

# Simple polarisation phase-shifting scatterplate interferometry

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## ABSTRACT

Simple polarised light implementation of the phase shifting method for analysing fringe patterns formed in Burch's common-path scatterplate interferometer has been proposed. The system employs circularly polarised beam at the input end, a quarter wave plate to change the rotation direction of circular polarisation of the reference beam, and a rotating analyser placed in two counter-rotating circular polarisation beams at the output of the interferometer. The simplicity of our approach is proved theoretically and experimentally. Problems related to non-ideal light modulation at the interferometer output and the ways to overcome them are discussed.

**Keywords:** optical testing, scatterplate interferometry, polarized light interferometry, phase shifting

## 1. INTRODUCTION

Common path interferometers are attractive for out-of-laboratory inspections. Because the test and reference beams traverse the same or almost the same path these interferometers are substantially insensitive to vibrations and air turbulence. The scatterplate interferometer (SI) invented by Burch<sup>1,2</sup> serves as one of the best examples. Besides solving the environmental limitations its attractiveness is due to relatively simple structure, low requirements on the interferometer illumination and observation optics, and averaging many measurements at one time<sup>3,4</sup>.

Unfortunately, in SI it is difficult to have the beams separated for phase shifting – one of the methods for computer fringe pattern processing. The solution is to use polarised light approach. Till now three proposals to implement phase shifting in SI were reported<sup>5-7</sup>. In the first two ones an auxiliary polarization optics was placed near the test mirror; the third one uses a birefringent scatterplate. All these solutions use orthogonal linear polarizations of the test and reference beams at the output. The phase between them is shifted with an electro-optic or liquid crystal retarder located at the input end of the interferometer. For the sake of completeness of reviewing the computer-aided fringe analysis methods, the FFT approach requiring high contrast carrier fringes should be quoted<sup>8</sup>.

In this paper a simplified fringe shifting method with polarisation modulated interferometer output is reported. By mere in-plane polariser rotation an arbitrary fringe displacement can be introduced. The test and reference beams should have orthogonal elliptical or circular polarisations<sup>9,10</sup>. Among advantages of our proposal its simplicity (limited number of auxiliary mechanical and optical elements), easy adjustment and calibration can be mentioned. Theoretical explanation and experimental verification for the configuration using a single scatterplate with inversion symmetry are presented. The influence of the errors in the polarization modulated output and the ways of their counterbalancing are discussed.

## 2. SCATTERPLATE INTERFEROMETRY

The scatterplate interferometer for testing concave spherical mirrors in its most common version using a single scatterplate with inversion symmetry transmittance is schematically shown in Fig. 1. A quasi-point source is imaged at the center of the mirror under test. Let us assume that the test optics does not introduce aberrations in

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the paraxial zone. Then the part of the beam illuminating the scatterplate and passing through it directly (without being scattered), and subsequently scattered on the second passage through the plate, is used as the reference beam. On the other hand, the radiation scattered by the plate on its first passage through it fills the whole aperture of the optics under test. Upon the second passage through the plate the unscattered part of that beam will form the object beam. Besides the two interferogram creating beams just mentioned two parasitic beams: doubly unscattered and doubly scattered ones are encountered. The first one forms the so-called “hot spot” (source image) in the detection plane, and the second one contributes the speckle bias into the interferogram. Since the exit pupil of the imaging/observation optics coincides with the scatterplate plane, the lateral dimensions of the beam-splitter and imaging objective must be large enough to avoid vignetting in the output plane.

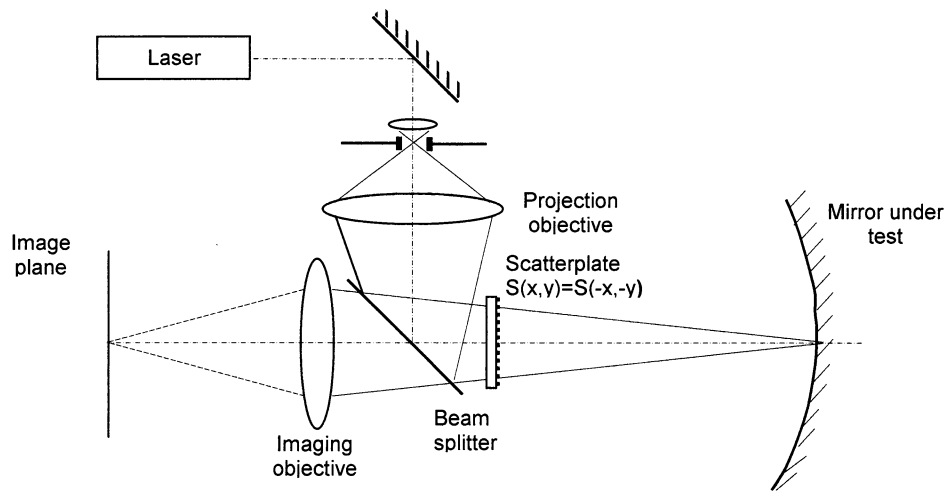


Fig. 1 Optical schematic of an autostigmatic scatterplate interferometer for testing concave mirrors.

