FACULTY OF ENGINEERING AND ARCHITECTURE

Efficient Optimization Based on Local Shift-Invariance for Adaptive SPECT Systems

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Workshop on Small Animal SPECT

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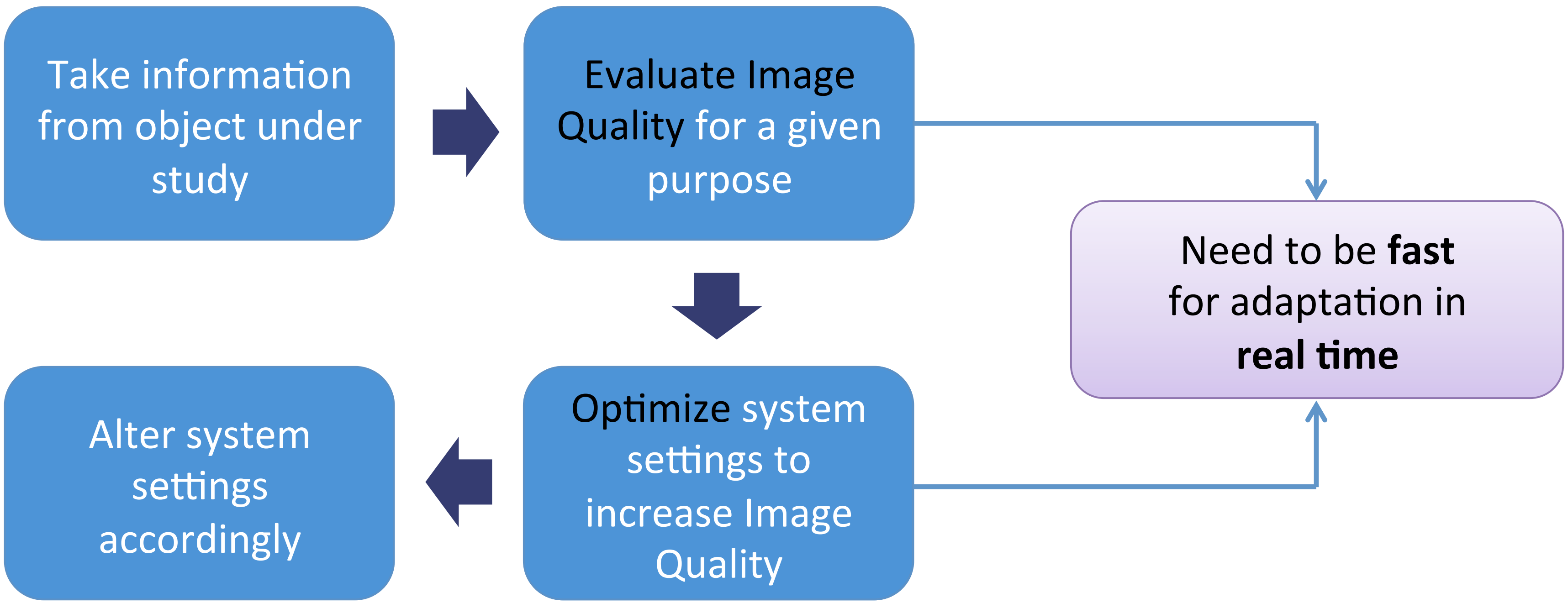
Overview

- Introduction
- Method
  - Image Quality
  - Optimization
- Application
  - Case 1: Single-Pinhole Rotating SPECT
  - Case 2: Multi-Pinhole Stationary SPECT (U-SPECT-II)
- Conclusion

