Rapid Constructions of Circular-Orbit Pinhole SPECT Imaging System Matrices by Gaussian Interpolation Method Combined with Geometric Parameter Estimations (GIMGPE)

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Outline

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- **Procedures of GIMGPE**
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  - Image Parameterization
  - Imaging Properties Database
  - Projection Centroids Model
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Imaging System Matrix

Linear Digital-Imaging System:

$$\mathbf{g} = \mathbf{Hf} + \mathbf{n}.$$

g — **Image**, ***Discrete***, ***pixel***

H — **Imaging System Matrix**, ***Discrete***, ***voxel-into-pixel***

f — **Object**, ***Discrete***, ***voxel***

n — **Noise Vector**.

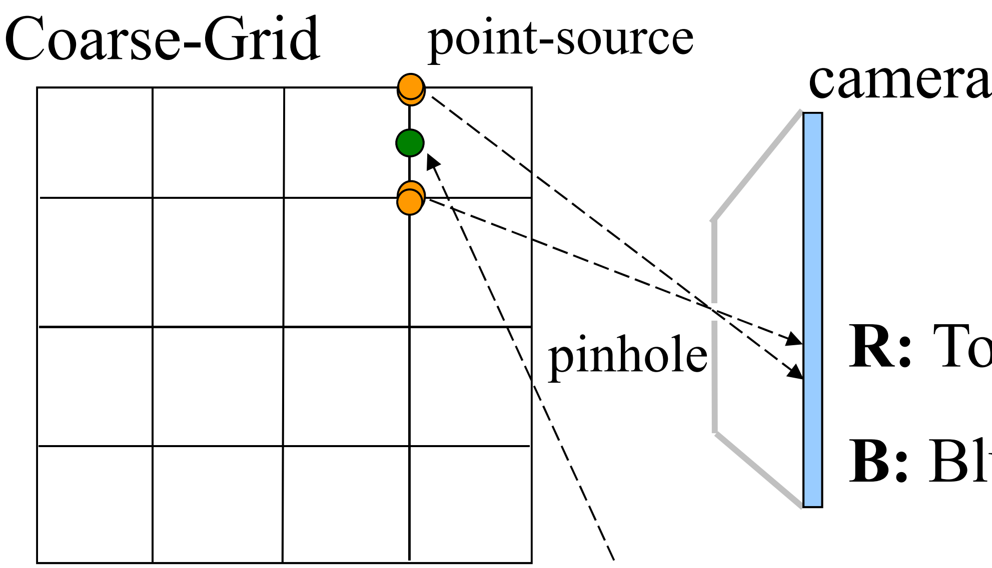
3. Combination

1. Measurement
2. Simulation

$$\mathbf{H} = [\mathbf{H}^{detector}\ \mathbf{H}^{geom}] \odot \mathbf{A}$$

Interpolation of Imaging System Matrices

- The interpolation of **H** matrix was firstly proposed by Rowe *et al.* [1].



$$\mathbf{H} = \mathbf{R} + \mathbf{B}$$

R: Total counts & centroid locations

B: Blur functions on the camera

A finer-grid H matrix can be interpolated.

R is a *slowly varying* function of point-source locations.

B is a function of centroid locations.

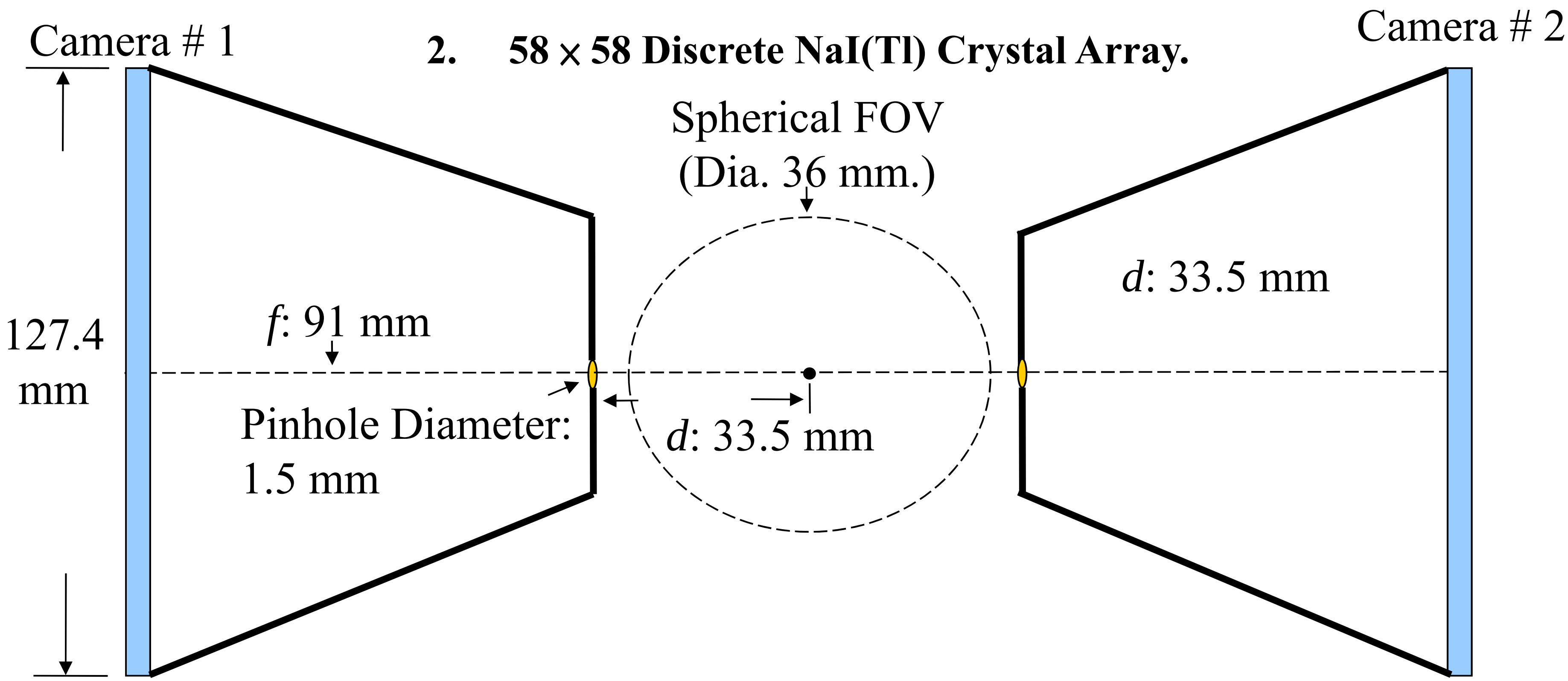
[1] *R. K. Rowe et al., Journal of Nuclear Medicine, vol. 34, no. 3, pp. 474-480, 1993.*

From Stationary to Circular Orbit

Previously, the proposed GIMGPE has tested with FASTSPECT II [2].

X-SPECT System:

1. **Dual- Headed Gamma Camera,**
2. **58 × 58 Discrete NaI(Tl) Crystal Array.**

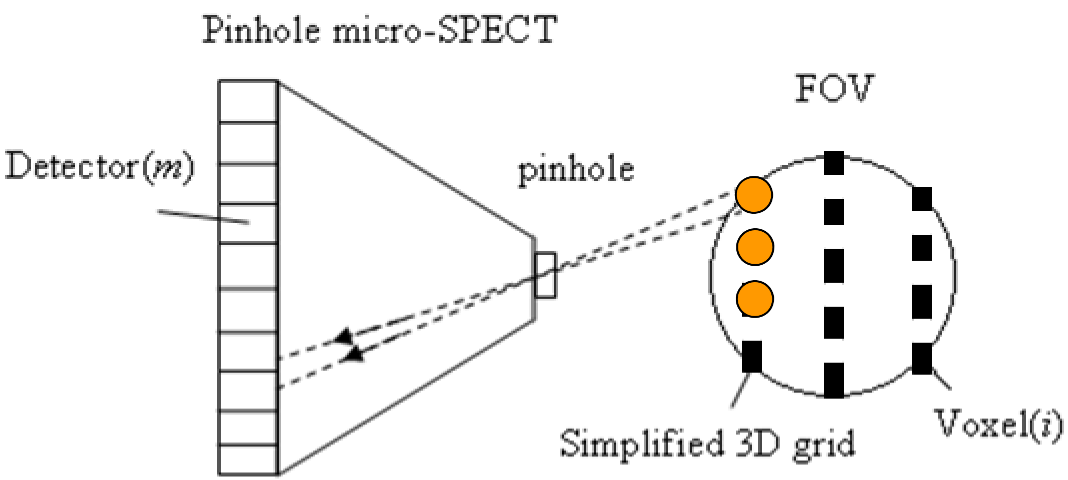


[2] M.-W. Lee and Y.-C. Chen, 2011 IEEE NSS/MIC Conference Record, pp. 2664 – 2667, 2011.