The scatterplate is placed in the plane passing through the center of curvature of the mirror under test. The mirror forms the image of the scatterplate back on it with lateral magnification equal to  $-1$ . This is why the scatterplate should possess the inversion symmetry, i.e.,  $S(x, y) = S(-x, -y)$ ;  $S(0, 0)$  corresponds to the center of symmetry. Such a plate can be obtained by doubly exposing a photosensitive material with a scattered light field or chemical etching. Between exposures the plate with photosensitive material has to be very precisely in-plane rotated by 180 degrees. Axial displacement of the scatterplate from the center of curvature plane of the test mirror changes the radius of the reference beam. The tilt between the interfering beams can be introduced by lateral translation of the scatterplate, i.e., displacing  $S(0, 0)$  from the interferometer optical axis.

### 3. POLARIZATION SHIFTING WITH PHASE MODULATION AT THE OUTPUT OF THE INTERFEROMETER

Quantitative analysis of interferograms is achieved using one of the computer aided fringe analysis methods (see, for example, [11]). Among them the phase shifting method is the most accurate one. In scatterplate interferometry (SI) it is difficult, however, to change the phase of one beam because of the common-path propagation of the two beams. The only solution is to apply the polarization approach. Till now three methods to implement phase shifting in SI were reported<sup>5-7</sup>. In all of them the phase shifting device in the form of an electro-optic or liquid crystal retarder was placed at the input end of the interferometer to introduce the phase shift between two orthogonal linear polarizations at the interferometer exit.

To simplify the arrangement and reduce its cost we propose to apply another polarization modulation solution, Fig. 2. Its foundations can be found in literature<sup>9,10</sup>, but to the authors' best knowledge its adaptation to scatterplate interferometry is novel.

The light beam entering the interferometer has a circular polarization. Weakly scattering plate does not change the state of polarization. A small quarter wave plate is placed just in front of the mirror being tested; the azimuth of its fast axis is oriented at 45 deg with respect to incidence plane. It is assumed that the quarter wave plate acts on the reference beam only and the influence of convergence/divergence of the beam passing through it is negligible (see the discussion of experimental results below). Upon double passage of the reference beam through the quarter wave plate the direction of circular polarization of this beam is inverted. Two beams of orthogonal circular polarizations, therefore, are found at the output and they form fringes behind the analyser. By rotating the analyser about its normal the fringes displace laterally (their phase changes).

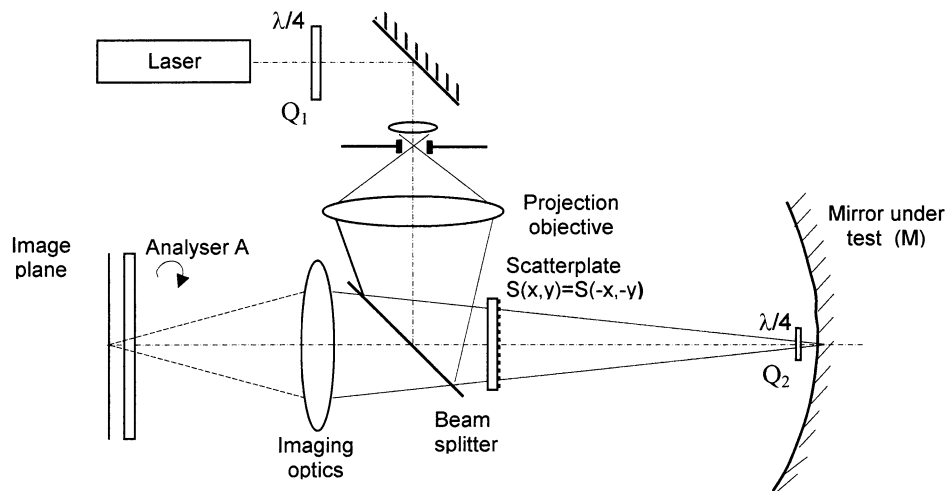


Fig. 2 Optical schematic of a scatterplate interferometer with polarisation modulated output for phase shifting.

