Image Quality and Adaptive Imaging

Matthew A. Kupinski
Associate Professor
College of Optical Sciences
University of Arizona
Tucson, Arizona

November 7, 2012

Introduction

- Imaging equation
- The need for objective measures of image quality
- Task-based assessment of image quality
- Adaptive and multimodality imaging
- Summary

Imaging Equation

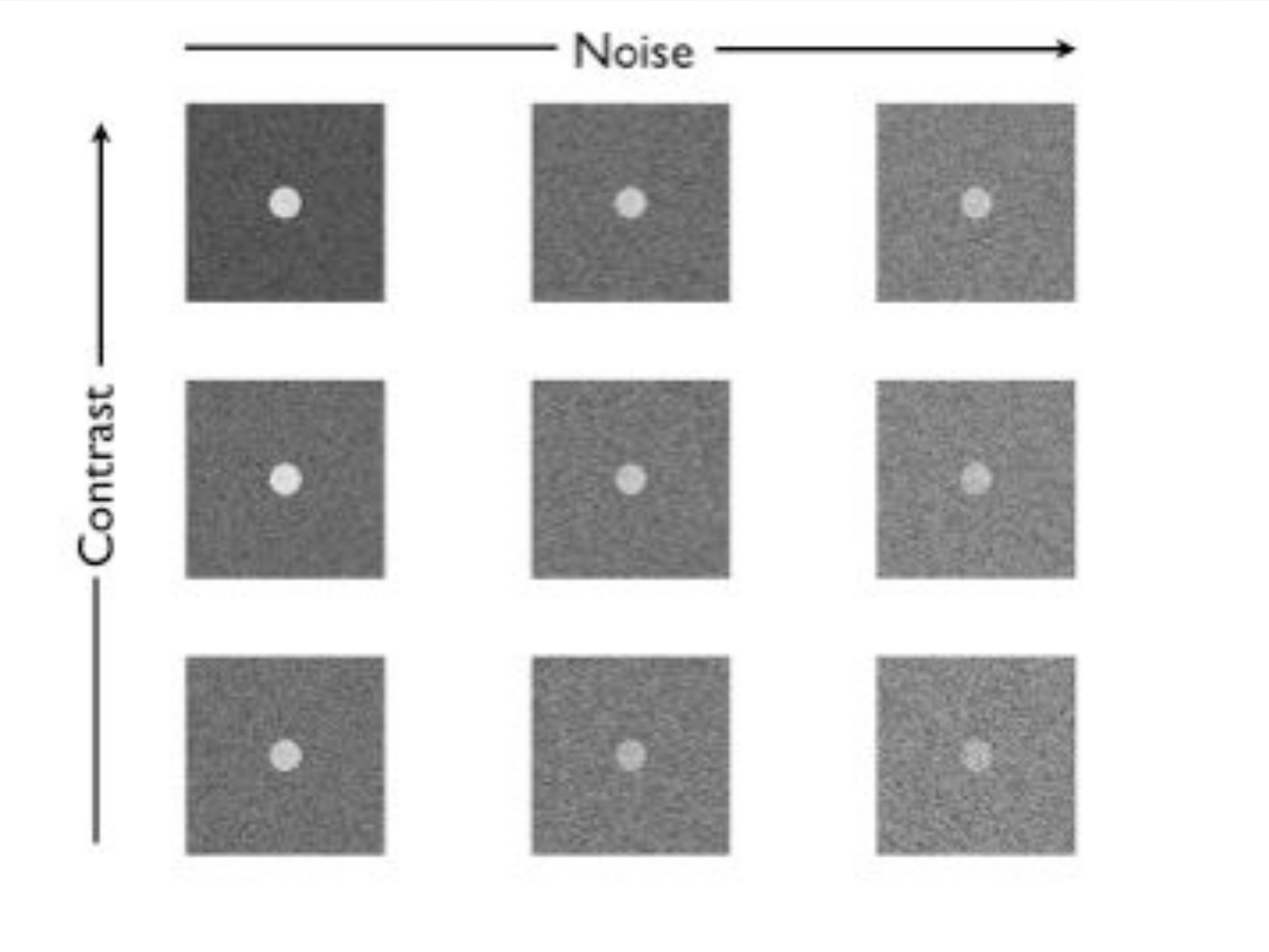
$$\boldsymbol{g} = \mathcal{H}\boldsymbol{f} + \boldsymbol{n}$$

- $\boldsymbol{f}$: Continuous function representing the distribution of the radiotracer
- $\boldsymbol{n}$: Noise. Not necessarily additive
- $\mathcal{H}$: Imaging operating
- $\boldsymbol{g}$: Discrete image data

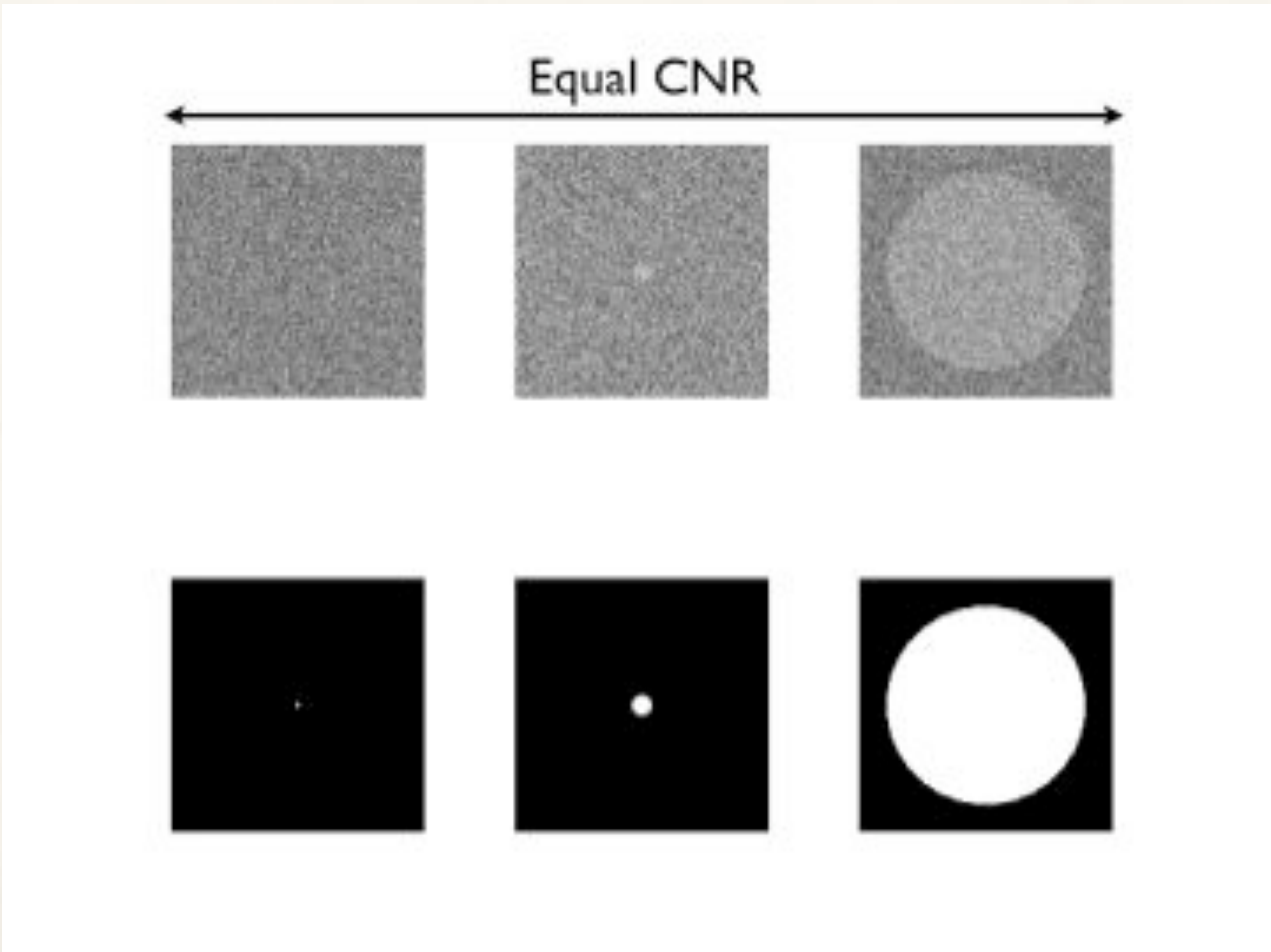
Potential methods

- Qualitative
  - Visual inspection using a single image
  - Visual inspection using a series of images
  - Visual inspection by committee
- Quantitative
  - Noise, resolution, contrast, etc.