Adaptive SPECT



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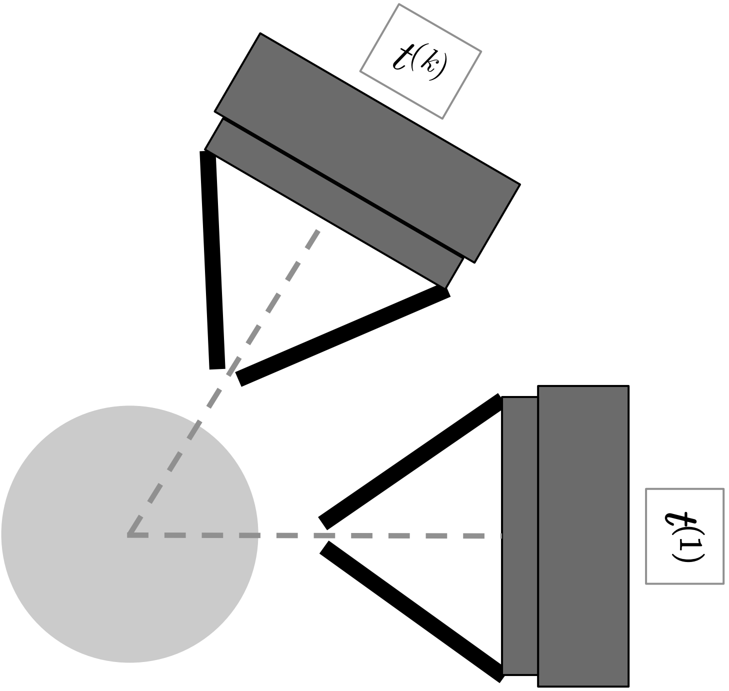
Adaptive Angular Sampling for SPECT Imaging

Nan Li and Ling-Jian Meng



- Rotating single-head SPECT
- Adapt time $t^{(k)}$ spent at each angle $k \in \{1,\ldots,K\}$, for a given total imaging time

[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)
[2] B. L. Franc et al., *J. Nucl. Med.* 49 (2008)

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Adaptive Angular Sampling for SPECT Imaging

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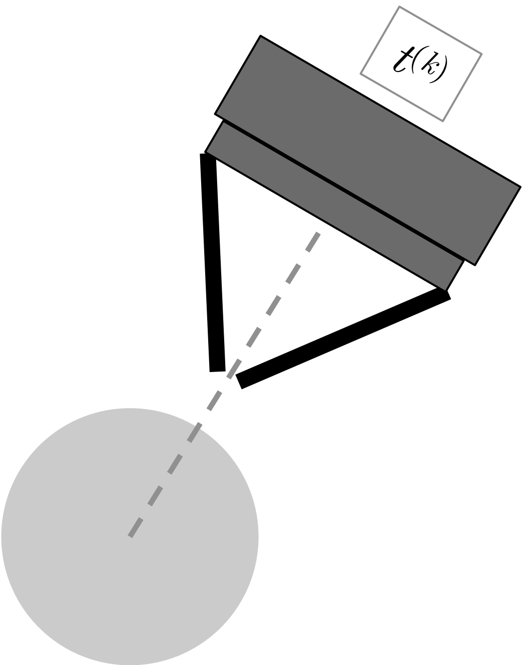

Mean projection data vector

Fisher Information Matrix

- $\overline{\mathbf{y}} = \mathbf{H}\mathbf{x}$ ➡ $\mathbf{F} = \mathbf{H}^T diag\left(\frac{1}{\overline{y}_i}\right)\mathbf{H}$
- For angle k: $\overline{\mathbf{y}}^{(k)} = t^{(k)}\mathbf{H}^{(k)}\mathbf{x}$

$$\mathbf{F} = \sum_{k=1}^{K} t^{(k)}\mathbf{F}^{(k)}$$

$$\mathbf{F}^{(k)} = \mathbf{H}^{(k)T} diag\left(\frac{1}{\left(\mathbf{H}^{(k)}\mathbf{x}\right)_i}\right)\mathbf{H}^{(k)}$$

[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)

Evaluation of Image Quality

- Analytical formulas for FOM at POI, based on Fisher Information Matrix $\mathbf{F}$
- To increase computational efficiency, $\mathbf{F}$ can be simplified:
  - [1]: Non-Uniform Object-Space Pixelation approach[3]
  - Our method: Local-shift invariance approximation[4,5]

Optimization

- Fast optimization based on the gradient of the FOM

$$Grad_j^{(k)} = \frac{\partial(FOM)}{\partial t^{(k)}}$$

Application

- Adapt time per angle in single-head single-pinhole rotating SPECT system[1]
- Adapt time per bed position in stationary multi-pinhole SPECT system (MILabs U-SPECT-II)

[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)
[3] L.-J. Meng and N. Li, *IEEE Trans. Nucl. Sci.* 56 (2009)
[4] J. Qi and R. M. Leahy, *IEEE Trans. Med. Imag.* 19 (2000)
[5] K. Vunckx et al., *IEEE Trans. Med. Imag.* 27 (2008)

