SIGNAL PROCESSING IN THE PRESENCE OF SIGNAL-DEPENDENT NOISE

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SIGNAL PROCESSING IN THE PRESENCE OF SIGNAL-DEPENDENT NOISE

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I hereby recommend that this dissertation prepared under my direction by John Gary Thunen entitled Signal Processing in the Presence of Signal-Dependent Noise be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy

Roland V. Shack — 3 JUNE 1970
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After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

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SIGNED: John Thomson

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ABSTRACT

The significance of signal-dependent noise is discussed in general. Particular emphasis is placed on the specific type of multiplicative noise which is present in the density variations in a photographic emulsion.

A theoretical treatment of the effect of multiplicative noise on signal detection and signal discrimination problems is presented. It is found that for the detection of a known signal in the presence of multiplicative Gaussian noise, the optimum processing of a sampled message is obtained by generating the test statistic given by

$$\Lambda = \sum_{i=1}^{N} s_i^2 \cdot \frac{u_i - u_o}{u_i}$$

Where the known signal is described by the N values $\{u_i\}$, the sampled message is described by $\{s_i\}$, and u_o is the background level when no signal is present. When the multiplicative noise is described by Poisson statistics, the optimum test statistic is found to be

$$\Lambda = \sum_{i=1}^{N} s_i \cdot \log_e(u_i/u_o)$$

When discriminating between two signals, $\{s_{1i}\}$ and $\{s_{2i}\}$, the optimum test statistics become

$$\Lambda = \sum_{i=1}^{N} s_i^2 \cdot \left(\frac{1}{u_{2i}} - \frac{1}{u_{1i}}\right)$$

for multiplicative, Gaussian statistics and

$$\Lambda = \sum_{i=1}^{N} s_i \cdot \log_e(u_{1i}/u_{2i})$$

for Poisson statistics.

An investigation of the limitations of these theoretical models is presented. Two-dimensional signal fields in the presence of multiplicative noise are simulated in a computer and processed for optimum signal detection according to the two derived methods. These results are compared to the results of processing according to the assumption of stationary noise statistics. This comparison reveals that modest improvements (20-30%) in the detection rate are obtained when the signal-dependent nature of the noise statistics are considered. The effects of signal-to-noise ratio, signal structure, and changing background level are also investigated.

An example of optimum signal discrimination using circles and squares as signals in multiplicative noise is reported. An improvement in the percentage of correctly identified signals is again observed when the proper test statistic is used.

A coherent optical processing model that can be used to conceptualize the spectral characteristics of a message containing noise described by any arbitrary form of signal dependence is proposed.

Finally, two examples of signal filtering in the presence of signal-dependent noise are included. The first concerns the processing of a real star field to determine the location of weak stars. The second is an illustration of the signal information contained in the noise spectrum of a message recorded on a common photographic film.

CHAPTER 1

INTRODUCTION

The aim of science, in at least one sense, is the understanding of the nature of the real world based on simplified, artificial models. The laws and relationships that evolve from these idealized models are approximations. For most real world processes, the simpler models must be altered slightly or supplemented as the precision of the measurements on the process is improved. Frequently, there is a trade-off between the utility of these simplified models and the accuracy of more complex ones.

While this philosophical picture is, itself, an elementary model of the nature of science, it is a useful one for appreciating the significance of research on signal-dependent noise. Consider the results of a measurement on some physical system. Define these results as a message that contains some signal, which is of interest, and some noise, which is not. (This approach is more fully explained in Chapter 2.) When incorporating this message into some model, the easiest approach is to ignore the presence of the noise altogether. While this approximation can be justified for most messages there are, of course, many processes that require an accounting of the noise. The next level of complexity is to assume that noise is an independent factor and can be added to the model accordingly. Again, considering the simplicity of the approximation, this assumption is remarkably satisfactory in explaining most cases involving noise.

As expected, however, there are still many physical processes for which the concept of independent noise is inadequate. In most cases, one must decide whether the benefits of a more accurate model can justify the additional complication of letting the noise depend in some way on the signal. In practice, only a few isolated problems have been treated using noise that is signal dependent.[1-4]

The motivation for the study presented here stems from a real problem in which it was decided that the independent noise assumption was inadequate. The problem involved the detection of faint star images recorded on a photographic plate. It was soon discovered that very little information is available concerning the practical problems involved in applying the principles of detection theory to two dimensional signals recorded on photographic film. Furthermore, the existing techniques for optimum signal detection are based on signals recorded in the presence of additive, signal-independent, stationary noise. Unfortunately, the noise statistics for photographic film do not obey this model. For these reasons, a study was begun with the three-fold intent of : (1) developing a better understanding of the significance of signal-dependent noise, (2) deriving statistical tests for the optimum detection of signals recorded on photographic film, and (3) exploring the practical limitations of these tests.

One of the difficulties in working with signal-dependent noise is the lack of appropriate mathematical tools. A result of this is that solutions tend to be highly specialized. Parameters, such as the type of noise distribution, the nature of the signal, and the type of message processing desired, all affect the validity and usefulness of the solution. Consequently, a decision was made to study the general tools and techniques appropriate to an entire class of problems, rather than to concentrate on a specific system to obtain results which are of little or no value in a slightly different application. This approach could best be realized by simulating two-dimensional signal detection problems on a computer. Using this method, it was possible to answer the relevant questions without introducing additional, extraneous parameters.

A typical problem to which the tools developed in this study might be applied is that of data storage on photographic film. Techniques for recording signals on film and retrieving the unprocessed message by sampling the resulting film density are well known. The performance of a system of this type is usually limited by one's ability to retrieve the original signal from the sampled, noisy message. Assuming that the noise statistics of the recording medium are known, questions which must be answered to determine the performance of any proposed system include the following. (1) What is the optimum test statistic to be used in determining the presence of a signal (code symbol)? (2) What is the effect of the signal structure and size on its detectability? (4) What is the significance of a change in the noise statistics? (5) What is the loss in assuming additive, signal-independent noise statistics? And, (6) What is the effect of adding a constant background level to the film? These are among the questions this study is attempting to answer.

CHAPTER 2

SOME PROPERTIES OF SIGNAL-DEPENDENT NOISE

The purpose of this chapter is not to present a comprehensive study of multiplicative noise but simply to review a few of the concepts and definitions which will be helpful in following the later theoretical developments in this report.

Signal-Independent Noise Versus Signal-Dependent Noise

Definition of Noise

In general, noise is only defined in the context of an experiment. Consider, for example, a recording of density variations as a function of location on a grainy photographic emulsion. If the desired measurement is an estimate of the mean background density, then the graininess is a source of noise. If one is studying the properties of film granularity, however, then the variations in the mean background level become the noise source.

To avoid this ambiguity, it is necessary to exercise some care in making a generalized definition of noise. If one begins by hypothesizing the existence of a signal--even though its functional form may be unknown--then the noise can be defined as that which alters the measurement of the hypothesized signal. Thus, for a simple one dimensional message function, $s(x)$, and a hypothetical signal, $u(x)$, it is always possible to write

$$s(x) = u(x) + n(x)$$

where n(x) is the noise.

Although n(x) is typically a random process, it is also possible to have n(x) deterministic. In either case, the distribution of values of n(x) can be described by some joint probability density function, $p_n(\bar{n})$, where $\bar{n} = (n(x_1),\ n(x\ ),\text{--}n(x_N)\ )$ is a complete set of N sample readings of n(x). In the case of a deterministic n(x), $p_n(\bar{n})$ is simply a multidimensional delta function. It should also be noted that $p_n(\bar{n})$ may depend in some way on the presence of the signal s(x).

Signal Independent Noise

The preceding definition of noise may seem awkward. This, of course, is due to its generality. Fortunately, there are several simplifying conditions that can frequently be applied to this model.

A fundamental assumption which will be used throughout this study is that of <u>independent noise samples</u>. This condition states that none of the sampled noise values, $n(x_i)$, depend on the values of any other samples, $\{n(x_j)\}$. This condition permits the joint probability density function for n(x) to be written as the product of the probability density functions of the individual noise samples, for example,

$$p_n(\bar{n}) = \prod_{i=1}^{N} p_{n_i}(n(x_i)).$$

The physical origins of this condition will be discussed at greater length in the section on Spectral Characteristics in this chapter.

A second major assumption is that of stationarity. A random process is said to be stationary if the statistics of the process are unaffected by a translation of the origin. In the case of $n(x)$, which is assumed to consist of independent samples, stationarity simply means that

$$p_{n_i}(n(x_i)) = p_{n_j}(n(x_j)) \quad \text{all } i, j.$$

Therefore, the joint probability distribution can be written as

$$p_n(\overline{n}) = [p_{n_i}(n_i)]^N$$

where N is the number of sample points of $n(x)$.

Ergodicity is another fundamental assumption which is of particular importance in the measurement, or estimation, of distribution functions. If a function is ergodic its statistics can be determined from a single, infinitely long, sample whereas averages over an ensemble of samples are normally required. Although this property is infrequently required in this study it will be assumed true for all stationary processes unless otherwise stated.

The conditions of stationarity and ergodicity of a noise source are extremely powerful and are usually assumed to be true.

An additional property of noise is that of signal dependence. If the noise statistics are unaffected by the presence (or absence) of the signal then the noise is said to be signal independent. This property is necessary, but not sufficient, to permit the simplifications made for stationary and ergodic processes.

Signal Dependent Noise

The issue of signal dependence is the crux of this study. Although the rewards gained in terms of a simpler noise model are good incentives for making the signal independent noise assumption, it is not unreasonable to expect that any system limited by noise originating at the signal source will have some signal dependence in the noise. The seriousness of this approximation depends not only on the nature of the dependence but on the use to which the model will be applied. For example, most phototubes have relative constant noise properties over low or moderate light levels, while for very high light levels, the detector noise increases with increased light. Fortunately, this is of justifiably little concern since at these high light levels the noise characteristics can usually be ignored. For other detectors such as photomultipliers and photographic emulsion this problem is not so easily dismissed.

The treatment of signal dependent noise is very difficult since it is neither stationary nor ergodic. In the general case, one knows only that

$$s(x) = u(x) + n(x)$$

and that

$$n = n(x, s(x))$$

There are, of course, many possible functional forms of $n(x, s(x))$. For example, a common type of dependence is

$$n(x) = [u(x)]^{p} \cdot n'(x) \qquad u(x) > 0$$

where p is some real number and $n'(x)$ is a noise process which does not depend on $u(x)$. A noise process of this form is frequently referred to as multiplicative noise. For the remainder of this report, however, the term multiplicative noise will refer specifically to the case where $p = \frac{1}{2}$. That is, a signal will be said to be in the presence of multiplicative noise if the measured message, $s(x)$, can be expressed as

$$s(x) = u(x) + [u(x)]^{\frac{1}{2}} \cdot n(x)$$

where $n(x)$ is now a stationary, signal independent random process. The motivation for concentrating on this particular relationship will be discussed further in the section Physical Origins in this chapter.

Two important, frequently used properties of this type of multiplicative noise are (1) the mean, or expected value, of $s(x)$ is given by

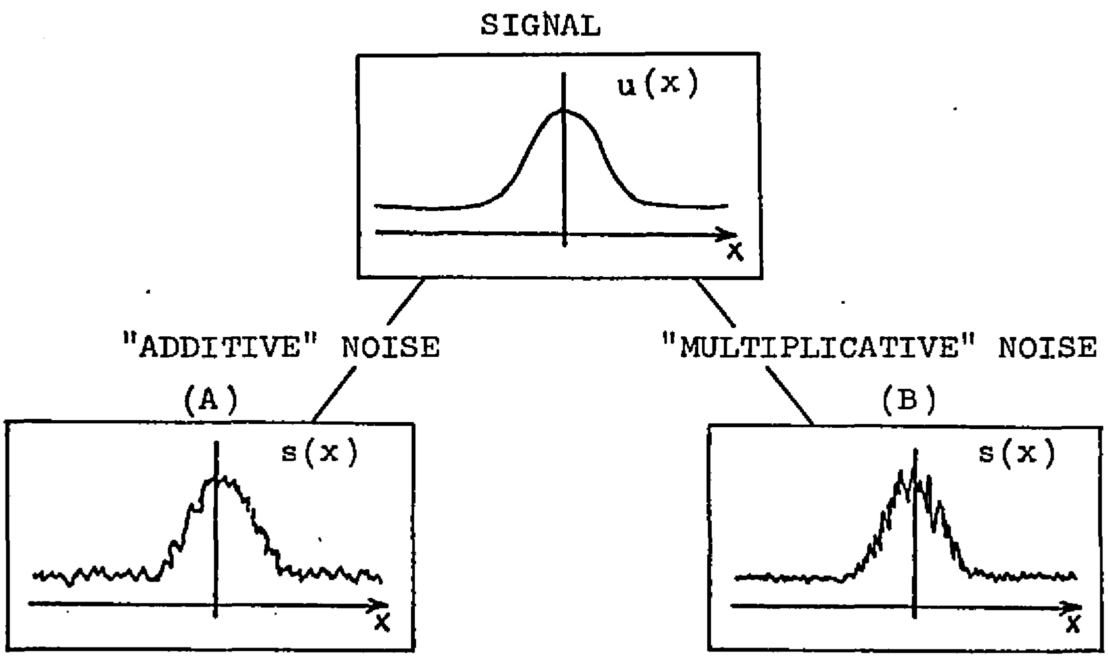
$$E\{s(x)\} = u(x) + [u(x)]^{\frac{1}{2}} \cdot E\{n(x)\}$$

and (2) the variance, σ_f^2, is given by

$$\sigma_f^2 = k \cdot u(x)$$

where k is a proportionality constant determined by the noise source.

The term "additive noise" will be used here as shorthand for stationary, signal independent noise. The use of the descriptors additive and multiplicative can be misleading but they are commonly used in practice and will be retained here. To help appreciate the qualitative difference between additive and multiplicative noise, Fig. 1 shows a gaussian signal in the presence of the two different noise types. In (A) the variance of the noise is constant for all x whereas in (B) the variance increases as u(x) increases. In both cases, the mean value of the noise is zero.



$f(x) = s(x) + n(x)$ $\quad$ $f(x) = s(x) + [s(x)]^{1/2} \cdot n(x)$

Fig. 1. Comparison of Additive and Multiplicative Noise.

Probability Distributions

Common Single-Variable Distributions

So far, the form of the probability density function p_n has been left arbitrary. Although there are an infinite number of possible distributions, only a few are commonly used in practice. The three distributions which will be used in this study are among the most common.[5,6] They are the Gaussian (or normal), the Poisson, and the log normal distributions.

The Gaussian distribution is the most common. For a signal given by $s(x) = u(x) + n(x)$ the signal is said to obey Gaussian statistics at the point x_o if

$$p_s(s(x_o)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\tfrac{1}{2}\left(s(x_o) - u(x_o)\right)^2/\sigma^2\right] \quad .$$

It should be noted that for the processes being considered in this study, it will be necessary to restrict this distribution to positive values of $s(x_o)$. The significance of this fundamental limitation is discussed in Chapter 5, Conclusions.

The Poisson distribution is given by

$$p_s(s(x_o)) = \exp\left(-\alpha u(x_o)\right) \left[\frac{\alpha u(x_o)}{s(x_o)!}\right]^{s(x_o)}$$

where α is an arbitrary proportionality constant. This distribution frequently occurs in many natural processes and is particularly important in photomultiplier and photographic film noise.

The third distribution of interest in this study is the log normal distribution. This is given by

$$p_s(s(x_o)) \propto \exp[p_G(s(x_o))]$$

where $p_G(s(x_o))$ is a Gaussian distribution. Detailed studies of these statistical forms are available in many texts.

Fig. 2 shows a comparison of these three functions along with plots of typical messages ($s(x) = u_o + n(x)$) with statistics determined by each of the three distributions.

It is of interest to note that in the limit as $u_o \to \infty$, all three distributions become identical.

It should also be pointed out that the Poisson and log normal distribution are inherently non-stationary. Only in the Gaussian case is it possible to write a distribution function for the noise contribution which does not change as $u(x)$ changes. In particular, the noise variance, σ^2, is a function of u_o, for the Poisson and log normal distributions.