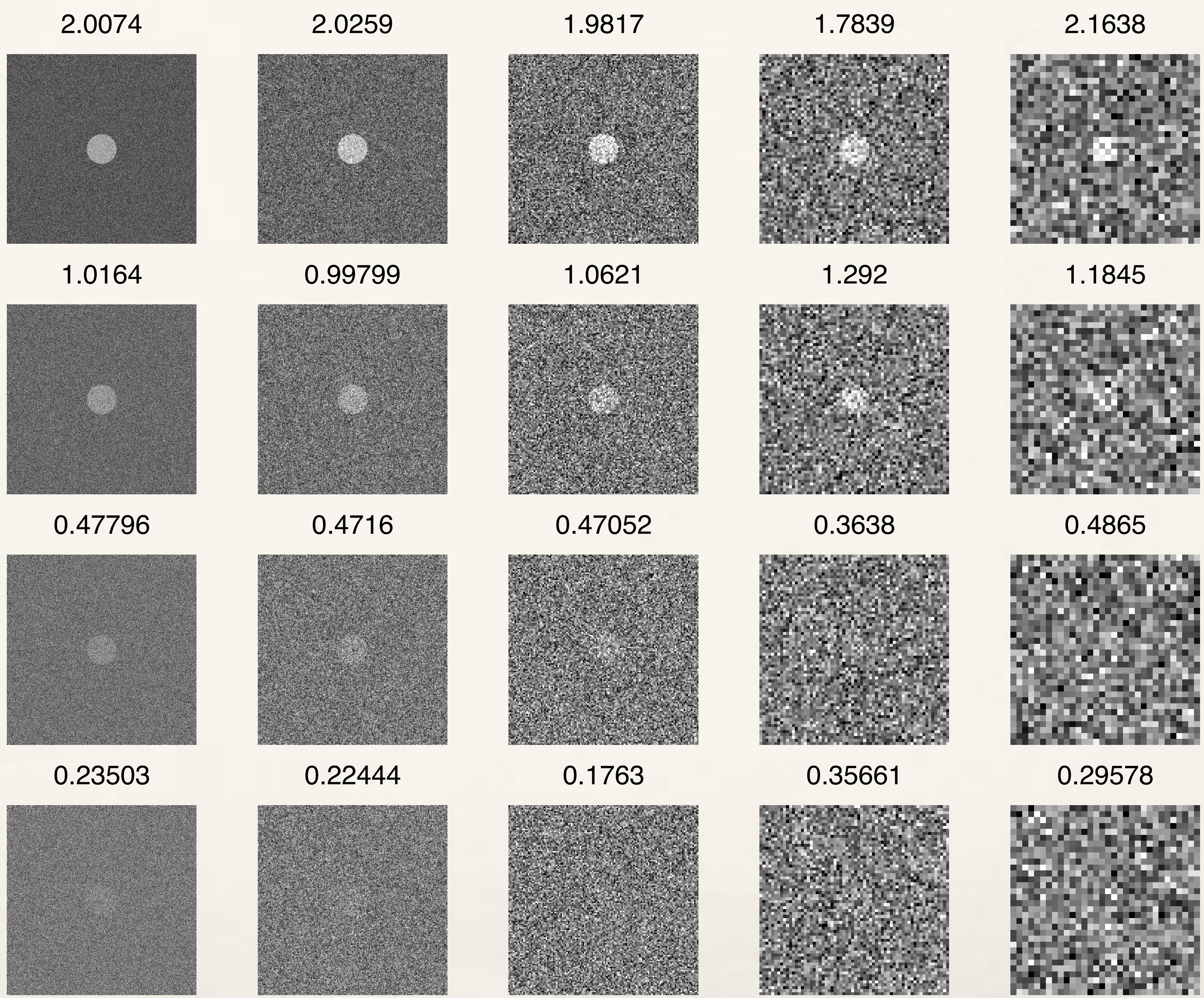
CNR



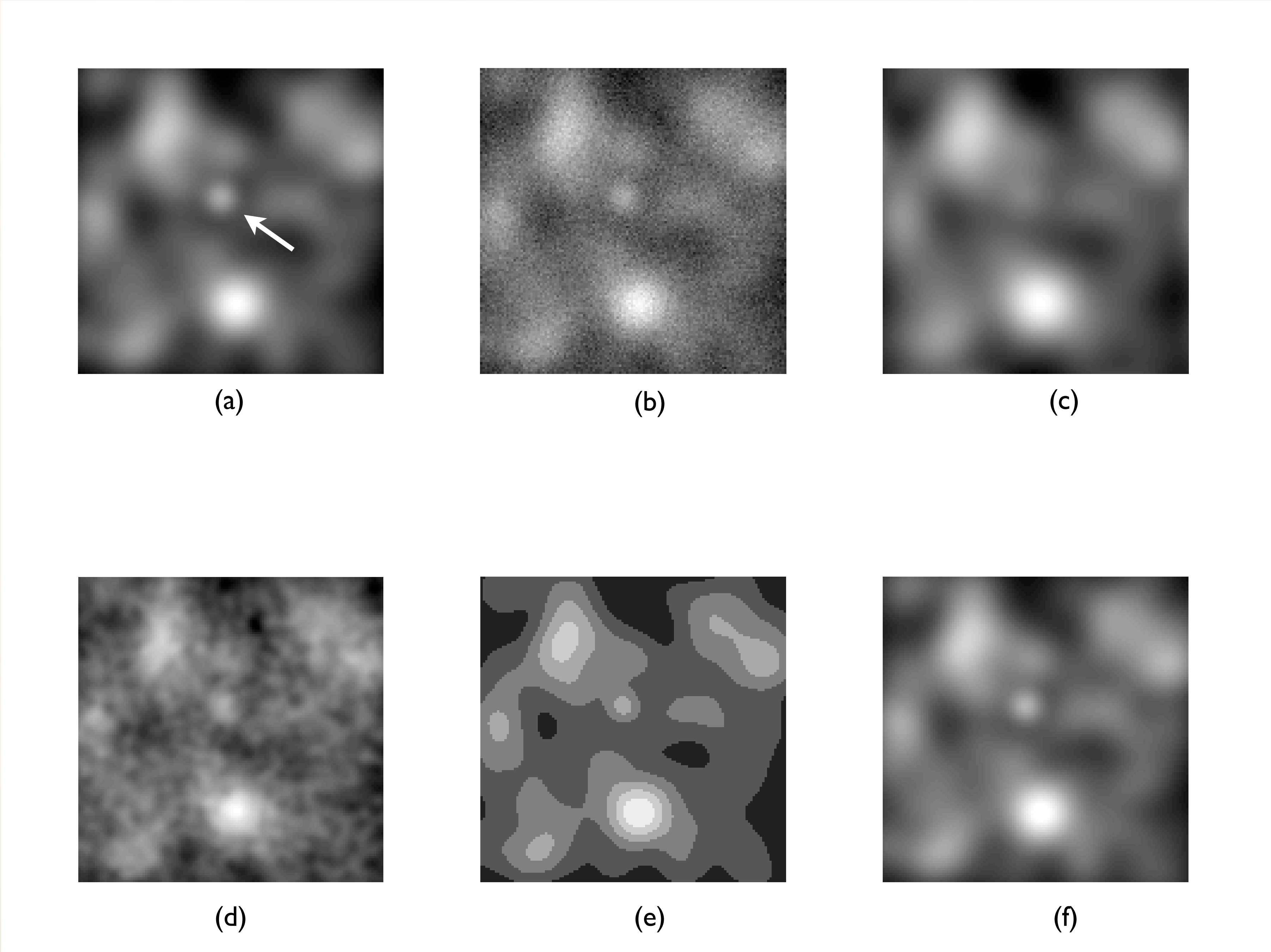
CNR



CNR


2.0074
2.0259
1.9817
1.7839
2.1638
1.0164
0.99799
1.0621
1.292
1.1845
0.47796
0.4716
0.47052
0.3638
0.4865
0.23503
0.22444
0.1763
0.35661
0.29578

MSE



Task-based assessment

Task-based assessment of image quality

- Task
  What is the image to be used for?

- Observer
  Who is performing the task?

- Objects
  What are you imaging?

Measure the ability of the observer to perform the task

Task-based assessment

Task-based assessment of image quality

- Task
  What is the image to be used for?

- Observer
  Who is performing the task?

- Objects
  What are you imaging?

Measure the ability of the observer to perform the task

Tasks

- Classification
  - Signal present vs signal absent
- Estimation
  - Estimation of cardiac ejection fraction
- Combined tasks
  - Detection and localization of abnormalities

Figures of merit

Need a scalar figure of merit for comparisons

- Detection tasks
  - Area under ROC Curve or SNR
- Estimation
  - EMSE (not pixel-based!) or Bayes risk
- Combined
  - EROC analysis

