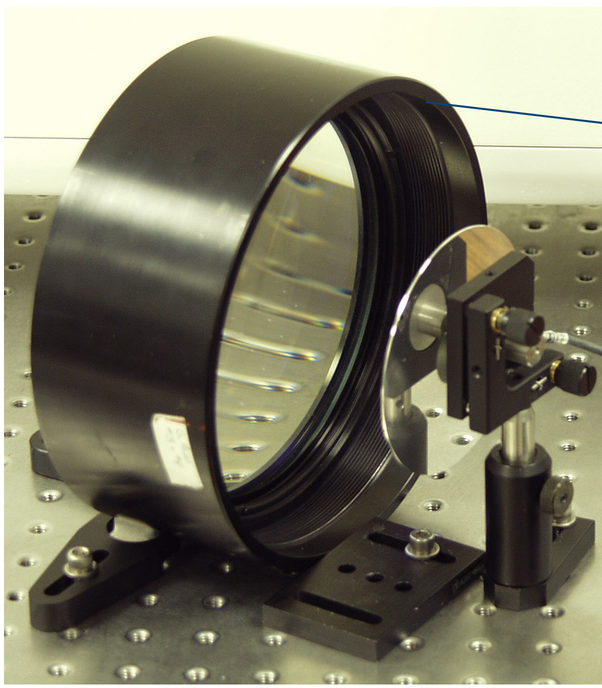
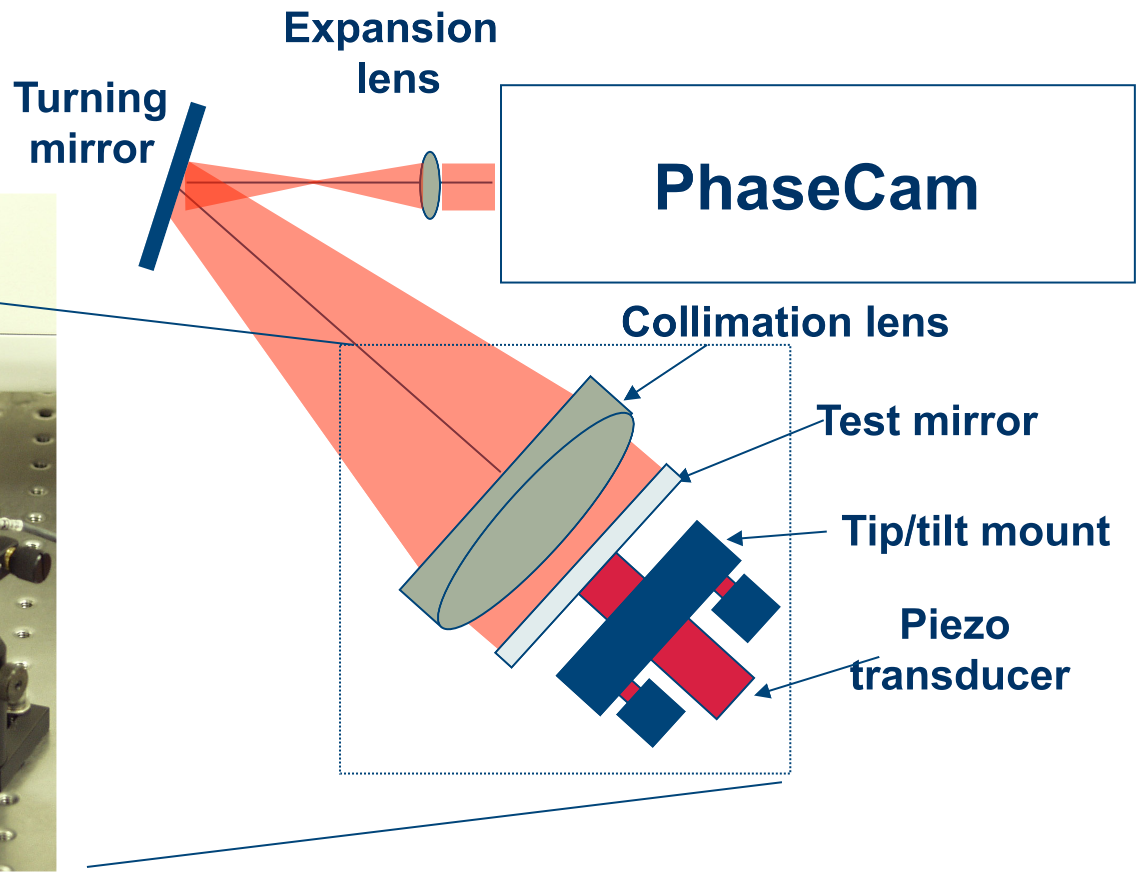
Measuring Vibration

Test articles: aluminum flat (0.7mm thick)
Si wafer (0.2mm thick)