Multi-Variable Distributions

When dealing with signal detection problems, one must know not only the distribution of noise values at a single point, but also the joint probability density function for the entire signal array. If the signal consists of k discrete points labeled s_1, s_2 --- s_k, ($s_1 = s(x_1)$, $s_2 = s(x_2)$, etc.), and if the noise values at each of

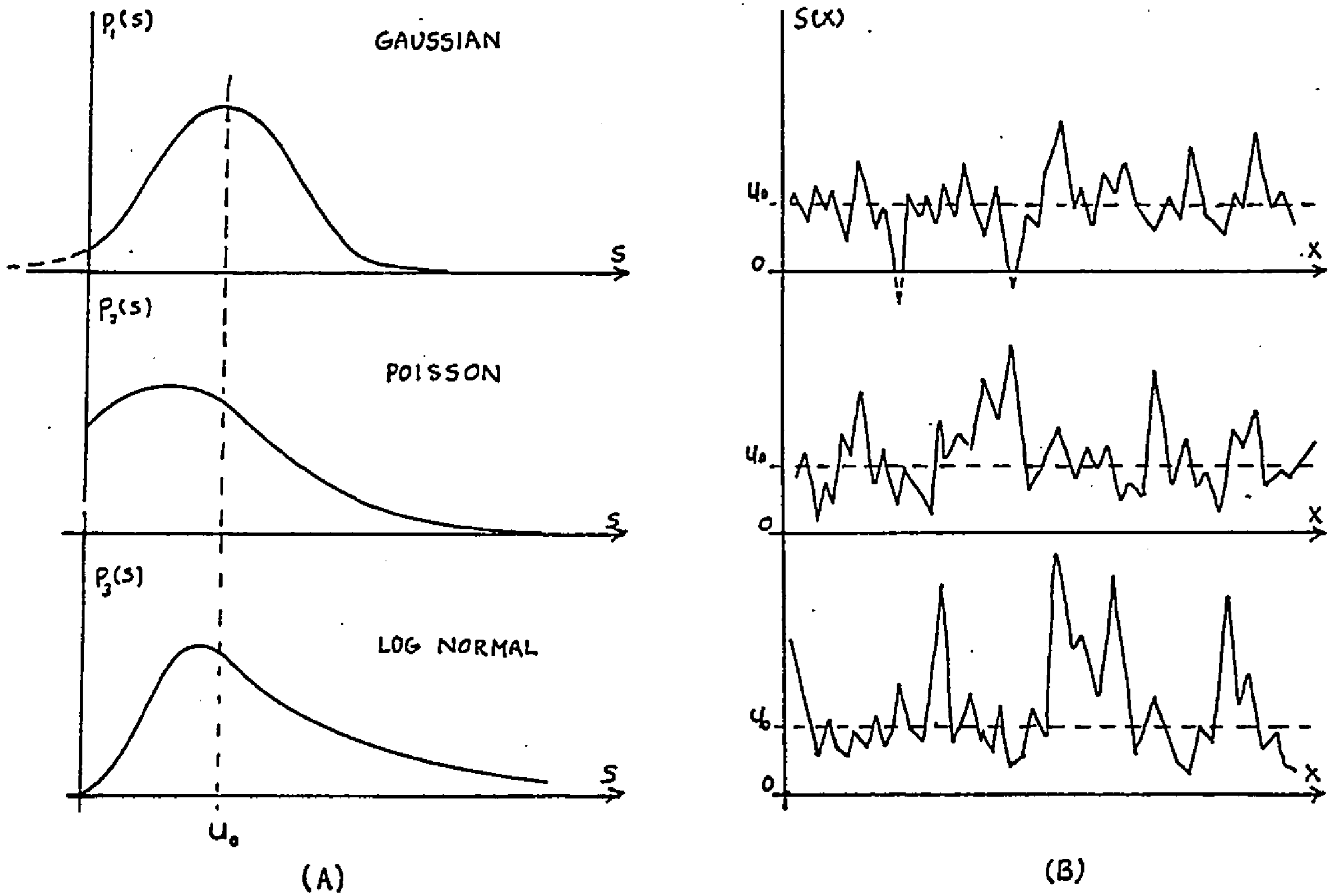


Fig. 2. Comparison of Gaussian, Poisson and Log Normal Noise Distributions.

(A) Compares probability distributions for variable with mean and standard deviation given by u_0. (B) illustrates typical message samples.

these points are statistically independent, then it is possible to express the joint probability density function $p_{\bar{s}}(s_1, s_2 --- s_k)$ as the product of the individual functions $p_{s_i}(s_i)$.

In the case of a stationary process, the Gaussian distribution for a k dimensional signal becomes simply

$$p_{\bar{s}}(\bar{s}) = \prod_{i=]}^{k} \{(\frac{1}{2\pi\sigma^2})^{\frac{1}{2}} \exp[-\tfrac{1}{2}(s_i - u_i)^2/\sigma^2]\}$$

$$= (\frac{1}{2\pi\sigma^2})^{k/2} \exp[\frac{-1}{2\sigma^2}\sum_{i=1}^{k}(s_i - u_i)^2]$$

In the case of a signal in the presence of noise which is signal dependent, the Gaussian model becomes

$$P_{\bar{s}}(\bar{s}) = \prod_{i=1}^{k}(\frac{1}{2\pi\sigma_i^2})^{\frac{1}{2}} \cdot \exp[-\tfrac{1}{2}\sum_{i=1}^{k}(s_i - u_i)^2/\sigma_i^2]$$

Similarly, the Poisson distribution becomes

$$p_{\bar{s}}(\bar{s}) = \prod_{i=1}^{k}\frac{e^{-\alpha u_i}[\alpha u_i]^{s_i}}{s_i!}$$

These expressions will be used to derive optimum signal detection techniques in Chapter 3.

Spectral Characteristics

White Noise

The frequency spectrum is an important tool in the study of signal processing. The spectral analysis of deterministic signals is extremely fundamental and is presented in many elementary textbooks. When noise (statistical uncertainty) is present, it is of interest to examine the power spectral density. Qualitatively, this is a measure of the noise power density per unit frequency interval. For a real stationary random process $s(x)$, the power spectral density $P_s(f)$ is given by

$$P_s(f) = \int_{-\infty}^{\infty} R_{ss}(x) e^{-j2\pi fx} dx$$

where $R_{ss}(x)$ is the ensemble autocorrelation function, i.e.,

$$\begin{aligned} R_{ss}(x) &= E\{s(x')s(x' + x)\} \\ &= \int_{-\infty}^{\infty} s(x')s(x' + x)\, p_{ss}(s(x'), s(x' + x)) \,.\, d(s(x'))d(s(x' * x)) \end{aligned}$$

The term "white noise" can now be defined as any noise process which has a constant $P_s(f)$ for all f of interest. Or, a process for which the autocorrelation function is effectively a delta function. It should be noted that the power spectral density function $P_s(f)$ also contains the signal power spectrum as an ind-, ent additive function.

Effects of Signal Dependent Noise

The preceding expressions were based on the assumption that the autocorrelation function is independent of shifts in the x origin. The autocorrelation is still easily defined for a nonstationary (signal dependent) random process as

$$R_{ss}(x_1, x_2) = E\{s(x_1).s(x_2)\}$$

Unfortunately, the simple picture of the power spectral density being the Fourier transform of R_{ss} is now lost. This problem will be further touched on in the section "Simple Filtering" in Chapter 3.

Physical Origins

Probability Theory

If the probability that an event occurs on any given trial is p, then the probability, $p_n(k)$, that the event occurs k times in n trials is given by

$$\begin{aligned} p_n(k) &= \binom{n}{k} p^k q^{n-k} \\ &= \frac{n!}{k!(n-k)!} p^k q^{n-k} \qquad q = 1-p \end{aligned}$$

The Poisson Theorem states that if

$$n >> 1 \quad \text{and} \quad p << 1$$

then $p_n(k)$ can be approximated by

$$p_n(k) \simeq \frac{e^{-np}(np)^k}{k!}$$

This is the well-known Poisson distribution and has the properties

$$E\{k\} = np$$

and

$$\mathrm{Var}\ \{k\} \equiv \sigma^2 = np$$

If in addition to the above restrictions, we also have $np >> 1$, the DeMoivre-Laplace Theorem may be used to give

$$P_n(k) \simeq \frac{1}{\sqrt{2\pi npq}} \exp[-\tfrac{1}{2}(k-np)^2/npq]$$

This is a Gaussian distribution with the properties

$$E\{k\} = np$$

and

$$\mathrm{Var}\ \{k\} = npq \simeq np\ .$$

It is significant that this Gaussian distribution is simply a special case a Poisson process and that both distributions are possible models for the same physical process.

Photomultiplier Tube

A familiar example of the many applications of these theorems is the photomultiplier tube. If a PM tube has an integrating time constant τ, then the current is proportional to the number of electrons, k, which arrive during the interval τ. By subdividing τ into many intervals $\Delta\tau$, each interval becomes an event, and the probability of an electron arrival during that interval is $p = \alpha\Delta\tau$ where α is the average number received per unit time. The number of events, n, is given by $\tau/\Delta\tau$. By making $\Delta\tau$ sufficiently small, the two constraints, $n >> 1$ and $p << 1$ can be satisfied. Thus, the probability of measuring a photomultiplier tube current of i at some sample time is

$$P_\tau(i) = \frac{e^{-\alpha\tau}(\alpha\tau)^i}{i!}$$

If $\alpha\tau >> 1$ the distribution becomes Gaussian.

The mean and variance for both distributions is $\bar{i} = \sigma_i{}^2 = \alpha\tau$.

Clearly, the noise statistics for this process are a function of the signal level.

Photographic Film

The noise characteristics of a signal recorded on a photographic emulsion are exceedingly complex. However, it is possible to make remarkably good predictions of some simple properties through the use of a very elementary model. If the density measured in a sample area, A, on an emulsion can be assumed proportional to the number of exposed grains in A, then the problem becomes analagous to the photomultiplier tube example. If E is the average number of developed grains per unit area (proportional to exposure) then the distribution of density values can be expressed by

$$P_A(D) = \frac{e^{-EA}(EA)^D}{D!}$$

As the exposure (or sample area) is increased, the quantity EA increases and the distribution becomes gaussian with $\bar{D} = EA = \sigma_D{}^2$.

Since the recorded signal, u(x), is proportional to E it is possible to approximate the variance of the recorded message as $\sigma_D^2(x) = k\, u(x)$. This property will be used extensively in Chapter 3.

A review of the many studies of photographic film noise tends to suggest that no single simple model will ever give good results under all conditions.[7,8] Although the above model is admittedly crude, the results have been found to give reasonably good agreement with most empirical measurements. In any case, it is a far better model than assuming the grain noise statistics to be entirely independent of the signal density as is frequently done.

CHAPTER 3

PROCESSING METHODS

For signals in the presence of multiplicative noise, as well as for stationary processes, there are many types of processing that are of interest. In this chapter, theoretical developments for three different processing problems are given. They are 1) filter theory, 2) signal detection, and 3) signal discrimination. It is assumed that the reader already has some familiarity with these subjects.[9] Although the majority of this report is concerned only with these three subjects, it should not be concluded that these are the only cases where the presence of non-stationary noise is of potential concern. This chapter is limited to these subjects only because it is felt that a contribution to the existing literature can be made in these areas.

Signal Detection

Statement of the Problem

Consider the following typical signal detection problem. An encoded message is stored as a one-dimensional (for simplicity) array on a photographic emulsion. The code is binary with a "1" being indicated by the presence of a signal pulse u'(x). A typical form for u'(x) would be a rectangle--band limited by the recording optics and the photographic medium. A "O" is indicated by the absence of a signal pulse. The signals are recorded only at intervals of x_o where x_o is slightly larger than the width of the pulse. A typical message segment is shown in Fig. 3.

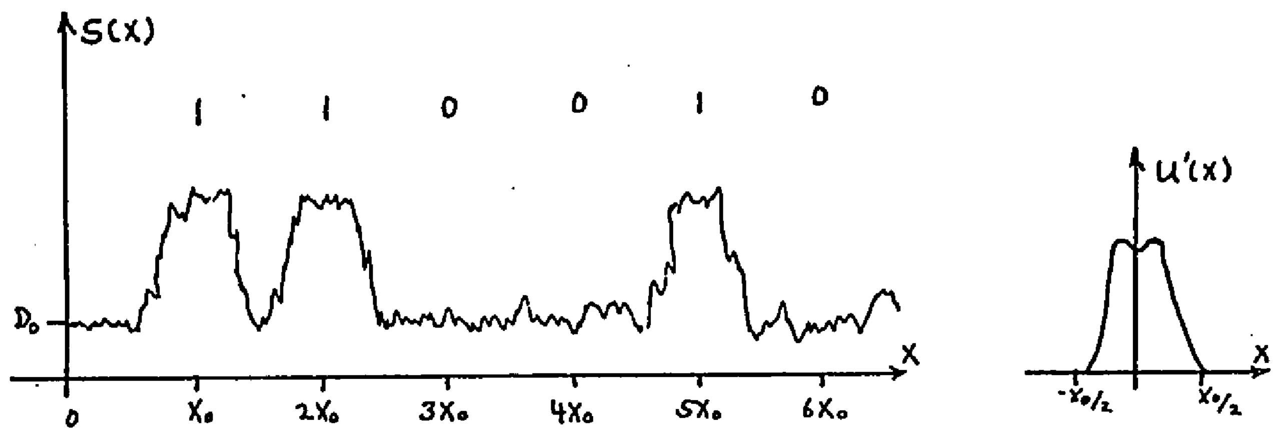


Fig. 3. Typical Binary Message Segment.

The pulse, u'(x) can only occur at centers given by nx_o. s(x) is always $\geq$ 0.

Note that the message, s(x), contains noise, n(x), and a background density, u_o, in addition to the signal pulses, $s(x-nx_o)$. The background density can be incorporated either into the noise or into the signal. Unfortunately, either choice can lead to some notational difficulties. To minimize these problems, the following definition will be used

$$u(x) \equiv u'(x) + u_o$$

Both u(x) and u'(x) will be used throughout the remainder of this discussion.

What is now desired for this problem is a detection scheme. For computational convenience, assume that s(x) has been sampled at a rate such that the noise samples are independent but no signal aliasing has occurred. Suppose the signal, $\{s_i\}$, consists of a set of N samples (s_1, s_2 . . . s_N) for each interval $nx_o \pm x_o/2$. We now wish to process these N values in each interval in a way that permits the fewest wrong decisions as to whether the intended symbol was a "0" or a "1".

General Solution

To find an optimum solution to this problem, we would like to compare

$P\ (H_1|\{s_i\})$ = Prob [$u'(x-nx_o)$ occurred | given that $\{s_i\}$ was received]

to

$P\ (H_o|\{s_i\})$ = Prob [$u'(x-nx_o)$ did not occur | given that $\{s_i\}$ was received]

In general, it is not possible to write simple, analytic expressions for these probabilities. However, with the aid of Bayes' Theorem, the expressions can be rearranged into something easier to handle. Bayes' Theorem gives the relationship between two conditional probabilities as

$$P(A|B)\ \ P(B)\ \ = P(B|A)\ P(A)$$

Using this relationship, the desired probabilities become

$$P(H_i|\{s_i\}) = \frac{P(H_1)}{P(\{s_i\})}\ \ P(\{s_i\}|H_1)$$

and

$$P(H_o|\{s_i\}) = \frac{P(H_o)}{P(\{s_i\})} \cdot P(\{s_i\} \mid H_o)$$

To eliminate the unknown probability $P(\{s_i\})$, evaluate the ratio of $P(H_1|\{s_i\})$ and $P(H_o|\{s_i\})$. This gives no loss of information since the desired decision will be based on which of the two situations was most likely to have occurred.

$$\frac{P(H_1|\{s_i\})}{P(H_o|\{s_i\})} = \frac{P(H_1)}{P(H_o)} \cdot \frac{P(\{s_i\}|H_1)}{P(\{s_i\}|H_o)}$$

Note that $P(H_1)$ and $P(H_o)$ are apriori probabilities and are constant for any given system. Thus,

$$\frac{P(H_1|\{s_i\})}{P(H_o|\{s_i\})} \propto \frac{P(\{s_i\}|H_1)}{P(\{s_i\}|H_o)} \equiv \Lambda(nx_o)$$

The quantity $\Lambda(nx_o)$ can, in principle, be evaluated for any hypothetical signal location. The higher the value of $\Lambda(nx_o)$, the more likely it is that a signal (pulse) occurred at nx_o. The choice of a threshold value of $\Lambda(nx_o)$ for making a decision is a problem in decision theory which is discussed in Chapter 4.