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1. Fisher Information-based approximation for **Post-Filtered MLEM** ran to convergence[5,6,7,8], at voxel j:

$$CRC_j \approx \mathbf{e}^{jT}\mathbf{P}\mathbf{G}\mathbf{F}\mathbf{e}^j, \qquad Var_j \approx \mathbf{e}^{jT}\mathbf{P}\mathbf{G}\mathbf{F}\mathbf{G}^T\mathbf{P}^T\mathbf{e}^j \quad \rightarrow \quad CNR_j = \frac{CRC_j}{\sqrt{Var_j}}$$

Post-Filter

Pseudoinverse of $\mathbf{F}$ → computational challenge

2. Local Shift-Invariance assumption on $\mathbf{F}^{(k)}$ [4,5,8]:

$$\mathbf{F}^{(k)} \approx circ\left\{\mathbf{F}^{(k)}\mathbf{e}^j\right\} = \mathbf{Q}^T \mathrm{diag}\left[\lambda_i^{F^{(k)}}\right]\mathbf{Q}$$

3D DFT

Block-Circulant Matrix

Eigenvalues – obtained using $\mathbf{Q}$

[4] J. Qi and R. M. Leahy, *IEEE Trans. Med. Imag.* 19 (2000)
[5] K. Vunckx et al., *IEEE Trans. Med. Imag.* 27 (2008)
[6] J. A. Fessler, *IEEE Trans. Image Process.* 5 (1996)
[7] J. A. Fessler and W. L. Rogers, *IEEE Trans. Image Process.* 5 (1996)
[8] J. Nuyts et al., *Methods* 48 (2009)

4. F and G also become block-circulant, with:

$$\mathbf{F} = \sum_k t^{(k)} \mathbf{F}^{(k)} \Rightarrow \mathbf{F} \approx \mathbf{Q}^T diag\left[\lambda_i^F\right]\mathbf{Q}, \qquad \lambda_i^F = \sum_k t^{(k)} \lambda_i^{F(k)}$$

$$\mathbf{G} \approx \mathbf{Q}^T diag\left[\lambda_i^G\right]\mathbf{Q}, \qquad \lambda_i^G = \begin{cases} 0, & \lambda_i^F = 0 \\ \frac{1}{\lambda_i^F}, & \lambda_i^F \neq 0 \end{cases}$$

5. In the Optimization we use:

$$\left.\begin{aligned} CRC_j &\approx \mathbf{e}^{jT}\mathbf{P}\mathbf{Q}^T diag\left[\lambda_i^G \lambda_i^F\right]\mathbf{Q}\mathbf{e}^j \\ Var_j &\approx \mathbf{e}^{jT}\mathbf{P}\mathbf{Q}^T diag\left[\lambda_i^G\right]\mathbf{Q}\mathbf{P}^T\mathbf{e}^j \end{aligned}\right\} \; CNR_j = \frac{CRC_j}{\sqrt{Var_j}}$$

$$Grad_j^{(k)} = \frac{\partial CNR_j}{\partial t^{(k)}} \propto \mathbf{e}^{jT}\mathbf{P}\mathbf{Q}^T diag\left[\lambda_i^{F(k)} \cdot \left(\lambda_i^G\right)^2\right]\mathbf{Q}\mathbf{P}^T\mathbf{e}^j$$

Computations reduced to element by element multiplications and FFTs

[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)