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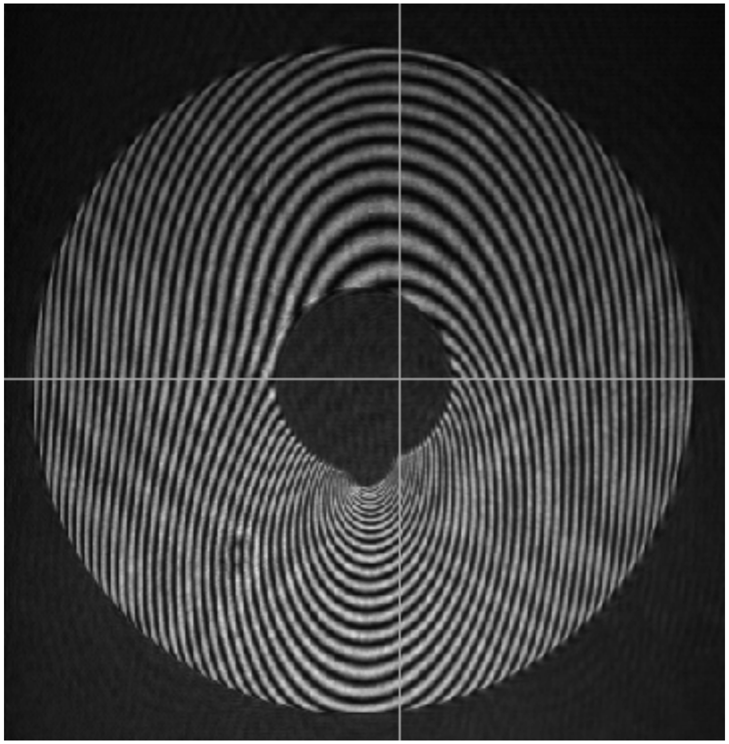
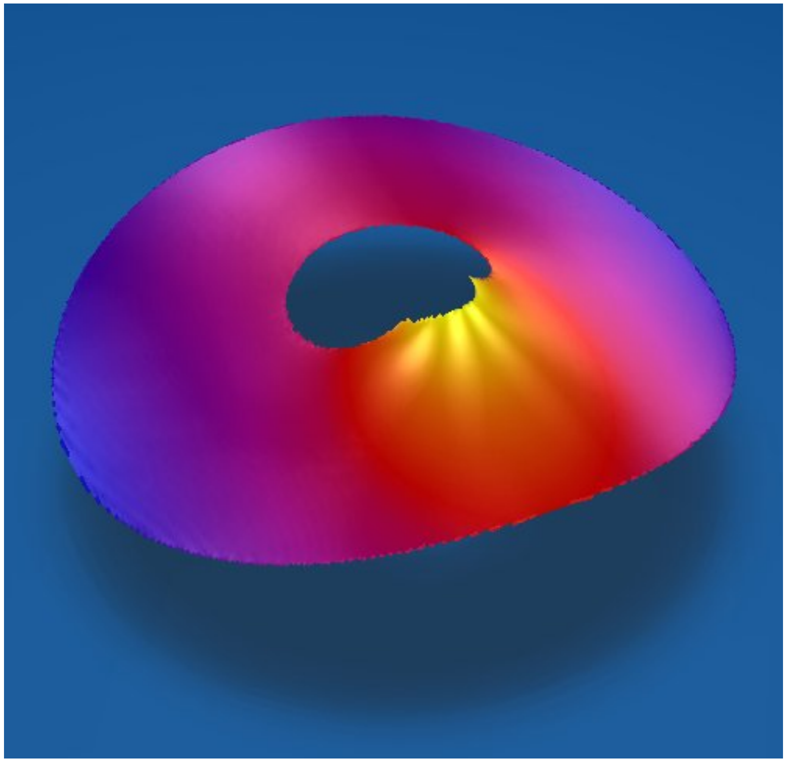
Static Shape of Al Mirror

