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Short Course

on

Image Quality

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Schedule

- Intro - KJ Myers
- Stochastic Models of Objects and Images - HH Barrett
- Classification Tasks - KJ Myers
- Estimation Tasks - MA Kupinski
- Psychophysics: Experimental Methods and Data Analysis - BD Gallas
- Computational Methods – MA Kupinski
- Applications in Nuclear Medicine - EC Frey

Objective Assessment of Image Quality

- What information is desired from the image?
- What objects will be imaged?
- How will the information be extracted?
- What measure of performance will be used?

What information is desired from the image?

- We call this the TASK
- TASK: Given an image, make an inference about the patient

Two types of inferences

- Classification = labeling
  - Tumor detection (present vs. absent)
  - Malignant vs. benign lesion
  - Is object from H_1 or H_2 ?
- Parameter estimation = quantitation
  - Tumor size, tumor localization
  - Tracer uptake
  - Degree of stenosis

What measure of performance will be used?

- Figure of merit
- Desirable properties:
  - Quantitative
  - Objective
  - Scalar
  - Calculable

Figures of Merit for Classification

- Receiver Operating Characteristic curve
- Average error rate
- Average cost of decisions made
  - Bayes' risk
- Error bounds

Figures of Merit for Estimation

- Bias, variance
- Mean-square error (MSE)
- Ensemble MSE
- Error bounds

How will the information be extracted?

- Observer = decision-making strategy
- Options
  - Human
  - Anthropomorphic model
  - Computer-aided diagnosis (CAD)
  - Best linear detector/estimator
  - Ideal detector/estimator

Ideal Classifier

- Captures all information in the data
- Adds no noise or additional uncertainty
- Test statistic is the likelihood ratio:

$$\Lambda(\mathbf{g}) = \frac{\mathrm{pr}\,(\mathbf{g}\,|\,H_2)}{\mathrm{pr}\,(\mathbf{g}\,|\,H_1)}$$

- Test statistic can be a nonlinear function of the data

Ideal Estimators

- θ: vector of unknown parameters
- ML estimation: $\hat{\theta}_{ML} = \arg\max_{\theta} \, \mathrm{pr}\,(\mathbf{g} \,|\, \theta)$
- Estimator can be nonlinear in $\mathbf{g}$

Ideal observers

- Estimation

$$\hat{\theta}_{ML} = \arg\max_{\theta} \, \mathrm{pr}\,(\mathbf{g} \,|\, \theta)$$

- Classification

$$\Lambda(\mathbf{g}) = \frac{\mathrm{pr}\,(\mathbf{g} \,|\, H_2)}{\mathrm{pr}\,(\mathbf{g} \,|\, H_1)}$$

Both require knowledge of pdf for data conditioned on the object class!

Optimal linear classifier

- Hotelling observer
- Computes test statistic t

$$t = \mathbf{w}_{Hot}^{t}\mathbf{g} \qquad (13.197)$$

where

$$\mathbf{w}_{Hot} = \mathbf{K}_{\mathbf{g}}^{-1}\Delta\overline{\mathbf{g}} \qquad (13.177)$$

Optimal linear estimator

- Generalized Wiener estimator
- Computes linear estimate $\hat{\theta}$

$$\hat{\boldsymbol{\theta}} = \overline{\boldsymbol{\theta}} + \mathbf{W}^{t}\left[\mathbf{g} - \overline{\overline{\mathbf{g}}}\right]$$

where

$$\mathbf{W}^{t} = \mathbf{K}_{\boldsymbol{\theta},\overline{\mathbf{g}}}\mathbf{K}_{\mathbf{g}}^{-1}\,. \qquad (13.391)$$

Optimal linear observers

- Estimation

$$\hat{\theta} = \overline{\theta} + \mathbf{W}^t \left[\mathbf{g} - \overline{\overline{\mathbf{g}}}\right]$$

$$\mathbf{W}^t = \mathbf{K}_{\boldsymbol{\theta},\overline{\mathbf{g}}} \mathbf{K}_{\mathbf{g}}^{-1}$$

- Classification

$$t = \mathbf{w}_{Hot}^t \mathbf{g}$$

$$\mathbf{w}_{Hot} = \mathbf{K}_{\mathbf{g}}^{-1} \Delta \overline{\mathbf{g}}$$

Both require knowledge of mean, covariance of $\mathbf{g}$!

