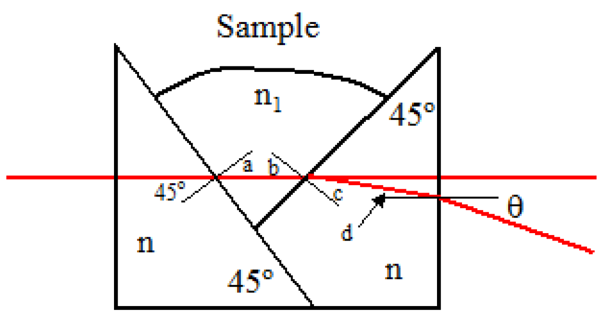
Optical Material Qualification

OM-1

Show that for the Hilger-Chance refractometer the relationship between the refractive index n_1 of the sample and the refractive index n of the V-block is given by

$$n_1 = \left(n^2 - \mathrm{Sin}[\theta] \left(n^2 - \mathrm{Sin}[\theta]^2\right)^{1/2}\right)^{1/2}.$$

Solution



$$n\,\mathrm{Sin}[45°] = n_1\,\mathrm{Sin}[a] \quad \Longrightarrow \quad \mathrm{Sin}[a] = \frac{n}{n_1}\mathrm{Sin}[45°]$$

$$a + b = 90°, \qquad c + d = 45°, \quad \mathrm{Sin}[\theta] = n\,\mathrm{Sin}[d]$$

$$n_1\,\mathrm{Sin}[b] = n\,\mathrm{Sin}[c] \quad \Longrightarrow \quad n_1\,\mathrm{Cos}[a] = n\,\mathrm{Sin}[45° - d] \qquad (1)$$

From (1)

$$n_1 \sqrt{1 - \frac{n^2}{{n_1}^2}\mathrm{Sin}[45°]^2} = n\,\frac{\mathrm{Cos}[d] - \mathrm{Sin}[d]}{\sqrt{2}}$$

$$\sqrt{1 - \frac{n^2}{2\,n_1^2}}\;n_1 = n\,\frac{-\frac{\mathrm{Sin}[\theta]}{n} + \sqrt{1 - \frac{\mathrm{Sin}[\theta]^2}{n^2}}}{\sqrt{2}}$$

$$\left(\sqrt{1 - \frac{n^2}{2\,n_1^2}}\;n_1\right)^2 = \left(n\,\frac{-\frac{\mathrm{Sin}[\theta]}{n} + \sqrt{1 - \frac{\mathrm{Sin}[\theta]^2}{n^2}}}{\sqrt{2}}\right)^2$$

$$-\frac{n^2}{2}+n_1^2=\frac{1}{2}n^2\left(-\frac{\mathrm{Sin}[\theta]}{n}+\sqrt{1-\frac{\mathrm{Sin}[\theta]^2}{n^2}}\right)^2$$

$$\mathbf{Solve}\left[-\frac{n^2}{2}+n_1^2==\frac{1}{2}n^2\left(-\frac{\mathrm{Sin}[\theta]}{n}+\sqrt{1-\frac{\mathrm{Sin}[\theta]^2}{n^2}}\right)^2,\ n_1\right]$$

$$\left\{\left\{n_1\to-\sqrt{n^2-n\,\mathrm{Sin}[\theta]\sqrt{1-\frac{\mathrm{Sin}[\theta]^2}{n^2}}}\right\},\left\{n_1\to\sqrt{n^2-n\,\mathrm{Sin}[\theta]\sqrt{1-\frac{\mathrm{Sin}[\theta]^2}{n^2}}}\right\}\right\}$$