12.4 micron P-V



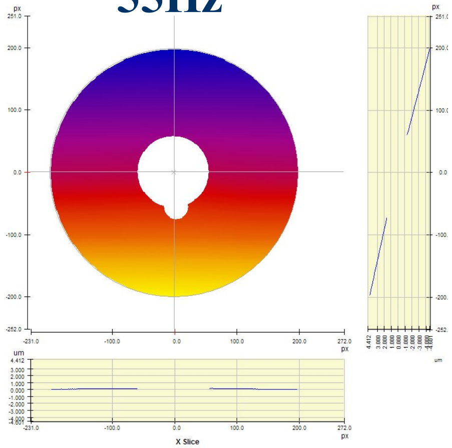
Interferogram



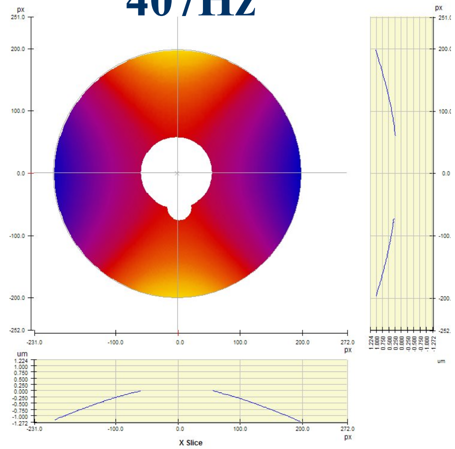
Surface profile

Several Resonant Modes

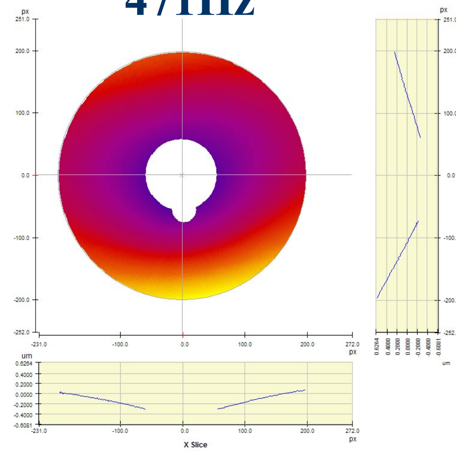
55Hz



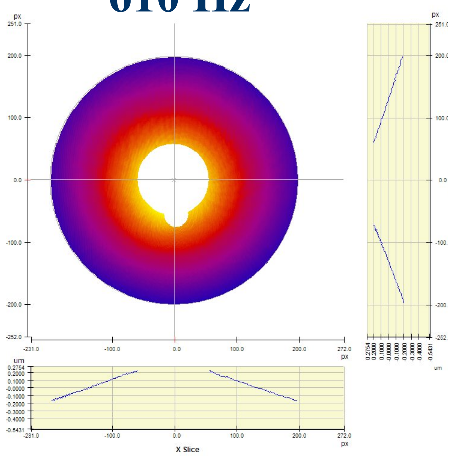
407Hz



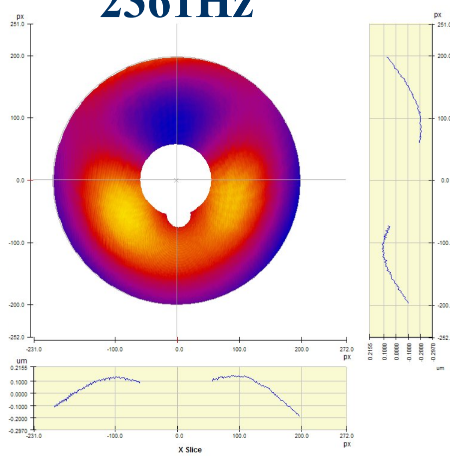
471Hz



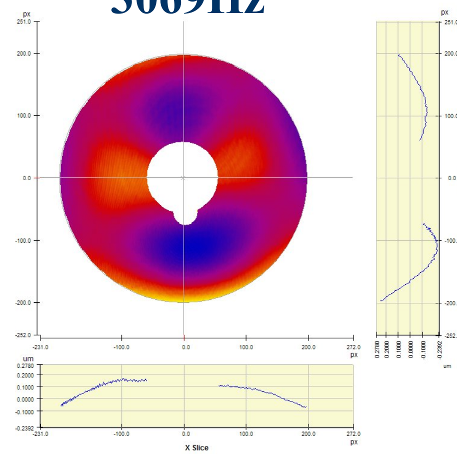
610 Hz



2361Hz



3069Hz



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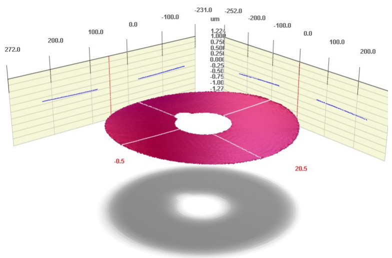
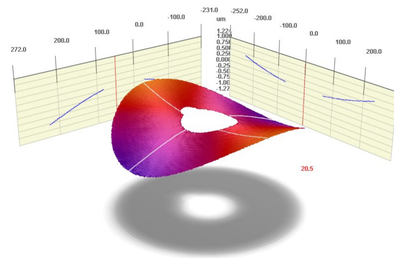
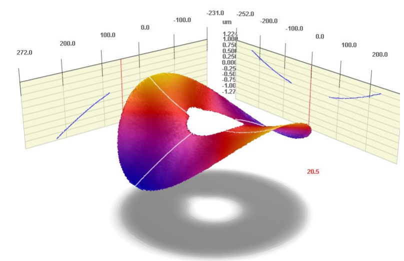
Phase Sweep at 408 Hz



