November 23, 2021

Homework 12
OPTI 507
(due November 30, 2021)

Problem 1:
This problem illustrates the mathematical approach to the quantum confined Stark effect. Consider an electron in a quantum well with infinite barriers ($-L_z/2 < z < L_z/2$) in an external dc electric field.
Show that the Schroedinger equation

$$\left\{-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial z^2} + eE_z z\right\}\zeta(z) = \varepsilon\zeta(z)$$

can be re-written as

$$\left\{\frac{\partial^2}{\partial \eta^2} + \eta\right\}\xi(\eta) = 0$$

Assume the force $F \equiv -eE_z$ to be positive.

(10 points)

Problem 2:
From Problem 1 it follows immediately that $\xi(\eta)$ is a superposition of Airy functions $Ai(-\eta)$ and $Bi(-\eta)$. Assume now, for simplicity, that the field-induced energy drop across the well is much larger than the lowest confinement energy $\varepsilon_1^{(0)}$ of the un-biased well. Assume, accordingly, that the lowest eigenvalue is deep in the triangular part of the potential. In this case, one can assume that the wavefunction is given solely by $Ai(-\eta)$. Using the boundary condition of vanishing wavefunction at $z = L_z/2$, show that the lowest eigenvalue is given by

$$\frac{\varepsilon_1}{\varepsilon_1^{(0)}} = \eta_1 \pi^{-2/3}\left(\frac{FL_z}{\varepsilon_1^{(0)}}\right)^{2/3} - \frac{1}{2}\frac{FL_z}{\varepsilon_1^{(0)}}$$

with $\eta_1 \approx 2.5$ being the first zero of the Airy function. Does this result indicate a field-induced shift towards higher or lower energies? Explain your answer.

(10 points)

Problem 3:
Consider a two-dimensional (2D) electron system with a 2-fold (for spin) degenerate parabolic and isotropic band with effective mass $m_e = 0.05m_0$ ($m_0 =$ electron mass in vacuum). Assume the carriers to occupy the band according to a Fermi function at zero temperature with a carrier density of 4×10^{12} cm^{-2} . Determine the magnitude of the Fermi wave vector and of the Fermi energy in units of eV. Instructions: Use direct k-integration (no credit will be given if you are using the concept of density of states to perform the k-integration) to derive the relevant formula.

(10 points)