The heuristic description given above can be presented in simple mathematical terms. Jones matrixes of polarisation components of the interferometer are as follows:

- a) Quarter wave plates,  $Q_1$  and  $Q_2$ , set at  $45^\circ$

$$Q_{1,2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}, \quad (1)$$

- b) Mirror under test M (reflection)

$$M = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \quad (2)$$

- c) Quarter wave plate  $Q_2$ , second passage after reflection at M

$$Q_{2,2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}. \quad (3)$$

Assuming horizontal linear polarisation of the laser beam entering the interferometer, i.e.,  $E = [1,0]$ , we get the following polarisation state descriptions of the object and reference beams, respectively, just before the analyser:

Object beam:

$$O = M \cdot Q_{1,2} \cdot E = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}; \quad (4)$$

Reference beam:

$$R = Q_{2,2} \cdot M \cdot Q_{1,2} \cdot Q_{1,2} \cdot E = -\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}. \quad (5)$$

Orthogonal circular polarisation beams are encountered just before the analyser A. The minus sign appearing in the reference beam denotes mutual phase shift of  $\pi$  between the beams. It has no influence, however, on the phase stepping process implemented by rotating the analyser. According to the analyses given, for example in [9] and [10], analyser adjacent angular settings separated by  $\pi/4$  provide fringe patterns with mutual phase shift of  $\pi/2$ .

General mathematical description of this effect is as follows. The amplitudes and phases of circularly polarized reference and object beams just before analyzer A can be described by

$$E_r = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} a_r e^{i\varphi_r}, \quad (6)$$

$$E_o = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} a_o e^{i\varphi_o}, \quad (7)$$

where  $a_r$ ,  $a_o$  and  $\varphi_r$  and  $\varphi_o$  are amplitudes and phases of reference and object beams, respectively. After passing through the analyzer A, these beams interfere and the resulting amplitude and phase are given by

$$E_\alpha = (E_o + E_r) \mathbf{A}_\alpha = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} \{ a_o e^{i(\varphi_o + \alpha)} - a_r e^{i(\varphi_r - \alpha)} \}, \quad (8)$$

where  $\mathbf{A}_\alpha$  is the Jones matrix of analyzer A rotated by angle  $\alpha$

$$\mathbf{A}_\alpha = \begin{bmatrix} \cos^2 \alpha & \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha & \sin^2 \alpha \end{bmatrix}. \quad (9)$$

Next the intensity distribution can be calculated as

$$I_\alpha = E_\alpha E_\alpha^* = \frac{1}{2} (a_o^2 + a_r^2) - a_o a_r \cos(\Delta\varphi + 2\alpha), \quad (10)$$

where:  $\Delta\varphi = \varphi_0 - \varphi_r = 2k\Delta w$ ,  $k = 2\pi/\lambda$  and  $\Delta w$  is a function describing the shape deformation of the mirror under test M. According the equation (10), analyzer adjustment angular settings separated by  $\alpha = \pi/4$  provide fringe patterns with mutual phase shift  $2\alpha = \pi/2$ .

#### 4. EXPERIMENTAL WORK

The system was set up and aligned according to the description given above. Four-image algorithm with a phase step of  $\pi/2$  between the frames was selected for automatic interferogram analysis. The four component interferograms are shown in Fig. 3. The spherical mirror under test had a diameter of 200 mm and the radius of curvature of 1100 mm.

Because of slightly non-circular polarisations at the interferometer output some intensity variations were observed between the frames. The reasons of this effect might be: the reflections at metallic surfaces, slightly convergent/divergent reference beam passing through  $Q_2$ , polarisation element departures from theoretical parameters, and the wedge-type beam splitter. Intensity differences between the frames introduce errors in further fringe pattern analysis. To avoid or minimise them average intensity levels of all four fringe patterns were equalised by calibration. For this purpose visual histogram estimation was used and, in result, intensities of images in Figs. 3a and 3d were multiplied by a factor of 1.2.