Solutions for Three Different Noise Distributions

The evaluation of $\Lambda(nx_o)$ depends on the probability distribution of the noise. If, for example, one makes the common assumption that the noise is stationary, independent and aussian, the needed probabilities become

$$P(\{s_i\}|H_1) = \left(\frac{1}{2\pi\sigma_o^2}\right)^{N/2} \exp\left[-\tfrac{1}{2}\sum_{i=1}^{N} (s_i-u_i)^2/\sigma_o^2\right] \cdot (\Delta s_i)^N$$

and

$$P(\{s_i\}|H_o) = \left(\frac{1}{2\pi\sigma_o^2}\right)^{N/2} \exp\left[-\tfrac{1}{2}\sum_{i=1}^{N} (s_i-u_o)^2/\sigma_o^2\right] \cdot (\Delta s_i)^N$$

Solving for the ratio of these probabilities, one obtains

$$\Lambda(nx_o) = \exp\left\{\frac{1}{2\sigma_o^2}\sum_{i=1}^{N} [2s_i(u_o-u_i) - u_i^2 - u_o^2]\right\}$$

or

$$\log[\Lambda(nx_o)] = \frac{1}{\sigma_o^2}\sum_{i=1}^{N} s_i u_i' - \frac{1}{2\sigma_o^2}\sum_{i=1}^{N} (u_i^2 - u_o^2)$$

Note that the only term in this expression which depends on the measured message values is $\sum_{i=1}^{N} s_i u_i'$. The second term is a bias term which is constant for a given $u(x)$ and a constant background.

The bias term has the effect of normalizing the measurement to remove the signal energy as a variable. This is important in decision theory and in applications where more than one type of signal is present, but in this simple example it is of little significance.

The significance of this result is that if the measured message $s(x)$ is convloved with the known signal shape and evaluated at $x = nx_o$ the value obtained is proportional to the probability that $u'(x - nx_o)$

was actually present. This development can be recognized as a derivation of the matched filter.[10] It should be remembered, however, that this result--as well as the use of the matched filter--is only an optimum process when the noise is stationary and independent.

Consider the case where the distribution of message values, s_i, is determined by Poisson statistics. The conditional probabilities are now

$$P(\{s_i\}|H_1) = \prod_{i=1}^{N} \frac{e^{-\alpha u_i}[\alpha u_i]^{s_i}}{s_i!}$$

$$P(\{s_i\}|H_o) = \prod_{i=1}^{N} \frac{e^{-\alpha u_o}[\alpha u_o]^{s_i}}{s_i!}$$

where α is a normalizing constant given by $\alpha = u_o/\sigma_o{}^2$. With the Poisson distribution the mean and variance are equal. This makes normalization by α necessary since few real process variables have this property.

The ratio of these probabilities, $\Lambda(nx_o)$, is

$$\Lambda(nx_o) = \prod_{i=1}^{N} e^{-\alpha(u_i - u_o)}[u_i/u_o]^{s_i}$$

or

$$\log[\Lambda(nx_o)] = \sum_{i=1}^{N} s_i \cdot \log(u_i/u_o) - \sum_{i=1}^{N} \alpha u_i^!$$

Comparing this result to that obtained for stationary noise it is seen that both contain a bias term and a processing signal which is convolved with $s(x)$ and evaluated at $x = nx_o$. The fundamental difference between the two noise cases is in the structure of the processing signal. The significance of these differences in structure has been investigated experimentally and is discussed in Chapter 4.

Finally, consider the case where the noise is Gaussian distributed but has a variance which is proportional to the signal level, $u(x)$. That is,

$$\sigma_i^2 = ku_i$$

The proportionality constant can be determined from a knowledge of the variance σ_o^2 of the background in the presence of the signal.

$$\sigma_o^2 = ku_o \quad \text{or,} \quad k = \sigma_o^2/u_o$$

$$\sigma_i^2 = \frac{\sigma_o^2}{u_o} \cdot u_i$$

The ratio of probabilities for a Gaussian process with a signal dependent variance is given by

$$\Lambda(nx_o) = \prod_{i=1}^{N} (\sigma_o/\sigma_i) \cdot \exp\{-\tfrac{1}{2}[(s_i-u_i)^2/\sigma_i^2 - (s_i-u_o)^2/\sigma_o^2]\}$$

Using the above expression for σ_i, one obtains

$$\Lambda(nx_o) = \prod_{i=1}^{N} [u_o/u_i]^{\frac{1}{2}} \cdot \exp\{\frac{-1}{2\sigma_o^2}[s_i^2(\frac{u_o}{u_i} - 1) + u_o(u_i-u_o)]\}$$

or,

$$\log \Lambda(nx_o) = -\tfrac{1}{2}\sum_{i=1}^{N} \log(u_i/u_o) + \frac{1}{2\sigma_o^2}\sum_{i=1}^{N} s_i^2(\frac{u_i'}{u_i}) - \frac{u_o}{2\sigma_o^2}\sum_{i=1}^{N} u_i'$$

Here again are bias terms and a convolution term. This time, however, the processing signal is not convolved with the message directly but with s_i^2.

Shown below is a comparison of optimum signal detection techniques for the three different noise distributions.

Quantity	Stationary Gaussian	Poisson	Multiplicative Gaussian
Message used in processing	$\{s_i\}$	$\{s_i\}$	$\{s_i^2\}$
Processing signal	$\frac{1}{\sigma_o^2}\{u_i\}$	$\{\log(u_i/u_o)\}$	$\frac{1}{2\sigma_o^2}\{u_i'/u_i\}$
Bias terms	$\frac{1}{2\sigma_o^2}\Sigma(u_i^2-u_o^2)$	$\Sigma\alpha(u_i-u_o)$	$\frac{1}{2}\Sigma\log(u_i/u_o) + \frac{u_o}{2\sigma_o^2}\Sigma u_i'$

Decision Theory

Test statistics for three different noise distributions have been developed. These tests, when applied to a received message, yield values which are proportional to the probability that one of the signals was present at the message point in question. The problem of taking these statistical values and selecting a threshold for making the best decision as to whether or not the signal was there has not been discussed. Although this study is primarily concerned only with detection processing, a few of the principles of decision theory are presented here as an aid in understanding the processing methods.

Fig. 4 illustrates the schematic relationship between decision theory and detection theory in the monitoring of some arbitrary system.

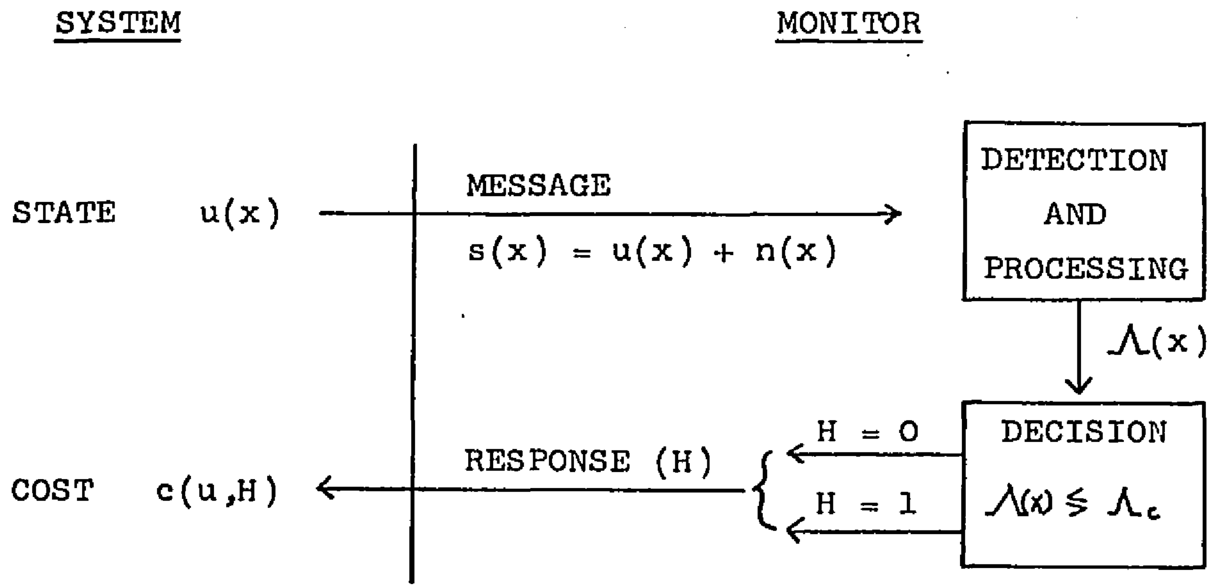


Fig. 4. Schematic Diagram of Decision Theory Model.

In the system shown, only two possible states are considered. Either u(x) is present (hypothesis H = 1) or it is not (hypothesis H=0). Information about the system is detected by the monitoring unit and processed in some optimum manner to provide the statistic, $\Lambda(x)$. The value of Λ at any point x is a measure of the probability that the correct hypothesis is either H = 0 or H = 1. Depending on the value of the threshold, Λ_c, one of the two hypotheses is accepted and an appropriate response is made.

In this simple two state system, four different situations are possible. They are (1) u(x) occurs and decision H = 1 is made (correct decision), (2) u(x) occurs and decision H = 0 is made (miss), (3) u(x) does not occur and decision H = 1 is made (false alarm), and (4) u(x) does not occur and decision H = 0 is made (correct decision).

A relative cost to the system, $C(u,H)$ is assigned to each of four possible situations. The parameter u is either 1 or 0 depending on whether $u(x)$ did or did not occur.

It is now possible to find an optimum value for the threshold Λ_c . It is desired to select Λ_c so that the expected value of the cost of the system is a minimum. The expected cost is given by

$$E\{C(u_1 s)\} = C(0,0) \cdot \text{Prob } (u=0,\ H=0) + C(0,1) \cdot \text{Prob } (u=0,\ H=1)$$
$$+ C(1,0) \cdot \text{Prob } (u=1,\ H=0) + C(1,1) \text{ Prob } (u=1,\ H=1) \ .$$

It is possible to find a minimum for this expression by using relationships similar to

$$\text{Prob } (u=0,\ H=0) = p_o \int_{-\infty}^{s_c(x)} \cdot \ p(s_c(x)/u=0)\ d(s(x))$$

where p_o is the a priori probability that $u(x)$ will not occur and $s_c(x)$ is some decision boundary in the multidimensional space spanned by the message $s(x)$(See Chapter III of Ref. 9). Following this approach one finds that the minimum cost is given by

$$\frac{p(s_c/u=1)}{p(s_c/u=0)} = \frac{p_o}{1-p_o} \cdot \frac{C(0,1) - C(0,0)}{C(1,0) - C(1,1)} \equiv \Lambda_c .$$

Thus, Λ_c is an optimum decision threshold for the test statistic, $\Lambda(x)$, based on the a priori signal occurrance probability and the relative costs of the four possible decision situations.

Detection Plots

In evaluating the performance of a system such as shown in Fig. 4, it should be noted that only two parameters are required to characterize the four possible decision situations. The most commonly chosen parameters are the detection probability given by

$$\beta = \int_{s_c}^{\infty} p(s(x)/u=1)ds(x)$$

and the false alarm probability given by

$$\alpha = \int_{s_c}^{\infty} p(s(x)/u=0)ds(x)$$

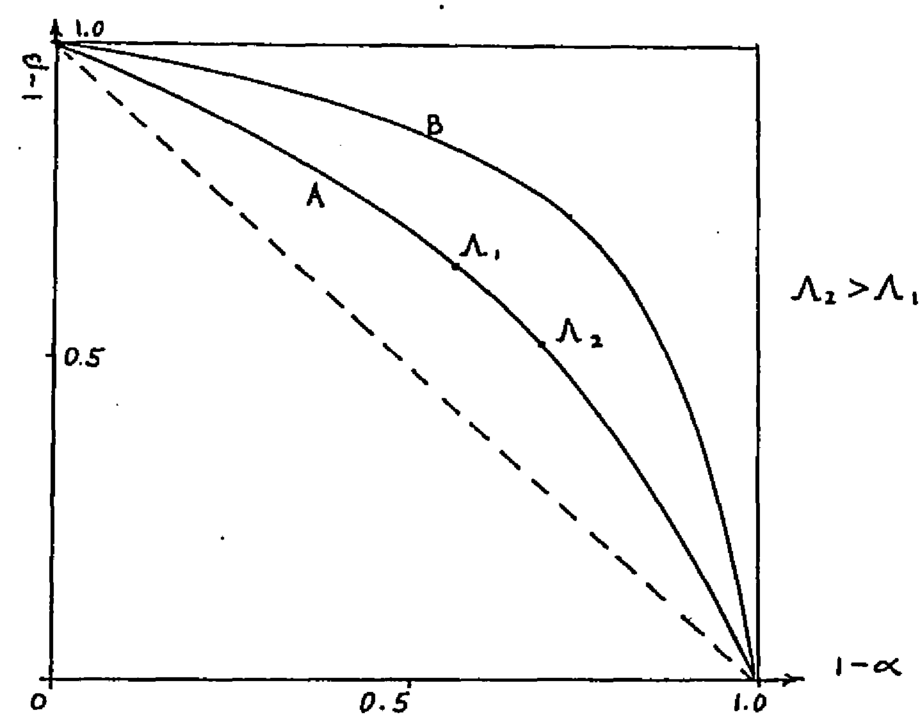


Fig. 5. Typical Detection Curve.

Plot p(H=1/u=1) versus p(H=0/u=0) as a function of Λ_c. Curve B is better performance than A.

The effectiveness of the monitoring system can be displayed by plotting detection and false alarm probabilities as a function of the decision threshold, Λ_c. Shown in Fig. 5 is a typical plot of type used in this study. Each curve is the locus of points obtained as Λ_c is changed. Note that as the probability of detection is increased by decreasing the threshold from Λ_{c1} to Λ_{c2} the probability of a correct decision when no signal is present $(1 - \alpha)$ decreases. The performance indicated by curve B is better than that of curve A. This may be due to a better processing method or it may simply represent the same receiver operating at a higher signal-to-noise ratio. The dotted straight line represents the worst case where decisions are made purely on the basis of chance.

Signal Discrimination

General Solution

In theory, the problem of signal discrimination is exactly like the detection problem. Conceptually, the two situations may be quite different. While signal detection decides whether a known signal is present at a particular location or not, signal discrimination decides which of two (or more)known signals is present. Signal detection may be thought of as the special case of signal discrimination where one of the two possible signals is the null signal.

Following the development pattern in the preceding section, consider the following two probabilities.

$P(H_1/\{s_i\})$ = Prob[$u_i'(x-nx_o)$ occured/given that $\{s_i\}$ was received]

and

$P(H_2|\{s_i\})$ = Prob $[u_2'(x-nx_o)$ occurred$|$given that $\{s_i\}$ was received]

where

$$u_1'(x) = u_1(x) - u_o$$

and

$$u_2'(x) = u_2(x) - u_o$$

are the two possible signals occurring at locations given by nx_o.

The ratio of these probabilities is formed and found to be

$$\frac{P(H_1|\{s_i\})}{P(H_1|\{s_i\})} \propto \frac{P(\{s_i\}|H_1)}{P(\{s_i\}|H_1)} = \Lambda(nx_o)$$

Again, $\Lambda(nx_o)$ is a test statistic which depends on the noise distribution and can be calculated for a received message $\{s_i\}$. In actually making the decision as to which of the signals was present, decision theory is used to establish some optimum threshold value for $\Lambda(nx_o)$.

Solutions for Three Different Noise Distributions

Solutions for $\Lambda(nx_o)$ are obtained in the same manner as for the signal detection case. Thus, without repeating the expressions given for the detection case, the results for the same three noise distributions are as follows.