Simplified Grid-Scan Experiment

A **simplified grid-scan pattern** contains

- The ***boundary of Field-of-View***, and
- ***Three inner planes*** that are parallel to the camera plane.



Reducing ~75% voxels of a full 3D grid-scan pattern

at 0° projection angle

Image Parameterization

Circular Gaussian Function

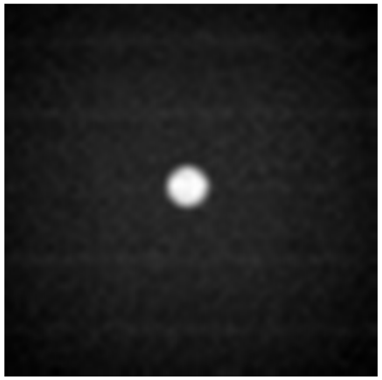
$$h_{i\theta}(u,v)=\frac{A}{2\pi\sigma^2}\exp[-\frac{(u-u_c)^2+(v-v_c)^2}{2\sigma^2}],$$

A : Amplitude,

u_c and v_c : Centroid coordinates,

σ : Standard deviation.

Photon Counts: ~ 6000 (counts)



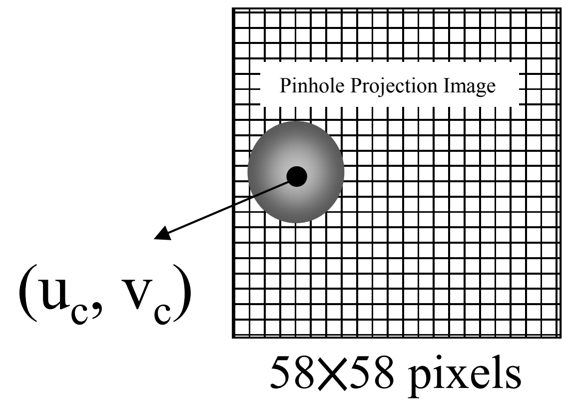
Projection of a point source stepped on a voxel within the FOV of a pinhole SPECT system.

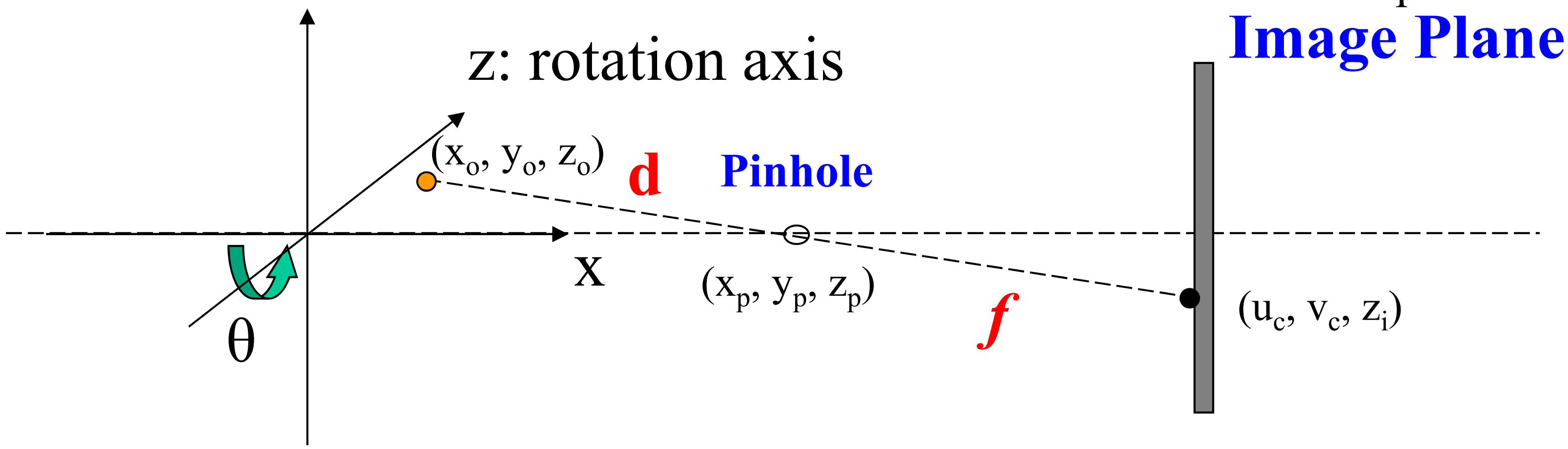
Circular Orbit Pinhole SPECT System

$$d = \sqrt{(x_p - x_o)^2 + (y_p - y_o)^2 + (z_p - z_o)^2} \text{ (mm)},$$

$$f = \sqrt{(x_p - u_c)^2 + (y_p - v_c)^2 + (z_p - z_i)^2} \text{ (mm)},$$

$D = f + d$ and $M = f / d$.





Amplitude $(D)_m$ around the same pixel

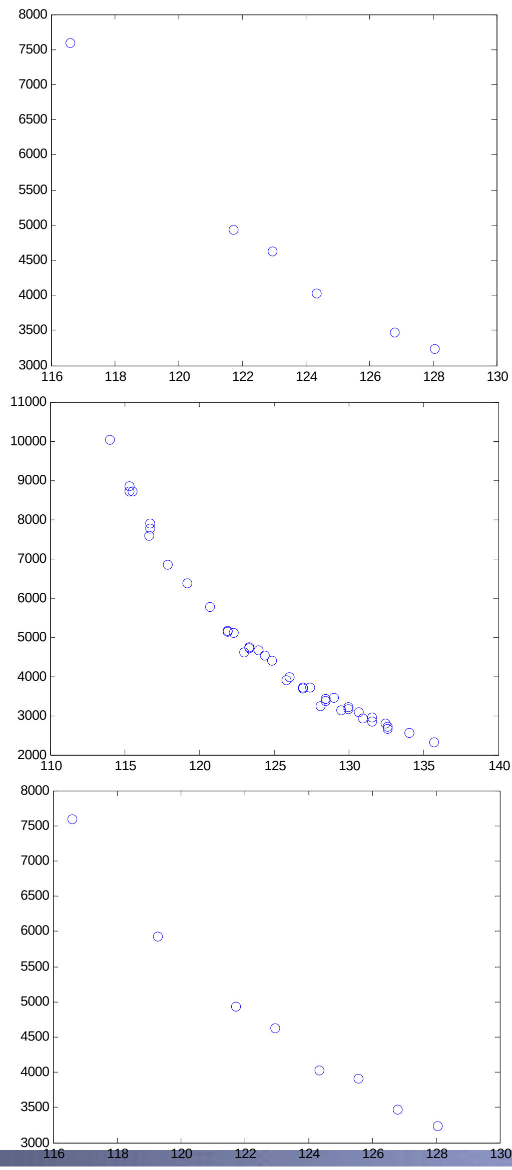
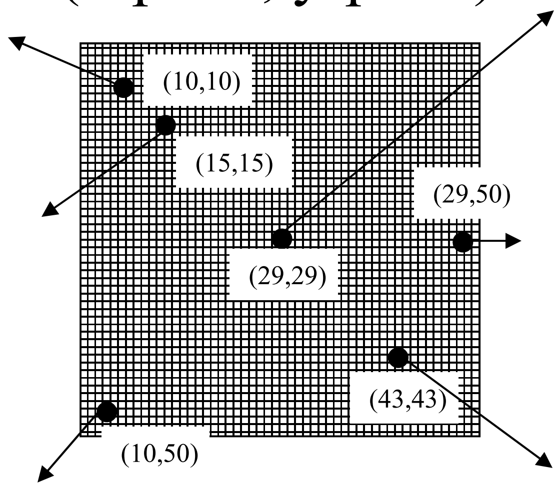
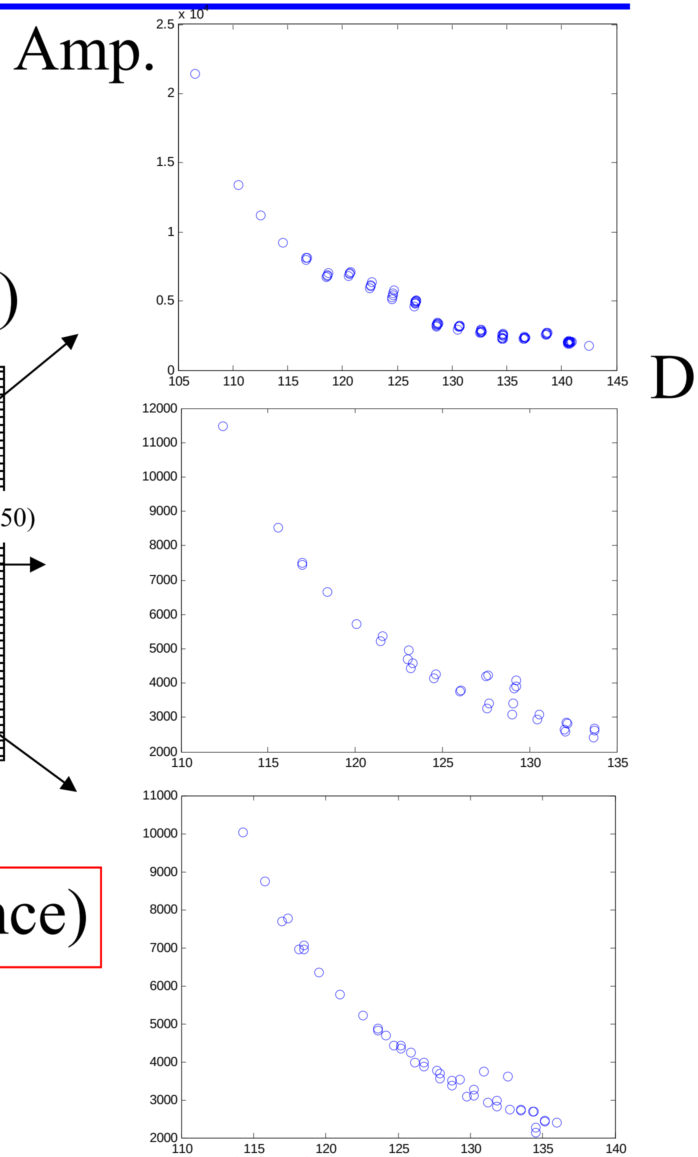