Decision Variables $t = T(g)$

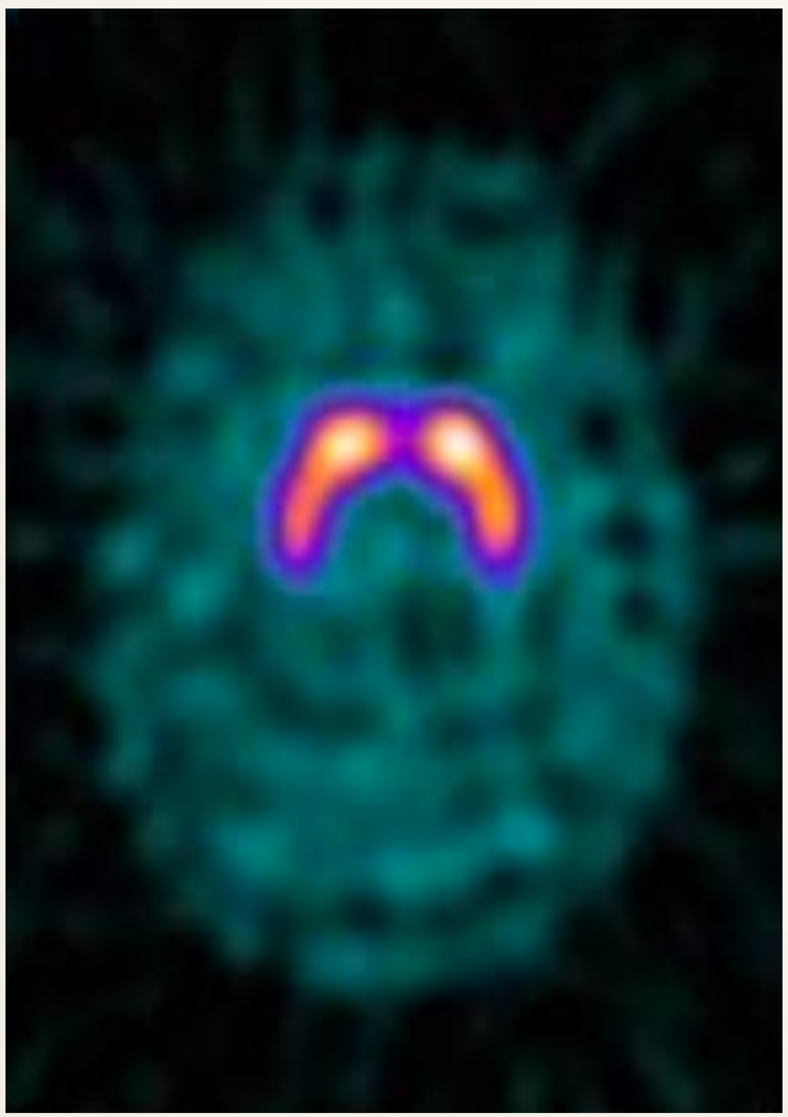


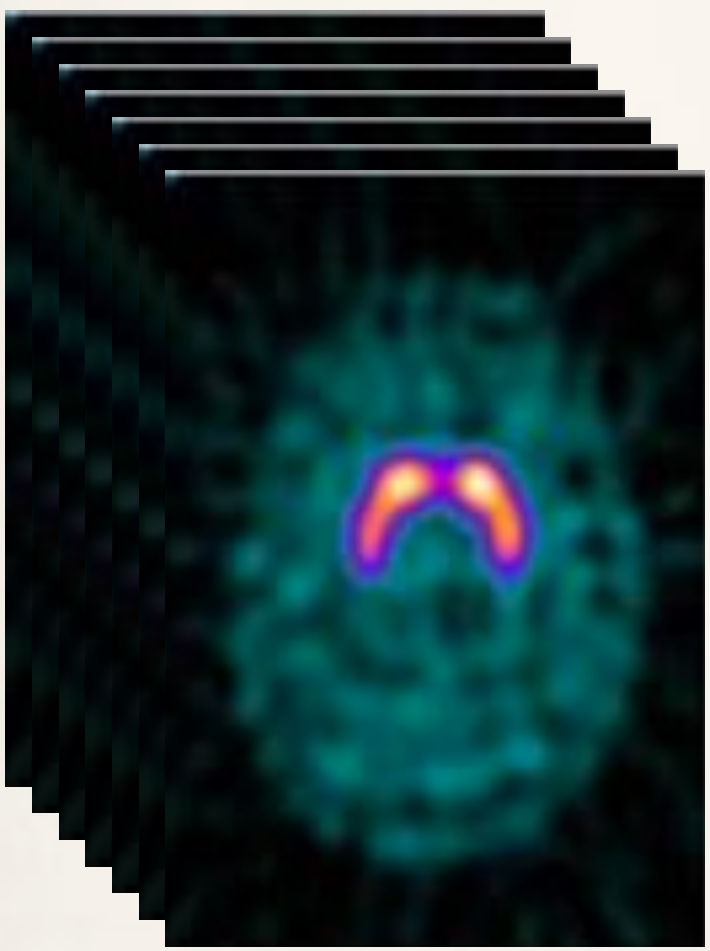
Image g

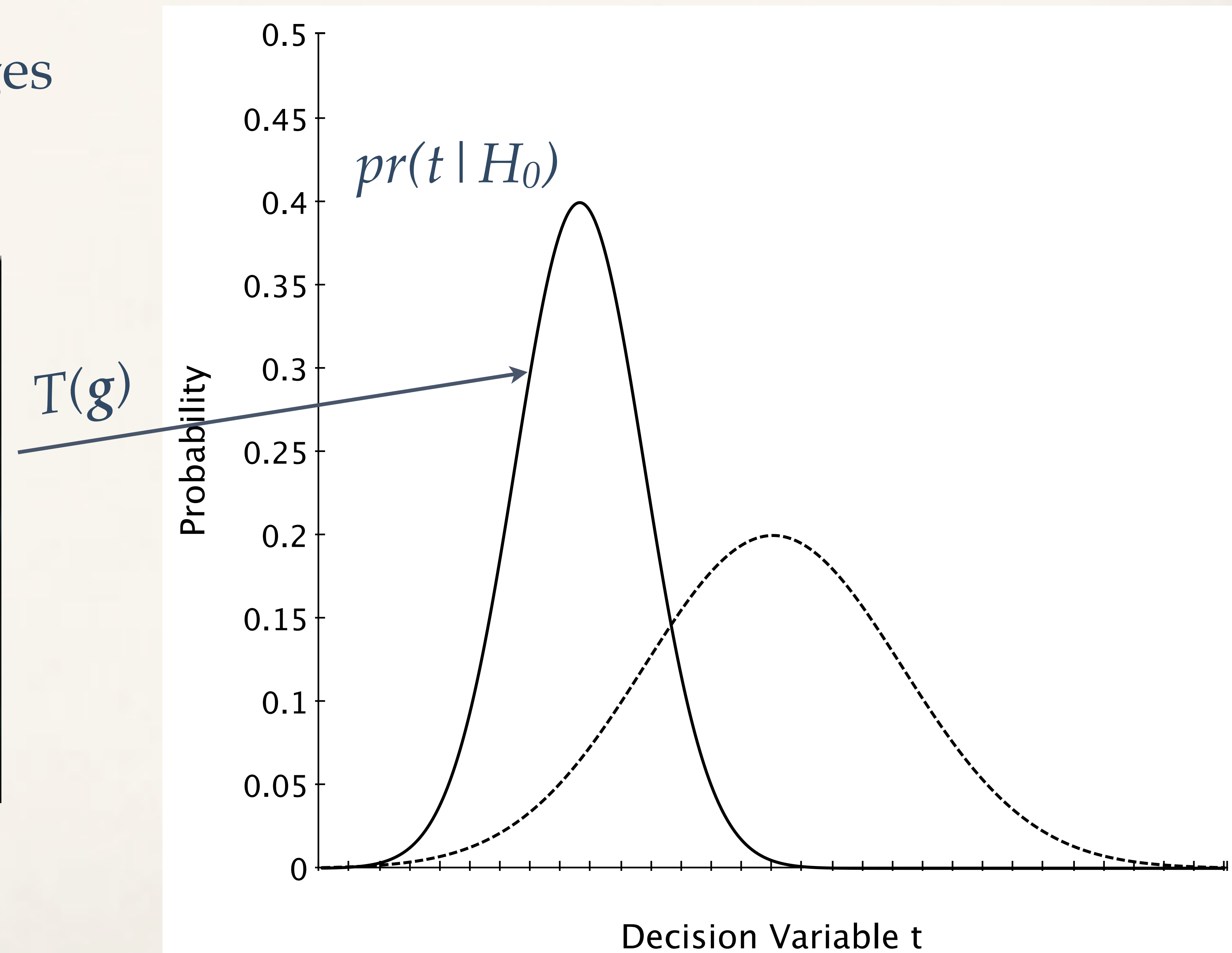
$T(g)$

Decision Variable t

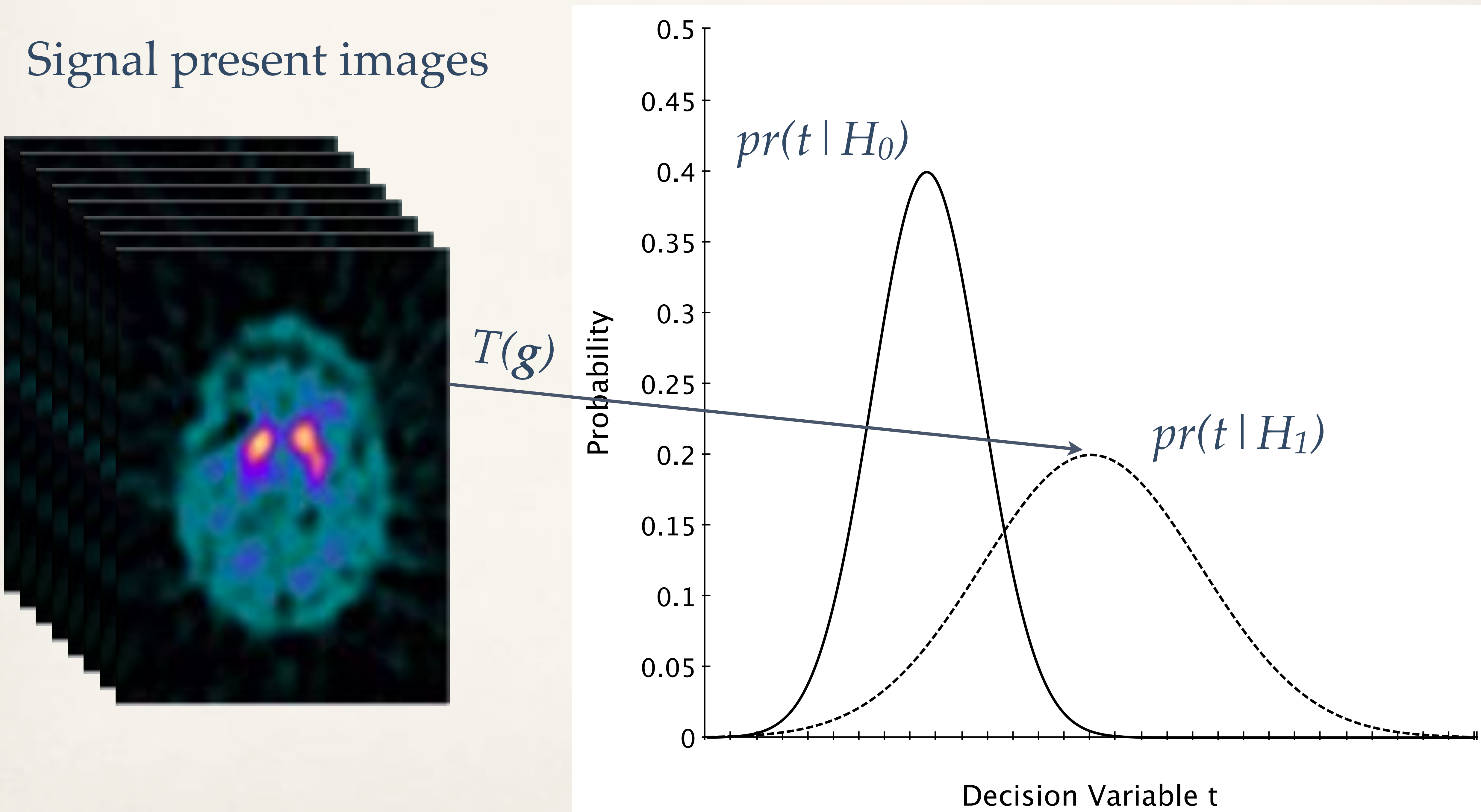
Decision Variables

Signal absent images

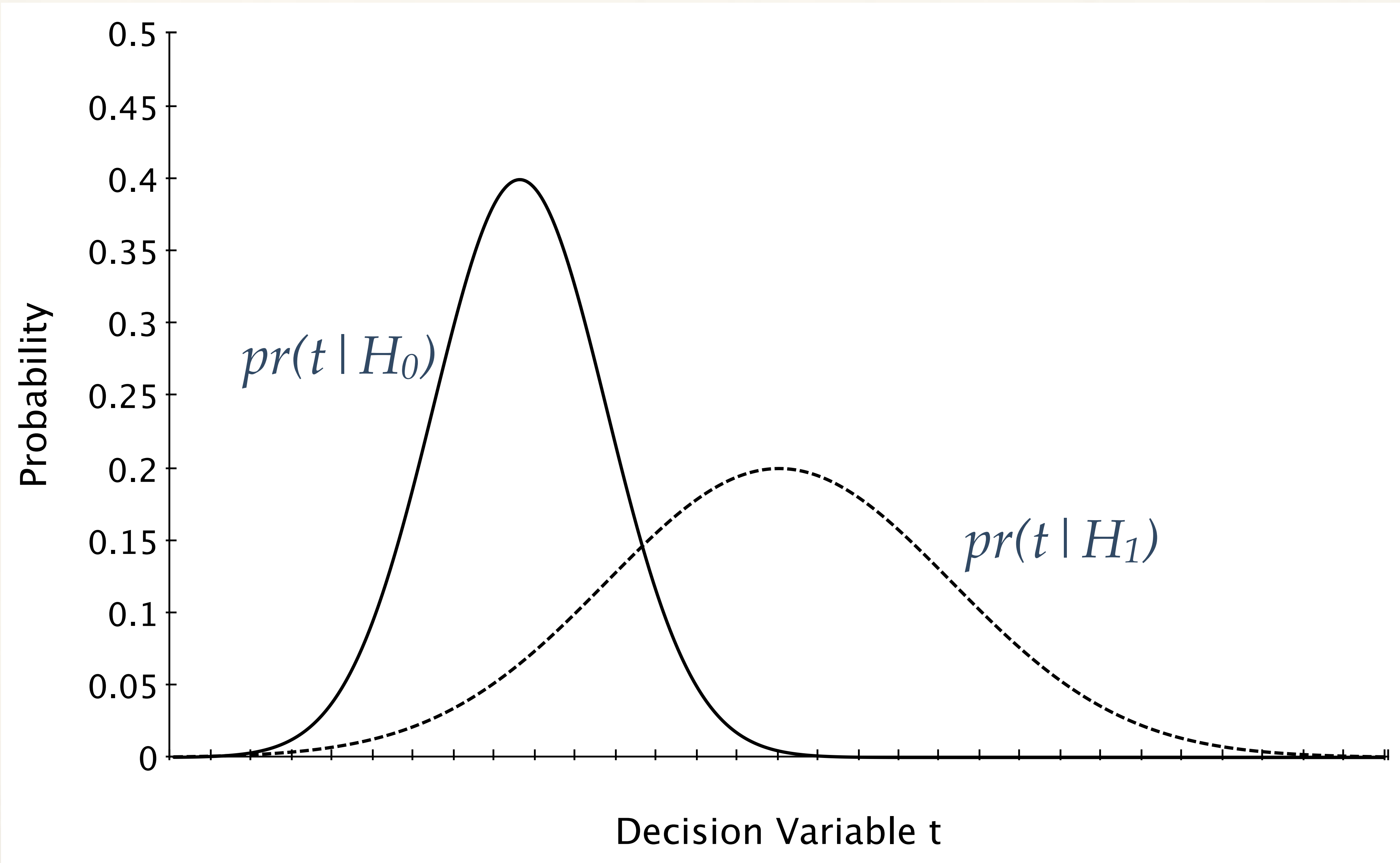




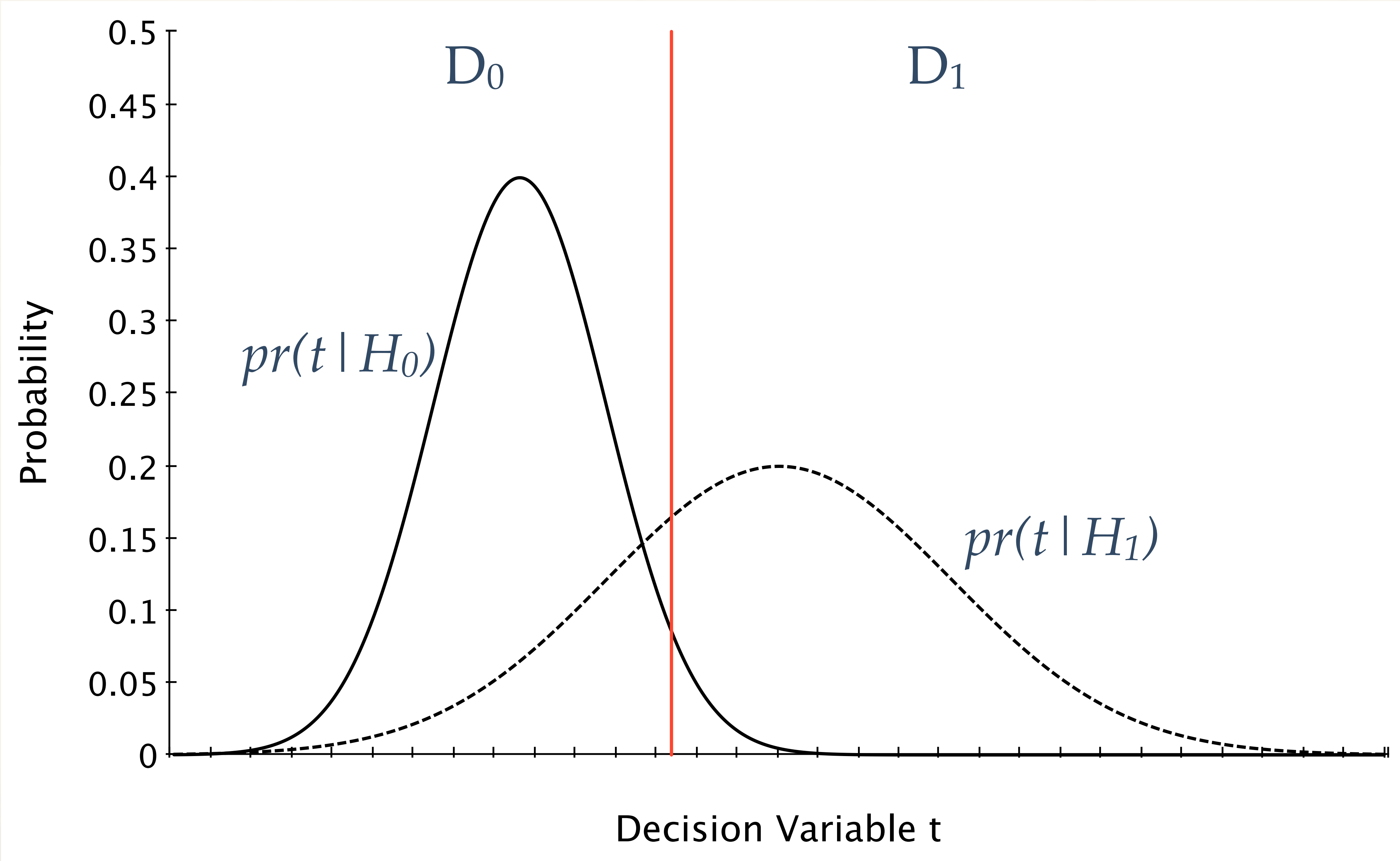
Decision Variables