$$n_1=\sqrt{n^2-\mathrm{Sin}[\theta]\sqrt{n^2-\mathrm{Sin}[\theta]^2}}$$

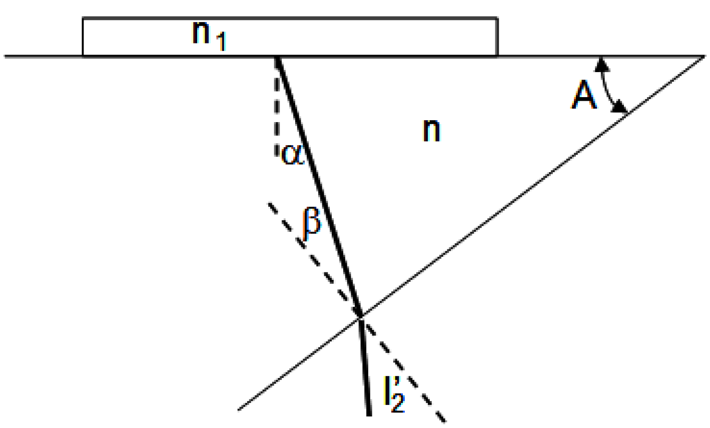
OM-2

Show that for the Pulfrich refractometer

$$n_1=\mathrm{Sin}[A]\sqrt{n^2-\mathrm{Sin}[I_2']^2}\ \text{-}\mathrm{Cos}[A]\ \mathrm{Sin}[I_2']$$

where n $_1$ is the refractive index of the sample, n is the refractive index of the instrument prism, A is the prism angle, and I_2' is the angle at which the limiting ray emerges from the second face of the instrument prism.

Solution



$90° - \alpha + A + 90° - \beta = 180°$

Therefore

$\beta = A - \alpha$

$$n_1 = n\,\text{Sin}[\alpha]; \quad \text{Sin}[I_2'] = n\,\text{Sin}[\beta]$$

$$n_1 = n\,\text{Sin}[\alpha] = n\,\text{Sin}[A-\beta] = n\,(\text{Cos}[\beta]\,\text{Sin}[A] - \text{Cos}[A]\,\text{Sin}[\beta])$$

$$n_1 = n\,\text{Sin}[A]\sqrt{1-\text{Sin}[\beta]^2} - \text{Cos}[A]\,\text{Sin}[I_2']$$

$$\boxed{n_1 = \text{Sin}[A]\sqrt{n^2-\text{Sin}[I_2']^2} - \text{Cos}[A]\,\text{Sin}[I_2']}$$

OM-3

Apart from taking precautions against thermal and like errors, if the prism angle and angle of deviation between incident and emerging rays of a prism spectrometer are measured to an accuracy of 1 arc-second, the refractive index n can be measured to an accuracy on the order of 1 part in the 5th decimal place.

(A) Determine if this statement is correct. What is the optimum range of prism angle?
(B) Which angle measurement is more critical, the prism angle or angle of deviation?

How does the answer depend upon prism angle? For all calculations assume the refractive index is approximately 1.5.

Solution

For minimum deviation

$$\mathbf{n} = \frac{\text{Sin}\left[\frac{a+d}{2}\right]}{\text{Sin}\left[\frac{a}{2}\right]};$$

where a is the prism angle and d is the angle of deviation.

$$\mathbf{dnda} = \partial_a(\mathbf{n})$$

$$\frac{1}{2}\text{Cos}\left[\frac{a+d}{2}\right]\text{Csc}\left[\frac{a}{2}\right] - \frac{1}{2}\text{Cot}\left[\frac{a}{2}\right]\text{Csc}\left[\frac{a}{2}\right]\text{Sin}\left[\frac{a+d}{2}\right]$$

$$\mathbf{dndd} = \partial_d(\mathbf{n})$$

$$\frac{1}{2}\text{Cos}\left[\frac{a+d}{2}\right]\text{Csc}\left[\frac{a}{2}\right]$$

but

$$\text{Sin}\left[\frac{a+d}{2}\right] = n\,\text{Sin}\left[\frac{a}{2}\right]$$

and

$$\text{Cos}\left[\frac{a+d}{2}\right] = \sqrt{1-n^2\,\text{Sin}\left[\frac{a}{2}\right]^2}$$

n = .