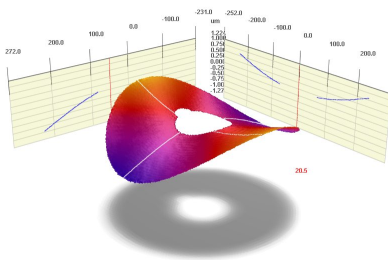
0 deg



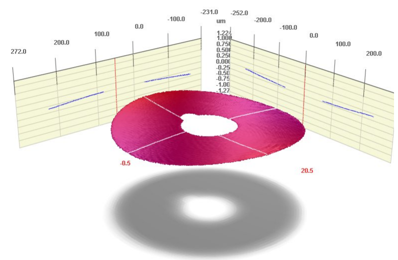
44 deg



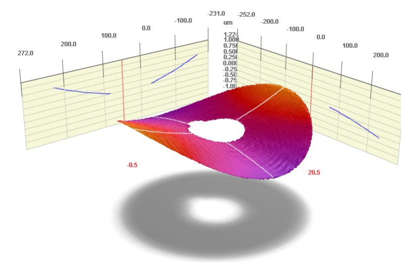
88 deg



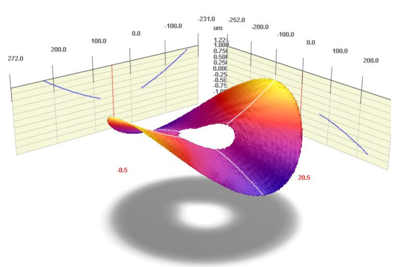
122 deg



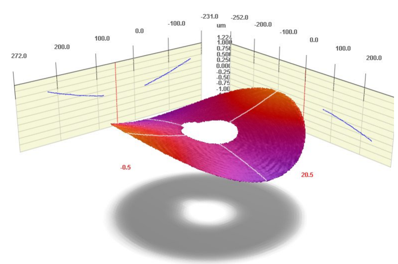
168 deg



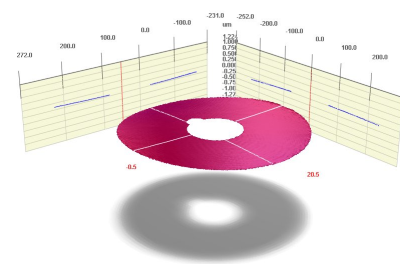
212 deg



266 deg



310 deg



360 deg

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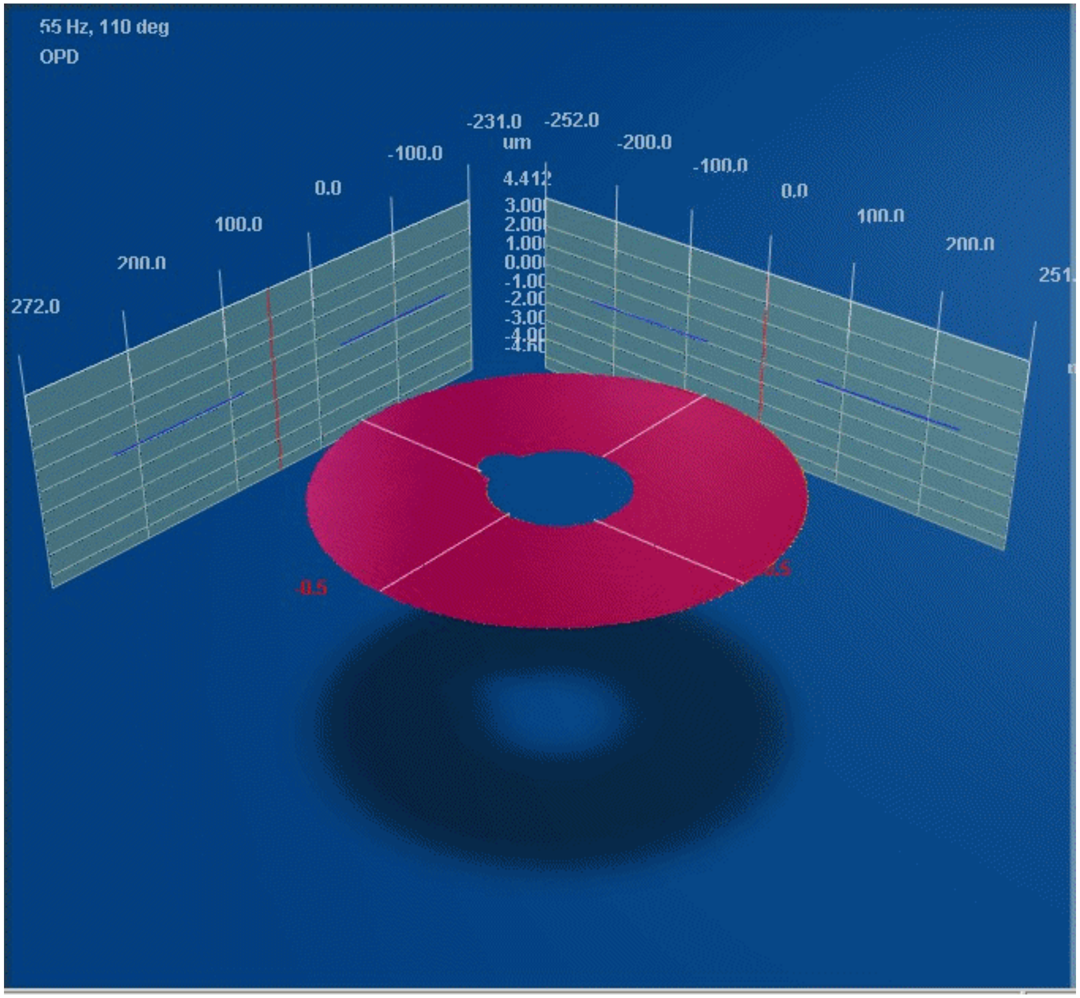
Al Mirror, 55 Hz, First Order Mode



