October 31, 2019

Homework 9
OPTI 507
(due November 7, 2019)

**Problem 1:**
Starting from the following expression for the absorption coefficient of a two-band semiconductor,

\[ \alpha(\omega) = \frac{8\pi\hbar\omega}{n_b c} s_d \frac{1}{V} \left| d_{ck,\nu k} \right|^2 \frac{\pi}{2\hbar} \delta(\epsilon_{cv}(k) - \hbar \omega) \]

where \( s_d \) is the spin degeneracy factor (which in "the book" is taken to be 2), derive an analytical expression for \( \alpha \). Assume the transition to be first-class dipole allowed (i.e. the dipole matrix element is independent of \( k \)), and the transition energy to be parabolic, i.e. \( \epsilon_{cv}(k) = \epsilon_{c,k} - \epsilon_{v,k} = \frac{\hbar^2 k^2}{2m_r} + E_g \) where \( \frac{1}{m_r} = \frac{1}{m_e} + \frac{1}{m_h} \) is the inverse reduced mass.

First, write down an expression for \( \alpha(\omega) \) as a product where one factor is the so-called joint density of states, \( g_{cv}(\hbar \omega) = \frac{1}{V} \sum_k \delta(\epsilon_{cv}(k) - \hbar \omega) \). Determine the joint DOS for the parabolic transition energies used in this problem through a simple comparison of the known DOS of parabolic bands by appropriately replacing parameters. You don't need to derive the expression for the DOS of a parabolic band.

(10 points)

**Problem 2:**
Sketch the result from Problem 1 for the case of GaAs, assuming \( s_d = 2 \), \( r_{cv} = 0.31 \text{ nm} \), \( m_e = 0.062 \text{ } m_b \), \( m_h = 0.32 \text{ } m_b \) (\( m_b = \text{ electron mass in vacuum} \)), \( n_b = 3.3 \), \( E_g = 1.50 \text{ eV} \).

The frequency axis should range from 1.35 eV to 1.8 eV. Use the fine structure constant in Gauss units, \( e^2 / \hbar c = \frac{1}{137} \), to eliminate the squared electron charge \( e^2 \).

(10 points)

**Problem 3:**
Same as Problem 1, but now do **not** use the concept of density of states. Evaluate the \( k \)-sum directly.

(10 points)