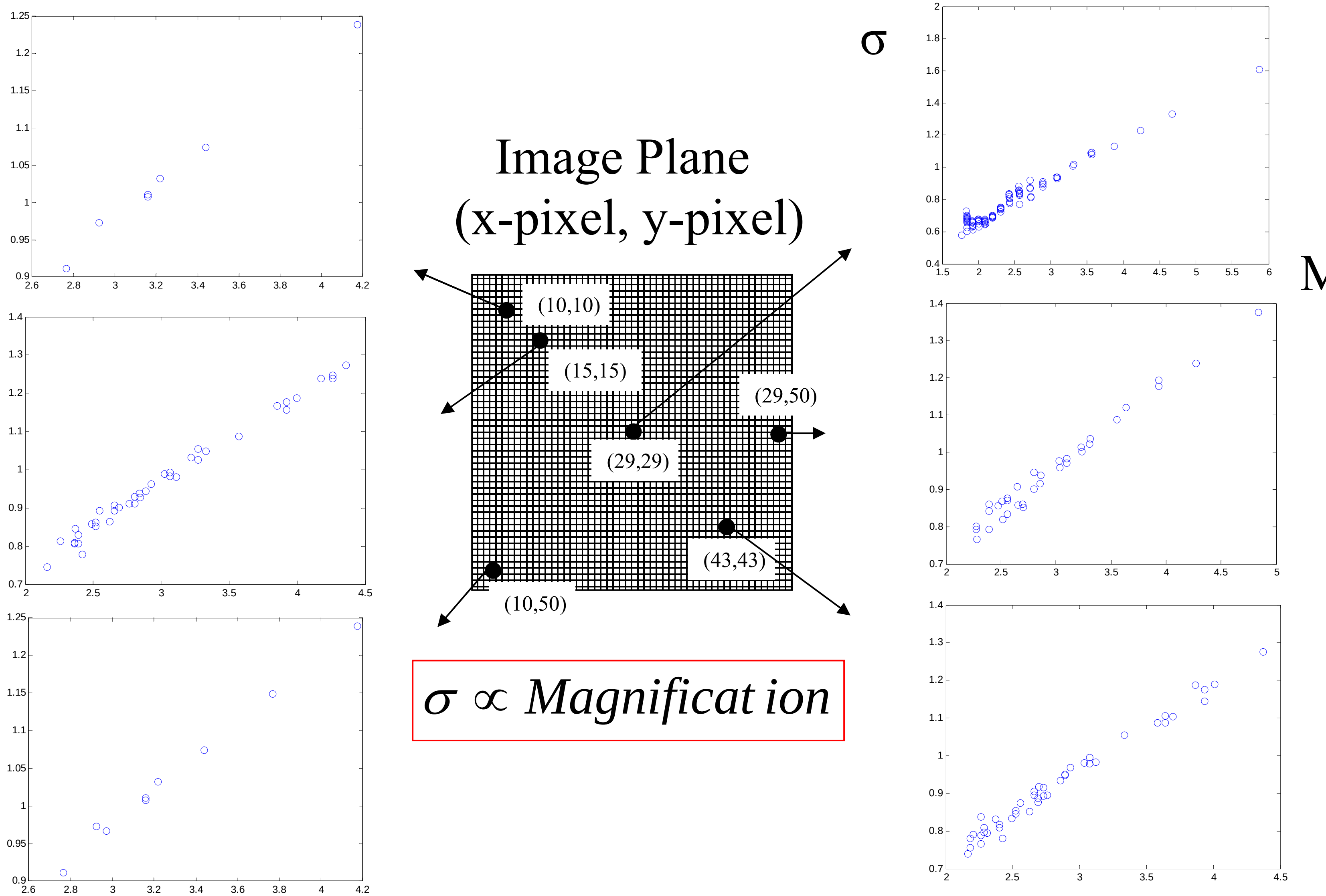
Image Plane
(x-pixel, y-pixel)



$$Amp. \propto (1/\text{Distance})$$



$\sigma(M)_m$ around the same pixel



$$\sigma \propto Magnification$$

Imaging Properties Database

Fitting the imaging property curves:

- 4th-order Laurent polynomial for $\text{Amplitude}(D)_m$.
- Linear function for $\sigma(M)_m$.

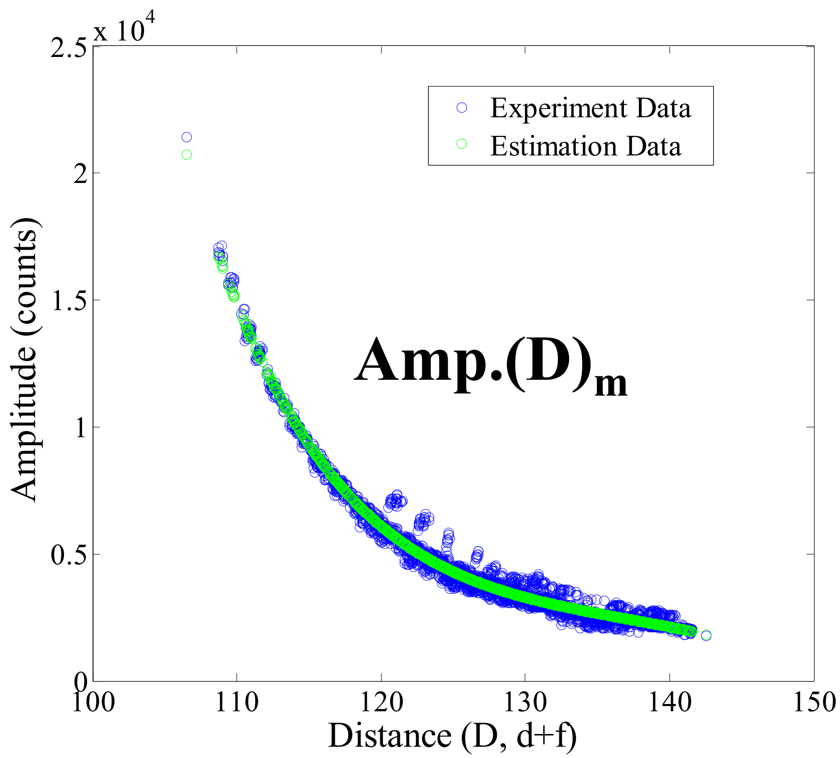
$$Amplitude(D)_m = a_1 \frac{1}{D^3} + b_1 \frac{1}{D^2} + c_1 \frac{1}{D} + d_1,$$