(i) Stationary noise

$$\Lambda(nx_o) = \exp\{\frac{-1}{2\sigma_o^2}\sum_{i=1}^{N} [(s_i-u_{1i})^2-(s_i-u_{2i})^2]\}$$

or,

$$\log[\Lambda(nx_o)] = \frac{1}{\sigma_o^2}\sum_{i=1}^{N} s_i(u_{1i}-u_{2i}) - \frac{1}{2\sigma_o^2}\sum_{i=1}^{N} (u_{1i}^2-u_{2i}^2)$$

(ii) Poisson noise

$$\Lambda(nx_o) = \prod_{i=1}^{N} e^{-\alpha(u_{1i}-u_{2i})}[u_{1i}/u_{2i}]^{s_i}$$

or

$$\log[\Lambda(nx_o)] = \sum_{i=1}^{N} s_i.\log[u_{1i}/u_{2i}] - \alpha\sum_{i=1}^{N} (u_{1i}-u_{2i})$$

(iii) Multiplicative Gaussian noise $(\sigma_i^2 = \frac{\sigma_o^2}{u_o} . u_i)$

$$\Lambda(nx_o) = \prod_{i=1}^{N} (\frac{s_{2i}}{s_{1i}})^{\frac{1}{2}} \exp\{\frac{-u_o}{2\sigma_o^2} [(s_i-u_{1i})^2/u_{1i}-(s_i-u_{2i})^2/u_{2i}]\}$$

or,

$$\log[\Lambda(nx_o)] = -\tfrac{1}{2}\sum_{i=1}^{N} \log(s_{1i}/s_{2i}) + \frac{u_o}{2\sigma_o^2}\sum_{i=1}^{N} s_i^2 (\frac{1}{u_{2i}} - \frac{1}{u_{1i}})$$
$$- \frac{u_o}{2\sigma_o^2}\sum_{i=1}^{N} (u_{1i}-u_{2i})$$

Shown below is a comparison of these results according to their use in a discrimination problem.

Quantity	Stationary Gaussian	Poisson	Multiplicative Gaussian
Message used in processing	(s_i)	(s_i)	(s_i^2)
Processing signal	$\frac{1}{\sigma_o^2}\{u_{1i}-u_{2i}\}$	$\{\log(u_{1i}/u_{2i})\}$	$\frac{u_o}{2\sigma_o^2}\{\frac{1}{u_{2i}}-\frac{1}{u_{1i}}\}$
Bias Terms	$\frac{1}{2\sigma_o^2}\Sigma(u_{1i}^2-u_{2i}^2)$	$\alpha\Sigma(u_{1i}-u_{2i})$	$\frac{1}{2}\Sigma\log(s_{1i}/s_{2i})$ $+\frac{u_o}{2\sigma_o^2}\Sigma(u_{1i}-u_{2i})$

The results in this and the preceding section were applied to a range of signal detection and discrimination problems. This work and its significance are discussed in Chapter 4.

Filter Theory

Optical Filtering Analogy

The analogy between electrical and optical systems has been studied extensively in recent years.[11,12] The use of coherent light, made practical by the discovery of the laser, has provided motivation for much of the exchange between electrical and optical theories. This analogy has proven to be of mutual benefit to both sciences. In particular, the exploitation of coherent optical processing was greatly facilitated by the existence of the appropriate analytic tools in electrical systems analysis. Similarly, the ability to observe and manipulate such things in optics as frequency and power spectra has provided insight to the study of classical filter theory.

The electrical-optical analogy can be best appreciated by referring to Fig. 6. Fig. 6 illustrates a typical coherent optical image forming system.

Briefly, if the field to the right of plane P, is given by $u+(x_1,y_1) = u_1^- (x_1,y_1) \cdot t_1(x_1,y_1)$ then the optical equivalents to the system in Fig. 3-A are given by:

Electrical		Optical
Complex input signal $u_1(t)$	$\leftrightarrow$	Complex field $u_1^+ (x_1,y_1)$
Complex output signal $u_2(t)$	$\leftrightarrow$	Complex field $u_3(x_3,y_3)$
Filter frequency spectrum $H(\omega)$	$\leftrightarrow$	Exit pupil transmission $H_2(x_2,y_2)$
Signal power spectrum $P_{ss}(\omega)$	$\leftrightarrow$	Observed intensity $I_2(x_2,y_2)$
System impulse response $h(t)$	$\leftrightarrow$	Point spread function $u_3(x_3,y_3)$ for point source at $u_1(0,0)$

Noise Analogy

The optical system shown in Fig. 6 is usually applied only to coherent incident fields. When incoherent light is used, the system is generally reformulated so that intensity ($|u(x,y)|^2$) becomes the linear variable. In the context of the coherent system, however, incoherent light takes on a different significance. It plays the role of additive, uncorrelated noise.[13]

Assuming this to be true, it is then possible to express any message as the sum of a signal and a noise term as defined in Chapter 2

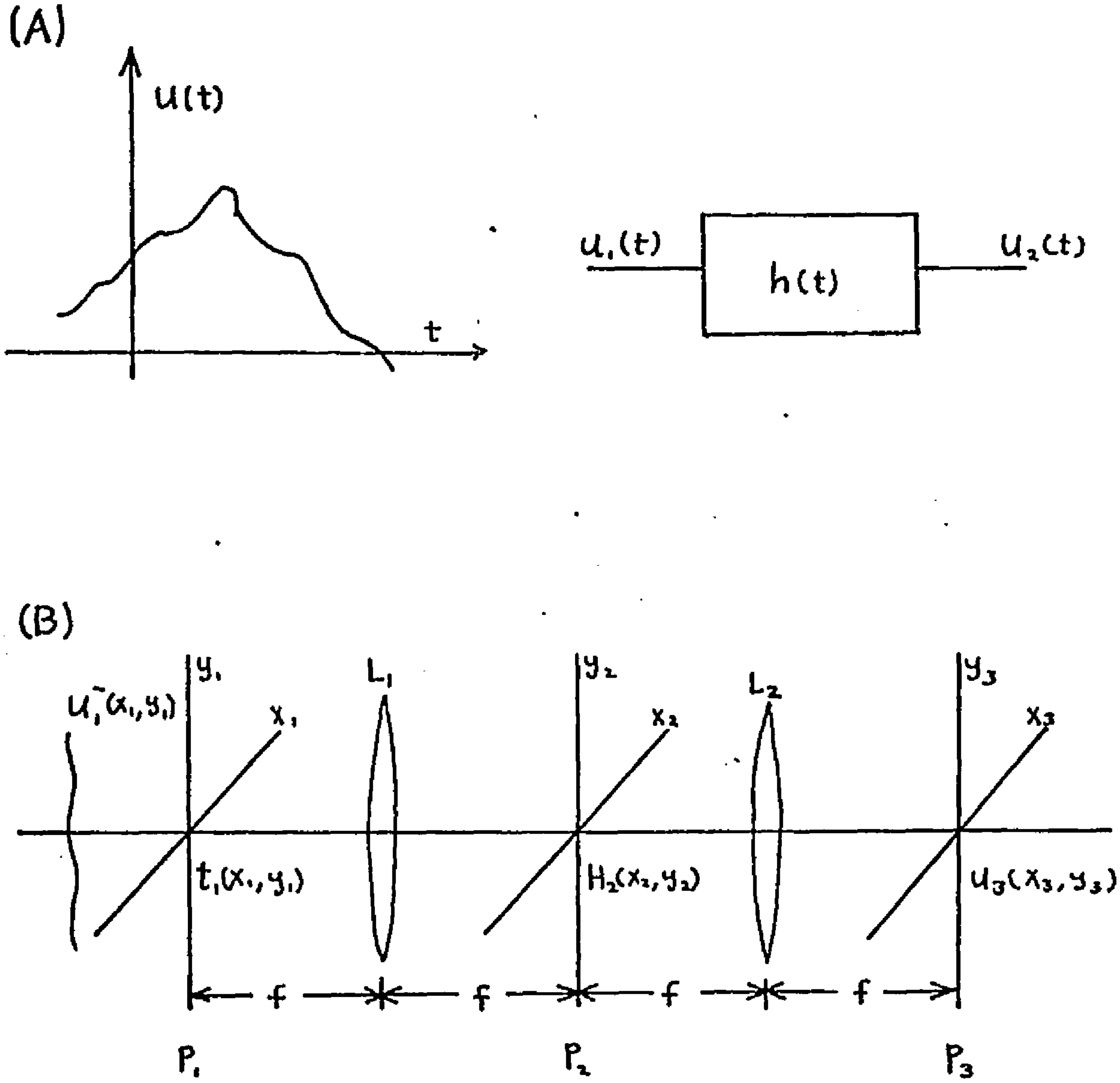


Fig. 6. Comparison of Optical and Electrical Systems.

(A) Passive system characterized by impulse response, h(t). $u_1(t)$ and $u_2(t)$ are the input and output signals (B) Optical equivalent to A.[2] $u_1(x_1, y_1)$ is incident complex field and $t_1(x_1, y_1)$, $H_2(x_1, y_2)$ are amplitude transmittion functions.

by

$$u_1^+(x_1,y_1) = u_{coh}^+(x_1,y_1) + u_{inc}^+(x_1,y_1)$$

$$= [u_{coh}^-(x_1,y_1) + u_{inc}^-(x_1,y_1)] \cdot t_1(x_1,y_1)$$

As discussed in Chapter 2, it is possible to have noise which is not everywhere uncorrelated. This means that $u_{inc}^+(x_1,y_1)$ is, in fact, partially coherent. While partially correlated noise is not an unreasonable physical assumption (all real noise sources as well as real field disturbances have some small correlation length), this report will continue to be restricted to the case of independent noise samples.

It is also possible to have noise (u_{inc}^+) which depends on the signal (u_{coh}^+). Note that u_1^+ is given by $u_1^+ = u_1^- \cdot t_1$. Thus if the signal is defined as some feature in the amplitude screen, t_1, then u_{inc}^+, which has also passed through the screen, will be modulated by t_1 and given noise statistics which are dependent on the signal. If, on the other hand, the signal information is contained in $u_{coh}^-(x_1,y_1)$ and $t_1(x_1,y_1)$ is a clear aperture (or, perhaps, a field stop) then the noise, u_{inc}^+, will not be influenced by the presence of the signal u_{coh}^+.

The field in plane p_2 is described by the Fourier transform of the field in plane p_1. Thus, the intensity distribution, $I_2(x_2,y_2) \propto |u_2(x_2,y_2)|^2$ is proportional to the power spectral density, $P_{u_1u_1}$.

Physically, p_2 contains the power spectrum from $u^+_{coh}(x_1,y_1)$ plus contributions from $u^+_{inc}(x_1,y_1)$. If $u^+_{inc}(x_1,y_1)$ is stationary, the simple relationships discussed in the section "Spectral Characteristics" in Chapter 2 apply. If $u^+_{inc}(x_1,y_1)$ is effectively uncorrelated, then the intensity distribution $I_2(x_2,y_2)$ is uniform and the noise is described as "white".

It is, however, in the case where the noise is not stationary that this model is of greatest benefit. Although the intensity (P_{uu}) in the p_2 plane is uniform for u_{inc} uncorrelated, there may still be some information in the amplitude distribution $u_2(x_2,y_2)$ (frequency spectrum) before it is squared. If $u^+_{inc}(x_1,y_1)$ is considered to be the product of an uncorrelated uniform intensity field, $|u^-_{inc}(x_1,y_1)|^2$, and an aperture with transmission given by $t_1(x_1.y_1)$ then the problem can be solved by the application of optical partial coherence theory. For the conditions given in this problem the Zernike-Van Cittert Theorem states that the field in the p_2 plane from $u^+_{inc}(x_1,y_1)$ will be partially coherent and that the coherence function will be proportional to $T(\frac{x_2}{\lambda_f}, \frac{y_2}{\lambda_f})$, which is the Fourier transform of $t_1(x_1,y_1)$ obtained by irradiating the aperture t_1 with coherent light.

If, for example, the noise is signal dependent (the signal information is contained in the aperture $t_1(x_1,y_1)$ and u^-_1 consists of $u^-_{coh} + u^-_{inc}$, then it is possible to filter out the signal spectrum from u^-_{coh} and yet still extract information about the signal due to the partial coherence of the noise spectrum in the p_2 plane. Partial coherence theory also states that this information can be observed in the p_3 plane through the proper choice of filters in the p_2 plane.

Thus, for a signal in the presence of signal dependent noise, more information about the input signal can potentially be extracted from the filtered signal than if the noise were signal independent.

Multiplicative Noise Model

As this report is concerned almost exclusively with the type of noise generated by a Poisson process, it is of interest to determine what the equivalent optical system for such a process would be.

In the Gaussian approximation to the Poisson process, it is clear that the noise can be considered as a stationary noise source modulated by the square root of the signal. That is, in terms of the notation used for a general message,

$$s(x) = u(x) + n(x)$$

where $n(x) = (u(x))^{\frac{1}{2}} \cdot n'(x)$ and $n'(x)$ is a stationary noise distribution. Under these conditions, both the mean and variance of $s(x)$ are proportional to $u(x)$.

This process can be simulated by the system shown in Fig. 3 if

$$u^{-}_{coh}(x_1) = (u(x))^{\frac{1}{2}}$$

$$u^{-}_{inc}(x_1) = n'(x)$$

and

$$t_1(x_1) = (u(x))^{\frac{1}{2}}$$

Under these conditions, the incident message is then given by

$$u^{+}_{1}(x_1) = (u^{-}_{coh} + u^{-}_{inc})\ t(x_1)$$

$$u_1^+(x_1) = u(x_1) + (u(x_1))^{\frac{1}{2}} \cdot n'(x_1)$$

This system is now a possible model for a Poisson process with mean value and variance equal to $u(x)$.

CHAPTER 4

EXPERIMENTAL RESULTS

All of the experiments performed in this study have certain features in common. They are concerned with some form of noisy signal array. The array is sampled and the data are then processed by several different methods. Conclusions are then drawn by comparing the results of these different processing methods.

The majority of the experiments deal with the problem of signal detection. Part of this emphasis is because it appears that this is the area of greatest promise for the application of the signal dependent noise model and part is because many of the results can be applied directly to other processing problems.

Experimental Methods

Computer Simulation

The greatest potential application of this work is probably in the processing of signals stored on a photographic emulsion. It would seem reasonable, therefore, to perform the experimental work on photographic emulsions. Unfortunately, there are some complicating factors that make such a plan impractical. These factors stem from the nature of the noise in photographic emulsions. As discussed in Chapter 2, Physical Origins of Noise Distributions, the simple multiplicative noise model is, at best, an approximation. Most of the questions about the signal dependent noise processing methods involve questions of <u>degree</u> of improvement. As this degree is typically small it was felt that the large uncertainties introduced by statistical variations in a photographic emulsion could not be tolerated. For example, differences between the stationary Gaussian and the multiplicative Gaussian models might be observed but it is doubtful that a distinction could be made between the multiplicative Gaussian and the multiplicative Poisson distributions. As photographic film is not the only possible medium for application of these theories, it was decided that these more subtle distinctions should be measured and that the complicating factor of noise distributions in a photographic emulsion could better be left as the subject of a separate study.

The best alternative medium that would permit the study of small differences in the processing methods was the computer. Through the use of the computer one could guarantee the nature of the noise distribution, the independence of the noise samples, the signal distribution, and an accurate knowledge of all other parameters used in any given experiment. Furthermore, the use of the computer for simulating data fields proved to be many times faster than recording a field on film and sampling with a densitometer. The latter method was successfully attempted, however. The results were processed by the computer and, except for the difficulties mentioned above, such a technique proved entirely feasible.

The computer used throughout this study was Control Data Corporation's model CDC 6400.

SIMULAT Program

A program was written in Fortran (extended version) to accomplish the task of taking a given signal shape, bandlimiting it, sampling it, reproducing an array of these signals on a background and adding noise according to some desired distribution. The program had to be altered frequently to accommodate the requirements of a particular experiment, such as in the simulation of circularly symmetric, non-separable signals, but the basic outline of the SIMULAT program remained unchanged.

A flow chart showing the essential features of SIMULAT is shown in Fig. 7. A copy of the actual program (used for producing bandlimited square signals in the presence of Gaussian multiplicative noise) is included as Appendix I.

There are several potential difficulties in simulating a signal field. First, all data are sampled. This is necessary regardless of whether the field is an actual densitometer trace of a photographic film or the output of a computer program because the processing methods have all been designed to be implemented on a digital computer.

