Wiener Estimation of Tumor Volume and Location in Nuclear Medicine

The process of selecting a system configuration for a particular imaging study is often guided by intuition and prior clinical experience. A quantitative approach uses task-based assessment of image quality to determine the relationship between system configuration and system performance.[1] When imaging is utilized for the purpose of estimating parameters of interest, the ensemble mean-squared error (EMSE) is a conventional choice for a figure of merit (FOM) to quantify system performance. The Wiener estimator minimizes the EMSE among all linear estimators. In this work, we perform extensive simulation studies that use the Wiener EMSE to rank order varying imaging-system configurations for the task of estimating parameters that describe a tumor model. Our simulation studies account for object randomness, sampling from a parameter ensemble, and detector noise. The SPECT system being evaluated is the multi-module, multi-resolution imaging system (M^3R). This system uses four pinhole plates, composed of a variety of pinhole patterns, which can be positioned at four different magnifications. Tomographic data are collected by rotating the object. For the current study, we consider four discrete object rotations for a total of 16 angular projections. Computation of the Wiener EMSE, given by

$$EMSE = tr\left[K_\theta\right] - tr\left[K_{\theta,g} K_g^{-1} K_{\theta,g}^t\right],$$

is a potential method for choosing among the various system configurations in order to maximize the performance of linear estimation.

In our current simulation study, the task is to estimate features about the location and volume of the tumor from the image data. Consider the quantities we wish to estimate to be the elements of a vector called θ. For the purposes of this simulation, we confine our treatment to signals that are related to θ by a parameterized tumor model

$$f^{tum}(r;\theta) = A\,\mathrm{sph}\left(\frac{r-c}{R}\right)$$

where A is the activity per unit volume, c is the three-dimensional location of the signal's center, and the signal's shape is defined by a boundary that is unity inside a sphere of radius R and zero outside. The parameter vector for this model is 5×1 and given by $\theta = (A, R, c_x, c_y, c_z)^t$.

Using the Wiener estimator EMSE as a FOM requires inverting the data covariance matrix and calculating the cross-covariance between the data and the parameters. To make these calculations more tractable, we choose prior probability density functions for the object ensemble that yield analytic forms for the cross-covariance. To further speed the calculation of EMSE among a set of candidate imaging configurations, we exploit the redundancy in this calculation. Given the assumption that detector saturation is not a consideration, SPECT imaging systems act as linear operators. Therefore, the above matrix expression for EMSE can be decomposed into a system-independent and a system-dependent component. The system-independent calculation involves the expectation of the parameters with respect to the object model. For system comparison, these expressions are projected into data space using the measured PSF of each aperture under evaluation.

Calculating and inverting the full data covariance is computationally prohibitive; the problems of data storage are only exacerbated for tomography studies. We sample from the data ensemble in order to estimate the data covariance and employ an exact decomposition that reduces the dimensionality of the matrix to be inverted. Currently, we are investigating the effect of sample size for the data covariance estimate on the resulting FOM.

[1] Harrison H. Barrett, Kyle J. Myers, Nicholas Devaney, and Christopher Dainty, Objective assessment of image quality: IV. Application to adaptive optics, JOSA A, Vol. 23, Issue 12, pp. 3080-3105.