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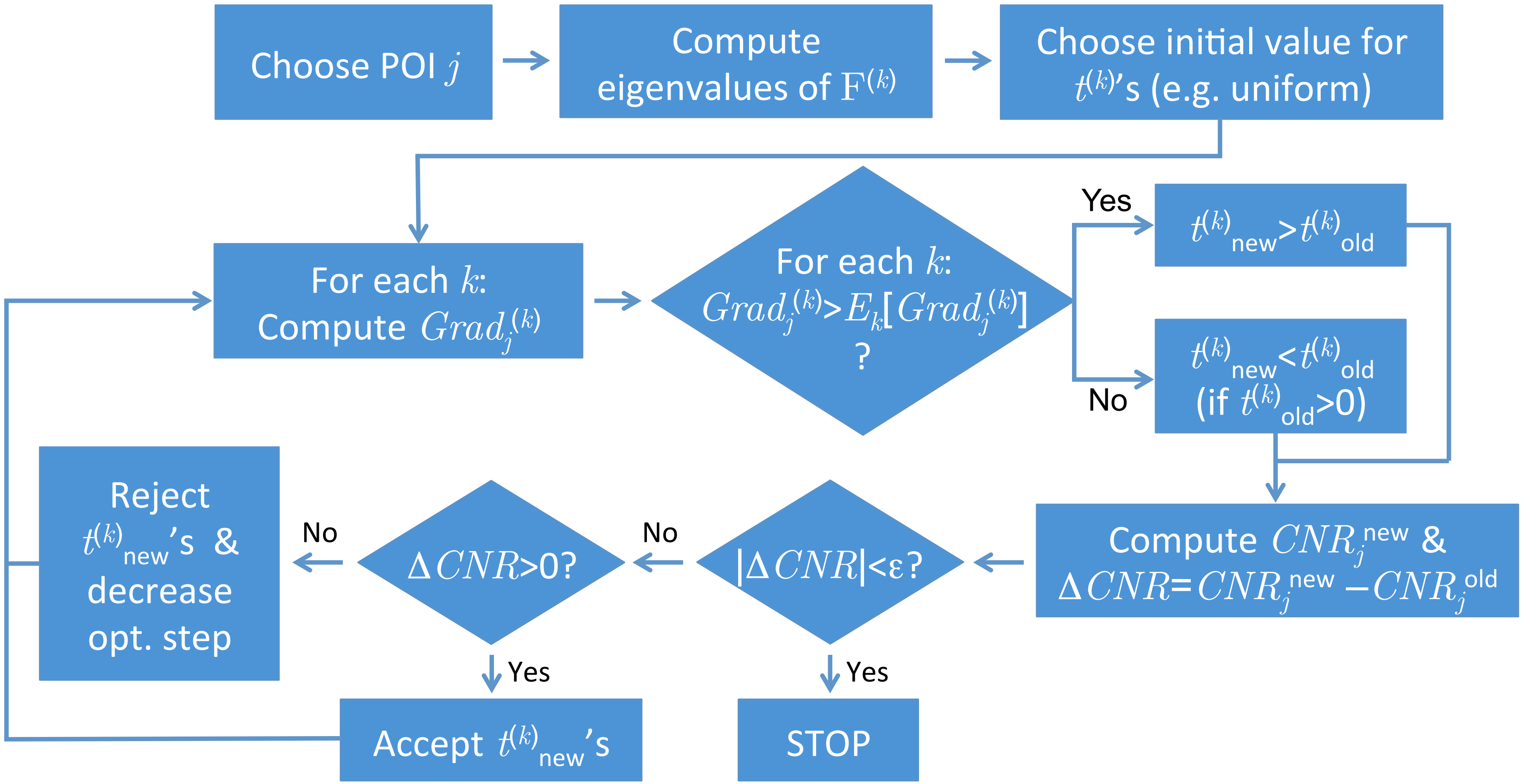
Optimization (maximize CNR_j)

$$Grad_j^{(k)} = \frac{\partial CNR_j}{\partial t^{(k)}}$$



[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)

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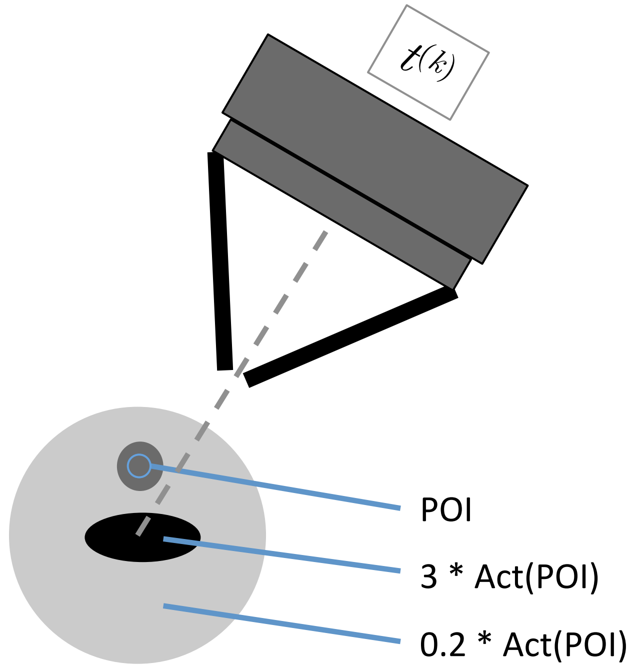
Case 1) Single-pinhole rotating SPECT system