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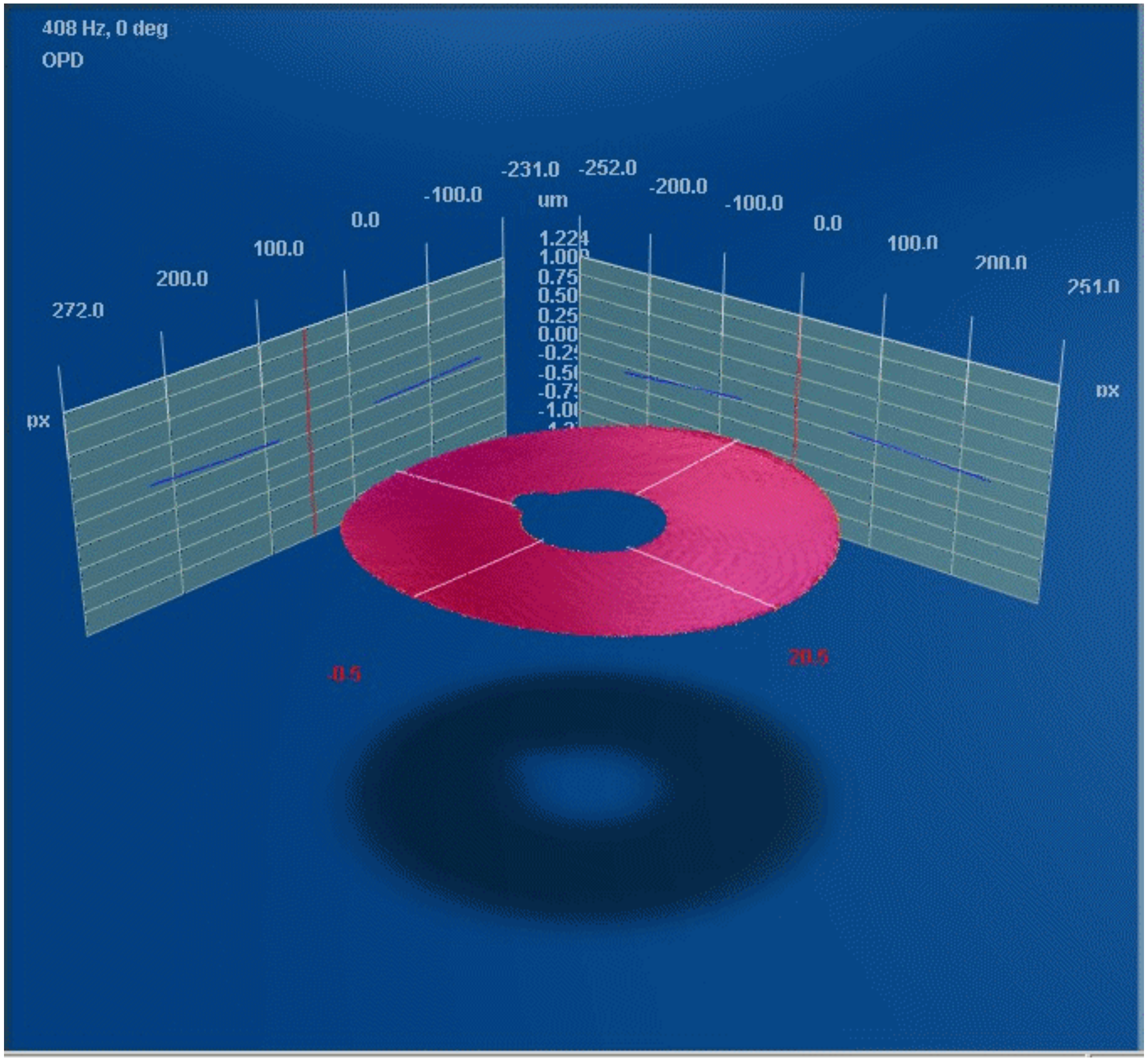
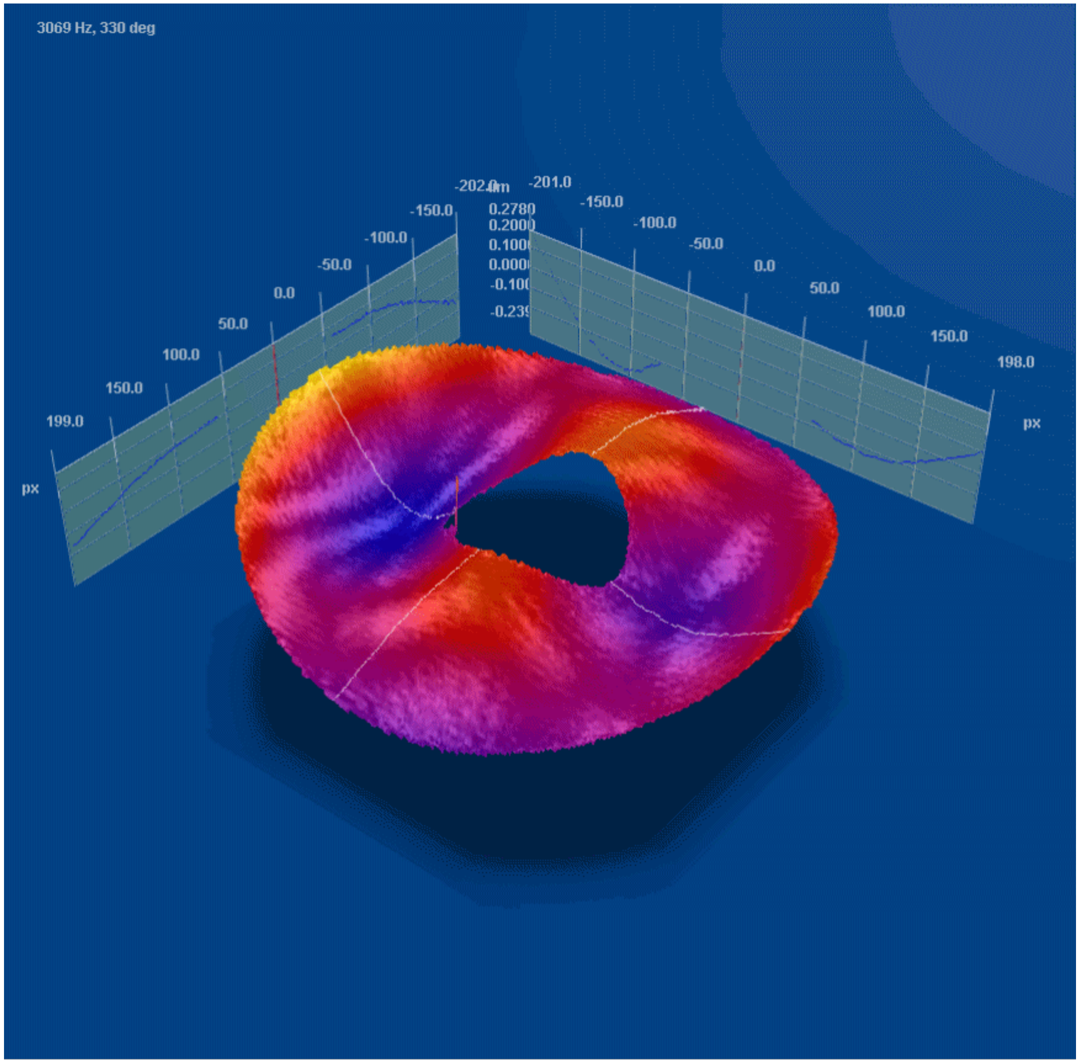
Al Mirror, 408 Hz