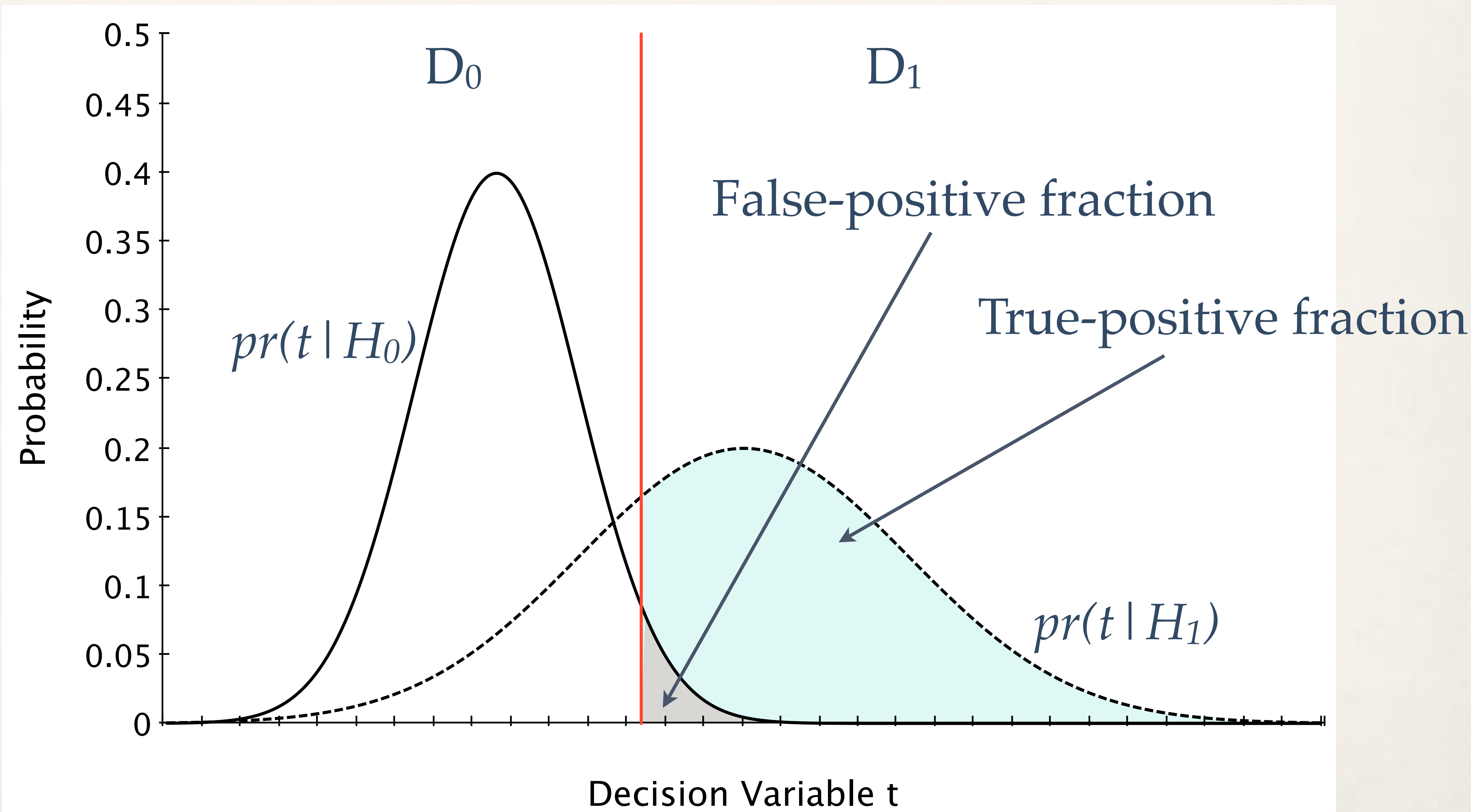
Decision Variables



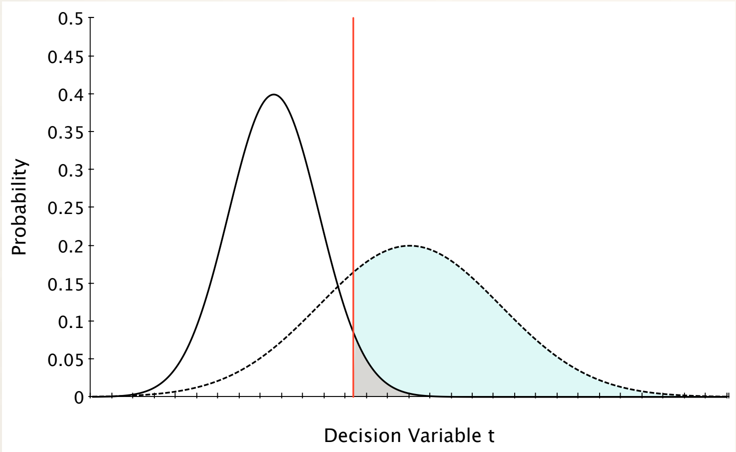
Decision Variables



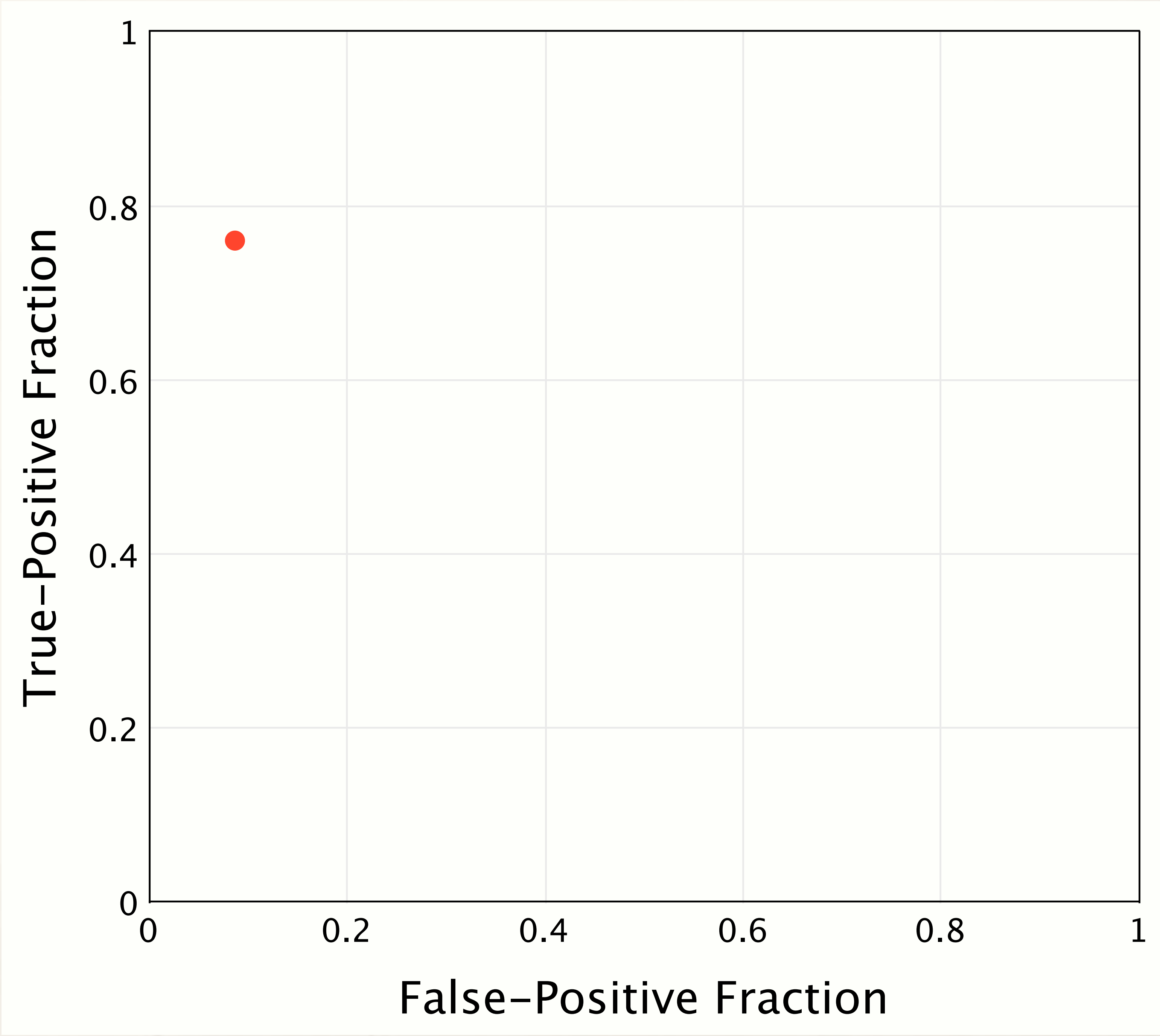
Decision Variables



ROC Curve

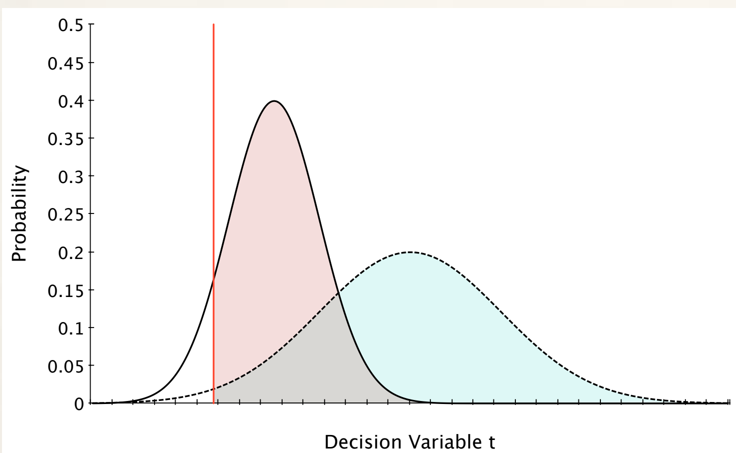


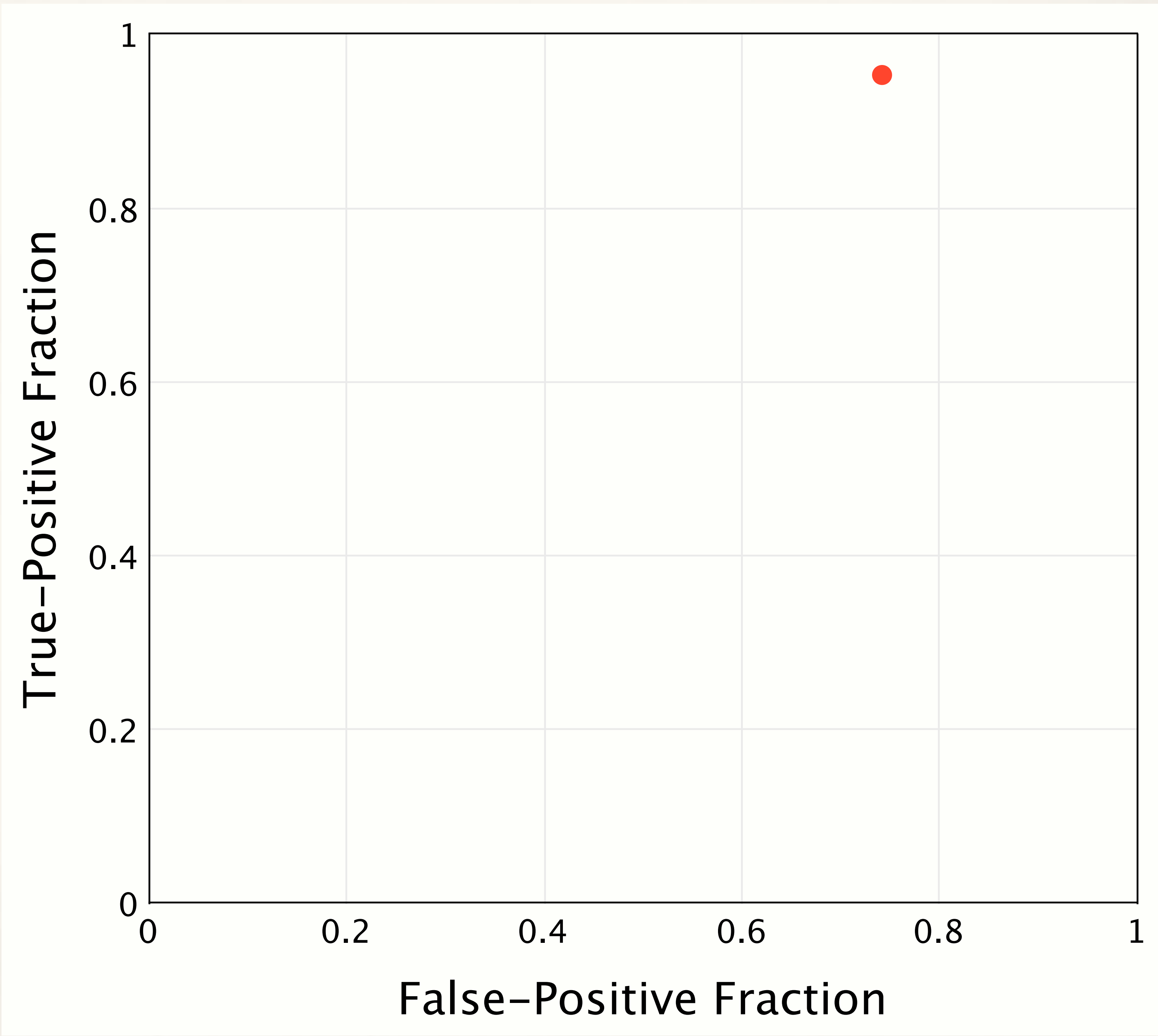
Probability
Decision Variable t
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0



