Problem 1:
This problem illustrates the mathematical approach to the quantum confined Stark effect. Consider an electron in a quantum well with infinite barriers \((-L_z/2 < z < L_z/2)\) in an external dc electric field.
Show that the Schrödinger equation
\[
\left\{ -\frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial z^2} + eE_zz \right\} \zeta(z) = \varepsilon \zeta(z)
\]
can be re-written as
\[
\left\{ \frac{\partial^2}{\partial \eta^2} + \eta \right\} \xi(\eta) = 0
\]
Assume the force \( F \equiv -eE_z \) to be positive.
(10 points)

Problem 2:
From Problem 1 it follows immediately that \( \xi(\eta) \) is a superposition of Airy functions \( Ai(-\eta) \) and \( Bi(-\eta) \). Assume now, for simplicity, that the field-induced energy drop across the well is much larger than the lowest confinement energy \( \varepsilon_1^{(0)} \) of the un-biased well. Assume, accordingly, that the lowest eigenvalue is deep in the triangular part of the potential. In this case, one can assume that the wavefunction is given solely by \( Ai(-\eta) \). Using the boundary condition of vanishing wavefunction at \( z = L_z/2 \), show that the lowest eigenvalue is given by
\[
\frac{\varepsilon_1}{\varepsilon_1^{(0)}} = \eta_1 \pi^{-2/3} \left( \frac{FL_z}{\varepsilon_1^{(0)}} \right)^{2/3} - \frac{1}{2} \frac{FL_z}{\varepsilon_1^{(0)}}
\]
with \( \eta_1 \approx 2.5 \) being the first zero of the Airy function. Does this result indicate a field-induced shift towards higher or lower energies? Explain your answer.
(10 points)

Problem 3:
Consider a two-dimensional (2D) electron system with a 2-fold (for spin) degenerate parabolic and isotropic band with effective mass \( m_e = 0.05m_0 \) \( (m_0 = \text{electron mass in vacuum}) \). Assume the carriers to occupy the band according to a Fermi function at zero temperature with a carrier density of \( 4 \times 10^{12} \) \( \text{cm}^{-2} \). Determine the magnitude of the Fermi wave vector and of the Fermi energy in units of eV.
Instructions: Use direct k-integration (no credit will be given if you are using the concept of density of states to perform the k-integration) to derive the relevant formula.
(10 points)