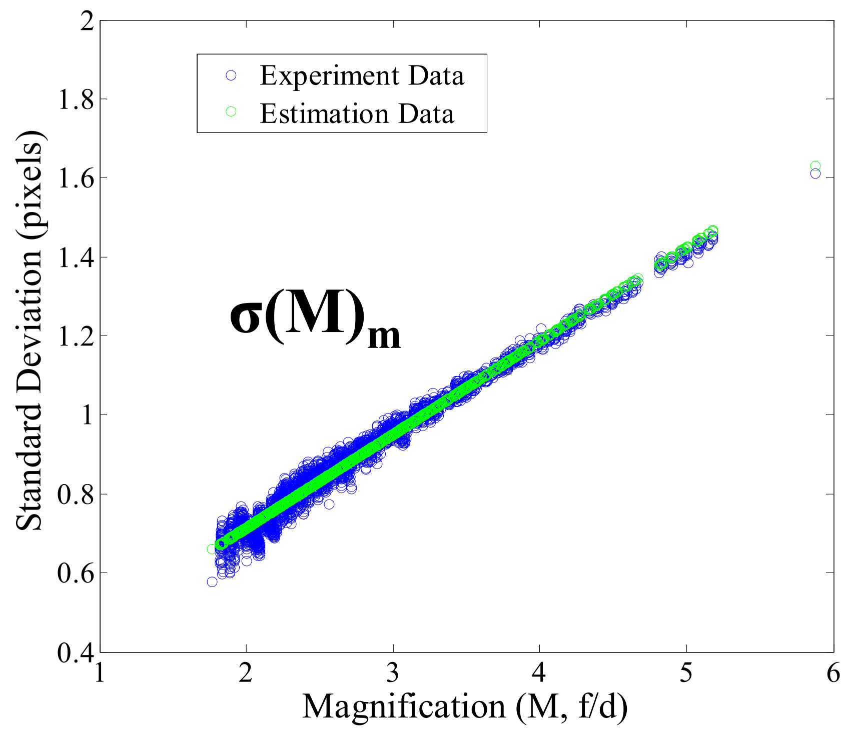
$$\sigma(M)_m = a_2 M + b_2,$$

where m denotes the m-th pixel.

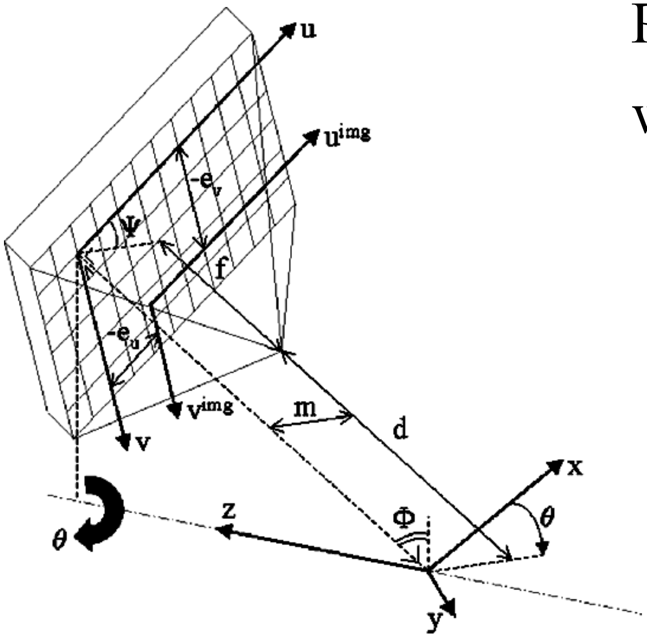
Imaging Properties Database

A full imaging properties database has [$58 \times 58 \times (4+2)$] elements: fitting coefficients of Amp.$(D)_m$ and $\sigma(M)_m$ curves.





Projection Centroids Model



Geometry of a pinhole SPECT imaging system [3].

Projection centroids are calculated as $(u_\theta^{img}, v_\theta^{img})$, where θ is the projection angle.

$$u_\theta^{img} = f\frac{m\cos\Psi - x'''(\theta,\Phi,\Psi)}{d + y'''(\theta,\Phi,\Psi)} + m\cos\Psi + e_u$$

$$v_\theta^{img} = f\frac{m\sin\Psi - z'''(\theta,\Phi,\Psi)}{d + y'''(\theta,\Phi,\Psi)} + m\sin\Psi + e_v$$

This projection model is proposed by D. Beque et al [3].

[3] D. Beque et al, IEEE Trans. Med. Imag., vol. 24, pp. 180-190, 2005.

Procedure of GIMGPE

Geometrical Calibration

↓

Projection Centroids Model

Simplified Grid-Scan Experiment

↓

Image Parameterization

↓

Imaging Properties Database:
Amp.(D)$_m$ and $\sigma(M)_m$

↓

16 projection angles

θ: 0° ~ 337.5°

H_θ

Rotate the Object Space at θ projection angle

↓

Estimate the Projection Centroids of each voxel

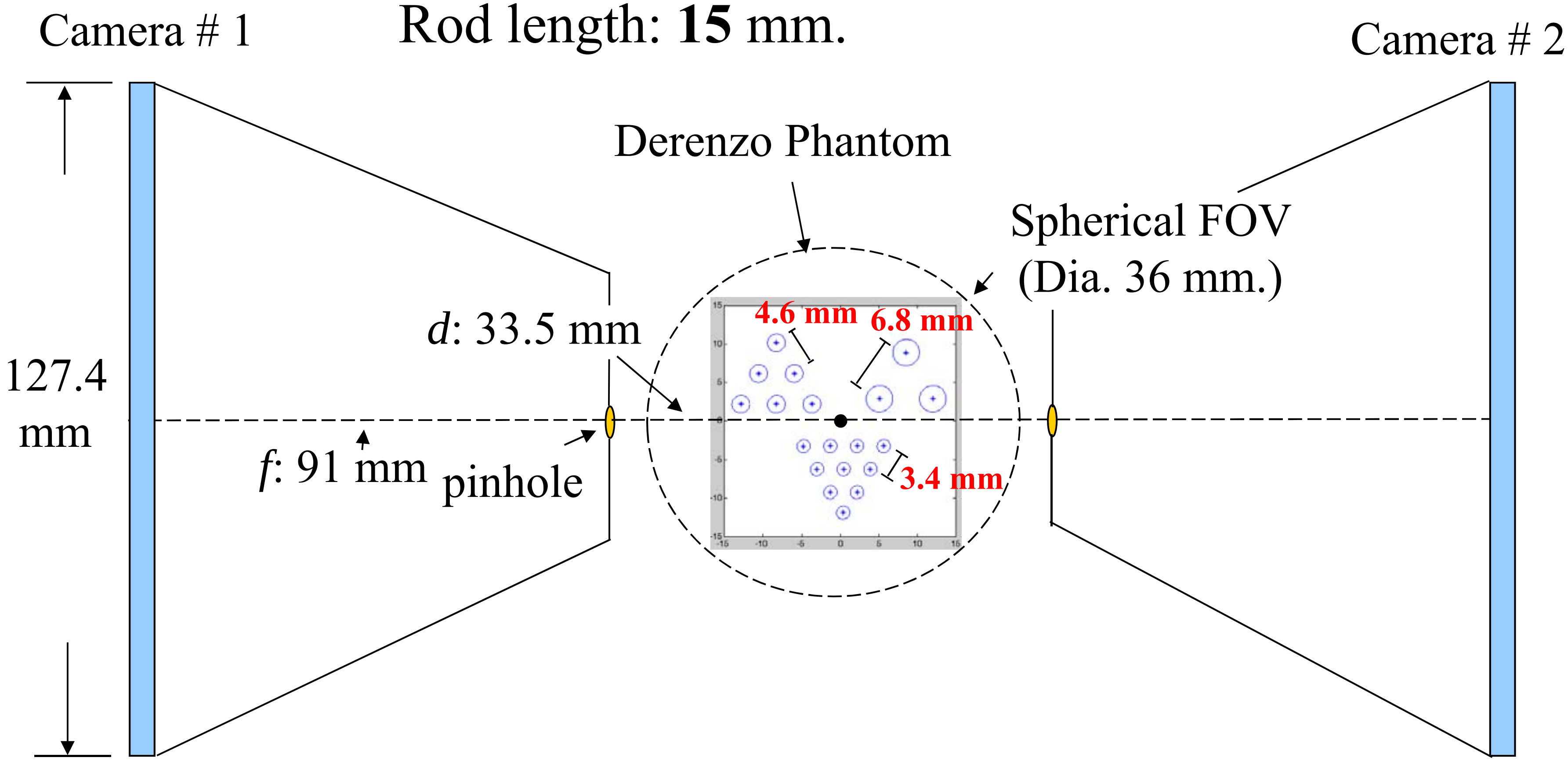
↓

Calculate the Blur Functions with the corresponding projection centroids, *D* and *M* of each voxel

