

September 16, 2021

HOMEWORK 4
OPTI 507
(due September 23, 2021)

Problem 1:

Using Fourier transformation, derive the dispersion relation of light in a temporally and spatially homogeneous (i.e. shift-invariant) medium, starting from the propagation equation

$$\left\{ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right\} \vec{E}(\vec{r}, t) = -\frac{4\pi}{c^2} \frac{\partial^2}{\partial t^2} \vec{P}(\vec{r}, t) \quad (1)$$

Present all necessary intermediate steps when performing the Fourier transform.

(10 points)

Problem 2:

Consider a medium characterized by a frequency-dependent and k -independent dielectric function $\varepsilon(\omega)$. For simplicity, assume in this and the following problem ω , $k(\omega)$ and $\varepsilon(\omega)$ to be real-valued.

(a) Write down a general expression for the phase (v_p) and group (v_g) velocity in the medium. Illustrate the difference with a plot in the k - ω plane (for the illustration, just assume some arbitrary dispersion relation $\omega(k)$). Name one example of a medium in which $v_p = v_g$ and one example of a medium in which $v_p \neq v_g$.

(b) Show that

$$\frac{v_g}{c} = \frac{1}{n + \omega \frac{\partial n}{\partial \omega}} \quad (2)$$

Here, n is the (real-valued) refractive index and c the speed of light in vacuum.

(10 points)

Problem 3:

Assume you have a medium with the dielectric function

$$\epsilon(\omega) = 1 - \frac{0.1}{\tilde{\omega} - 1 + i0.2} \quad (3)$$

where $\tilde{\omega} = \omega/\omega_0$, and the term that is off-resonant for positive frequencies has been neglected for simplicity.

Using the formula for the refractive index in terms of the dielectric function given in class (the one that has multiple square roots) and the formula derived in Problem 2, numerically evaluate and plot the refractive index (indicate the region of normal and anomalous dispersion), its frequency derivative, and the group velocity in units of c for $\tilde{\omega}$ from zero to 2. For the group velocity plot, use a vertical axis that shows v_g/c between -2 and $+2$. Indicate the regions where v_g is less than or greater than c , where it is negative, and where it is $\pm\infty$.

(10 points)