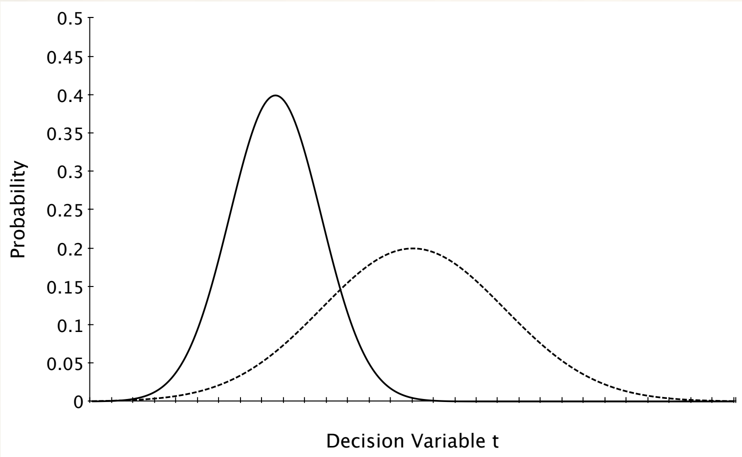
True-Positive Fraction
False-Positive Fraction
0
0.2
0.4
0.6
0.8
1

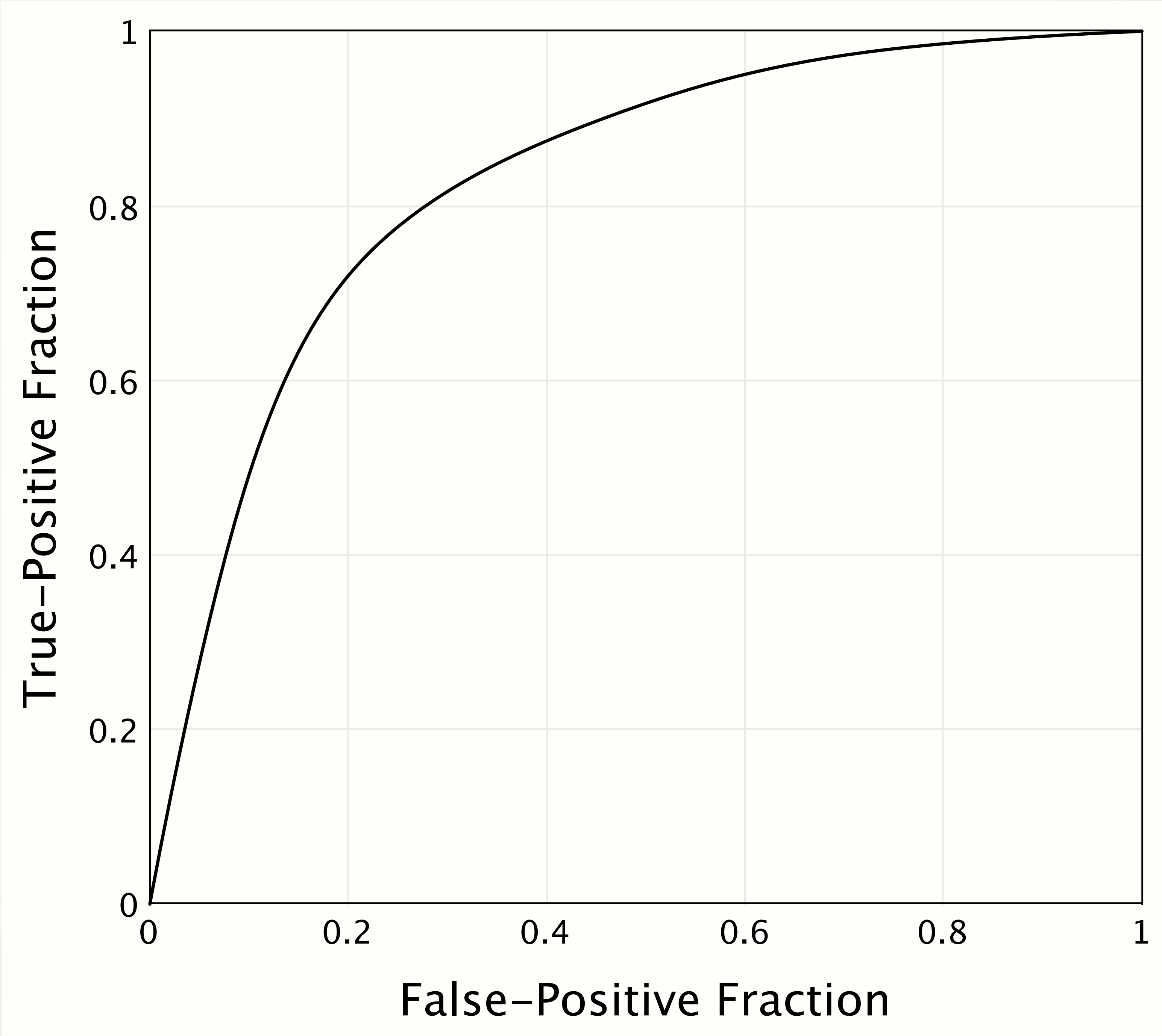
ROC Curve


Probability
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Decision Variable t


True-Positive Fraction
False-Positive Fraction
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

ROC Curve

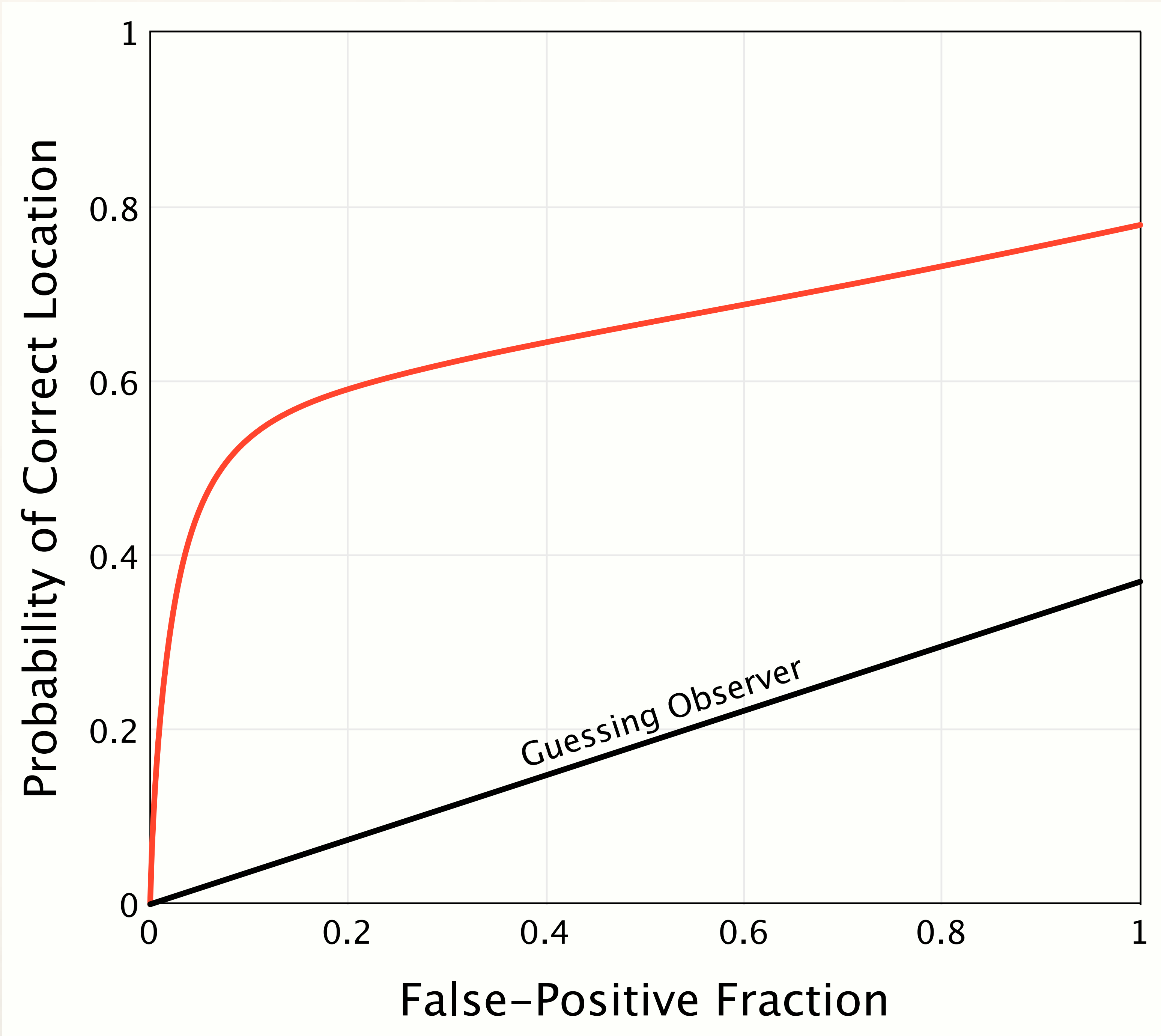

0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Probability
Decision Variable t


1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
True–Positive Fraction
False–Positive Fraction

Estimation

- $\boldsymbol{\theta}$: Parameters to be estimated
- $\widehat{\boldsymbol{\theta}}(\boldsymbol{g})$: Estimates for image $\boldsymbol{g}$
- $$\mathrm{EMSE} = \left\langle \left\langle |\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}|^2 \right\rangle_{\boldsymbol{\theta}|\boldsymbol{g}} \right\rangle_{\boldsymbol{\theta}}$$

Combined Tasks



Task-based assessment

Task-based assessment of image quality

- Task
  What is the image to be used for?

- Observer
  Who is performing the task?

- Objects
  What are you imaging?

Measure the ability of the observer to perform the task

Observers

- Classification

$$t = T(\boldsymbol{g})$$

- Estimation

$$\widehat{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}}(\boldsymbol{g})$$

- Combined

$$t = T(\boldsymbol{g})$$
$$\widehat{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}}(\boldsymbol{g})$$

Ideal Classifier

- Requires knowing the distributions of the image data
- Ideal observer maximizes the ROC curve

$$T(\boldsymbol{g}) = \Lambda(\boldsymbol{g}) = \frac{pr(\boldsymbol{g}|H_2)}{pr(\boldsymbol{g}|H_1)}$$

Ideal Estimator

- Posterior mean: $\widehat{\boldsymbol{\theta}}_{PM} = \int d\boldsymbol{\theta}\ \boldsymbol{\theta} pr(\boldsymbol{\theta}|\boldsymbol{g})$
- ML: $\widehat{\boldsymbol{\theta}}_{ML} = \arg\max_{\boldsymbol{\theta}} pr(\boldsymbol{g}|\boldsymbol{\theta})$
- MAP: $\widehat{\boldsymbol{\theta}}_{MAP} = \arg\max_{\boldsymbol{\theta}} pr(\boldsymbol{\theta}|\boldsymbol{g})$
- Estimators can be nonlinear in the image data

Ideal Observers

- Require knowledge of the PDF for the data conditioned on the object class!

Classification:

$$pr(\boldsymbol{g}|H_i)$$

Estimation:

$$pr(\boldsymbol{g}|\boldsymbol{\theta})$$

$$pr(\boldsymbol{\theta}|\boldsymbol{g}) \propto pr(\boldsymbol{g}|\boldsymbol{\theta})pr(\boldsymbol{\theta})$$

Ideal Linear Classifier

- Hotelling observer
- Computes test statistic t

  $$t = \boldsymbol{w}^{\dagger}\boldsymbol{g}$$

  where

  $$w = K_{\boldsymbol{g}}^{-1}\Delta\overline{\boldsymbol{g}}$$

Harold Hotelling

Ideal Linear Estimator

- Generalized Wiener estimator
- Computes linear estimate

$$\widehat{\boldsymbol{\theta}} = \overline{\boldsymbol{\theta}} + W^{\dagger}\left[\boldsymbol{g} - \overline{\overline{\boldsymbol{g}}}\right]$$

where

$$W^{\dagger} = K_{\boldsymbol{\theta},\overline{\boldsymbol{g}}} K_{\boldsymbol{g}}^{-1}$$

Norbert Wiener

Ideal Linear Observers

- Require only first- and second-order statistics of the image data
- Require the inversion of a large covariance matrix

Task-based assessment

Task-based assessment of image quality

- Task
  What is the image to be used for?
- Observer
  Who is performing the task?
- Objects
  What are you imaging?