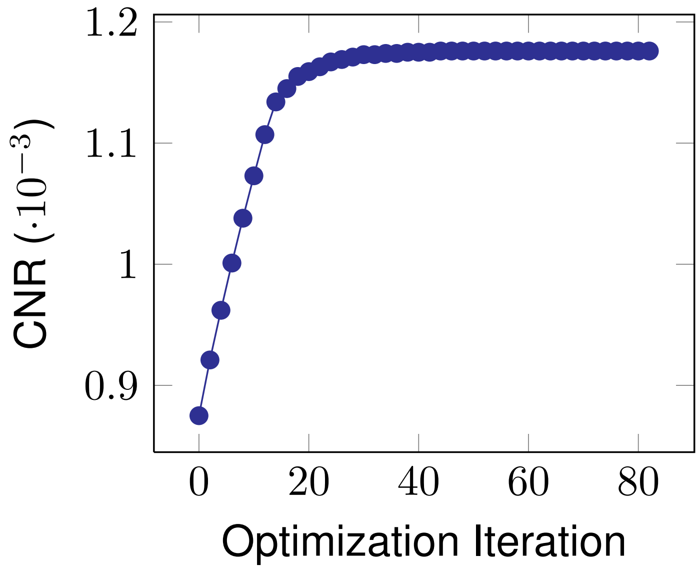
- Single-head single-pinhole SPECT
- Rotation: 32 Angles
- Image Size: 12.8mmx12.8mmx12.8mm
- Total Activity=18.5MBq * 64 minutes
- System Matrix modeled by ray-tracing (7-ray pinhole aperture subsampling[9])
- Reconstruction algorithm: PF-MLEM, Gaussian filter

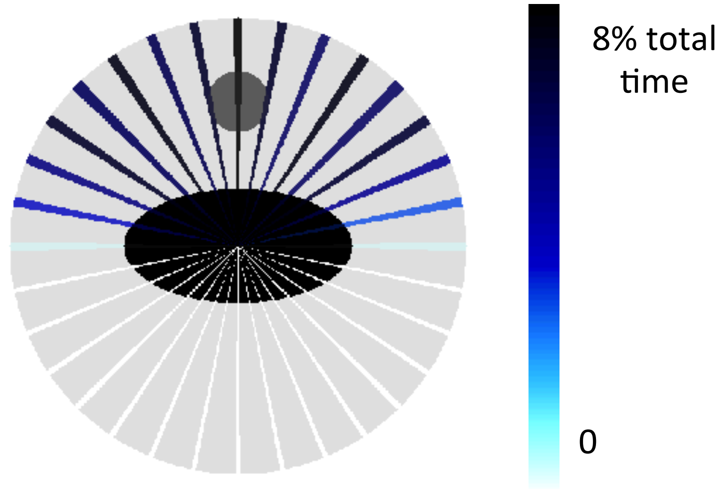


Original phantom[1] (transversal)

[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)
[9] C. Vanhove et al., *Eur. J. Nuc. Med. Mol. Imaging* 34 (2007)

