Homework #3
OPTI 370
1/26/2022
(due date: 2/2/2022)

Problem 1:

The first two problems in this assignment will help you to understand how a planar mirror resonator (Fabry-Perot resonator) works.

Consider two monochromatic waves travelling in z-direction, one wave traveling forward, $u_1(z,t) = a\cos(\omega t - k\,z)$, the other backward, $u_2(z,t)$, both having the same amplitude. Determine the total real-valued wave

$$u(z,t) = u_1(z,t) + u_2(z,t)$$

proceeding in two different ways: (i) using only the real-valued wave functions for the forward and backward travelling waves together with the well-known cosine addition formulas, and (ii) using the complex wave functions for the forward and backward travelling waves and deducing from its sum the real-valued total wave. Do you get the same answer in both cases?

(10 points)

Problem 2:

Continuing problem 1, assume now that the wave is traveling in vacuum ($c_0 = 3\times 10^8$ m/s), its wavelength is $1\mu m$ and the amplitude a=26 (units of $W^{1/2}$/cm). Write the two waves from the previous problem assuming that the time is given in fs and z is given in μm. Sketch the wave $u(z,t)$ in the spatial interval ranging from z_a = -1/4 to z_b = +1/4 for the following times (all in the same figure): t_n = 0, 5/6, 5/3, 2.5, 10/3. What do you notice? Is this a travelling wave? What is its amplitude at z_a and z_b?

(10 points)

Problem 3:

The following problem helps you to understand the nature of wave packets and light pulses. In this problem, the wave packet is trivial: it contains only two frequencies. In general, however, it would contain infinitely many frequencies. That case is then dealt with the help of the Fourier transform, which will be looked at in the next problem.

Consider a non-monochromatic wave (in this case a superposition of just two monochromatic waves) travelling in positive z-direction

$$u(z,t) = a\cos(2\pi\nu_1 t - k_1\, z) + a\cos(2\pi\nu_2 t - k_2\, z)$$

For simplicity, assume that you are monitoring the oscillation of the wave at z=0. Assume a=7.5 (units of $W^{1/2}$/cm), $\nu_1 = 1$ PHz, $\nu_2 = 1.05$ PHz, and assume that the time is given in units of fs. Make two plots. First, plot the two monochromatic oscillations as function of time in the interval between 0 and the "out-of-phase" time t_π. Next, plot the total oscillation $u(0,t)$ together with an "envelope" over the time interval 0 to $3t_\pi$.

(10 points)

Problem 4:

Using the general formula for the forward and backward Fourier transform given in class (see Appendix A of the book), determine the Fourier transform of the function

$$f(x) = \mathrm{rect}(x)$$

by evaluating the Fourier integral. Don't just use the table to get the result. Give every intermediate step needed to evaluate the integral. Do not use any electronic media for help with the integral. Note that the argument of the rect function has to be dimensionless, therefore the variable x cannot be time. In many cases it will be a ratio of two times (for example time divided by pulse duration). For the purpose of this problem, simply use the dimensionless variable x without specifying how it is related to time t (we will look at that in another homework problem).

(10 points)