

September 30, 2021

Homework 6
OPTI 507
(due October 7, 2021)

Problem 1:

In class, we discussed the dispersion of the polariton in a bulk dielectric system using the Lorentz oscillator model (for simplicity without damping). Show that the curves of the lower (LPB) and upper (UPB) polariton branch have indeed the shape and asymptotic values presented in the figure in class. In order to do that, follow the discussion from class and identify the stop band (i.e. the spectral region in which no real solutions exist) and the asymptotic behavior (including the slopes) of the LPB and UPB in the long and short wavelength limits.

(10 points)

Problem 2:

Consider again a polariton in a bulk dielectric system. Use the following parameter values: $\epsilon_0 = 8.6$, $\epsilon_\infty = 5.3$, $\hbar\omega_T = 2.3\text{eV}$. Assume you want to create a wave with a wavelength of 28 nm. Determine the corresponding frequency (in units of eV) on the LPB. Compare this to the frequency of an electromagnetic wave in vacuum with a wavelength of 28 nm. Specify the colors (if appropriate) of the two waves (inside the medium and in vacuum). Finally, determine the wavelength of light in vacuum with frequency $\hbar\omega = 2.3\text{eV}$ and briefly discuss potential benefits of the polariton system.

(10 points)

Problem 3:

Consider a surface plasmon polariton at a metal surface in vacuum within the Drude model without damping and with $\epsilon_\infty = 1$. Starting with the implicit form of the dispersion relation, $c^2 k_x^2 = \omega^2 \epsilon_1(\omega) \epsilon_2(\omega) / (\epsilon_1(\omega) + \epsilon_2(\omega))$, derive the explicit dispersion relation $\omega(k_x)$ that we discussed in class. Also, show that for small k_x , the dispersion is linear with $\omega \cong c k_x$ (it is sufficient and easier to consider the expression for ω^2 and then to show that $\omega^2 \cong c^2 k_x^2$).

(10 points)