Optimization Results



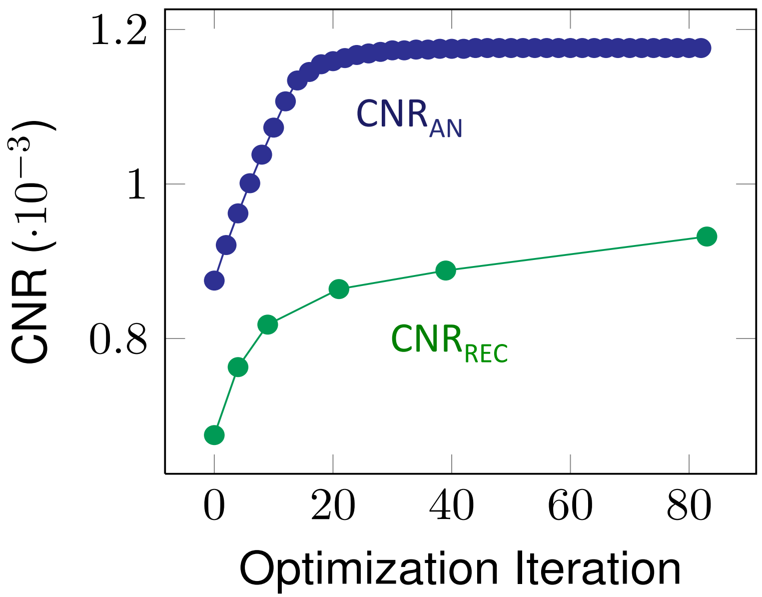


Final Time Distribution

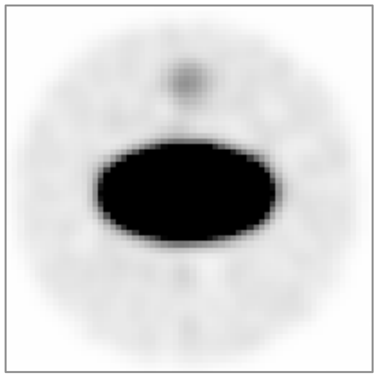
- 32 Angles
- Relative increase in CNR_j: **34% ➔ Optimization works and agrees with expectations**
- **Very fast**: ~1-2 minutes

[1] N. Li and L.-J. Meng, *IEEE Trans. Nucl. Sci.* 58 (2011)

Validation of Image Quality calculation



- CNR_{AN}: Analytical calculation
- CNR_{REC}: Computed from reconstructions:
  - 800 MLEM steps, Gaussian Post-Filtering
  - 600 noise realizations



Reconstruction realization from Uniform Time Distribution

- CNR_{AN} increase=34%
- CNR_{REC} increase=38%
- **Relative difference between CNR_{AN} and CNR_{REC} is ~ 30%**

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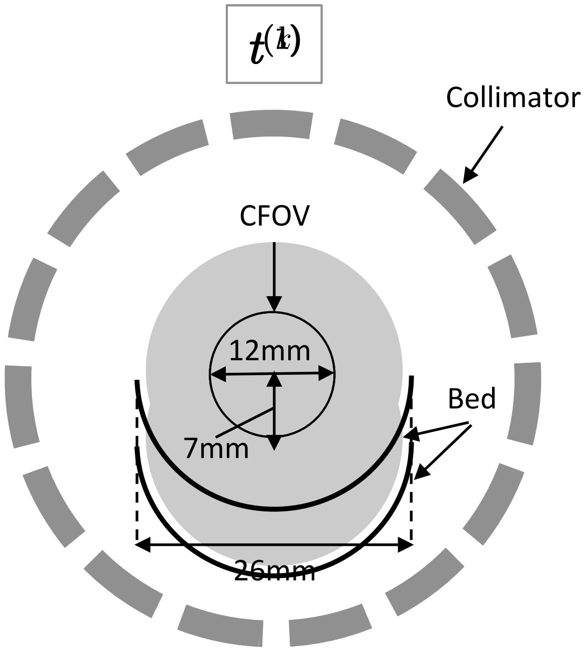
Case 2) U-SPECT-II system

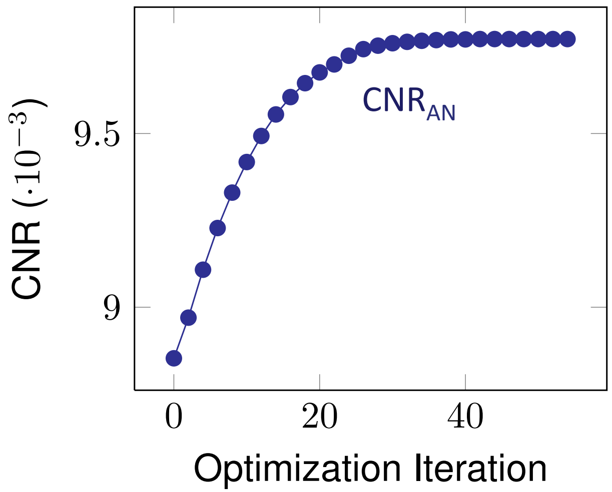
- MILabs U-SPECT-II:
  - Cylindrical collimator, 75 pinh.
  - Shielding cylinder
  - 3 stationary detectors



- MOBY Phantom[10] scaled to 89%, ^{99m}Tc-tetrofosmin
- Total Activity=75 MBq * 45 minutes
- System Matrix modeled by ray-tracing[9]
- Reconstruction algorithm: PF-MLEM, Gaussian filter

