Problem 1:
Starting from the following expression for the absorption coefficient of a two-band semiconductor,
\[
\alpha(\omega) = \frac{8\pi\hbar \omega}{n_p c} \left| d_{c,v}(0) \right|^2 s_d \frac{1}{V} \sum_k \frac{\pi}{2\hbar} \delta(\epsilon_{c,v}(k) - \hbar \omega)
\]
where \( s_d \) is the spin degeneracy factor (which in "the book" is taken to be 2), derive an analytical expression for \( \alpha \). Here, we have assumed the transition to be first-class dipole allowed (i.e. the dipole matrix element is independent of \( k \)), and we assume the transition energy to be parabolic, i.e.
\[
\epsilon_{c,v}(k) = \epsilon_{c,k} - \epsilon_{v,k} = \frac{\hbar^2 k^2}{2m_e} + E_g \quad \text{where} \quad \frac{1}{m_r} = \frac{1}{m_e} + \frac{1}{m_h}
\]
the inverse reduced mass. This expression for \( \alpha(\omega) \) contains a factor that is the so-called joint density of states, \( g_{c,v}(\hbar \omega) = \frac{1}{V} \sum_k \delta(\epsilon_{c,v}(k) - \hbar \omega) \). Determine the joint DOS for the parabolic transition energies used in this problem through a simple comparison of the known DOS of parabolic bands by appropriately replacing parameters. You don't need to derive the expression for the DOS of a parabolic band.

(10 points)

Problem 2:
Sketch the result from Problem 1 for the case of GaAs, assuming \( s_d = 2 \), \( s_v = 0.34 \text{ nm} \), \( m_e = 0.063 \text{ m}_0 \), \( m_h = 0.31 \text{ m}_0 \) (\( \text{m}_0 \) = electron mass in vacuum), \( n_p = 3.1 \), \( E_g = 1.47 \text{ eV} \). The frequency axis should range from 1.30 eV to 1.7 eV. Use the fine structure constant in Gauss units, \( \frac{e^2}{\hbar c} = \frac{1}{137} \), to eliminate the squared electron charge \( e^2 \).

(10 points)

Problem 3:
Same as Problem 1, but now do not use the concept of density of states. Evaluate the \( k \)-sum directly.

(10 points)