Sampling the final field, however, raises other problems. The signals must be bandlimited or potentially valuable information might be lost during the sampling process. Furthermore, it is desirable to keep the number of sample points per signal low to keep the time required to process the field of signals low. As these are two dimensional signals, a signal requiring only five sample points

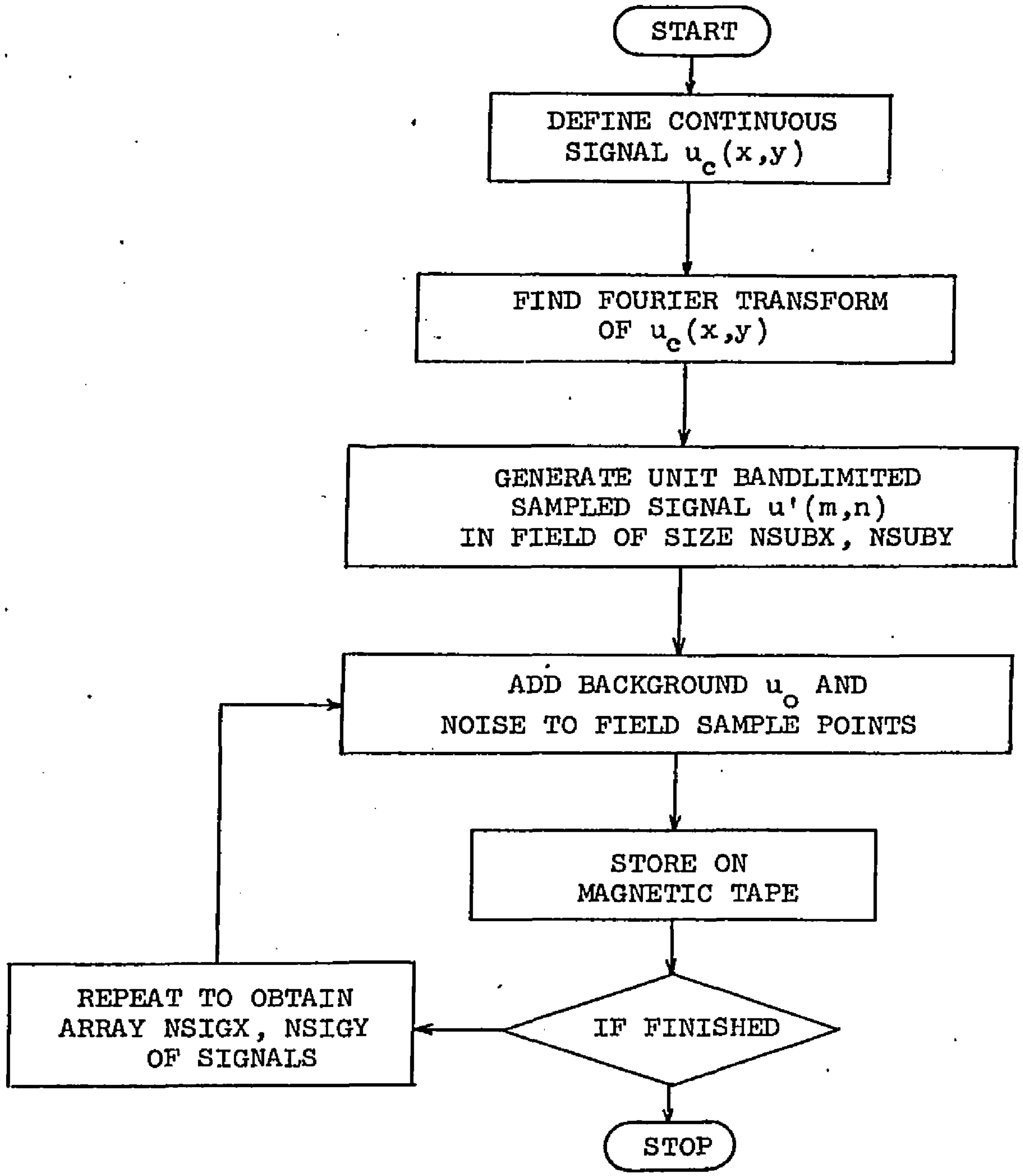


Fig. 7. Flow Chart for SIMULAT Computer Program.

This program is used to simulate a field of bandlimited, sampled signals in the presence of noise of a known distribution.

in one dimension will require a total of twenty-five sample points in two dimensions. This is also the size of the processing signal that must be convolved with the signal array. Since some arrays contain approximately 10^5 points it is important to keep the size of the processing signal relatively small. Thus, it is necessary to exercise some care in bandlimiting the continuous signal $u_c(x,y)$ and in selecting the sample spacing and field size. The relationship between these quantitites is shown in Fig. 8.

The bandlimiting process is actually accomplished by assuming that the given signal $u_c(x,y)$ is periodic (not an unreasonable approximation since $u_c(x,y)$ typically occurs in large arrays). By doing this, the signal spectrum can then be expressed as a sampled spectrum. Such a spectrum is not only easier to handle in the computer but it greatly facilitates reconstruction of the bandlimited signal $u'(m,n)$.

The sample interval is normalized to unity. This establishes the maximum signal frequency component at one-half to avoid aliasing. The frequency scale is determined by the scale of the signal which is limited by the field size, NSUBX. For most of the experiments, the field size, NSUBX, was 20 units and the signals ranged from 3 to 7 units in one dimension. Larger signals and field sizes means less signal distortion due to bandlimiting but more sample points to process. As the signal shape for most experiments is somewhat arbitrary, the distortion is relatively unimportant.

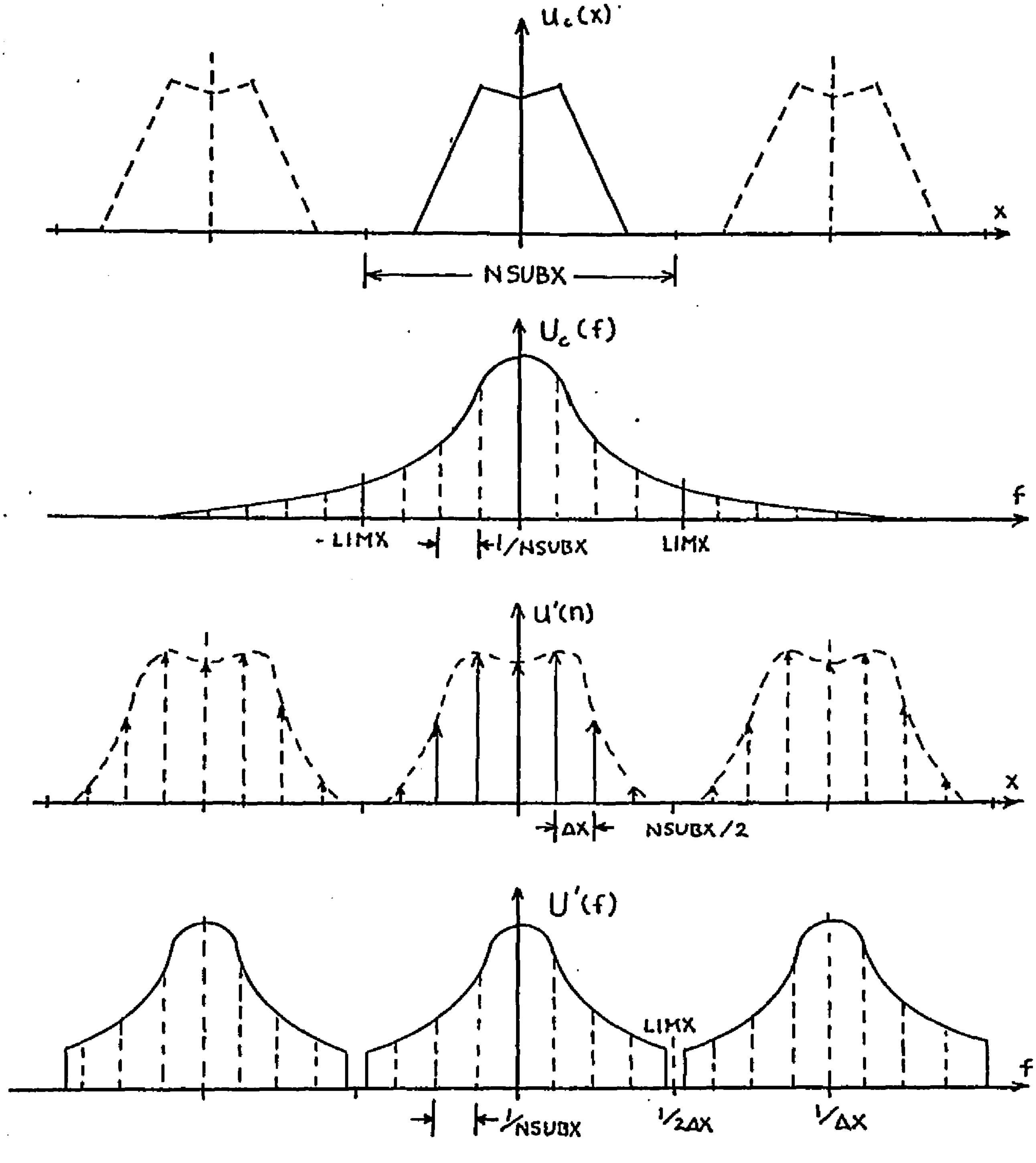


Fig. 8. Analysis of Signal Bandlimiting and Sampling Operation.

Given signal $u_c(x)$, it transform $U_c(f)$ is bandlimited so that the signal u'(m) can be sampled. The dotted samples and transforms are a result of the computer representation.

Fig. 9 illustrates the results of SIMULAT for a 4 x 4 square in a 20 x 20 field. The unit sampled field may be repeated as many times as desired before adding the background and noise.

The noise, which is added to each sample point, is generated in the computer to give the desired distribution. The CDC 6400 computer supplies a sequence of random numbers uniformly distributed between 0 and 1 on demand. These numbers can be converted to a Gaussian distribution by the following transformation.[5]

$$x_1 - x_0 = \sigma . \ (-2 \log_e y_i)^{\frac{1}{2}} \ . \ \cos(2\pi y_2)$$

$$x_2 - x_0 = \sigma . \ (-2 \log_e y_1)^{\frac{1}{2}} \ . \ \sin(2\pi y_2)$$

where y_1, y_2 are a pair of uniformly distributed random numbers and x_1, x_2 are the new pair of independent, Gaussian distributed numbers. The distribution has a variance given by σ^2 and a mean value of x_o.

The specific method used for implementing this transformation and adding the noise value to the signal plus background can be seen in the SIMULAT program list in Appendix I. This approach allows the use of either stationary or multiplicative Gaussian distributions.

Although Poisson noise was not used in the SIMULAT routine, tests were made using the log normal distribution--which is closely related to the Poisson distribution. The log normal distribution is easily generated by taking the $\log_e$ of x_1, x_2 in the above expressions.

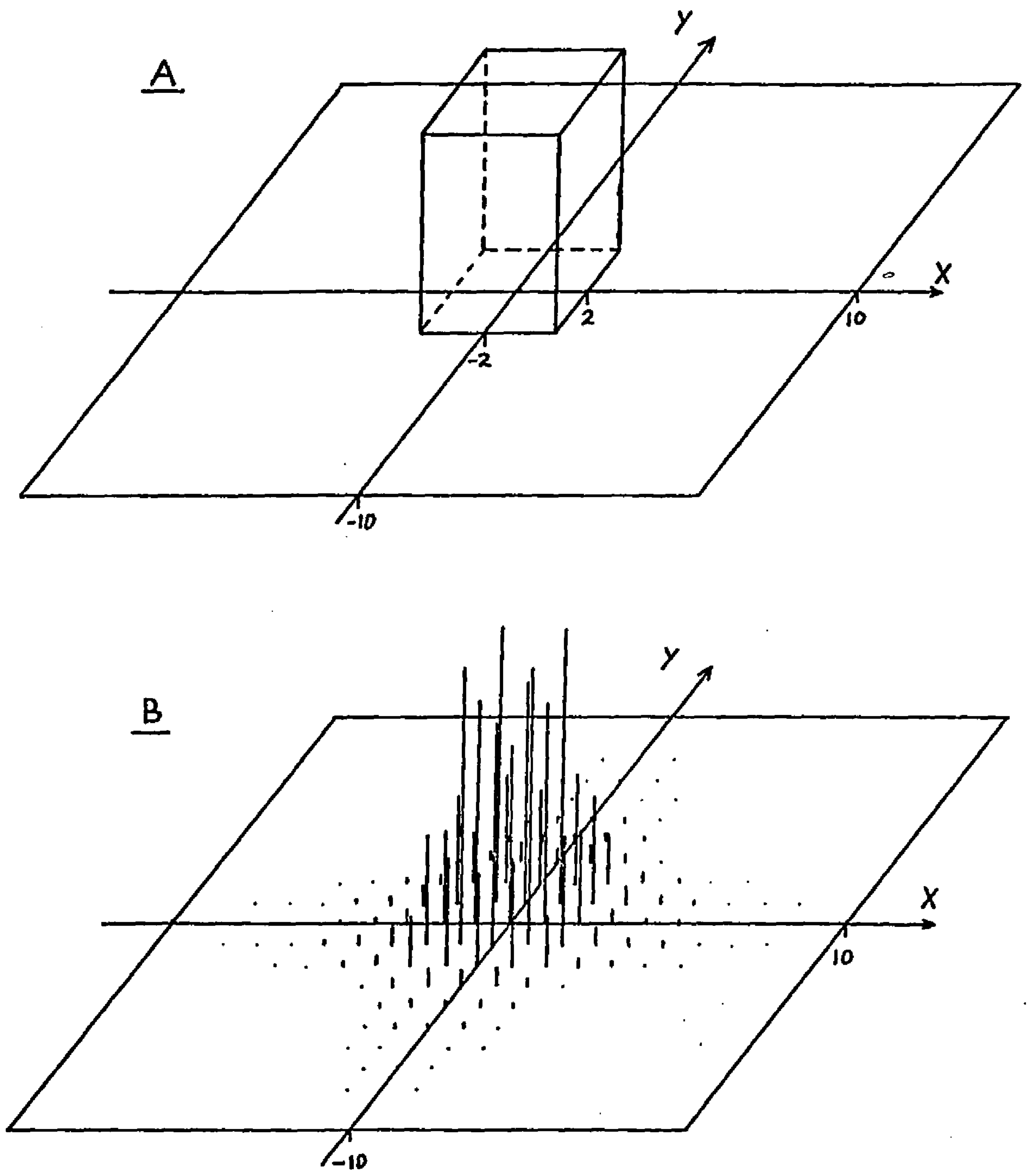


Fig. 9. Unit Bandlimited Signal Matrix From SIMULAT.

Signal is 4 x 4 square in 20 x 20 field bandlimited to permit unit sample interval.

Signal Detection

Processing Programs

In the experiments to determine the effects of multiplicative noise on signal detection methods, four different processing procedures were employed. Although separate computer programs were written to implement each of the methods, all of them followed the same general procedure illustrated by the flow chart in Fig. 10. The four programs are AVERAGE, MATCHED, MULTIPL, and POISSON. All the programs involve the convolution of some processing signal sp(m,n) with the message function, s(m,n). For the AVERAGE program the processing signal has unit amplitude over the domain of the hypothetical signal. This is the easiest and most common type of signal processing. The MATCHED program uses the definition of the signal itself as a weighting function. This is the classical matched filter and provides optimum processing when the noise is Gaussian distributed, additive and independent. The MULTIPL and POISSON programs use the optimum processing signals derived in Chapter 3. A copy of the MULTIPL program listing is included as Appendix II.