Al Mirror, 3069 Hz, Higher Order Mode



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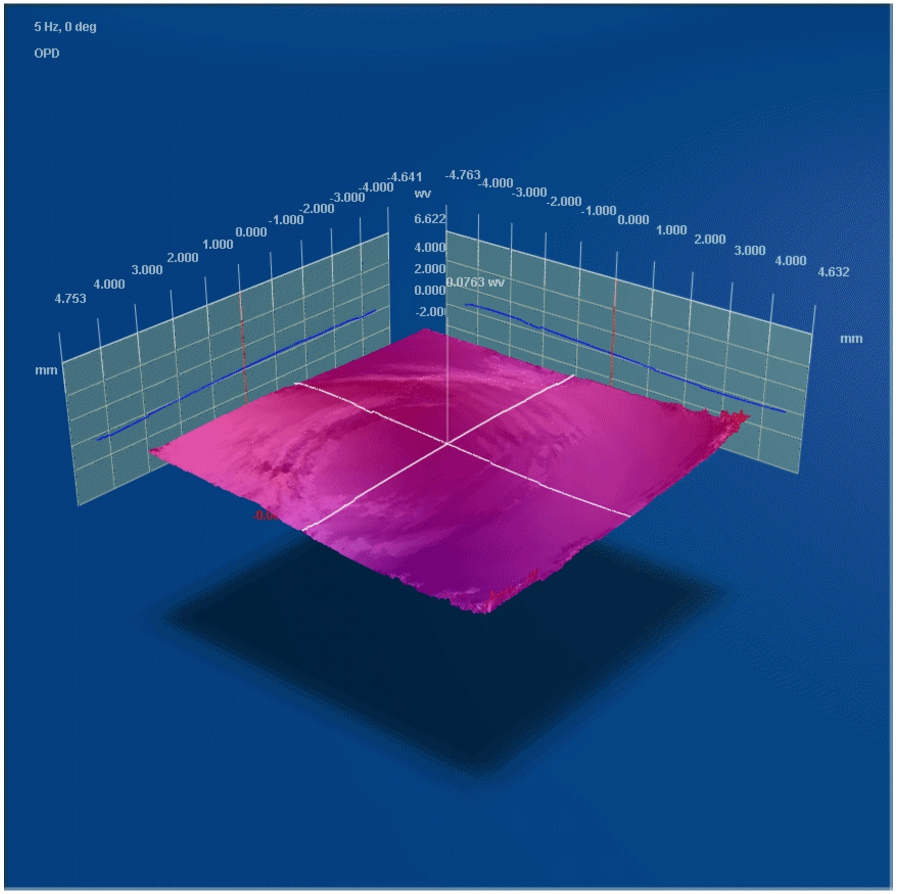
Magnetically Deformable Mirror