[9] C. Vanhove et al., *Eur. J. Nuc. Med. Mol. Imaging* 34 (2007)
[10] W. Branderhorst et al., *Phys. Med. Biol.* 57 (2012)
[11] W. P. Segars et al., *Mol. Imaging Biol.* 6 (2004)
[12] B. Vastenhouw and F. Beekman, *J. Nucl. Med.* 48 (2007)

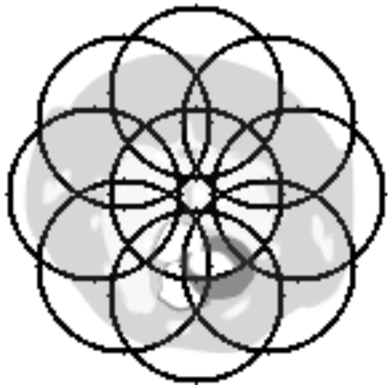
Optimization Results



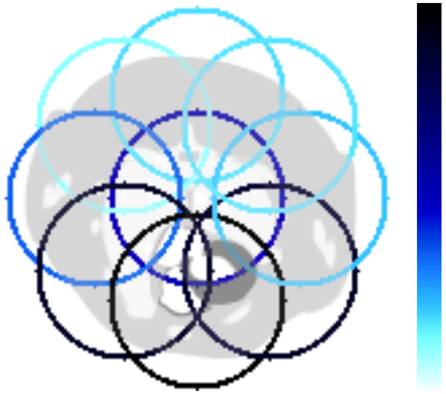
- POI in the heart
- Relative increase in CNR_j: **10%**
- ~ 5-10 minutes

9 Bed Positions

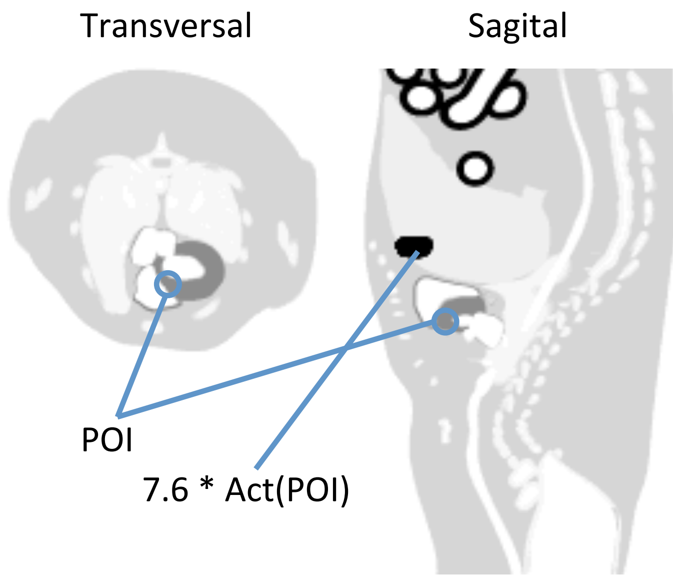
Uniform Time



Optimal Time

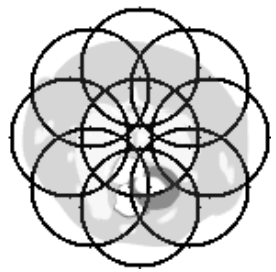


Validation of Image Quality calculation



- Same total time (45min)
- Uniform time per bed position
- Transversal bed shifts

CNR_j values (analytical)

Bed Positions / **Phantom**				
MOBY Phantom	3.4×10^{-3}	5.1×10^{-3}	8.9×10^{-3}	9.3×10^{-3}
MOBY Phantom Without extra-cardiac activity	5.5×10^{-3}	7.7×10^{-3}	13.5×10^{-3}	13.9×10^{-3}

[10] W. Branderhorst et al., *Phys. Med. Biol.* 57 (2012)

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Conclusion

- We presented an efficient method to:
  1. Evaluate Image Quality at a POI
  2. Optimize time per angle (Case 1)/bed position (Case 2)

- Optimization works in both cases studied, more impact in Case 1

- Case 1: analytical CNR values consistent with reconstructions

- Case 2: analytical CNR values for the full MOBY phantom seem to agree with literature[10], but further investigation is needed

[10] W. Branderhorst et al., *Phys. Med. Biol.* 57 (2012)

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