Measure the ability of the observer to perform the task

Objects are continuous functions

- Nuclear medicine: Object is 3D distribution of radiopharmaceutical; 4D if we consider time variation
- X-ray imaging: Object is 3D distribution of x-ray absorption and scattering coefficients (vector valued)
- Written as $f(\boldsymbol{r})$ or $f(\boldsymbol{r}, t)$ or $\boldsymbol{f}$

Multimodality and Adaptive Imaging

Ideal observers

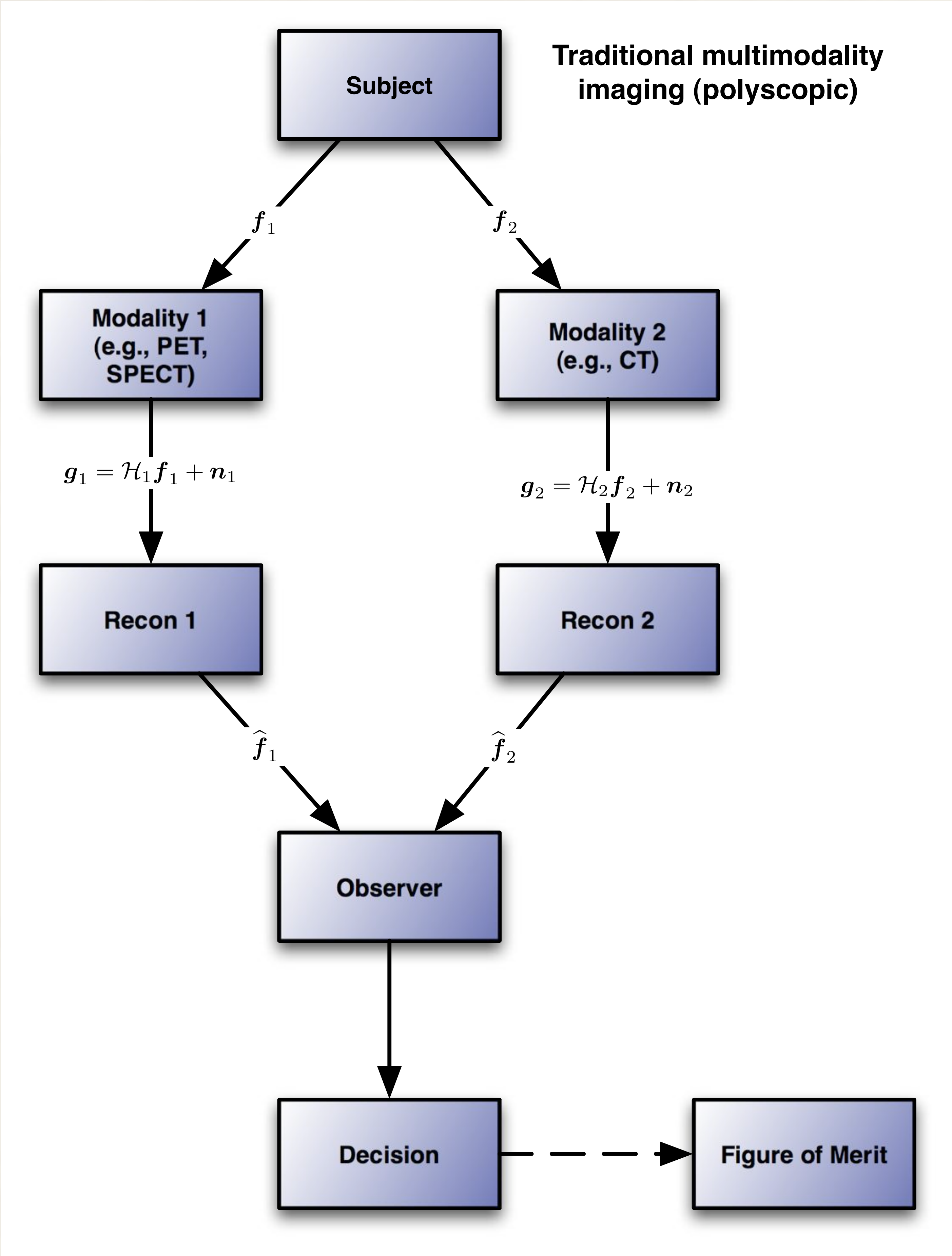
Detection	Estimation
$\Lambda(\boldsymbol{g}) = \frac{pr(\boldsymbol{g}\|H_2)}{pr(\boldsymbol{g}\|H_1)}$	$\widehat{\boldsymbol{\theta}}_{PM} = \int d\boldsymbol{\theta}\ \boldsymbol{\theta} pr(\boldsymbol{\theta}\|\boldsymbol{g})$
$pr(\boldsymbol{g}\|H_i)$	$pr(\boldsymbol{\theta}\|\boldsymbol{g}) \propto pr(\boldsymbol{g}\|\boldsymbol{\theta}) pr(\boldsymbol{\theta})$

Multimodality and Adaptive Imaging

Ideal observers

$$pr(\boldsymbol{g}|\boldsymbol{\theta})$$

$\Lambda(g)$	$\boldsymbol{\theta}|g)$
$pr(\boldsymbol{g}|H_i)$	$pr(\boldsymbol{\theta}|\boldsymbol{g}) \propto pr(\boldsymbol{g}|\boldsymbol{\theta})pr(\boldsymbol{\theta})$


Traditional multimodality imaging (polyscopic)
Subject
f_1
f_2
Modality 1 (e.g., PET, SPECT)
Modality 2 (e.g., CT)
$g_1 = \mathcal{H}_1 f_1 + n_1$
$g_2 = \mathcal{H}_2 f_2 + n_2$
Recon 1
Recon 2
$\hat{f}_1$
$\hat{f}_2$
Observer
Decision
Figure of Merit

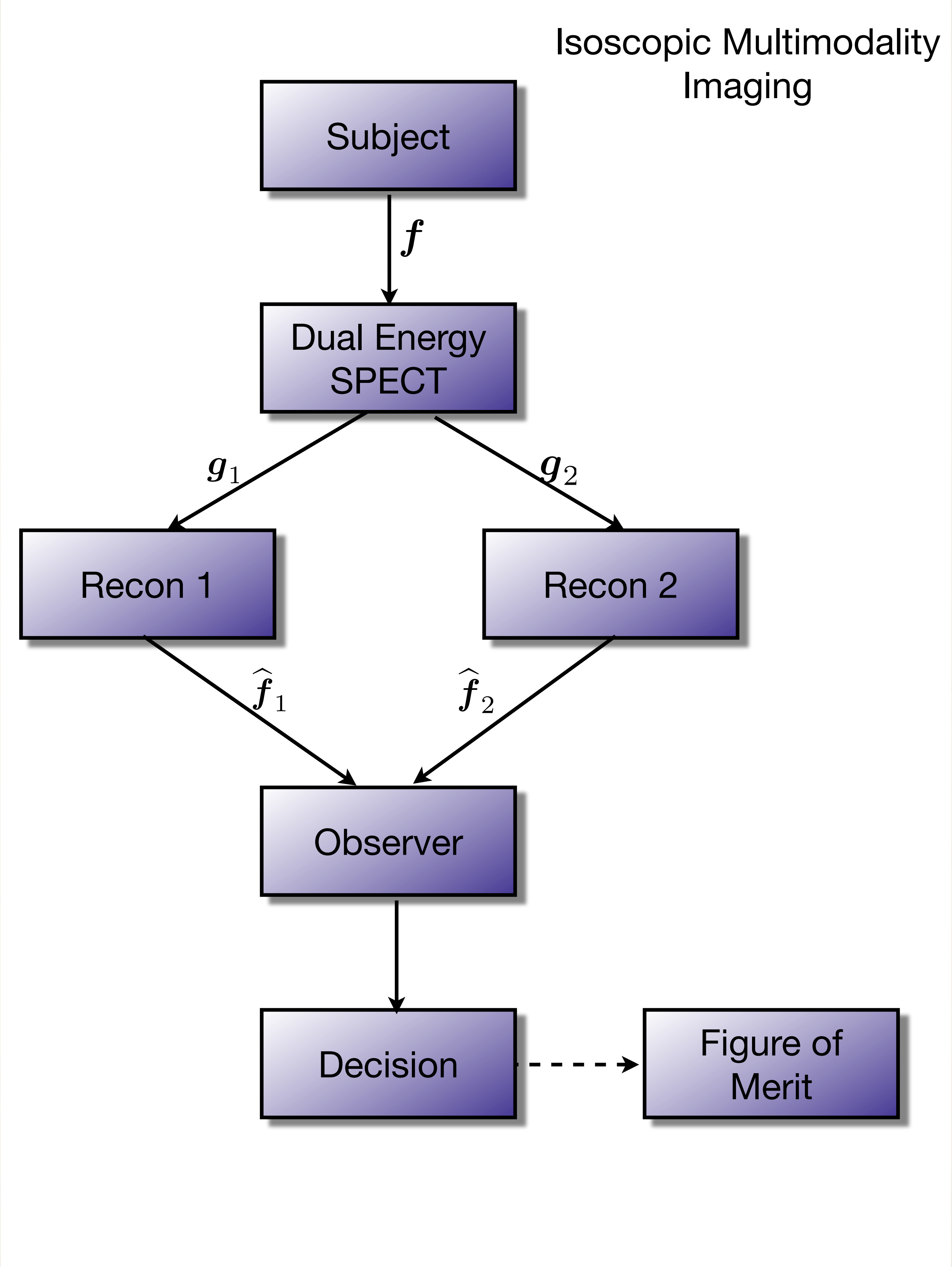
$$g_1$$

$$g_2$$

Traditional multimodality imaging

$$pr(\boldsymbol{g}_1, \boldsymbol{g}_2|\boldsymbol{\theta}) = \int\int pr(\boldsymbol{g}_1|\boldsymbol{f}_1)pr(\boldsymbol{g}_2|\boldsymbol{f}_2)pr(\boldsymbol{f}_1, \boldsymbol{f}_2|\boldsymbol{\theta})d\boldsymbol{f}_1 d\boldsymbol{f}_2$$

- Each system has independent noise
- The objects depend on one another

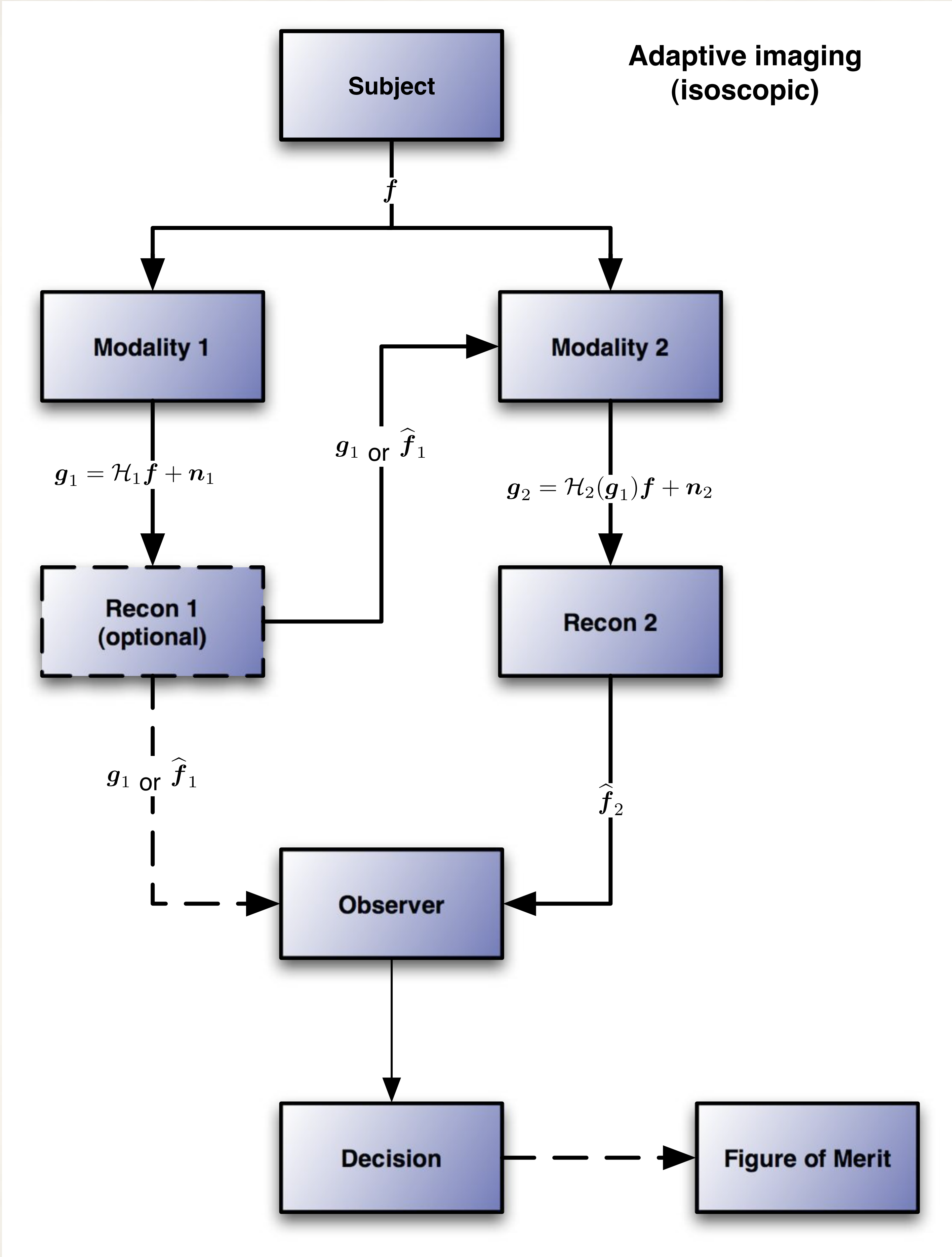


Isoscopic Multimodality
Imaging
Subject
$\boldsymbol{f}$
Dual Energy
SPECT
$\boldsymbol{g}_1$
$\boldsymbol{g}_2$
Recon 1
Recon 2
$\widehat{\boldsymbol{f}}_1$
$\widehat{\boldsymbol{f}}_2$
Observer
Decision
Figure of
Merit