dnda = dnda /. $\left\{\text{Sin}\left[\frac{a+d}{2}\right] \to n\,\text{Sin}\left[\frac{a}{2}\right],\ \text{Cos}\left[\frac{a+d}{2}\right] \to \sqrt{1-n^2\,\text{Sin}\left[\frac{a}{2}\right]^2}\right\}$

$$-\frac{1}{2}\,n\,\text{Cot}\left[\frac{a}{2}\right]+\frac{1}{2}\,\text{Csc}\left[\frac{a}{2}\right]\sqrt{1-n^2\,\text{Sin}\left[\frac{a}{2}\right]^2}$$

dndd = dndd /. $\text{Cos}\left[\frac{a+d}{2}\right] \to \sqrt{1-n^2\,\text{Sin}\left[\frac{a}{2}\right]^2}$

$$\frac{1}{2}\,\text{Csc}\left[\frac{a}{2}\right]\sqrt{1-n^2\,\text{Sin}\left[\frac{a}{2}\right]^2}$$

```
n = 1.5;
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TableForm[Table[{a, dnda, dndd}, {a, 10 °, 80 °, 5 °}],
 TableHeadings -> {None, {" a", "   dnda", "  dndd"}}]
```

a	dnda	dndd
10 °	-2.88492	5.68762
15 °	-1.94031	3.75651
20 °	-1.47347	2.77999
25 °	-1.19805	2.18498
30 °	-1.01871	1.78032
35 °	-0.894697	1.484
40 °	-0.805754	1.25485
45 °	-0.740797	1.06986
50 °	-0.693379	0.915001
55 °	-0.659684	0.781053
60 °	-0.6376	0.661438
65 °	-0.626376	0.550889
70 °	-0.626812	0.444299
75 °	-0.642607	0.334812
80 °	-0.687492	0.206323

a)

A prism angle of 60° or a little larger appears optimum. For either angle deviation error or prism angle error of 1 arc sec (5 x 10^{-6}radians) the error in n is of the order of 3 x 10^{-6}. Therefore, for 1 arc sec error in both prism angle and angle of deviation an accuracy of 1 part in the 5th decimal place is reasonable.

b)

For 60° prism, sensitivity to both prism angle and angle of deviation approximately the same. For smaller prism angles we have more sensitivity to angle of deviation errors and for larger prism angles there is more sensitivity to prism angle error.

OM-4

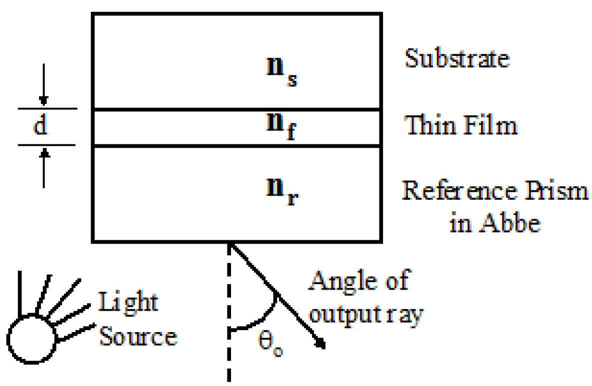
The Abbe refractometer is used to measure the thickness and refractive index of a thin film coated on a substrate having a higher refractive index than the film. When the thin film is contacted to the measuring prism and light is transmitted up through the measuring prism, the light reflected off the two surfaces of the thin film is observed. Interference fringes are seen. The first two dark interference fringes are designated n_1 and n_2 respectively, where n_1 is the one with apparently higher index of refraction. Show that n_f, the index of the film, and d, the film thickness, are given by

$$n_f = \sqrt{\frac{(4 n_1^2 - n_2^2)}{3}}, \qquad \text{and} \quad d = \frac{\lambda}{2} \sqrt{\frac{3}{(n_1^2 - n_2^2)}}.$$

Solution

The Abbe thin film measuring technique described below is useful for looking at film thicknesses between 0.05 and 0.0001 mm. The refractive index of the film must be within the range of the instrument and the sample must be flat and uniform in thickness. The sample is looked at in reflection.

