

Homework #4
OPTI 370
2/2/2022
(due date: 2/9/2022)

Problem 1:

In the last homework assignment, you determined the Fourier transform of the function

$$f(x) = \text{rect}(x)$$

Now assume you want to analyze a pulse with rectangular shape and width $\sigma_t = 413 \mu\text{s}$. Using the same strategy shown in class, namely writing the argument of the rect function as $x = t / \sigma_t$, using the result from last week's homework or Tab. A.2-1 of the book, and using the definition of the spectral width of a sinc function given in class, determine the spectral width of the given pulse. How does the spectral width change if you double the pulse's duration σ_t ?

(10 points)

Problem 2:

Consider a monochromatic wave of frequency $\nu = 53 \text{ THz}$ and amplitude $a=0.98$ (units of $\text{W}^{1/2}/\text{cm}$) travelling in z-direction in a medium of length $d = 24.5\text{cm}$, characterized by the absorption coefficient $\alpha(\nu) = 0.079\text{cm}^{-1}$ and refractive index $n(\nu) = 1.24$. The wave is given by

$$u(z,t) = a \cos(\omega t - k z) e^{-\frac{1}{2}\alpha(\nu)z}$$

Write down the corresponding complex wave functions $U(z,t)$ (which includes the time dependence) and $U(z)$ (which does not include the time dependence). Determine the wavelength in vacuum and in the medium, as well as input and output intensity. Plot the intensity as function of z.

(10 points)

Problem 3:

Consider a monochromatic wave of frequency $\nu = 548 \text{ THz}$ and amplitude $a=0.46$ (units of $\text{W}^{1/2}/\text{cm}$) travelling in z-direction in a medium of length $d = 13.2\text{cm}$, characterized by the gain coefficient $\gamma(\nu) = 0.081\text{cm}^{-1}$ and refractive index $n(\nu) = 1.35$. The wave is given by

$$u(z,t) = a \cos(\omega t - k z) e^{\frac{1}{2}\gamma(\nu)z}$$

Write down the corresponding complex wave functions $U(z,t)$ (which includes the time dependence) and $U(z)$ (which does not include the time dependence). Determine the wavelength in vacuum and in the medium, as well as input and output intensity. Plot the intensity as function of z .

(10 points)

Problem 4:

Consider a monochromatic oscillation with frequency $\nu_0 = 597$ THz. Let the Fourier transform of $U(t)$ be $V(\nu) = (a + jb) \delta(\nu - \nu_0)$ with $a=9.6$ and $b=6.2$ (units of $W^{1/2}/cm$). Determine the complex amplitude $U(t)$, the real-valued oscillation $u(t)$, and plot the latter over at least two optical cycles. Also, determine the intensity of the oscillation. Does the intensity change with time?

(10 points)