Isoscopic Multimodality Imaging

$$pr(\boldsymbol{g}_1, \boldsymbol{g}_2|\boldsymbol{\theta}) = \int pr(\boldsymbol{g}_1|\boldsymbol{f})pr(\boldsymbol{g}_2|\boldsymbol{f})pr(\boldsymbol{f}|\boldsymbol{\theta})d\boldsymbol{f}$$

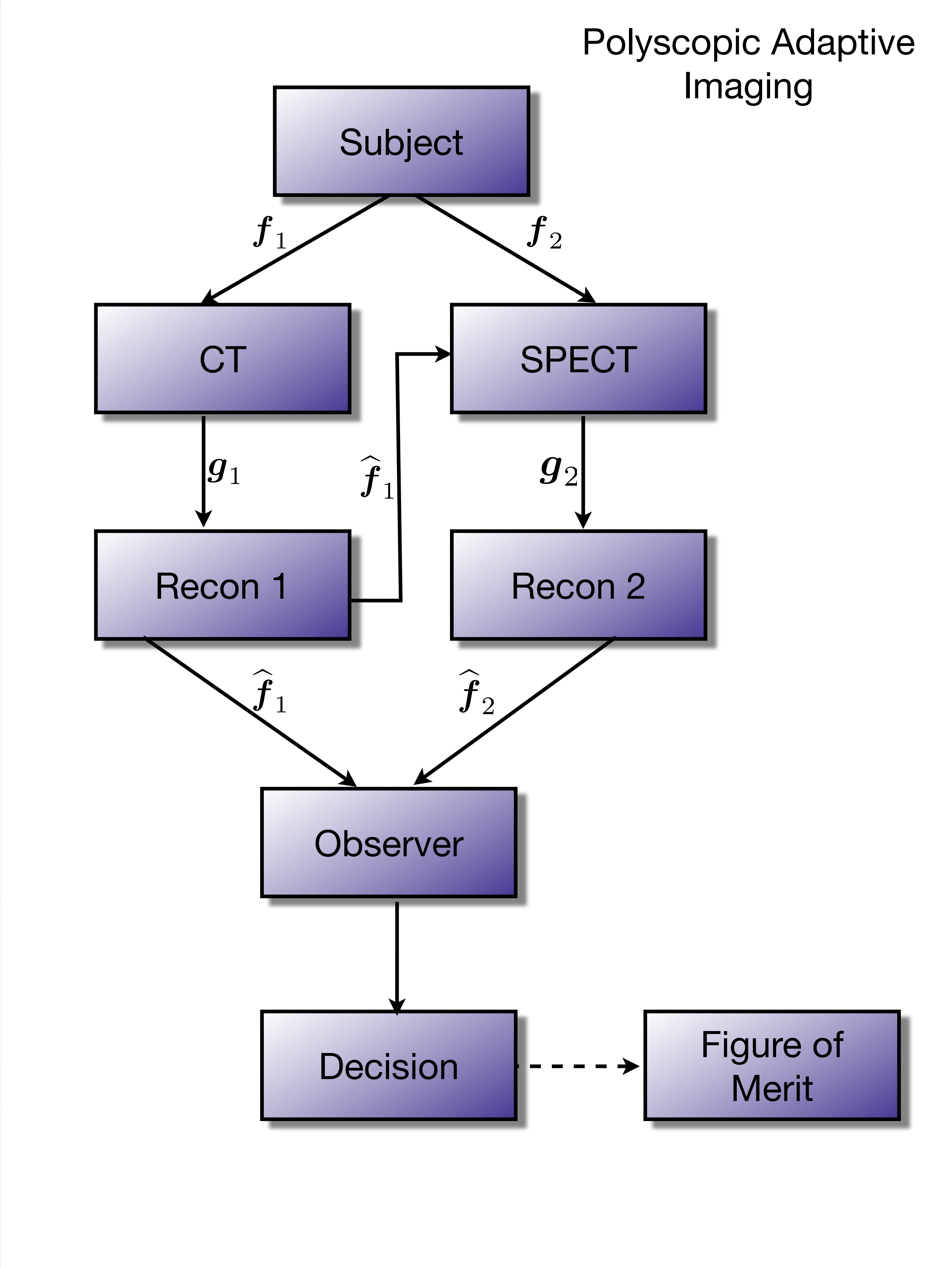
- Each system has independent noise
- Each image depends on the same object


Adaptive imaging
(isoscopic)
Subject
f
Modality 1
Modality 2
$g_1 = \mathcal{H}_1 f + n_1$
g_1 or $\hat{f}_1$
$g_2 = \mathcal{H}_2(g_1) f + n_2$
Recon 1
(optional)
Recon 2
g_1 or $\hat{f}_1$
$\hat{f}_2$
Observer
Decision
Figure of Merit

Isoscopic Adaptive Imaging

$$pr(\boldsymbol{g}_1, \boldsymbol{g}_2|\boldsymbol{\theta}) = \int pr(\boldsymbol{g}_1|\boldsymbol{g}_2, \boldsymbol{f})pr(\boldsymbol{g}_2|\boldsymbol{f})pr(\boldsymbol{f}|\boldsymbol{\theta})d\boldsymbol{f}$$

- The scout image g_2 affects the second acquisition g_1


Polyscopic Adaptive Imaging
Subject
$\boldsymbol{f}_1$
$\boldsymbol{f}_2$
CT
SPECT
$\boldsymbol{g}_1$
$\widehat{\boldsymbol{f}}_1$
$\boldsymbol{g}_2$
Recon 1
Recon 2
$\widehat{\boldsymbol{f}}_1$
$\widehat{\boldsymbol{f}}_2$
Observer
Decision
Figure of Merit

Polyscopic Adaptive Imaging

$$pr(\boldsymbol{g}_1, \boldsymbol{g}_2|\boldsymbol{\theta}) = \int\int pr(\boldsymbol{g}_1|\boldsymbol{g}_2, \boldsymbol{f}_1)pr(\boldsymbol{g}_2|\boldsymbol{f}_2)pr(\boldsymbol{f}_2, \boldsymbol{f}_2|\boldsymbol{\theta})d\boldsymbol{f}_1 d\boldsymbol{f}_2$$

Multimodality and adaptive imaging

Ideal linear observers

$$\boldsymbol{g} = \mathcal{H}\boldsymbol{f} + \boldsymbol{n}$$

Multimodality and adaptive imaging

Ideal linear observers

$$\boldsymbol{g} = \mathcal{H}_0 \mathcal{A} \boldsymbol{f} + \boldsymbol{n}$$

- The effects of the imaging aperture and detector are characterized by $\mathcal{H}_0$
- The patient-dependent effects of attenuation and scatter are characterized by $\mathcal{A}$

Multimodality and adaptive imaging

Ideal linear observers

$$\boldsymbol{g} = \mathcal{H}_0(\boldsymbol{g}_s)\mathcal{A}\boldsymbol{f} + \boldsymbol{n}$$

- The imaging system now adapts itself based on the scout measurements

Multimodality and adaptive imaging

Ideal linear observers

$$\boldsymbol{g} = \mathcal{H}_0(\boldsymbol{g}_s)\mathcal{A}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{w}(\boldsymbol{g}_s) = K^{-1}_{\boldsymbol{g}|\boldsymbol{g}_s}\mathcal{H}_0(\boldsymbol{g}_s)\overline{\mathcal{A}\boldsymbol{f}}_{\mathrm{sig}}$$

- $\overline{\mathcal{A}}$ is the average of the patient-specific portion of the imaging operator

$$w = K_{\boldsymbol{g}}^{-1}\Delta\overline{\boldsymbol{g}}$$

Multimodality and adaptive imaging

Ideal linear observers

$$\boldsymbol{g} = \mathcal{H}_0(\boldsymbol{g}_s)\mathcal{A}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{w}(\boldsymbol{g}_s) = K^{-1}_{\boldsymbol{g}|\boldsymbol{g}_s}\mathcal{H}_0(\boldsymbol{g}_s)\overline{\mathcal{A}\boldsymbol{f}}_{\text{sig}}$$

$$K_{\boldsymbol{g}|\boldsymbol{g}_s} = \overline{\overline{K}}^{(\text{noise})}_{\boldsymbol{g}|\boldsymbol{g}_s} + \overline{K}^{(\text{sys})}_{\overline{\boldsymbol{g}}|\boldsymbol{g}_s} + K^{(\text{obj})}_{\overline{\overline{\boldsymbol{g}}}|\boldsymbol{g}_s}$$

Multimodality and adaptive imaging

Ideal linear observers

$$\boldsymbol{g} = \mathcal{H}_0(\boldsymbol{g}_s)\mathcal{A}\boldsymbol{f} + \boldsymbol{n}$$

$$\widehat{\boldsymbol{\theta}}(\boldsymbol{g}, \boldsymbol{g}_s) = \overline{\boldsymbol{\theta}} + K_{\boldsymbol{\theta},\boldsymbol{g}|\boldsymbol{g}_s} K^{-1}_{\boldsymbol{g}|\boldsymbol{g}_s} \left[\boldsymbol{g} - \overline{\overline{\boldsymbol{g}}}(\boldsymbol{g}_s)\right]$$

$$\widehat{\boldsymbol{\theta}} = \overline{\boldsymbol{\theta}} + W^\dagger \left[\boldsymbol{g} - \overline{\overline{\boldsymbol{g}}}\right]$$

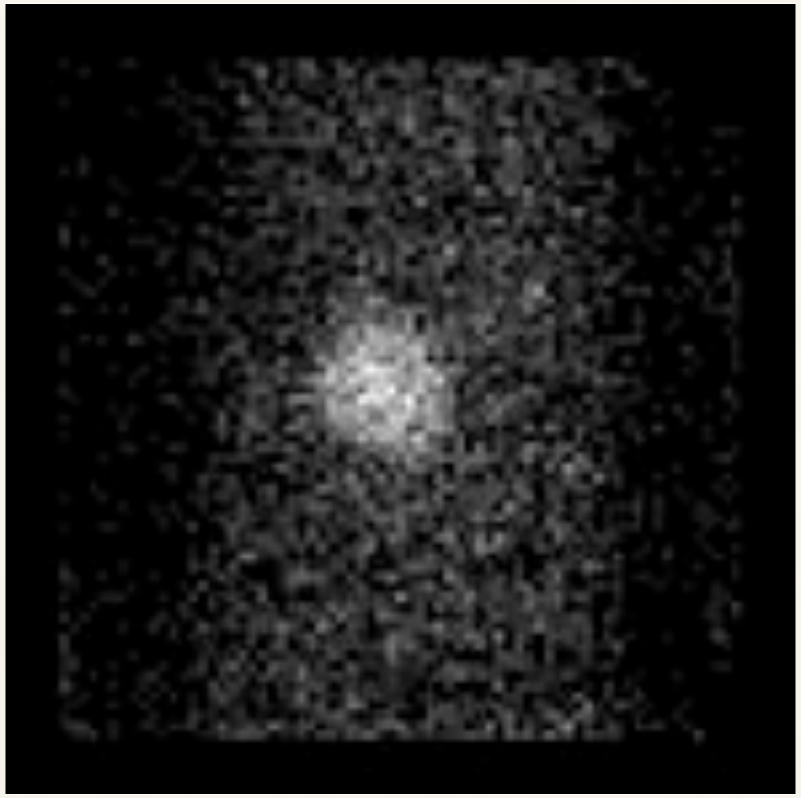
$$W^\dagger = K_{\boldsymbol{\theta},\overline{\boldsymbol{g}}} K^{-1}_{\boldsymbol{g}}$$

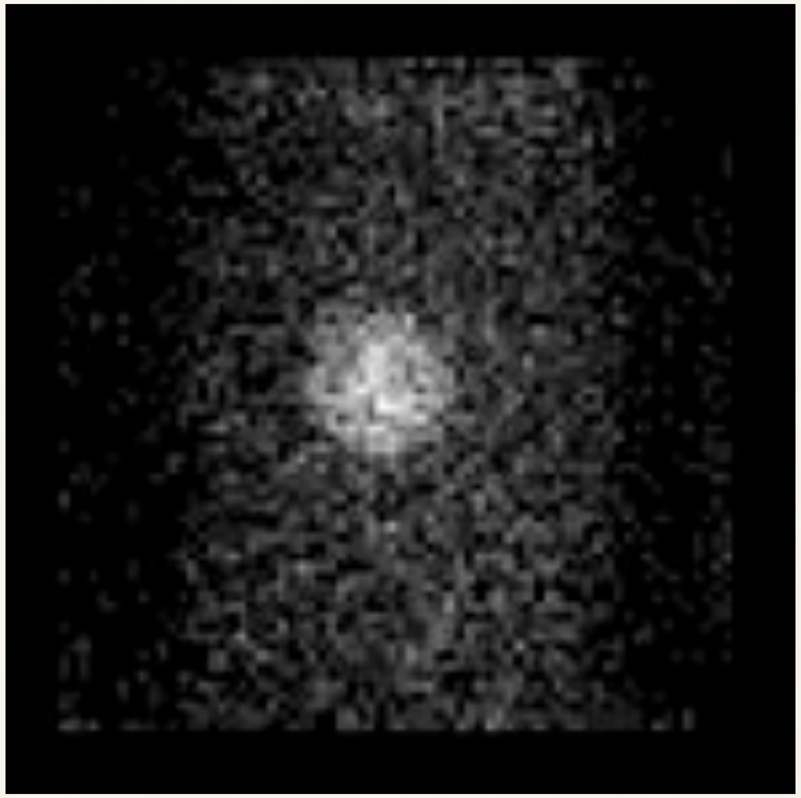
How to adapt?