Synchronous measurement with square-wave



Collaboration with Phil Laird, Liquid Optics Group, Laval University

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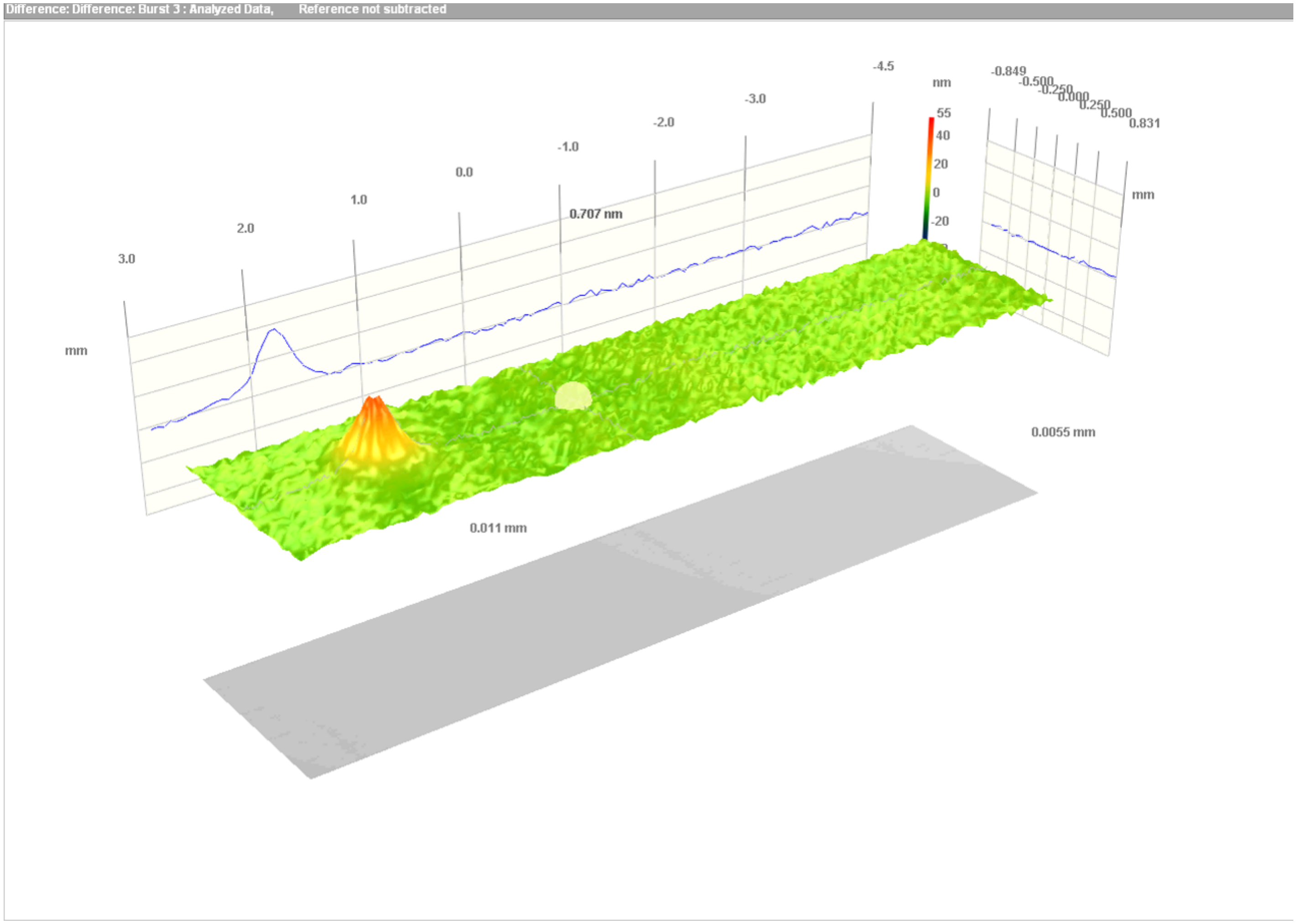
32 Element Deformable Mirror



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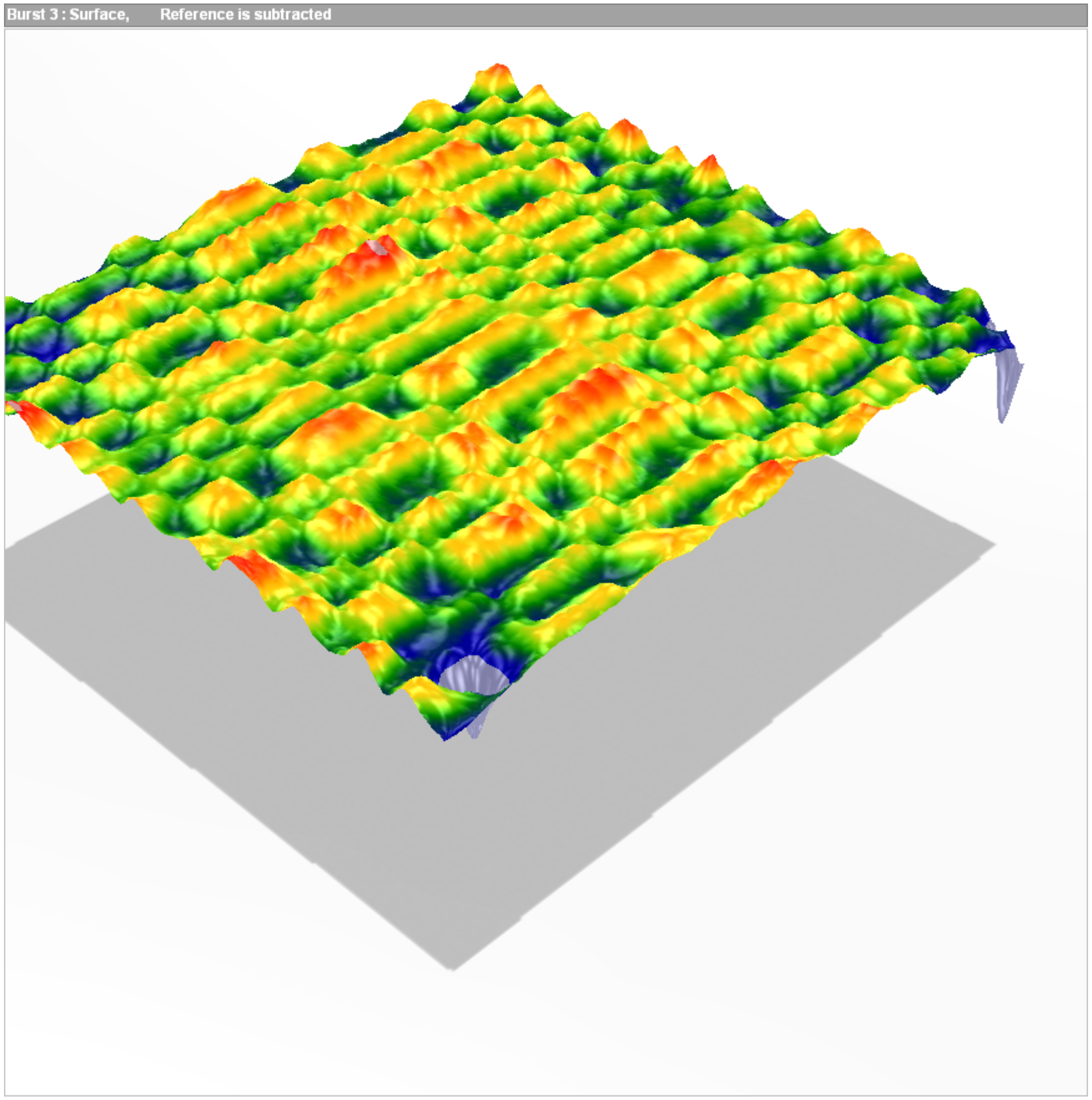
32 x 32 Element Deformable Mirror



