Homework #5

OPTI 370

2/9/2022

(due date: 2/16/2022)

Problem 1:

Consider a Fabry-Perot resonator with a mirror separation of $d = 11.3\text{cm}$ and mirror reflectivities $\mathcal{R}_1 = 1,\ \mathcal{R}_2 = 0.954$. Assume the resonator is filled with a medium that has an absorption coefficient of $\alpha_s = 0.0182\text{cm}^{-1}$ and refractive index 1.47 (here assumed to be independent of frequency). Calculate the total roundtrip loss coefficient (in the book called "distributed loss coefficient"), finesse, mode spacing (free spectral range), and linewidth. Are the lines narrow in the sense that their width is much less than the mode spacing, in other words, is this a good resonator? How large is the ratio of the linewidth over the mode spacing? On which side of the resonator would light come out of the resonator?

(10 points)

Problem 2:

Consider electromagnetic waves in a sourceless homogeneous medium, described by the four Maxwell equations

$$\nabla \times \mathcal{H} = \frac{\partial \mathcal{D}}{\partial t}\ , \quad \nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t}\ , \quad \nabla \cdot \mathcal{D} = 0\ , \quad \nabla \cdot \mathcal{B} = 0$$

and the definitions of the various auxiliary fields given in class. Derive the resulting propagation equation

$$\nabla^2 \mathcal{E} - \frac{1}{c_0^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathcal{P}}{\partial t^2}$$

To obtain this equation, you should assume that the magnetization density $\mathcal{M}$ is zero, and that medium is isotropic, in which case the susceptibility χ is a scalar, and hence the polarization $\mathcal{P}$ is simply related by a scalar factor to the electric field, $\mathcal{P} = \varepsilon_0 \chi \mathcal{E}$. You can then use the fact that ∇ is a vector operator with the vector identity $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$, and the fact that under the stated assumptions, the electric field is divergenless.

(10 points)

Problem 3:
Solve Exercise 10.2-1 on p. 381 of the book.

(10 points)

Problem 4:
Revisiting our original definition of irradiance (intensity) in terms of the real-valued wave $u(\vec{r},t) = a\cos(\omega t - k z)$, show that for a monochromatic plane wave at $z = 0$, the time-average of $u^2(0,t)$ over one optical cycle, $\langle u^2 \rangle$, eliminates the fast (2ω) oscillation and yields, for the definition of I used in class, $I = a^2$.

(10 points)