- Heuristic
- Task-based

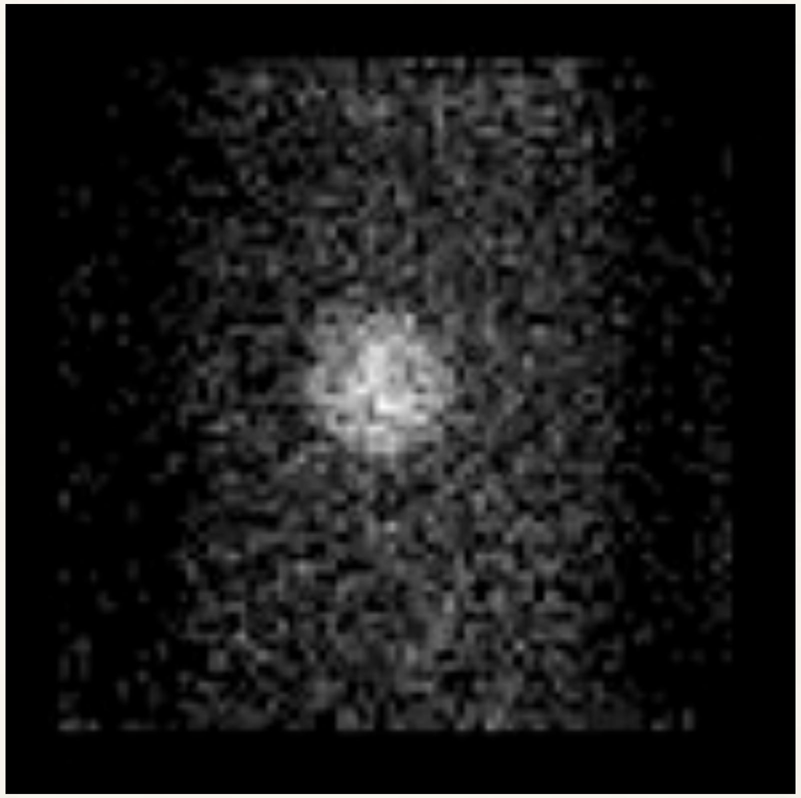
How to adapt?

How to adapt?

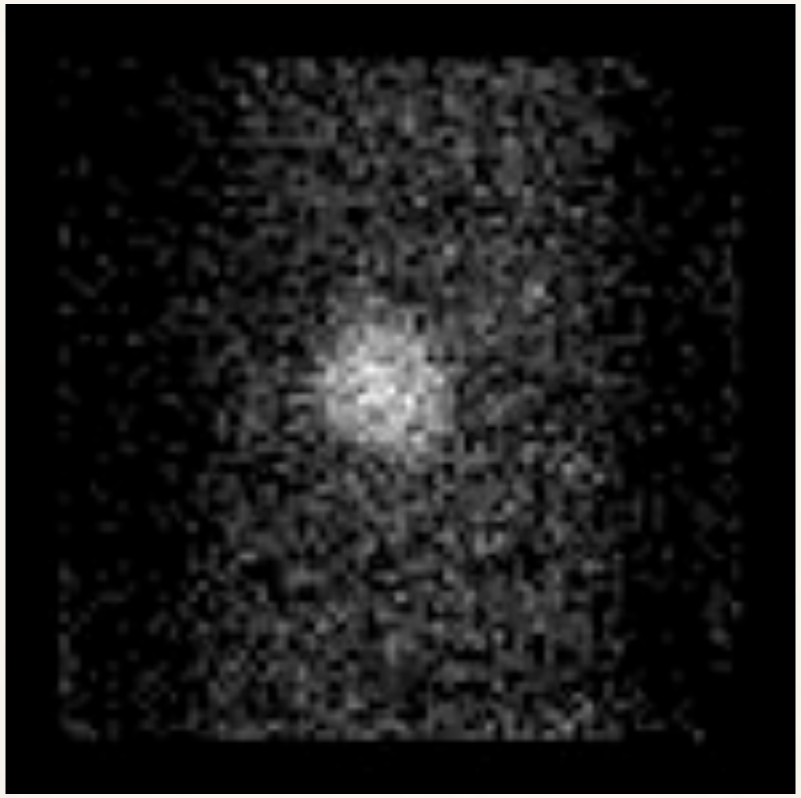


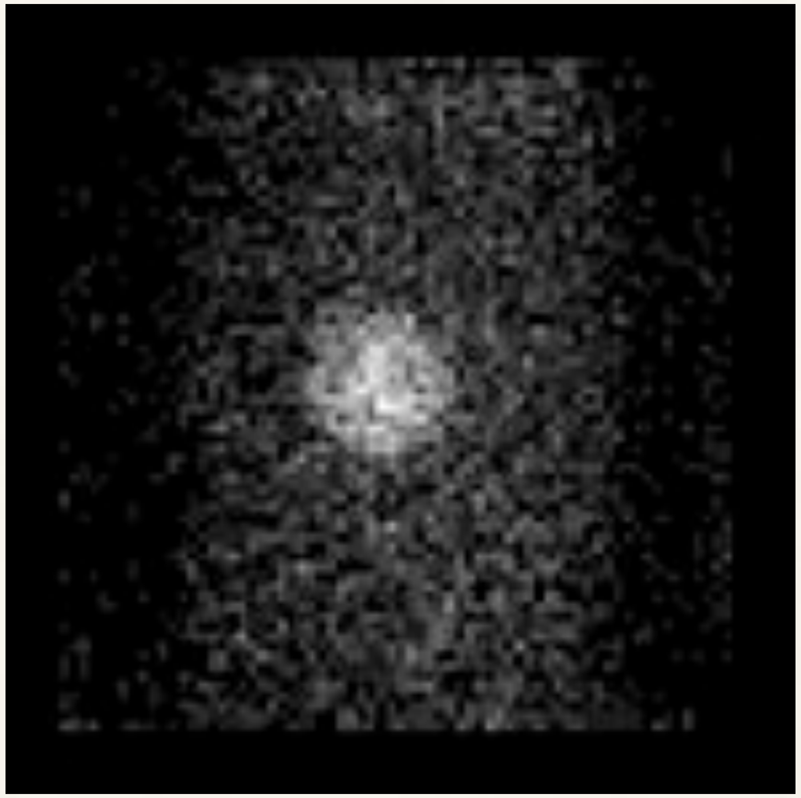


How to adapt?

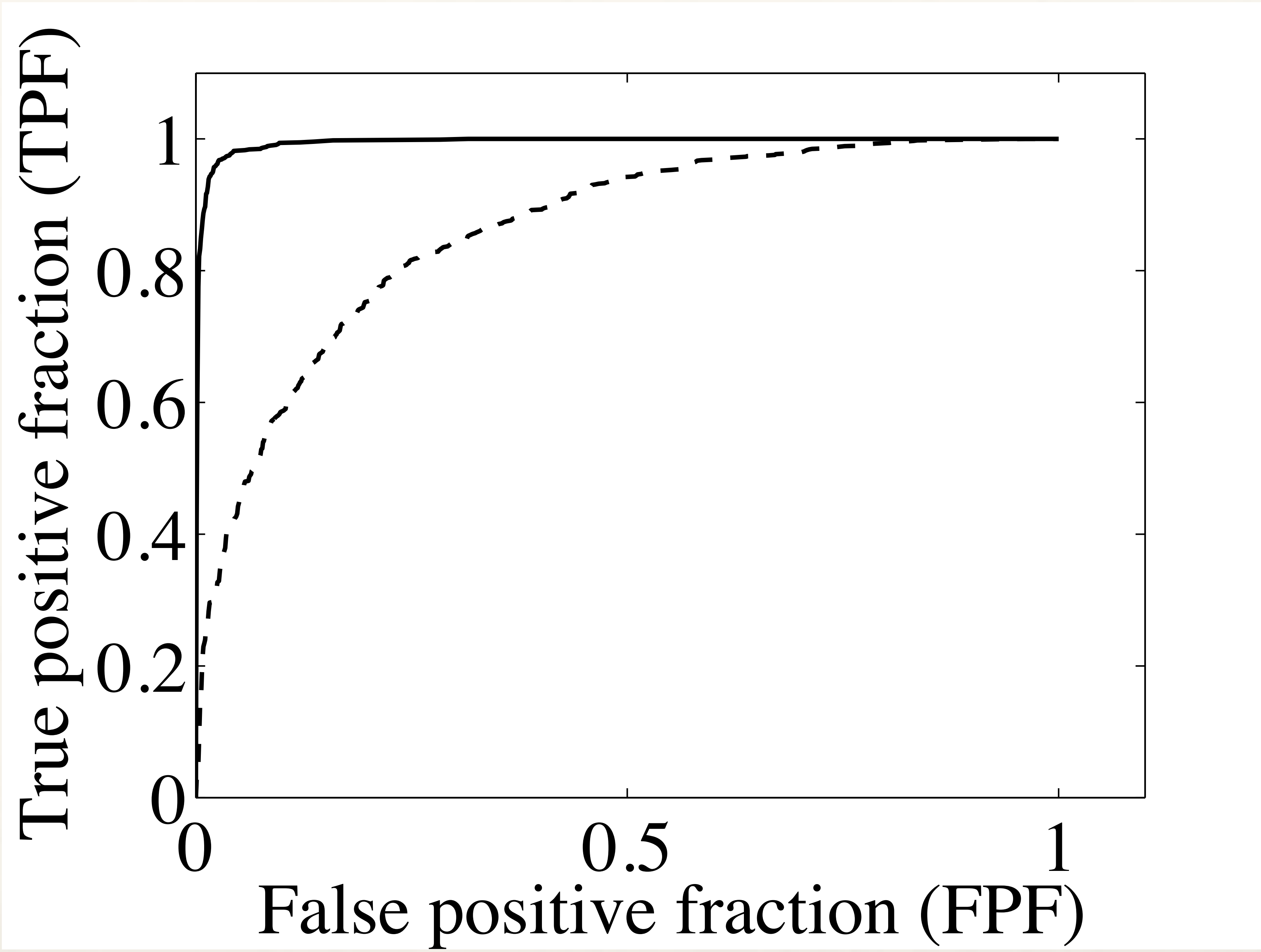


How to adapt?





How to adapt?



True positive fraction (TPF)
1
0.8
0.6
0.4
0.2
0
0
0.5
1
False positive fraction (FPF)

How to adapt?

How to adapt?

Task-based assessment of image quality

- Task
  What is the image to be used for?

- Observer
  Who is performing the task?

- Objects
  What are you imaging?

Measure the ability of the observer to perform the task

How to adapt?

Task-based assessment of image quality

- Task
  What is the image to be used for?

- Observer
  Who is performing the task?

- Objects
  What are you imaging?

Measure the ability of the observer to perform the task

How to adapt?

- Generate adaptation strategy for patients that are *consistent* with the scout data generated

How to adapt?

$$pr(\boldsymbol{f}|\boldsymbol{g}_s)$$

Summary

- Image quality measures should account for the task, the observer, and the patient population
- Knowledge of ideal observers helps define the limits of observer performance and can be used for hardware optimizations
- Adaptive imaging can be accomplished by analyzing patients consistent with the scout data