An important feature of these programs is that the message is read from magnetic tape on demand from the processing program. This keeps the required storage capacity of the computer at reasonably low level and permits arbitrarily long messages. Many experiments were run with messages of 80,000 words in length (100 x 800 matrix of samples). Processing on the CDC 6400 computer typically required

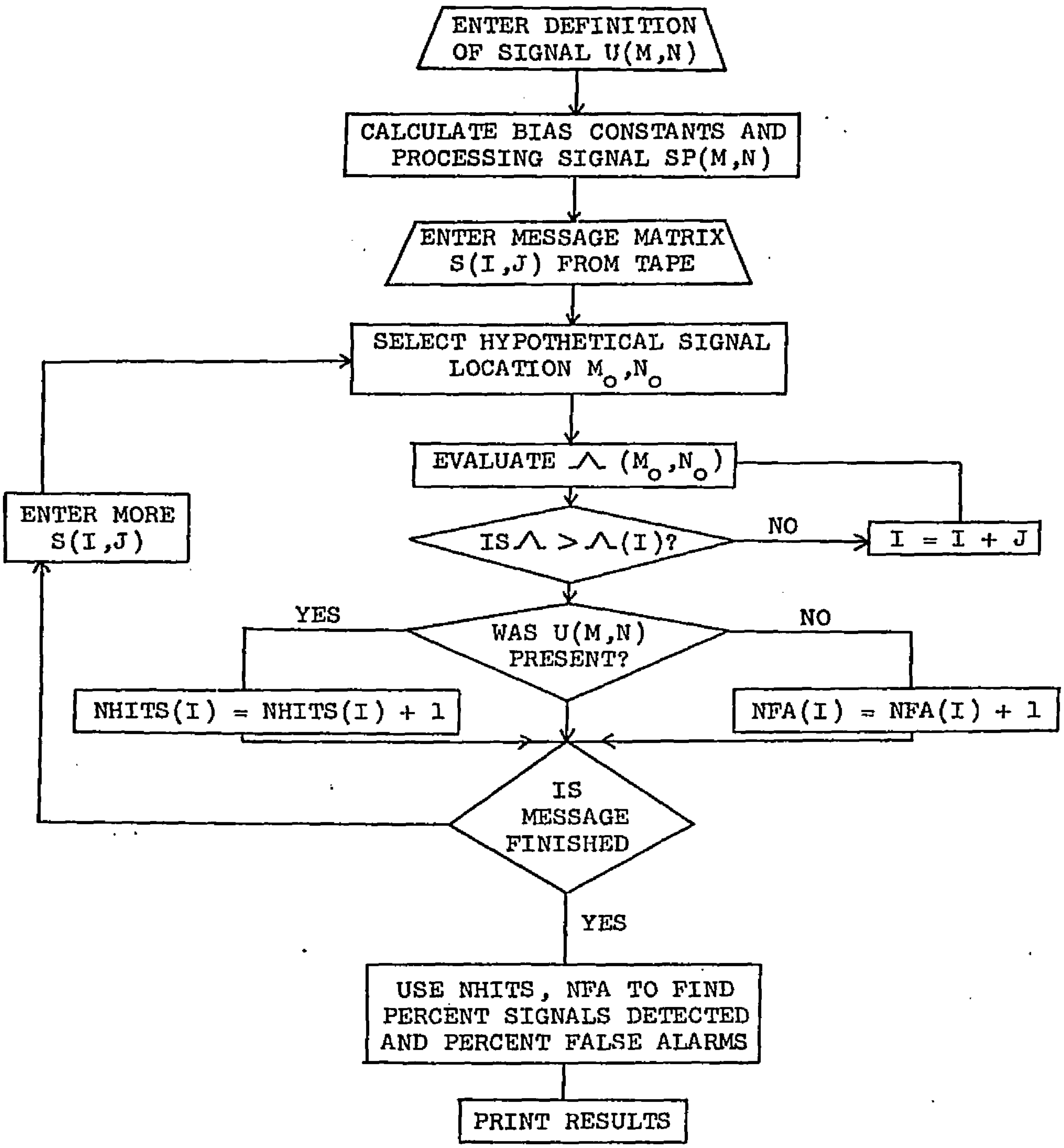


Fig. 10. Schematic Flow Chart for Typical Processing Program.

15 seconds of CPU time for the 80,000 word messages. Variation in processing time between the four programs was less than ten percent.

There were, typically, ten times as many independent hypothetical signal locations at which no signal was present as there were locations of actual signals. Messages with 80,000 words, for example, contained 200 signal locations and about 2500 other locations that were tested but had no signal present.

The value of the test statistic Λ, at each location was compared sequentially with as many as forty different threshold values--ranging from 10^{-1} to 10^{5}--to cover a wide range of decision levels.

The bias values (see section on Signal Detection in Chapter 3) were calculated differently for each of the four programs. Their inclusion is of little importance in most of the experiments.

A typical printed output of a processing program is shown in Appendix III.

Typical Comparison of Processing Methods

In an experiment of this type there are many parameters which must be specified. In particular, the signal type, noise distribution, background level, and signal-to-noise ratio all affect the results. While each of these parameters will be discussed, this section will be restricted to an experiment where the number of signals present is varied to determine the number needed for statistical validity. The rest of the variables are held constant. The procedures used are typical of those used in subsequent experiments.

The signals used were squares three sample units in dimension and bandlimited to avoid signal aliasing. The domain of the processing signal was chosen to be a 5 by 5 array of sample points. Points outside this region were all less than one percent of the peak signal value. Descriptions of the processing signals for the MATCHED program (which is also a description of the known signal), and the AVERAGE, MULTIPL and POISSON programs are shown below.

MATCHED

.0	.03	.03	.03	.0
.03	.93	.96	.93	.03
.03	.96	.99	.96	.03
.03	.93	.96	.93	.03
.0	.03	.03	.03	.0

AVERAGE

1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0

MULTIPL

.0	.03	.03	.03	.0
.03	.48	.49	.48	.03
.03	.49	.50	.49	.03
.03	.48	.49	.48	.03
.0	.03	.03	.03	.0

POISSON

.0	.03	.03	.03	.0
.03	.66	.67	.77	.03
.03	.67	.69	.67	.03
.03	.66	.67	.66	.03
.0	.03	.03	.03	.0

The size of the message was varied for each of three different runs. The first run contained 50 signals and 691 test locations not containing signals. The second run had 200 signals and 2821 test locations and the third run had 400 signals and 5642 test locations.

The noise in all three cases was Gaussian distributed with a variance proportional to the mean signal level. Specifically, if $u(m,n)$ is a description of the known signal (see MATCHED above) then the message at a signal location is given by

$$s(m,n) = u(m,n) + n(m,n)$$

where

$$u(m,n) = u'(m,n) + u_o$$

The background, u_o, was given by

$$u_o = 1.0$$

and the signal dependent noise variance by

$$\sigma_o^2\ (m,n) = u(m,n)$$

The purposes of this experiment are to see what effect, if any, the multiplicative noise model might have on a typical signal detection problem and to see what changes occur in these results as the number of signals tested is increased.

Using the detection curve discussed in Chapter 3 (see Fig. 6) the results of processing the 50, 200, and 400 signal messages using a matched filter based on independent Gaussian noise are shown in Fig. 11.

Note that in order to increase the graphical resolution, the detection curves are plotted on a log-log scale. The scale in the vertical direction extends a full decade more to reflect the increased

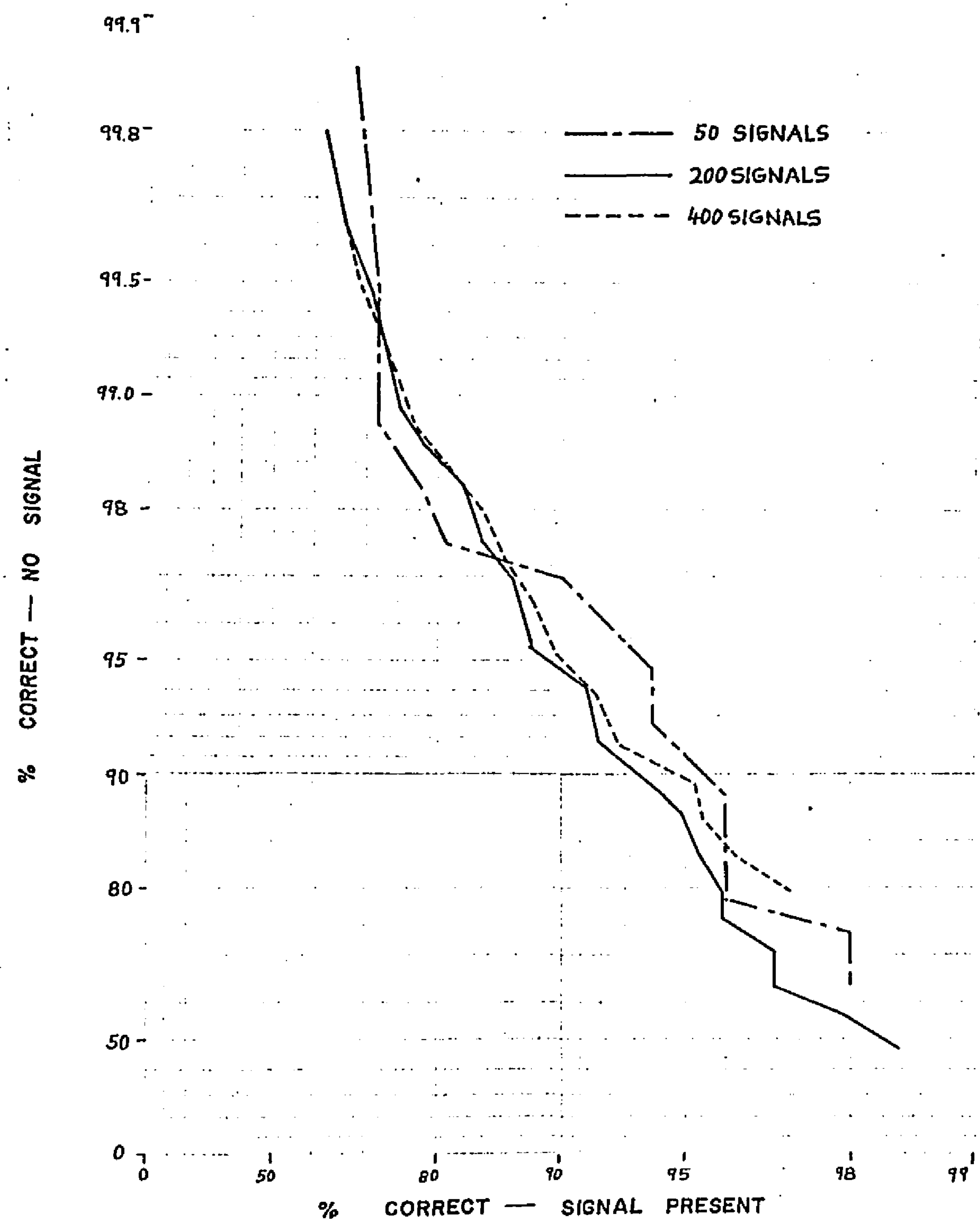


Fig. 11. Results of MTACHED Signal Detection on Different Message Sizes.

Processing parameters are identical for the three curves. Differences are due to inadequate numbers of signals for good statistical results.

precision due to the presence of ten times as many test locations without signal as with signals.

Before evaluating the statistical errors due to inadequate numbers it is first necessary to consider how well the plotted curves reflect what happened in the actual experiment. Fig. 12 shows an enlarged section of two typical detection curves. The solid connecting lines are purely hypothetical. Because of the nature of the digital computer processing, however, the points plotted for the different threshold values are essentially error free. Since the curves must be monotonically decreasing, the connecting line must lie somewhere between the dashed, rectangular error limit lines. Recalling that the coordinates can assume only a finite number of discrete values, the dotted line is a possible curve which might be obtained in the limit of a continuous range of threshold values. Thus, the ability to "resolve" two experimental curves can be determined by looking for overlap in the rectangular error limits between the plotted points.

Returning to the question of error due to insufficient statistics two conclusions can now be drawn from the curves shown in Fig. 11.

First, the agreement between curves is much worse at the ends than at the center. This is reasonable since only a small number of samples are involved in establishing these points. Second, to establish a curve over a reasonable range with a precision limited only by the errors due to the finite number of threshold values (sample points) used it is necessary to use a minimum of several hundred signals in the experiment.

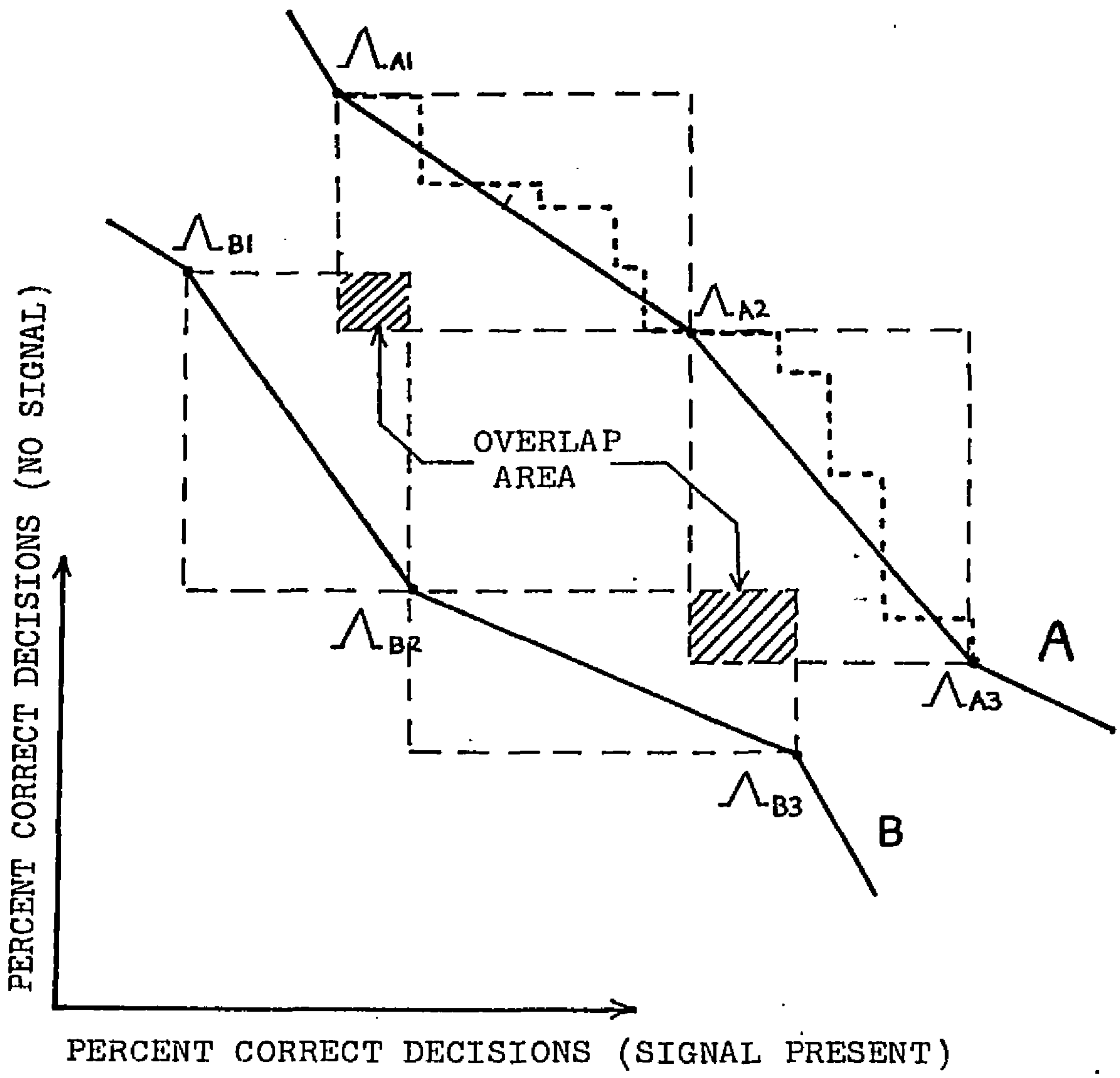


Fig. 12. Error Limits for Experimental Detection Curves.

Detection curve A can be said to be unequivocably better than B for this experiment over any range <u>not</u> containing any overlap area.

Before becoming resigned to the use of large numbers of signals, however, it should be noted that this analysis is only for establishing the absolute performance of one detection method applied to a message described by a particular set of noise statistics. If the goal is simply to find the relative performance under two different experimental conditions then the requirement on the number of samples needed is reduced.

Consider, for example, the relative performance of processing under the assumption of independent, Gaussian statistics (MATCHED program), processing assuming multiplicative Gaussian (MULTIPL), and multiplicative Poisson (POISSON) noise statistics. The results of these three programs applied to the message field containing only 50 signals is shown in Fig. 13. Because all of the methods are looking at the same noise values, it is possible to conclude that MATCHED and POISSON processing are essentially equal and that MULTIPL processing gives superior results for the parameters used in this one short experiment.

Finally, for a more convincing comparison of the three "optimum" signal detection methods plus processing by taking a simple average over the signal domain (AVERAGE program), the results of all four programs based on 400 signals is shown in Fig. 14. The dotted line represents the curve that would be obtained if decisions were made purely on a random basis.