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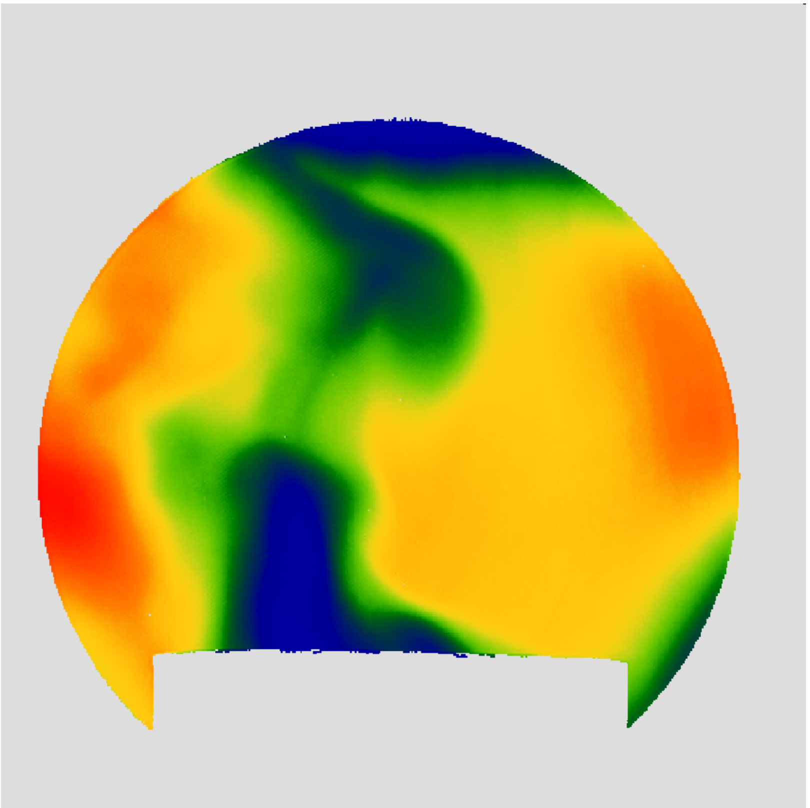
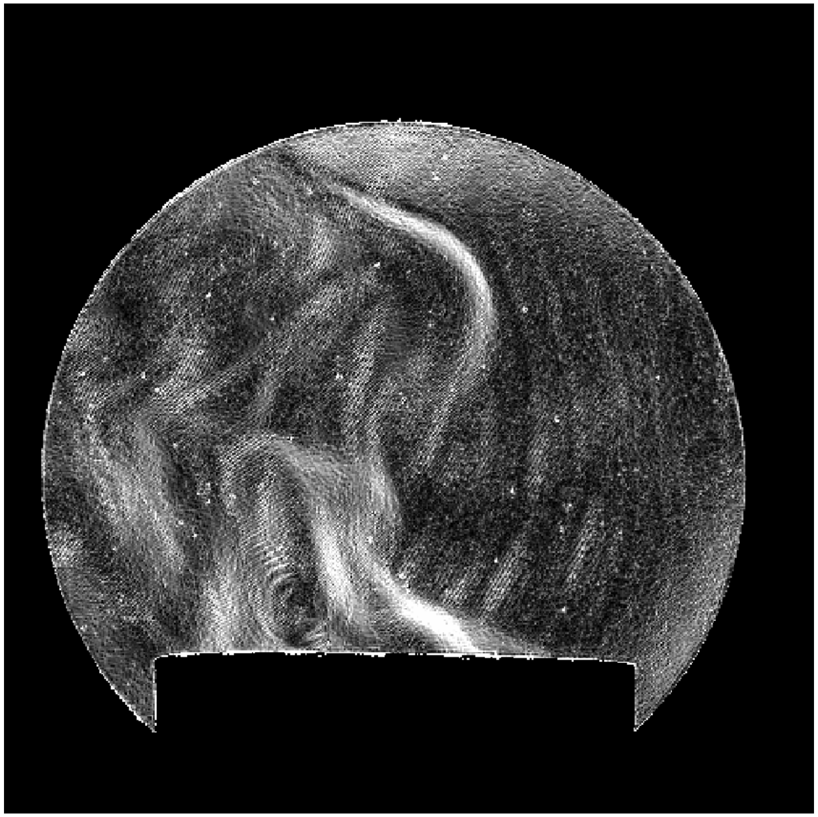
Heat Waves from Hot Coffee

OPD



Slope



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Conclusions - Single Shot Interferometer

- A dynamic single shot interferometer can greatly reduce the effect of vibration and averaging reduces the effect of air turbulence.

- Movies can be made showing how surfaces are vibrating.

- Once a person uses a dynamic phase-shifting interferometer it is hard to go back working with a temporal phase-shifting interferometer.