When the thin film is contacted to the measuring prism the usual critical angle dividing line disappears and a series of interference fringes are seen. The first two dark interference fringes are used. Figure 1 shows a schematic of the thin film and reference prism.



While the question only asked for the case where $n_s > n_f$, we will look at both the case where $n_s > n_f$ and the case where $n_s < n_f$. For both cases $n_r > n_f$.

Case I: $n_s > n_f$

$n_f \, \text{Sin}[\theta_f] = n_r \, \text{Sin}[\theta_r] = n_o \, \text{Sin}[\theta_o] = n_1$
$=$ index we read on Abbe scale

$$\text{Cos}[\theta_f] = \frac{\sqrt{n_f^2 - n_1^2}}{n_f}$$

Since $n_s > n_f$ we have a π phase change upon reflection.

For dark fringe

$2\, n_f\, d\, \mathrm{Cos}[\theta_f] = m\, \lambda$, m is an integer

For first dark fringe (other than $\theta_f = 90°$)

$2\, n_f\, d\, \mathrm{Cos}[\theta_{f1}] = \lambda$ where $\mathrm{Cos}[\theta_{f1}] = \frac{\sqrt{n_f^2 - n_1^2}}{n_f}$

For second dark fringe we have

$2\, n_f\, d\, \mathrm{Cos}[\theta_{f2}] = 2\, \lambda$ where $\mathrm{Cos}[\theta_{f2}] = \frac{\sqrt{n_f^2 - n_2^2}}{n_f}$

Therefore,

$2\, d \sqrt{n_f^2 - n_1^2} = \lambda$ and $2\, d \sqrt{n_f^2 - n_2^2} = 2\, \lambda$

filmIndex = n_f /. Solve[4 $(n_f^2 - n_1^2)$ == $n_f^2 - n_2^2$, n_f]

$$\left\{-\frac{\sqrt{4\, n_1^2 - n_2^2}}{\sqrt{3}}, \frac{\sqrt{4\, n_1^2 - n_2^2}}{\sqrt{3}}\right\}$$

n_f = filmIndex[[2]];

$$n_f = \frac{\sqrt{4\, n_1^2 - n_2^2}}{\sqrt{3}}$$

Next we will find the film thickness, d.

d = FullSimplify$\left[\frac{\lambda}{2} \frac{1}{\sqrt{n_f^2 - n_1^2}}\right]$;

$$d = \frac{\sqrt{3}\, \lambda}{2 \sqrt{n_1^2 - n_2^2}}$$

Case II: $n_s < n_f$ so no π phase change upon reflection.

For dark fringe

$2\, n_f\, d\, \mathrm{Cos}[\theta_f] = \left(m - \frac{1}{2}\right) \lambda$, m is an integer

For first dark fringe (other than $\theta_f = 90°$)

$$2\, n_f\, d\, \mathrm{Cos}[\theta_{f1}] = \frac{\lambda}{2}$$

For second dark fringe we have

$$2\, n_f\, d\, \mathrm{Cos}[\theta_{f2}] = \frac{3\, \lambda}{2}$$

Therefore,

$$2\,d\,\sqrt{n_f{}^2 - n_1{}^2} = \frac{\lambda}{2} \text{ and } 2\,d\,\sqrt{n_f{}^2 - n_2{}^2} = \frac{3\,\lambda}{2}$$

```
nf =.;
filmIndex = nf /. Solve[9 (nf^2 - n1^2) == nf^2 - n2^2, nf]
```

$$\left\{-\frac{\sqrt{9\,n_1^2 - n_2^2}}{2\,\sqrt{2}},\ \frac{\sqrt{9\,n_1^2 - n_2^2}}{2\,\sqrt{2}}\right\}$$

```
nf = filmIndex[[2]];
```

$$n_f = \frac{\sqrt{9\,n_1^2 - n_2^2}}{2\,\sqrt{2}}$$

Next we will find the film thickness, d.

```
d = FullSimplify[λ/4 1/Sqrt[nf^2 - n1^2]];
```

$$d = \frac{\lambda}{\sqrt{2}\,\sqrt{n_1^2 - n_2^2}}$$

OM-5

If the index matching oil film in an Abbe refractometer has a 1 minute wedge, what is the effect upon the index measurement if the oil has an index of 1.62, the prism has an index of 1.65, and the sample being measured has an index of 1.5?