Configuration of Derenzo Phantom

Rod diameter: **1.7**, **2.3**, and **3.4** (mm)

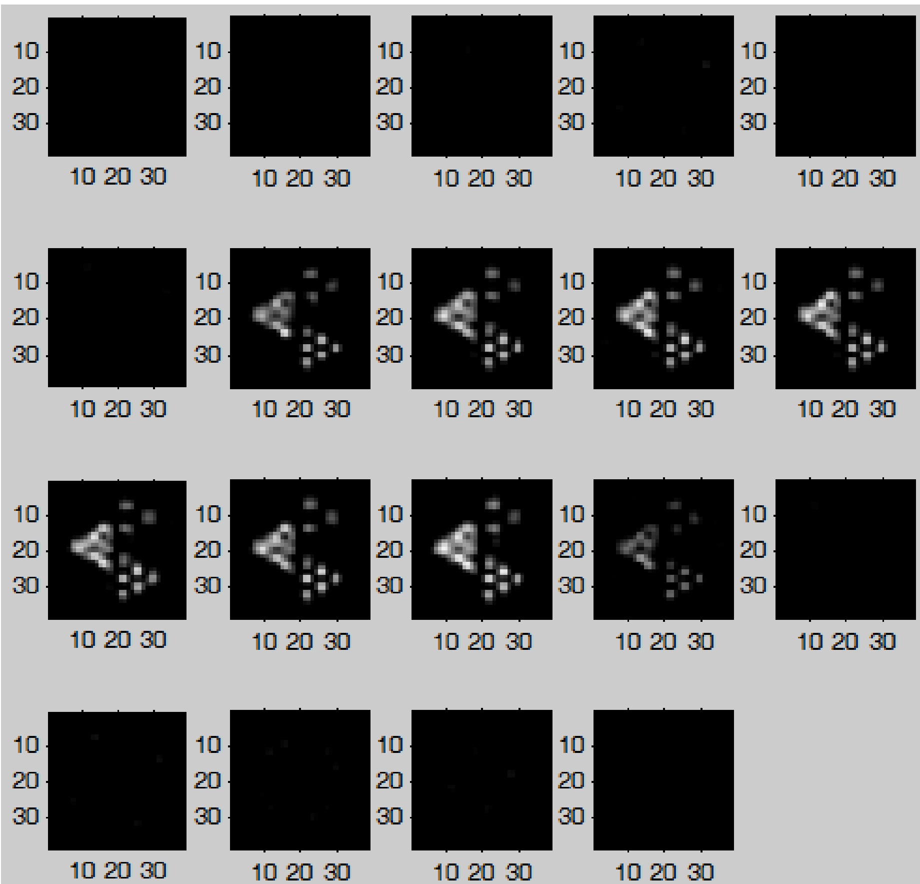
Rod length: **15** mm.



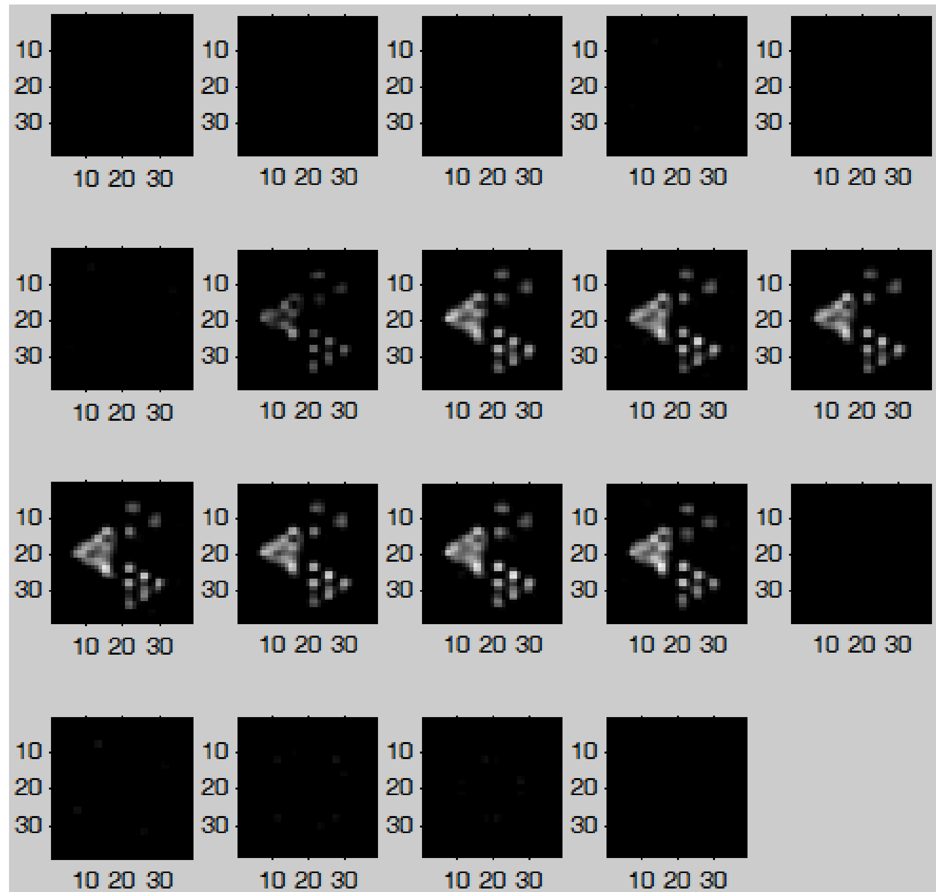
OSEM Reconstruction of a Derenzo Phantom

