October 21, 2021

Homework 8
OPTI 507
(due October 28, 2021)

Problem 1:

In our first discussion of phonons in class we considered the monatomic chain with nearest neighbor interaction, described by the restoring force *f*. Assume that, in addition to the nearest neighbor restoring force $f \equiv f_1$, you have an interaction between next nearest neighbors, f_2. Derive the dispersion relation $\Omega = \Omega_k$. Sketch the dispersion relation for two cases: $f_2 = 0.25 f_1$ and $f_2 = -0.25 f_1$. In which of the two cases are the acoustic long-wavelength modes softer or harder compared to the case of nearest neighbor interaction only? ("Soft modes" are those with a flat dispersion, and "hard modes" are those with a steep dispersion.)

Instruction: In the model of next nearest neighbor interaction, the restoring force acting on atom j contains the two terms proportional to the displacement differences between j and $j \pm 1$, and, similarly, two terms proportional to the displacement differences between j and $j \pm 2$.

(10 points)

Problem 2:

The term "restrahlen band" or "residual ray" band refers to the spectral filtering that can be achieved by reflecting a broad-band light beam several times from a resonant dielectric medium. Consider the case of a phonon dielectric function (for simplicity without damping),

$$\varepsilon(\omega) = 1 + \frac{\omega_{pl}^2}{\Omega_T^2 - \omega^2}$$

with $\hbar\Omega_T = 26\,\text{meV}$ $\quad \hbar\Omega_L = 35\text{meV}$, which is roughly appropriate as a model for the infrared optical response of CdS. Consider the frequency interval $0 \le \hbar\omega \le 50\text{meV}$ and an incoming light beam with intensity I_0 and with a spectrum that is flat over this interval. Define the once, twice, ... reflected intensity as I_1, I_2, ... Plot the spectrum of a beam that has been reflected 10 times, i.e. plot I_{10}. You may need some simple numerics to perform this task. Identify the pass-filter interval in your plot.

(10 points)

Problem 3:

The problem is meant to help your better understand the upcoming discussion of optical response of solids. Starting with the Schroedinger equation for a fixed $\vec{k}$, $i\hbar \frac{\partial}{\partial t} |\psi_{\vec{k}}(t)\rangle = [H_0 + H'(t)]|\psi_{\vec{k}}(t)\rangle$, and using the expansion of the time-dependent wave function in terms of eigenfunctions of H_0, derive an analytical expression for $a_n^{(1)}(t)$ (include all steps, not just the ones given in the Class Notes).

In class, we use the expression for $a_n^{(1)}(t)$ and derive the probability to find the system in state *n* at time *t*, given that it was in state *l* at time *t=0*, but this is not part of the homework problem.

(10 points)