Solution

At the first boundary

$$n_1\,\text{Sin}[\theta_1] = n_2\,\text{Sin}[\theta_2]$$

Due to a wedge of ϵ at the second boundary

$$n_2\,\text{Sin}[\theta_2 + \epsilon] = n_3\,\text{Sin}[\theta_3]$$

If $\epsilon = 0$ and $\theta_1 = 90°$ then $n_1 = n_3\,\text{Sin}[\theta_3] = 1.5$

If $\epsilon \neq 0$ then we still think $n_1 = n_2\,\text{Sin}[\theta_2 \pm \epsilon]$.

What is θ_2?

$$\text{Sin}[\theta_2] = \frac{1.5}{1.62} \quad \text{or} \quad \theta_2 = \text{ArcSin}\left[\frac{1.5}{1.62}\right] = 67.80839\,°$$

$$n_2\,\text{Sin}[\theta_2 + \epsilon] = 1.62\,\text{Sin}\left[67.80839\,° \pm \frac{1}{60}\,°\right]$$

$$\left\{1.62\,\text{Sin}\left[67.80839\,° + \frac{1}{60}\,°\right],\ 1.62\,\text{Sin}\left[67.80839\,° - \frac{1}{60}\,°\right]\right\}$$

{1.50018, 1.49982}

Thus we have an error of 2 parts in the 4th decimal place.

OM-6

An Abbe refractometer having a reference prism of refractive index 1.6 is used to measure the refractive index of a solid sample. During the measurement the maximum angle the light is from the normal in the reference prism is 76°.

a) Sketch the Abbe refractometer.

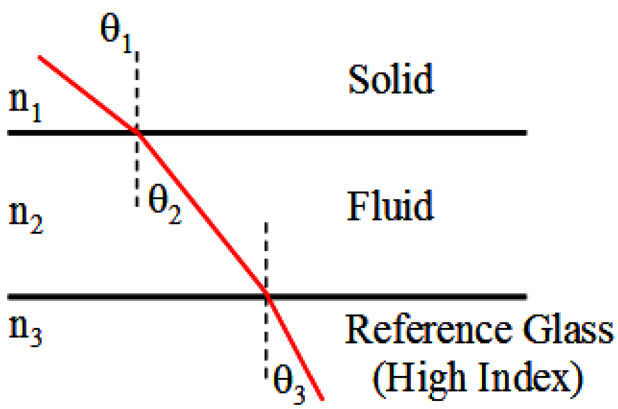
b) What is the refractive index of the solid sample being measured?

c) What is the minimum refractive index of the index matching fluid that can be used for this test.

Solution

a)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$



b)

$n_1 = 1.6\,\text{Sin}[76\,°]$

1.55247

c)

The refractive index of the index matching fluid must be greater than the index of the sample being measured. Therefore, the index must be greater than 1.55247.

OM-7

Give two factors which limit the maximum refractive index that can be measured using the Abbe refractometer.

Solution

The refractive index of the reference prism and the refractive index of the index matching fluid limit the refractive index that can be measured using the Abbe refractometer. In particular, the refractive index must be less than or equal to the lower of the refractive index of the index matching fluid or the reference prism.

OM-8

What influence will scratches on the reference prism of an Abbe refractometer have on the accuracy of the refractive index measurement? Other than re-polishing the surface, what can be done to minimize the effects of the scratches?