2-mm grid spacing Display scale: [10% max - 90% max] 1^{st} -19^{th} Slice

Full 3D Grid-Scan Experiment



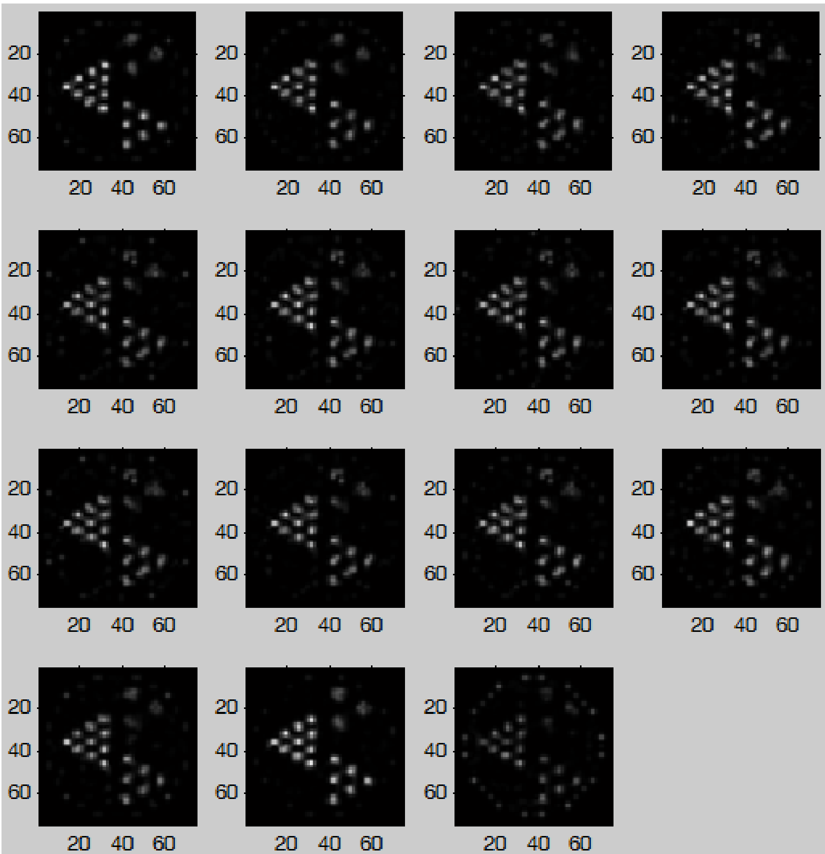
GIMGPE



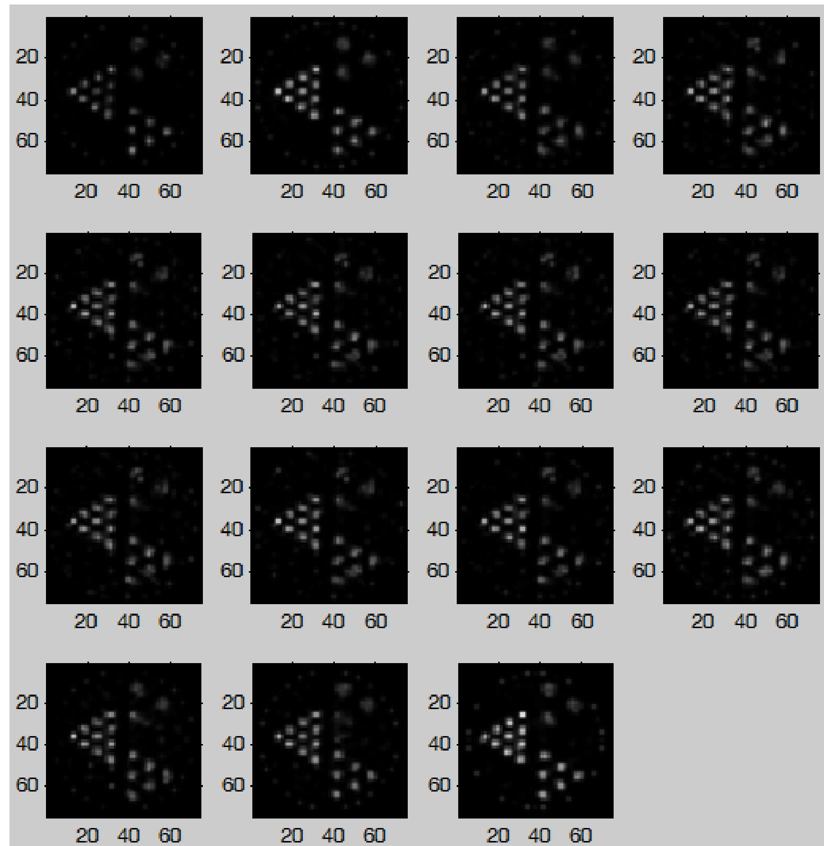
OSEM Reconstruction of a Derenzo Phantom

1-mm grid spacing Display scale: [0 - 90% max] 13th - 27th Slice

GIM[4]

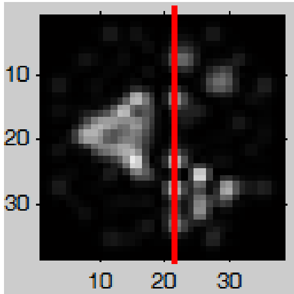


GIMGPE



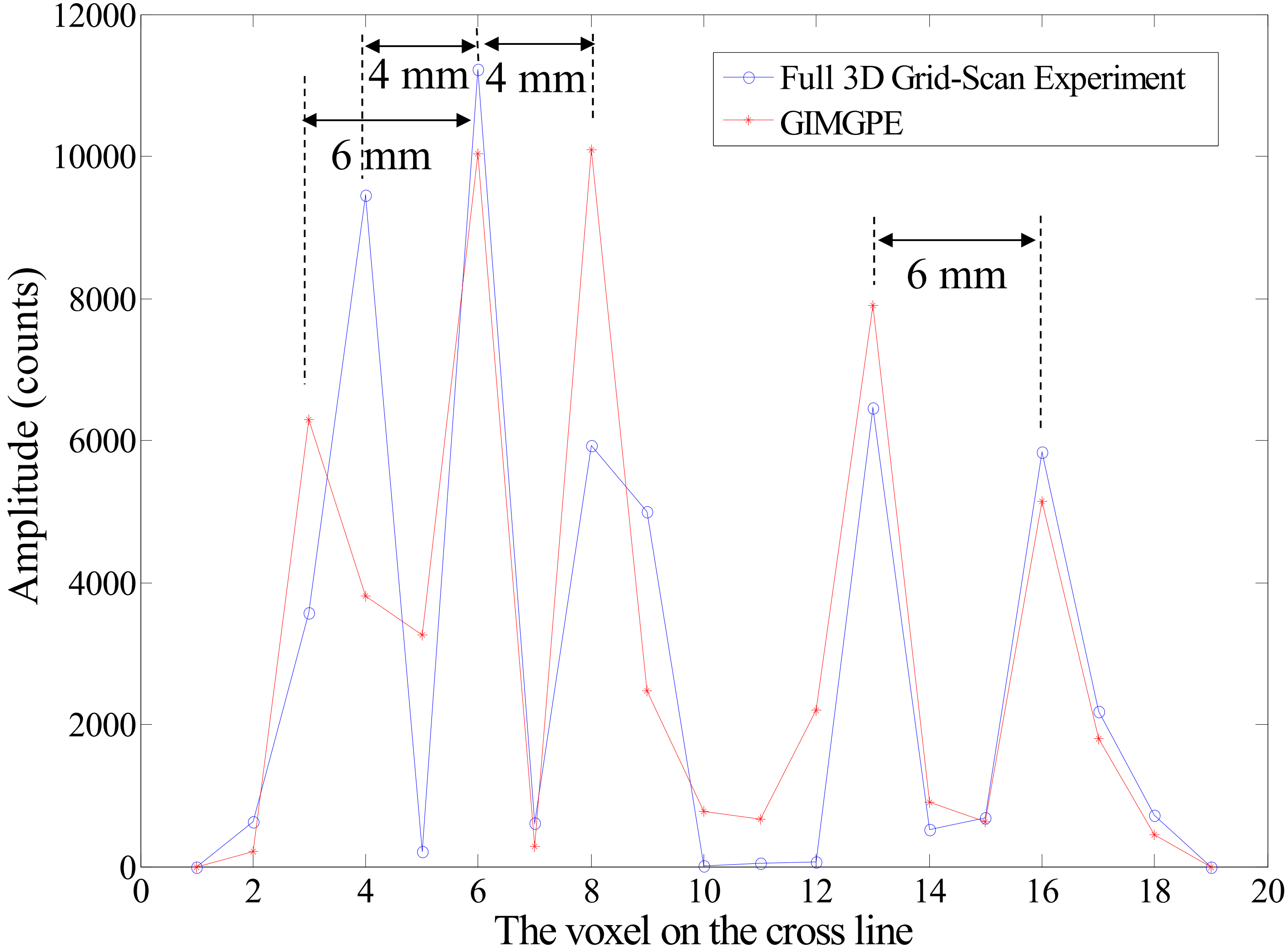
[4] Y. C. Chen et al, in Small-Animal SPECT Imaging, M. A. Kupinski and H. H. Barrett eds., Springer New York, pp. 195-201, June 2005.