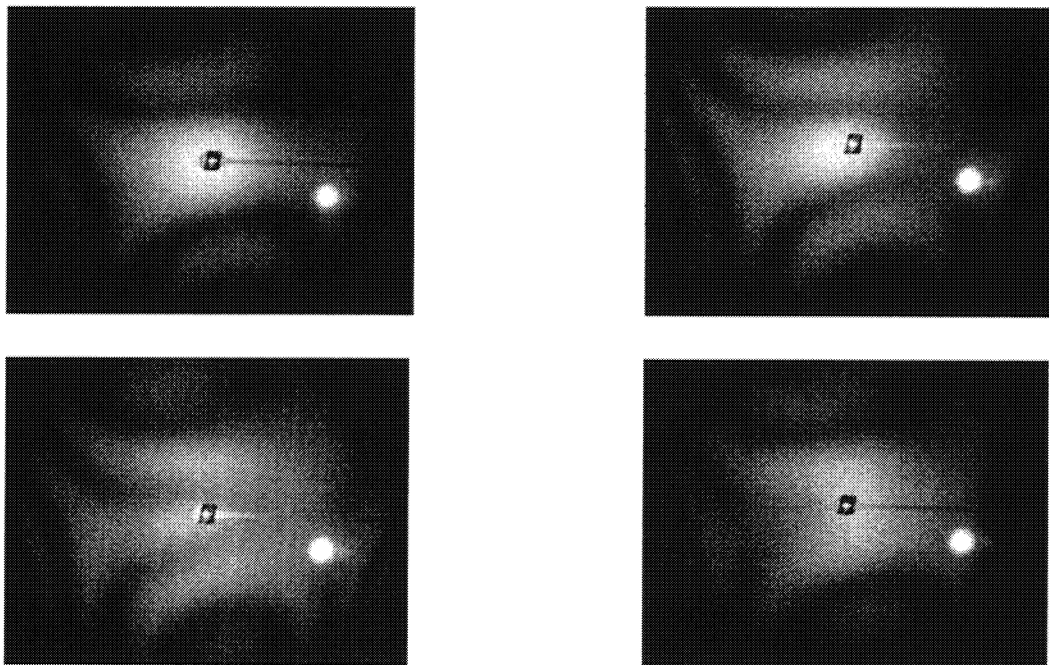


Fig. 3 Fringe patterns, mutually phase shifted by  $\pi/2$ , obtained in the scatterplate interferometer shown in Fig. 2

The phase modulo  $2\pi$  map was calculated and the minimum spanning tree algorithm was used for phase unwrapping. The wave front shape  $\Delta w$  at the output of the interferometer, proportional to the mirror surface shape, was determined according to scaling formulae  $\Delta w = \lambda \Delta\varphi/4\pi$ , where  $\Delta\varphi$  stands for the calculated phase distribution and  $\lambda$  is the light wavelength. Figure 4 shows an example of 3D result representation.

Astigmatism with little spherical aberration<sup>12</sup> can be deduced from interferograms, Fig. 3, and the calculated wave front shape, Fig. 4. The aberration of astigmatism has been introduced by non ideal mirror mounting. Further interferometer optimisation issues will be presented in the forthcoming paper.

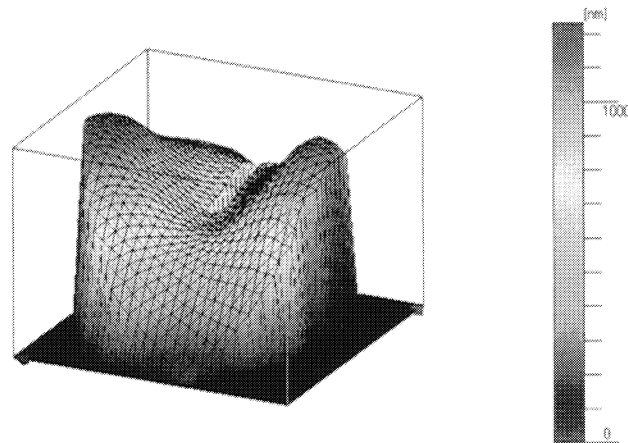


Fig. 4 3D representations of results obtained together with vertical and horizontal cross-sections.

## 5. CONCLUSIONS

A polarised light phase shifting approach for analysing fringe patterns obtained in the common-path interferometer with the scatterplate acting as the beam splitter and recombiner has been proposed. The system operates with circular polarisation at the input and output of the interferometer. The theoretical description has been verified by experiments. Some problems due to non-ideal polarisation optics performance were noticed and the ways of compensating them were undertaken. The simplicity of our solution with comparison to formerly proposed systems using more sophisticated devices for phase shifting between two mutually orthogonal linear polarisation components is to be emphasized.

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