Solution

Scratches will cause the dividing line between light and dark (critical angle boundary) to become fuzzy so the measurement accuracy is reduced. The effects of the scratches are minimized as the refractive index of the index fluid becomes closer to the refractive index of the reference prism.

OM-9

An Abbe refractometer is used to measure the refractive index of a transparent solid sample. The reference prism in the instrument has a refractive index of 1.62. The index matching fluid has a refractive index of 1.58.

a) What is the maximum refractive index that can be measured?

b) If the refractive index of the sample being measured is 1.55, what is the maximum angle the light makes with respect to the normal of the reference prism inside the prism?

c) What is the major advantage of the Abbe refractometer to a spectrometer for measuring refractive index?

Solution

a)

The maximum refractive index that can be measured is 1.58.

b)

$1.55 \, \text{Sin}[90°] = 1.62 \, \text{Sin}[\theta]$

$$\boxed{\theta = \text{ArcSin}\left[\frac{1.55}{1.62}\right] = 73.1°}$$

c)

A high quality prism does not need to be made from the sample being measured.

OM-10

Ellipsometry is used to measure the state of polarization of light reflected off a sample. What are the two properties of the state of polarization being measured?

Solution

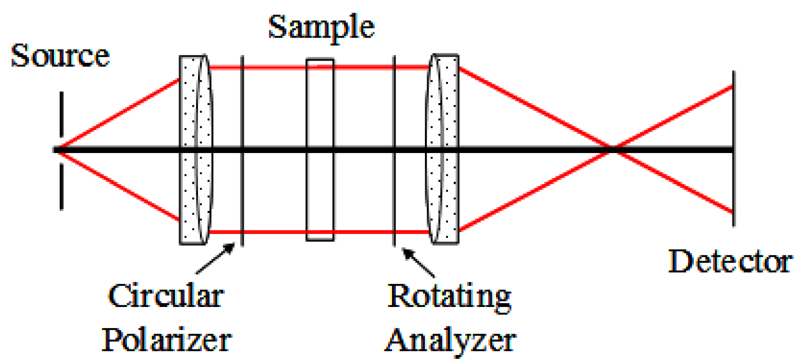
Since ellipsometry essentially measures the state of polarization of reflected or transmitted light it can be thought of as polarimetry. The state of polarization is defined by the phase and amplitude relationships between the two component plane waves into which the electric field is resolved. The wave having the electric field in the plane of incidence is called the p wave, and the wave having the electric field normal to the plane of incidence is called the s wave. In general, reflection causes a change in relative phases of the p and s waves and a change in the ratio of their amplitudes. The change in phase is characterized by the angle Δ, and the amplitude ratio change is characterized by Tan[ψ]. If the amplitudes of the incident and reflected beams are designated e and r, respectively, and phases of the incident and reflected beams are α and β, respectively

$$\text{Tan}[\psi] = \frac{|r_p|}{|r_s|}\,\frac{|e_s|}{|e_p|}$$

$$\Delta = (\beta_p - \beta_s) - (\alpha_p - \alpha_s)$$

OM-11

A source of wavelength 500 nm is used in the setup shown below to measure the birefringence of the sample.



In units of nm, what is the smallest amount of birefringence present in the sample if the irradiance at a detector point varies from I_{min} to I_{max} as the analyzer is rotated 360°, where I_{min} and I_{max} are

a)

$I_{min} = 1$ and $I_{max} = 1$;

b)

$I_{min} = 0$ and $I_{max} = 1$;

c)

What is the second smallest amount of birefringence such that $I_{min} = 1$ and $I_{max} = 1$?

Solution

a)

We must have circular polarization so the minimum birefringence in the sample is zero.

b)

To get an $I_{min}$of 0 we must have linear polarization. This means the sample must be a quarter-wave plate so the minimum birefringence is $\frac{500}{4}$ nm or 125 nm.

c)

To have $I_{max} = I_{min}$ we must have circular polarization. The second smallest amount of birefringence present would be a half wave plate or 250 nm.