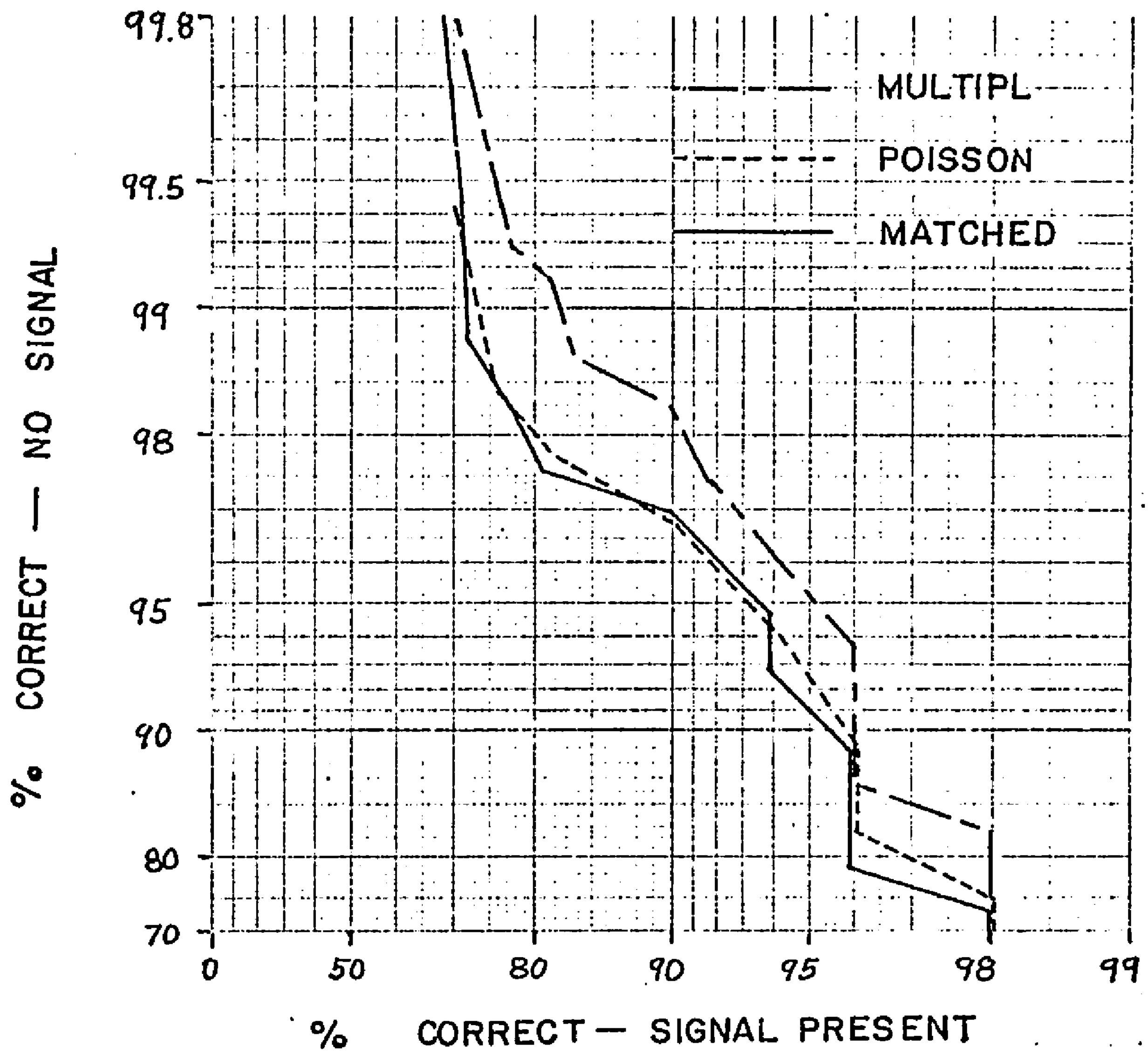


Fig. 13. Comparison of MATCHED, MULTIPL, and POISSON Processing for 50 Signals.

Relative performance is comparable to that obtained for much larger field sizes.

Conclusions based on the curves in Fig. 14 are: (1) all three "optimum" methods give better results than simple averaging, (2) matched filters based on independent Gaussian statistics and on multiplicative, Poisson statistics give essentially equal results, and (3) use of the matched filter based on multiplicative, Gaussian noise yields an improvement over the other methods. The magnitude of the improvement for the MULTIPL processing over the MATCHED processing is that for a given percentage of signals correctly detected, the false alarm rate will be about 25-30% lower for the MULTIPL processing on this type of message.

Effects of Signal-to-Noise Ratio

In general, the effects of increasing the signal-to-noise ratio in a signal detection problem are easy to predict. If the noise is signal independent, the increase in noise will have no effect on the method of processing or the shape of the matched filter. The detection curve will be shifted in location but no other changes should be expected. In the case of signal dependent noise, however, the situation is potentially more difficult. In addition to the shift in the detection curve as shown in Fig. 15, the shape of the processing signal also changes indirectly. Although there is no mathematical dependence of signal shape on the noise variance, the restrictions of the message to all positive values means that the only realistic way to have signal-to-noise ratios much less than one is to reduce the amplitude of the signal relative to the background level.

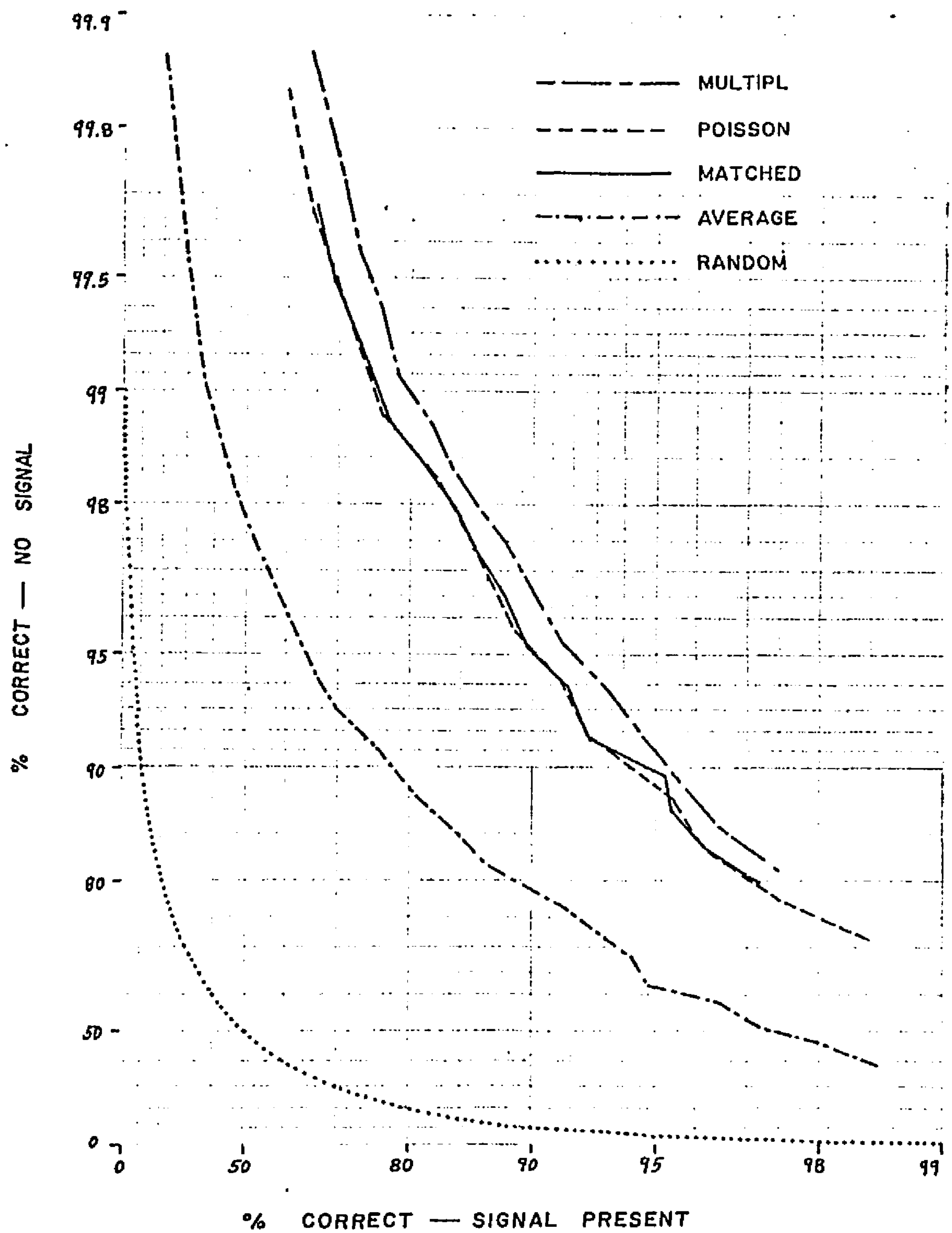


Fig. 14. Comparison of MATCHED, MULTIPL, POISSON, and AVERAGE Processing for a Large Signal Field.

Results are based on the processing of 400 signal locations and 5642 locations without signals. Dotted curve represents performance when decisions are made randomly.

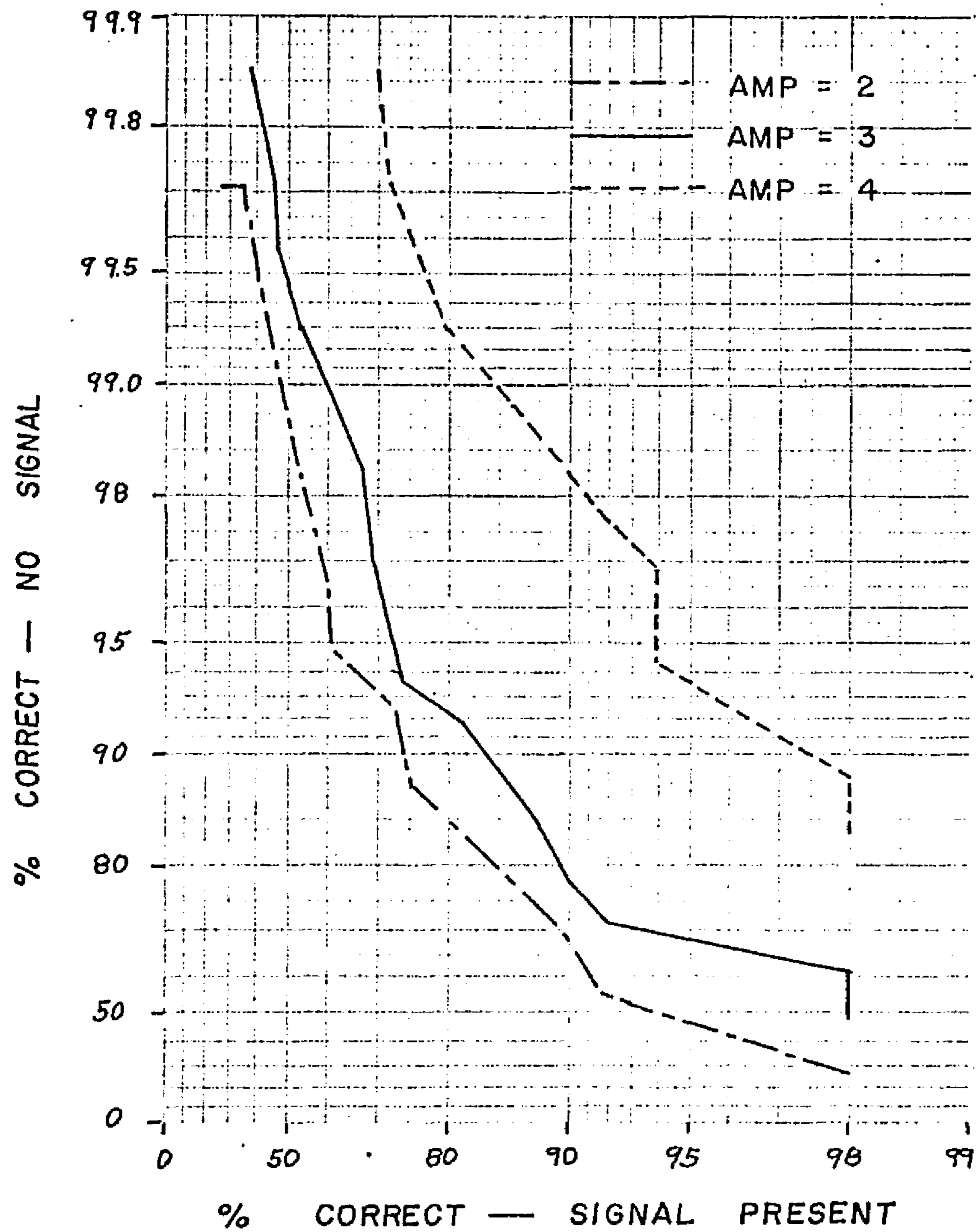


Fig. 15. Comparison of Detection Curves for Different Signal-to-Noise Ratios.

Processing of 50 bandlimited Gaussian signals ($\sigma = \sqrt{2}$). The processing signal is defined over a 5 x 5 square. AMP is the value of the aussian signal at its center. The background level and noise variance are both equal to one.

As the processing signal is defined by

$$sp(m,n) = u'(m,n)/(u'(m,n) + u_o)$$

it is clear that as the signal $u'(m,n)$ becomes small compared to the background, u_o, the processing signal simply becomes proportional to the signal itself. This is the same as optimum processing in the presence of signal independent, Gaussian noise. This is to be expected since when the signal-to-background ratio is small the message level--and hence the noise variance--is essentially constant.

From this argument it can be seen that the greatest difference in performance between the MATCHED, MULTIPL and POISSON programs should come when the signal-to-background ratio is much greater than one. This is unfortunate for two reasons. First, it is difficult to obtain reliable statistical information in detection problems where the signal-to-noise ratio is high because of the very low error probabilities, α and β (false alarm rate and miss rate). Secondly, this case is also the one of least importance in signal detection problems, as when the noise is very small, elaborate processing is not usually justified.

For these reasons, most of the experiments discussed in this report have a signal-to-noise ratio of approximately unity. The band-limited squares, for example, usually have unit amplitude and are recorded in the presence of noise with unit variance.

Effects of Signal Structure

Because of the shape of the optimum filter for signal detection changes according to the noise model, it is of interest to determine the dependence of signal shape on the effectiveness of different processing methods. The relationships between a known signal, $u'(x)$, and the processing signal which were developed in Chapter 3 for the three noise distributions are repeated below for convenience.

NOISE DISTRIBUTION	PROGRAM	PROCESSING SIGNAL
GAUSSIAN, INDEPENDENT	MATCHED	$u'(x)$
GAUSSIAN, MULTIPLICATIVE	MULTIPL	$\frac{u'(x)}{u'(x) + u_o}$
POISSON, MULTIPLICATIVE	POISSON	$\log_e\left[\frac{u'(x) + u_o}{u_o}\right]$

One might expect to observe the greatest difference in performance of the three methods when a signal is used that gives the greatest difference in the processing signals. The processing signals for the signal dependent noise models show the greatest distortion from the independent noise processing signal ($u'(x)$) when the known signal is large compared to the background, u_o. As discussed in the preceding section, however, the signals studied here will be restricted to those with magnitudes on the same order as the background.

The least difference in methods should be observed when the known signal has no structure. That is, a signal with a constant amplitude--but with an arbitrary domain--will transform into another constant amplitude signal over the same domain. Thus, the processing signals will be the same (except, possibly, for a difference in the bias level) for all three cases. Note that this does not necessarily mean that the processing programs will all give identical results. The MULTIPL program is operating on a message which has been squared.

Although most of the experiments in this study were conducted with bandlimited, three unit wide squares, a series of bandlimited, Gaussian signals (width $\sigma = \sqrt{2}$) was also investigated. Three runs of 50 Gaussian signals with amplitudes 2.0, 3.0 and 4.0 were processed and compared to the processing of 50, 3 x 3 bandlimited square signals. All four sets of data were recorded with the identical sequence of random noise values.

The purpose of this experiment was to determine if the advantage in the signal dependent noise processing observed with the 3 x 3 bandlimited squares is increased by the use of a signal with greater structure. A plot of the two basic signal shapes and their corresponding processing signal shapes is shown in Fig. 16. All signals are normalized to unity at the origin to permit a better comparison of their functional shapes. Note that the difference in the processing signals is greatest for the large amplitude Gaussian signal and least for the 3 x 3 bandlimited squares. Also, while the shape changes as a function of the amplitude of the Gaussian signal, this effect is relatively small compared to the differences due to the noise model.

