September 23, 2021

Homework 5
OPTI 507
(due September 30, 2021)

Problem 1:
Based on the Drude model including a phenomenological damping term, the dielectric function for a metal is given by

$$\epsilon(\omega) = \epsilon_\infty - \frac{(\omega_{pl}^2 \tau)/\omega}{\omega\tau + i} \ ,$$

where ω_{pl} is the plasma frequency of the metal. Consider the "penetration depth" (sometimes also called "skin depth") defined in terms of the absorption coefficient α as $d = 2/\alpha$ in the long-wavelength limit. Specifically, express d in terms of ω, the light velocity c, and the dc conductivity

$$\sigma_0 = \frac{\omega_{pl}^2 \tau}{4\pi}.$$

The long-wavelength assumption should be, in mathematical terms,

$$\omega \ll \tau^{-1} \ll \omega_{pl}.$$

The high-frequency dielectric constant ϵ_∞ is supposed to be real and of order unity. You have to show first that, in this limit, $|\epsilon''| \gg |\epsilon'|$.

In silver (Ag) the plasma frequency is approximately $\omega_{pl} = 1.5 \times 10^{16} s^{-1}$. Assuming τ to be 75 fs determine the ratio ω/σ_0 for $\omega = 0.8 \times 10^8 s^{-1}$ and estimate d.

(10 points)

Problem 2:
In the Lorentz oscillator model, the retarded susceptibility (subscript R) as a function of time is given as

$$\chi_R(t - t') = \frac{Ne^2}{m} \frac{1}{2\omega_0'} \ \theta(t - t') e^{-\gamma(t-t')} \ 2 \ \sin\left(\omega_0'(t - t')\right)$$

Prove that this expression for $\chi(t - t')$ is correct. Instructions: Use $\chi(t - t')$ and show that the corresponding $P(t)$ and its time derivatives obey the correct second-order differential equation.

(10 points)

Problem 3:
Using the complex refractive index defined as

$$\tilde{n} \equiv n + i\kappa = \frac{c}{\omega}(k' + ik'')$$

where n is the refractive index and κ the extinction coefficient, the normal incidence reflectivity can be written as

$$R = \left|\frac{1-\tilde{n}}{1+\tilde{n}}\right|^2$$

Determine R for ZnTe at $\lambda = 0.3\mu$m, and for the perovskite $CH_3NH_3PbI_3$ at $\lambda = 1.5\mu$m, using the website refractiveindex.info. Specify the 'path' you used in refractiveindex.info and the reference (for example 'Adachi et al. 1995') to get your information.

(10 points)