What objects will be imaged?

- What is at the input to the system?
  - What set(s) of patients?
  - Biology, chemistry, physics…

Properties we can image

- Acoustic reflectance
  - Medical ultrasound
- Concentration
  - Nuclear medicine
  - MRI (spin density)
  - MRS
- Field strength
  - Biomagnetic imaging
- Attenuation
  - Film densitometry
  - Transmission x-ray
- Scattering properties
  - medical ultrasound
- Electric, magnetic properties
  - Impedance tomography
  - MRI (magnetization
  - MRI (spin relaxation)
- Source strength
  - Fluorescence microscopy
- Index of refraction
  - Phase-contrast microscopy
- Gene expression
  - DNA chips, microarrays

Objects are continuous functions

- Nuclear medicine: Object is 3D distribution of radiopharmaceutical; 4D if we consider time variation
- X-ray imaging: Object is 3D distribution of x-ray absorption and scattering coefficients (vector valued)
- Written as $f(\mathbf{r})$ or $f(\mathbf{r},t)$ or $\mathbf{f}$

Digital images are discrete data sets

- We write the data as a vector $\mathbf{g}$
- Elements of $\mathbf{g}$ are written $\mathbf{g}_{ij}$ or $\mathbf{g}_{m}$
- Index contains voxel label, including detector location, projection angle, time, etc.

Imaging is a process that maps the object to the data

- We write this as: $\mathbf{g} = \mathcal{H}\,\mathbf{f} + \mathbf{n}$
- $\mathcal{H}$ is the mapping (system operator)
  - May be nonlinear
- $\mathbf{n}$ is the measurement noise (a vector)
  - Not necessarily additive

Which $\mathcal{H}$ is best?

- Need models/measures of $\mathcal{H}$ to characterize the data

Definitions of probability

- Frequentists
  - Relative frequency of occurrence
  - Games of chance (e.g., roll of a die)
  - Multiple experimental trials
- Bayesians
  - Not simply a frequency
  - No need to verify with multiple trials
  - Degree of belief

Conditional probability of the image data

- $\mathrm{pr}(\mathbf{g}|\mathbf{f})$: essential for optimal inference
- Describes randomness in data $\mathbf{g}$ for fixed object $\mathbf{f}$
- In principle, could repeat observations of same object to determine $\mathrm{pr}(\mathbf{g}|\mathbf{f})$

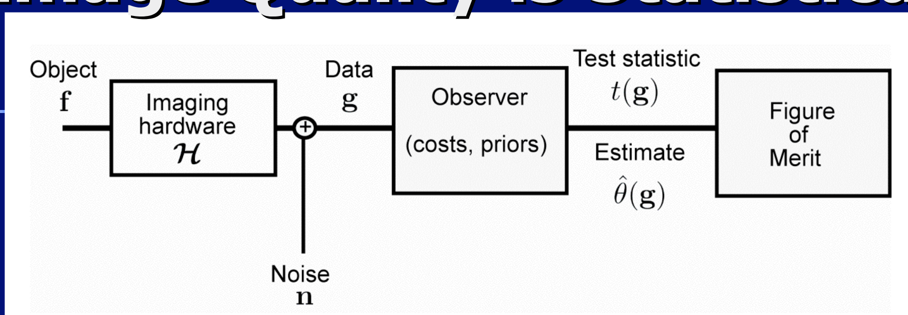
Role of Prior Information

- Prior: $\mathrm{pr}(\mathbf{f} \mid H_i)$ or $\mathrm{pr}(\theta)$
- State of knowledge before imaging
- Data are noisy; often have better inference by enforcing what we know about the object in advance

How to determine prior?

- Difficulty is dimensionality: How to determine $\mathrm{pr}(\mathbf{f})$ by frequentist means?
  – May make headway on lower-dimensional problem, e.g., rate parameter
- Bayesian answer:
  – Prior isn't a relative frequency
  – It captures a belief (e.g., smoothness)

Image Quality is Statistical



- The observer computes a test statistic or estimate: $t(\mathbf{g})$ or $\hat{\theta}(\mathbf{g})$. Inference may involve a Bayesian prior.
- The inferential process is repeated using many realizations of objects and noise.
- Analyze statistics of $t(\mathbf{g})$ or $\hat{\theta}(\mathbf{g})$ to determine a figure of merit. ➔ Frequentist performance measure!