Line Profile of One Slice

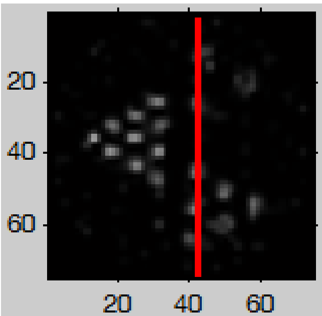


11th Slice

2-mm grid spacing

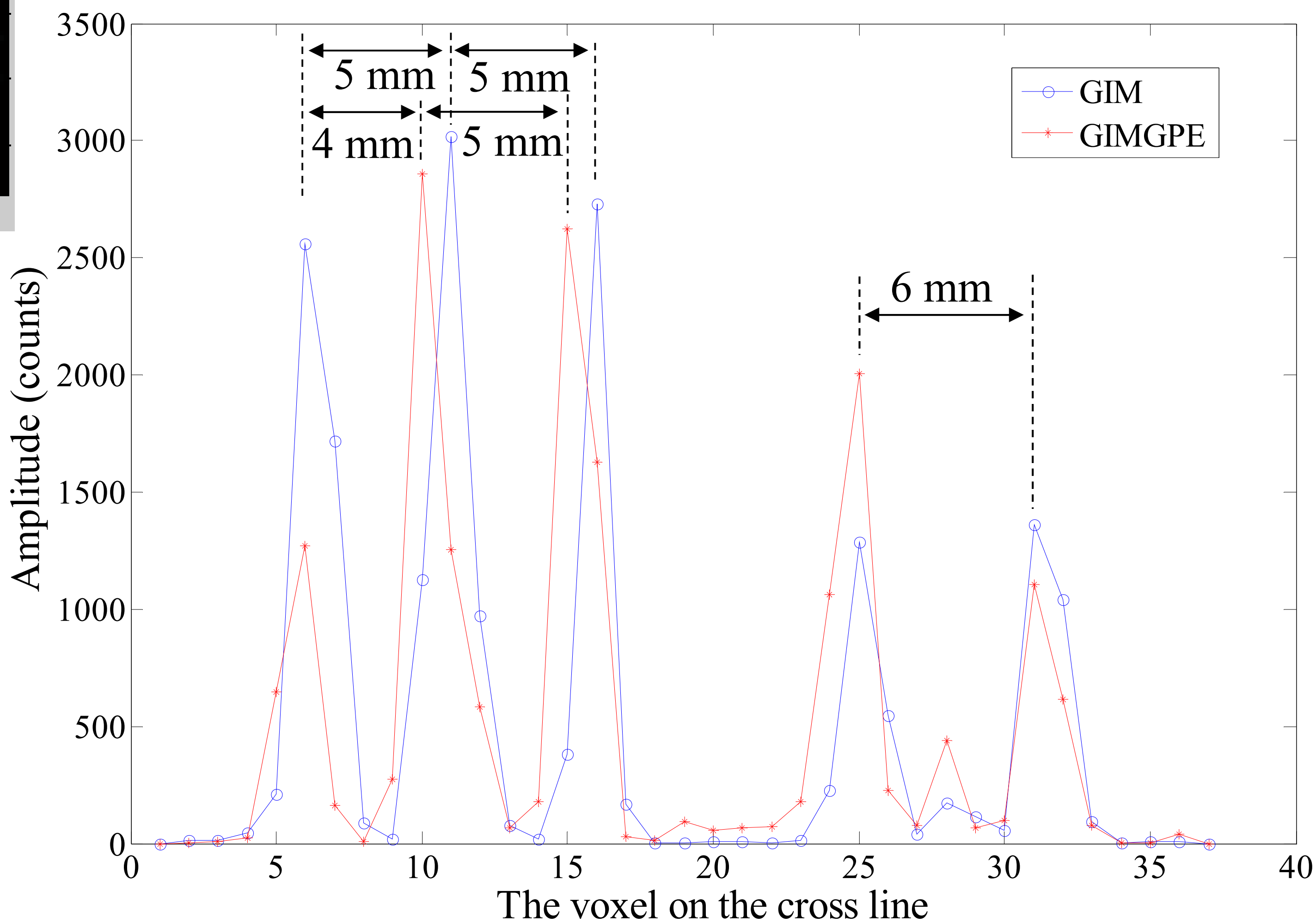


Line Profile of One Slice



20th Slice

1-mm grid spacing



Detectability

$\mathrm{H}_1\text{(signal absent)}: \quad \mathbf{g} = \mathbf{H}\mathbf{f}_\mathbf{b}(\mathbf{r}) + \mathbf{n} = \mathbf{b} + \mathbf{n},$

$\mathrm{H}_2\text{(signal present)}: \quad \mathbf{g} = \mathbf{H}[\mathbf{f}_\mathbf{b}(\mathbf{r}) + \mathbf{f}_\mathbf{s}(\mathbf{r})] + \mathbf{n} = \mathbf{b} + \mathbf{s} + \mathbf{n},$

$$SNR_\lambda^2 = \frac{\left[\sum_{m=1}^{M}(\overline{g}_{2m} - \overline{g}_{1m})\ln\left(\frac{\overline{g}_{2m}}{\overline{g}_{1m}}\right)\right]^2}{\frac{1}{2}\sum_{m=1}^{M}(\overline{g}_{2m} + \overline{g}_{1m})\ln^2\left(\frac{\overline{g}_{2m}}{\overline{g}_{1m}}\right)}.$$

Ideal Observer

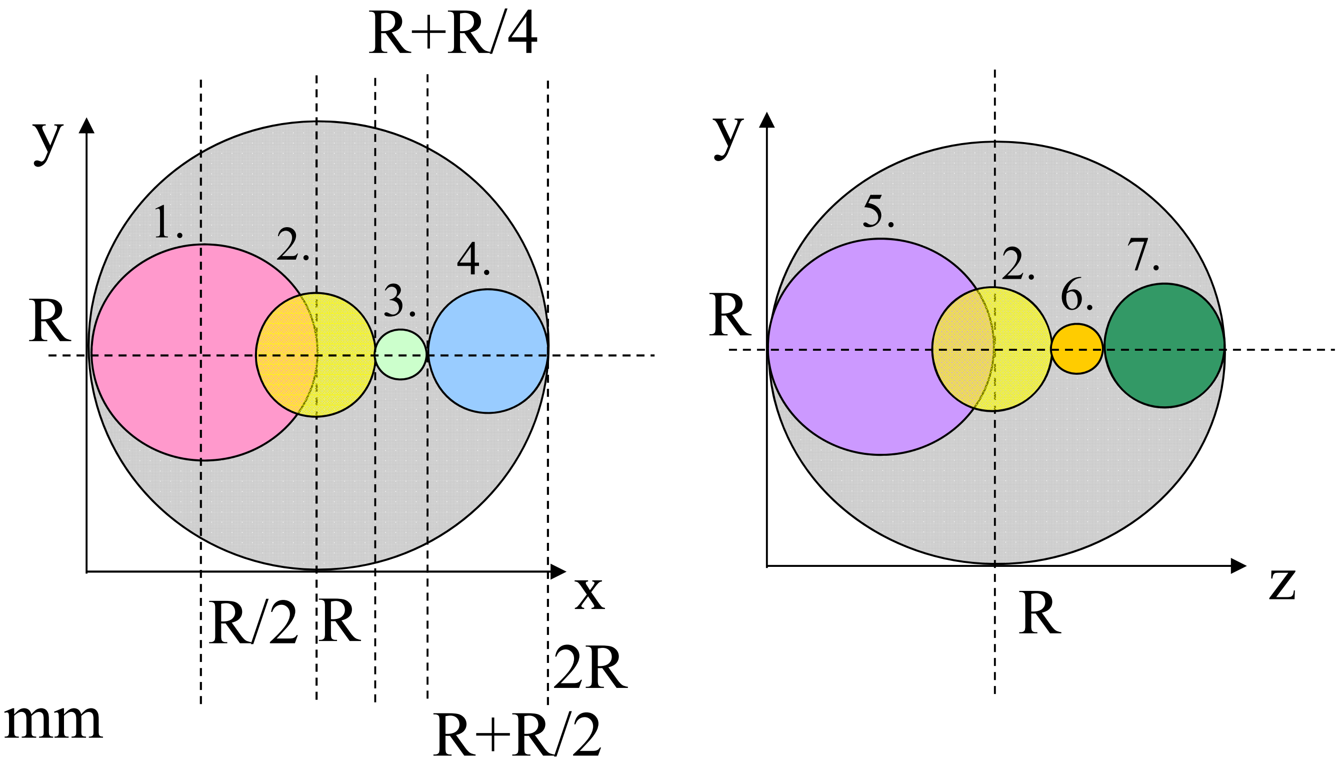
If consider a low-contrast signal,

$$SNR_\lambda^2 \cong d_A^2 = \sqrt{\sum_{m=1}^{M}\frac{(s_m)^2}{\overline{g}_m}}$$

SKE/BKE Tasks

Detectability Estimation:

- 7 **Uniform Sphere Phantom** Against a **Flat Background Noise**,
- 8 Contrast (A_S/A_B): [0.01; 0.02; 0.05; 0.10; 0.20; 0.50; 1.00; 2.00].



