Problem 1:
Based on the Drude model including a phenomenological damping term, the dielectric function for a metal is given by
\[
\epsilon(\omega) = \epsilon_\infty - \frac{\omega_{pl}^2 \tau}{\omega \tau + i},
\]
where \(\omega_{pl}\) is the plasma frequency of the metal. Consider the “penetration depth” (sometimes also called “skin depth”) defined in terms of the absorption coefficient \(\alpha\) as \(d = \frac{2}{\alpha}\) in the long-wavelength limit. Specifically, express \(d\) in terms of \(\omega\), the light velocity \(c\), and the dc conductivity \(\sigma_0 = \frac{\omega_{pl}^2 \tau}{4\pi}\).

The long-wavelength assumption should be, in mathematical terms,
\[\omega \ll \tau^{-1} \ll \omega_{pl}.
\]
The high-frequency dielectric constant \(\epsilon_\infty\) is supposed to be real and of order unity. You have to show first that, in this limit, \(|\epsilon''| \gg |\epsilon'|\).

In silver (Ag) the plasma frequency is approximately \(\omega_{pl} = 1.5 \times 10^{16} s^{-1}\). Assuming \(\tau\) to be 75 fs determine the ratio \(\omega/\sigma_0\) for \(\omega = 0.8 \times 10^8 s^{-1}\) and estimate \(d\).

(10 points)

Problem 2:
In the Lorentz oscillator model, the retarded susceptibility (subscript R) as a function of time is given as
\[
\chi_R(t - t') = \frac{N e^2}{m} \frac{1}{2\omega_0^2} \theta(t - t') e^{-\gamma(t-t')} 2 \sin (\omega_0(t - t'))
\]
Prove that this expression for \(\chi(t - t')\) is correct. Instructions: Use \(\chi(t - t')\) and show that the corresponding \(P(t)\) and its time derivatives obey the correct second-order differential equation.

(10 points)
Problem 3:
Using the complex refractive index defined as
\[ \tilde{n} \equiv n + i\kappa = \frac{c}{\omega}(k' + ik'') \]
where \( n \) is the refractive index and \( \kappa \) the extinction coefficient, the normal incidence reflectivity can be written as
\[ R = \frac{|1 - \tilde{n}|^2}{|1 + \tilde{n}|} \]
Determine \( R \) for ZnTe at \( \lambda = 0.3 \mu m \), and for the perovskite \( \text{CH}_3\text{NH}_3\text{PbI}_3 \) at \( \lambda = 1.5 \mu m \), using the website refractiveindex.info. Specify the 'path' you used in refractiveindex.info and the reference (for example ‘Adachi et al. 1995’) to get your information.

(10 points)