October 28, 2021

Homework 9
OPTI 507
(due November 4, 2021)

Problem 1:
Starting from the following expression for the absorption coefficient of a 3-dimensional two-band semiconductor,

$$\alpha(\omega) = \frac{8\pi\hbar\omega}{n_b c}\left|d_{c,\mathrm{v}}(0)\right|^2 s_d \frac{1}{V}\sum_{\vec{k}} \frac{\pi}{2\hbar}\delta(\varepsilon_{cv}(k) - \hbar\omega)$$

where s_d is the spin degeneracy factor (which in "the book" is taken to be 2), derive an analytical expression for α. Here, we have assumed the transition to be first-class dipole allowed (i.e. the dipole matrix element is independent of k), and we assume the transition energy to be parabolic, i.e. $\varepsilon_{cv}(k) = \varepsilon_{c,k} - \varepsilon_{\mathrm{v},k} = \frac{\hbar^2 k^2}{2m_r} + E_g$ where $\frac{1}{m_r} = \frac{1}{m_e} + \frac{1}{m_h}$ is the inverse reduced mass. This expression for $\alpha(\omega)$ contains a factor that is the so-called joint density of states, $g_{\mathrm{cv}}(\hbar\omega) = \frac{1}{V}\sum_{\vec{k}} \delta(\varepsilon_{cv}(k) - \hbar\omega)$. Determine the joint DOS for the parabolic transition energies used in this problem through a simple comparison of the known DOS of parabolic bands by appropriately replacing parameters. You don't need to derive the expression for the DOS of a parabolic band.

(10 points)

Problem 2:
Sketch the result from Problem 1 for the case of GaAs, assuming $s_d = 2$, $r_{\mathrm{cv}} = 0.35$ nm , $m_e = 0.065\ m_0$, $m_h = 0.32\ m_0$ (m_0 = electron mass in vacuum), $n_b = 3.2$, $E_g = 1.46$ eV . The frequency axis should range from 1.30 eV to 1.7 eV . Use the fine structure constant in Gauss units, $\frac{e^2}{\hbar c} \approx \frac{1}{137}$, to eliminate the squared electron charge e^2 .

(10 points)

Problem 3:
Same as Problem 1, but now do <u>not</u> use the concept of density of states. Evaluate the k-sum directly.

(10 points)