R = 18 mm

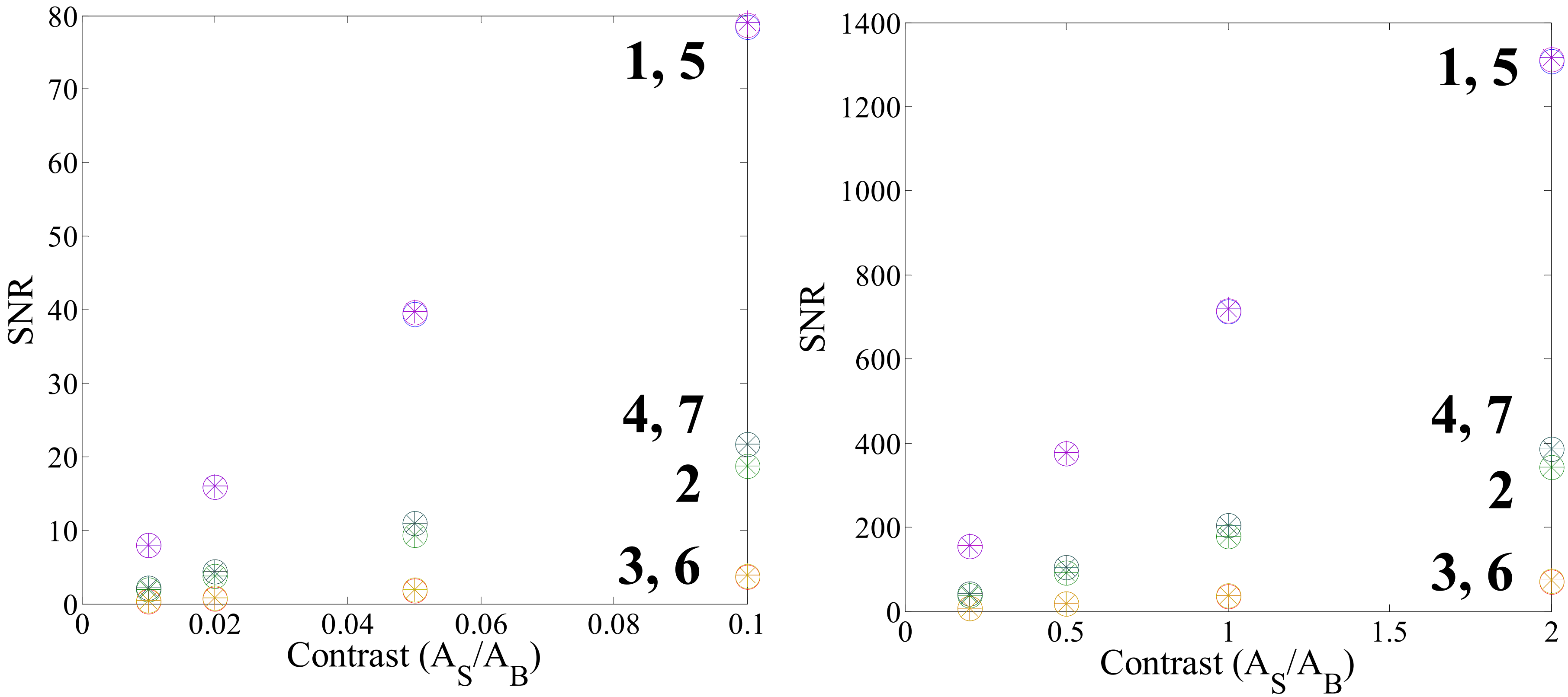
Detectability Performance

○: Full 3D Grid-Scan Experiment

$$\max\left(\frac{|SNR_{GIMGPE} - SNR_{Grid\text{-}Scan}|}{SNR_{Grid\text{-}Scan}} \times 100\right) = 1.16\%$$

∗: GIMGPE

2-mm grid spacing



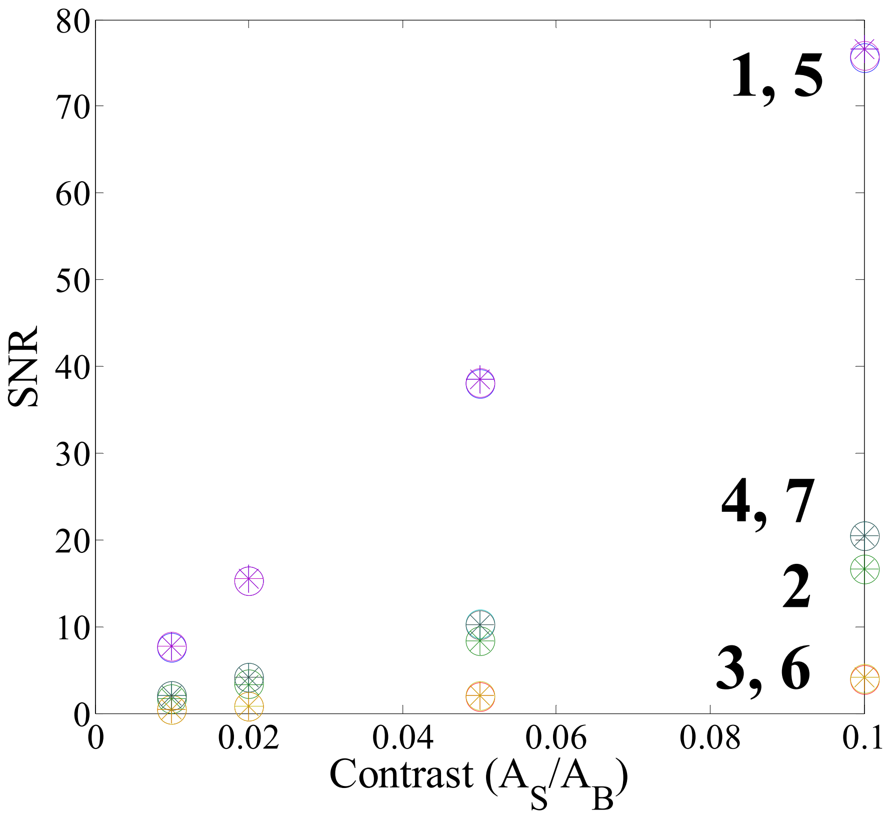
Detectability Performance

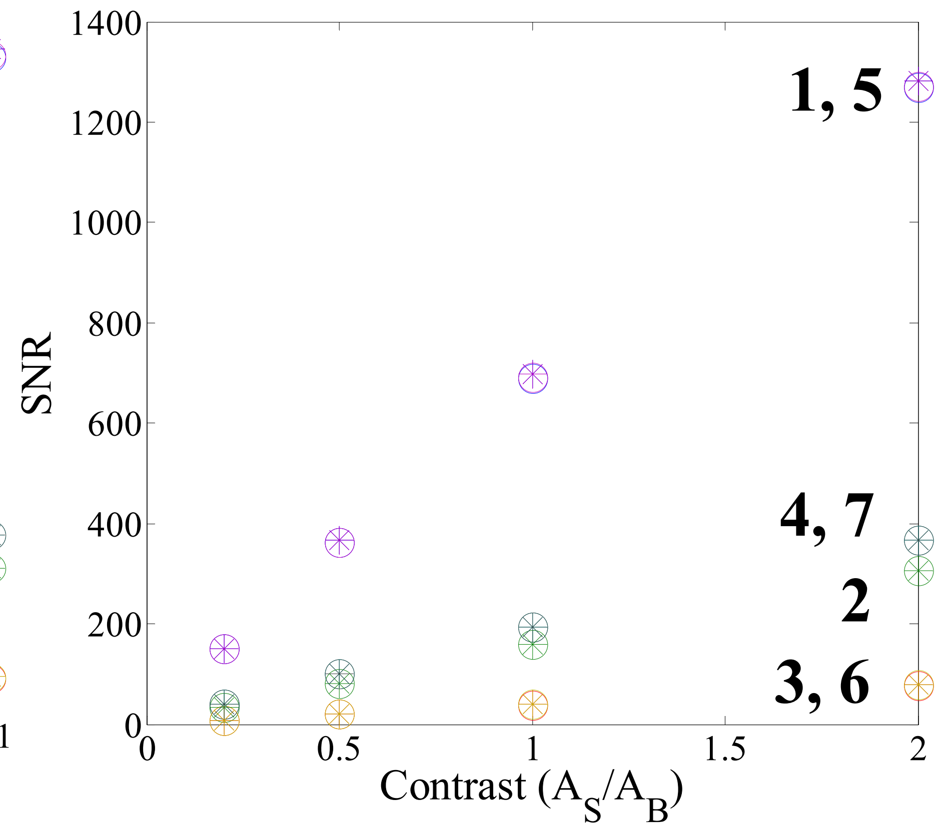
○: GIM

∗: GIMGPE

$$\max\left(\frac{\left|SNR_{GIMGPE} - SNR_{GIM}\right|}{SNR_{GIM}} \times 100\right) = 4.97\%$$

1-mm grid spacing





Conclusion

- The preliminary evaluation of GIMGPE method shortens the measurement time of a full ***2.0-mm grid H matrix*** about **64 times** and a full ***1.0-mm grid H matrix*** about **512 times**.
- The **OSEM reconstructed images** of a Derenzo phantom with the interpolated **H** matrices show **comparable resolution** and **similar line profiles** as that reconstructed with the full 3D grid-scan **H** matrix and the GIM finer-grid **H** matrix.
- The **SKE/BKE detection tasks** demonstrate the interpolated **H** matrices have the **same detectability level** as the full 3D grid-scan **H** matrix and the finer-grid GIM **H** matrix.
- Based on the **GIMGPE** method, further interpolations of the **H** matrix to **much finer spacing** and **more projection angles** could be easily done.

Acknowledgement

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