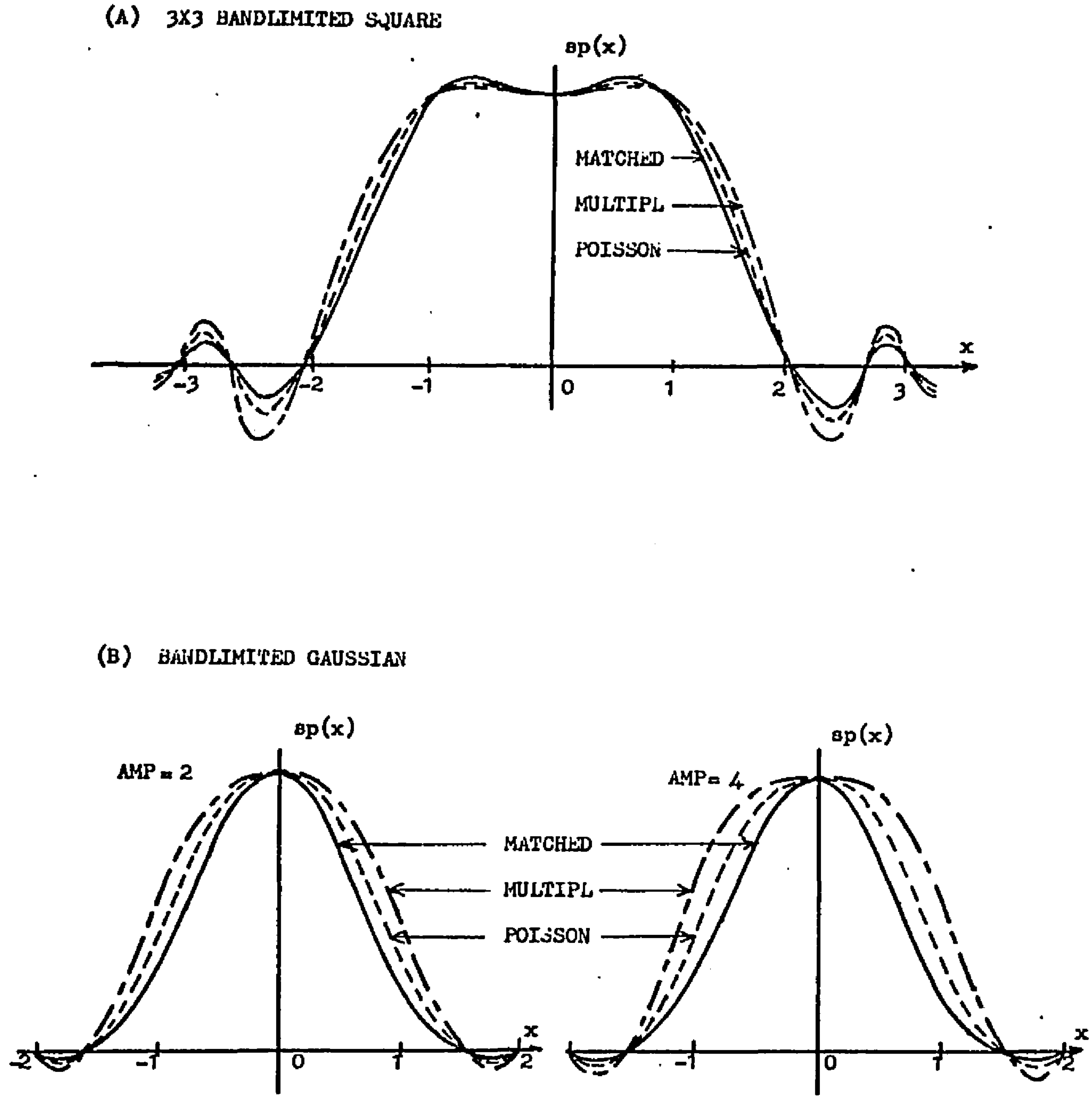


Fig. 16. Comparison of Processing Signal Shapes for Different Noise Models.

All curves are normalized to unity. The description of the known signal is the same as the MATCHED processing shape.

The results of processing these four sets of data were somewhat surprising. The detection curves for the 50 bandlimited squares were shown in Fig. 13 and are repeated here in Fig. 17 along with the curves for the 50 Gaussian signals with an amplitude of 3.0. The relative performance of the four programs was essentially the same for all three sets of Gaussian signals--hence, only one is shown. The most unexpected result is that the Gaussian signals show less distinction between processing methods instead of more, as was predicted. One explanation for this is that while there were nine nearly equally weighted points used in the definition of the bandlimited squares, most of the information about the presence of the Gaussian signal is concentrated in the single point at the origin. This is not important to the choice of the proper noise model to use for best detection, but it does mean that to obtain reliable statistical information about their detectability, many more test signals are required than are required for the bandlimited squares.

One conclusion does seem justified, however. The most important difference in optimum signal detection in the presence of multiplicative noise is not due to differences in the shape of the processing signals. This observation suggests that the advantage of the MULTIPL processing program which was observed in the section "Typical Comparison of Processing Methods", is due to the squaring of the message values before processing.

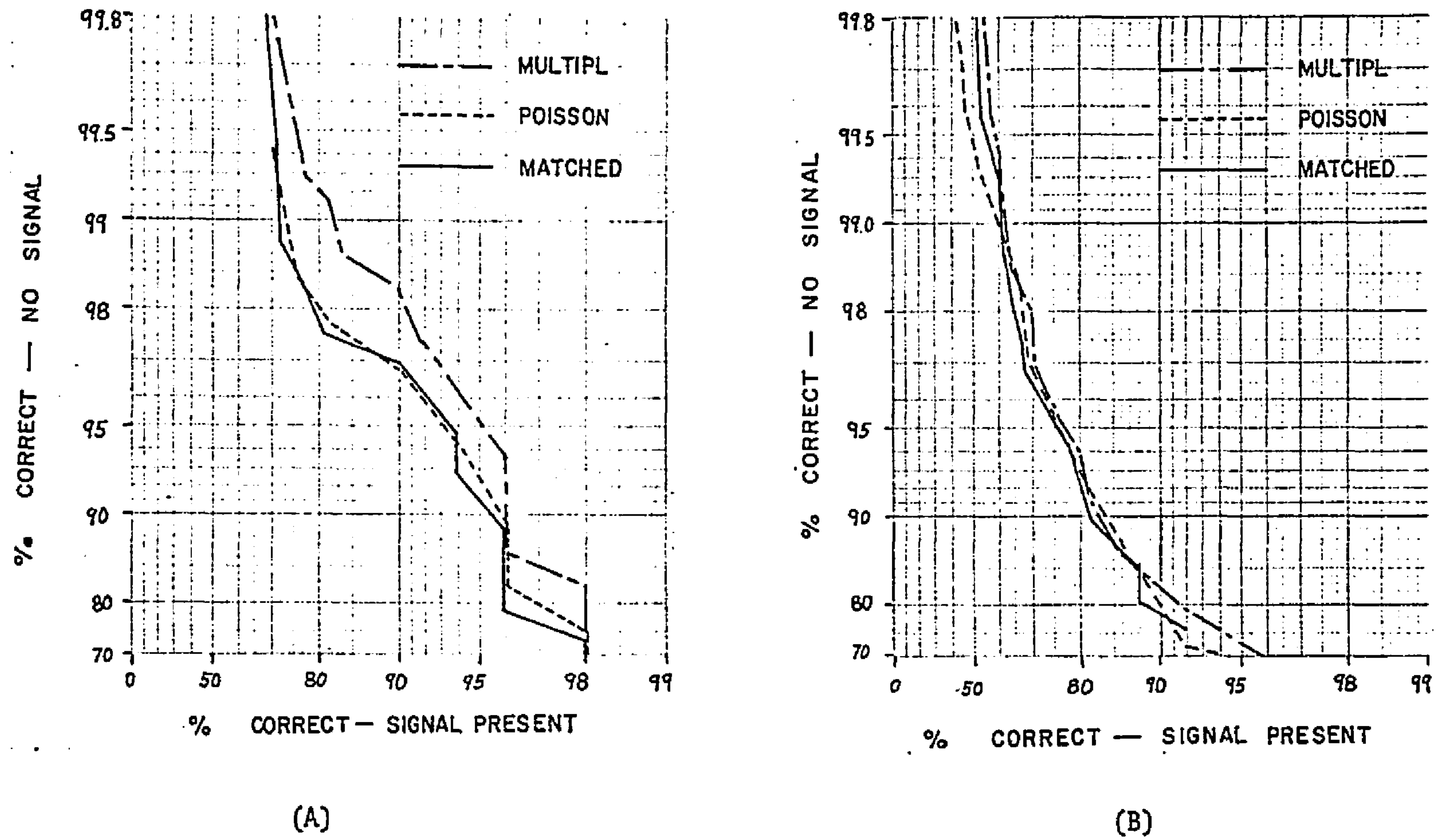


Fig. 17. Detection Curves for Two Different Signal Shapes.

(A) Shows processing results for 3 x 3 bandlimited square while (B) shows the results of the same processing methods for bandlimited gaussian signals ($\sigma = \sqrt{2}$).

Effects of Squaring Message

The hypothesis that the primary benefit in processing according to the multiplicative noise model comes in squaring the message amplitude before processing can be easily tested. A new processing program, SQUARED, was written that squared the message values as was done in MULTIPL, but then weighted all of the hypothetical signal values equally. This is equivalent to the AVERAGE program being applied to the squared message field. Any advantage over either the MATCHED or AVERAGE program demonstrated by the SQUARED program must be due only to the squaring operation.

The MATCHED, AVERAGE, and SQUARED processing programs were applied to a message field containing 400 3 x 3 bandlimited square signals. The resulting detection curves are shown in Fig. 18.

The detection curves show several interesting effects. First, the performance of the AVERAGE and SQUARED programs depend critically on the size of the area that is being averaged. For either of the two sizes used, however, the SQUARED program gives significantly better results than the AVERAGE program. When the averaged area was a 3 x 3 square the AVERAGE and MATCHED results are essentially equal and inferior to the SQUARED results. These observations confirm the hypothesis of the advantage of using a squared message field when in the presence of multiplicative noise and indicate that matching the processing signal shape exactly to the known signal is of minimal importance.

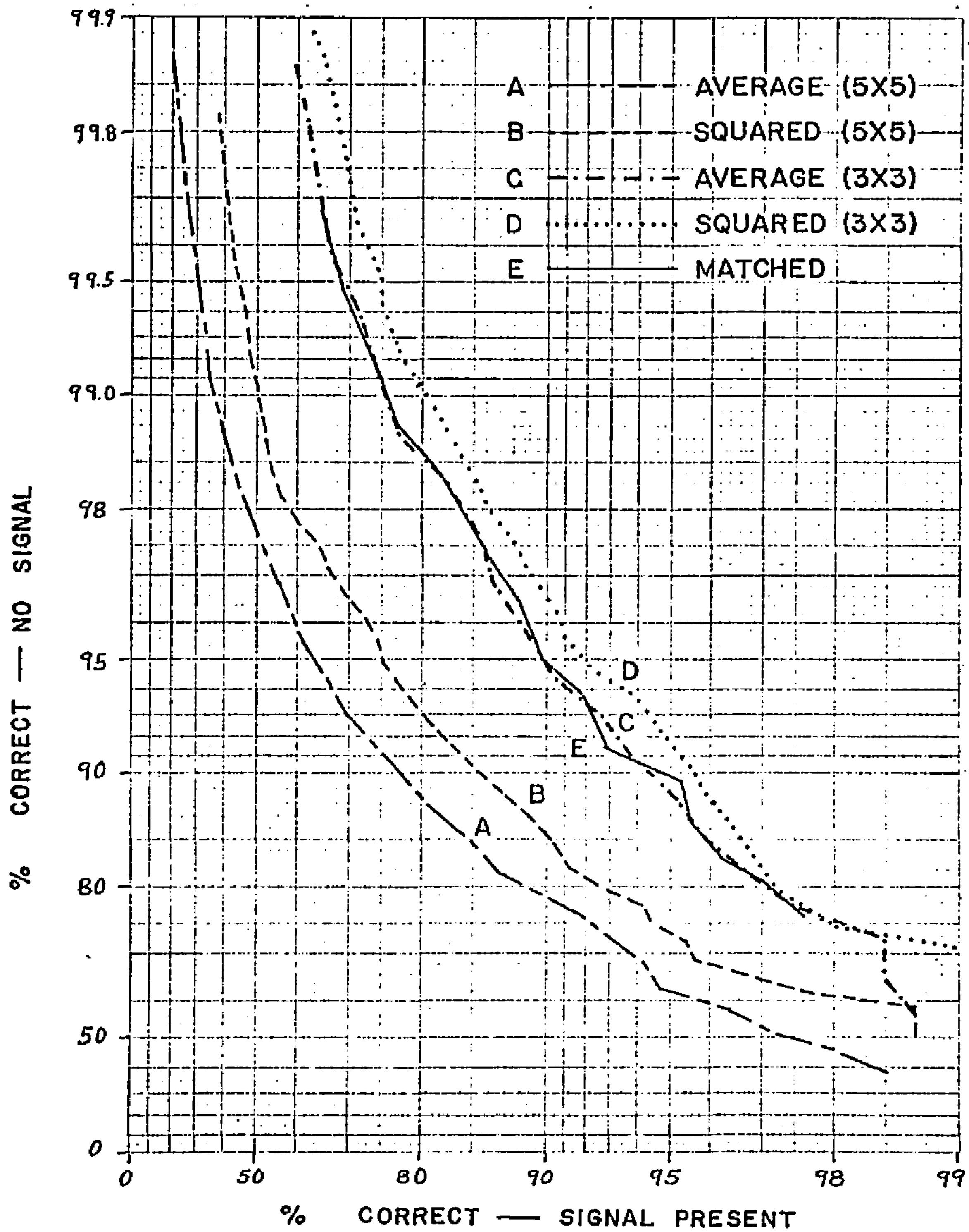


Fig. 18. Results of Processing by Averaging Squared Message Values.

Results are based on the processing of 400 3 x 3 bandlimited square signals. Curves A and B averages over a 5 x 5 square. Curves C and D are averages over a 3 x 3 square.

Effect of Changes in the Background Level

One of the problems that occurs when the signals are recorded on photographic emulsion is that the noise statistics change as the background density changes. This is true even when the individual signals are of low amplitude, and can be modeled by independent noise. The change in the background by itself can be compensated for by changing the bias term in the independent noise model but it is not clear what the effect of incorrectly estimating the new noise variance will be.

This question was investigated by simulating a field of 300 3 x 3 bandlimited square signals in the presence of multiplicative noise and with a changing background level. The exact field conditions were

	BACKGROUND u_o	NOISE VARIANCE σ_o^2	SIGNAL AMPLITUDE
FIRST 100 SIGNALS	0.5	0.163	0.2
SECOND 100 SIGNALS	0.75	0.200	0.2
THIRD 100 SIGNALS	1.0	0.231	0.2

These figures describe a field where the noise variance is everywhere given by

$$\sigma_N^2\ (x) \;=\; 0.231\ \bar{u}(x)$$

This field was processed by the four programs, AVERAGE, MATCHED, MULTIPL and POISSON. The MULTIPL and POISSON program always used the correct estimate of background and variance while the MATCHED program assumed a variance of 0.2 for the entire field. The results are shown in Fig. 19.

At first glance, the results appear inconclusive. In the regions of very low false alarm rate or very low miss rate the MULTIPL and POISSON processing appear to offer an advantage over the MATCHED processing. This advantage decreases in the region where the false alarm and miss rates are more nearly equal.

These observations are, in fact, predictable with the aid of some subtle arguments involving more decision theory than detection theory. Specifically, it can be shown that the effect of incorrectly estimating the noise variance is of no significance in a binary decision when the decision threshold has been chosen to make the two events equally probable. As one event becomes increasingly more likely, an error in the predicted decision boundary occurs and a slight decrease in the detection rate for that threshold follows. A more thorough explanation of this effect is outside the scope of this report. It should suffice to note that the behavior observed in the detection curves in Fig. 19 is supportable by theory and represents the magnitude of improvement that might be expected by using a multiplicative noise model for this detection problem.

Effects of Noise Distribution

In all of the preceding experiments the noise in the message obeyed a truncated, Gaussian distribution with a variance proportional

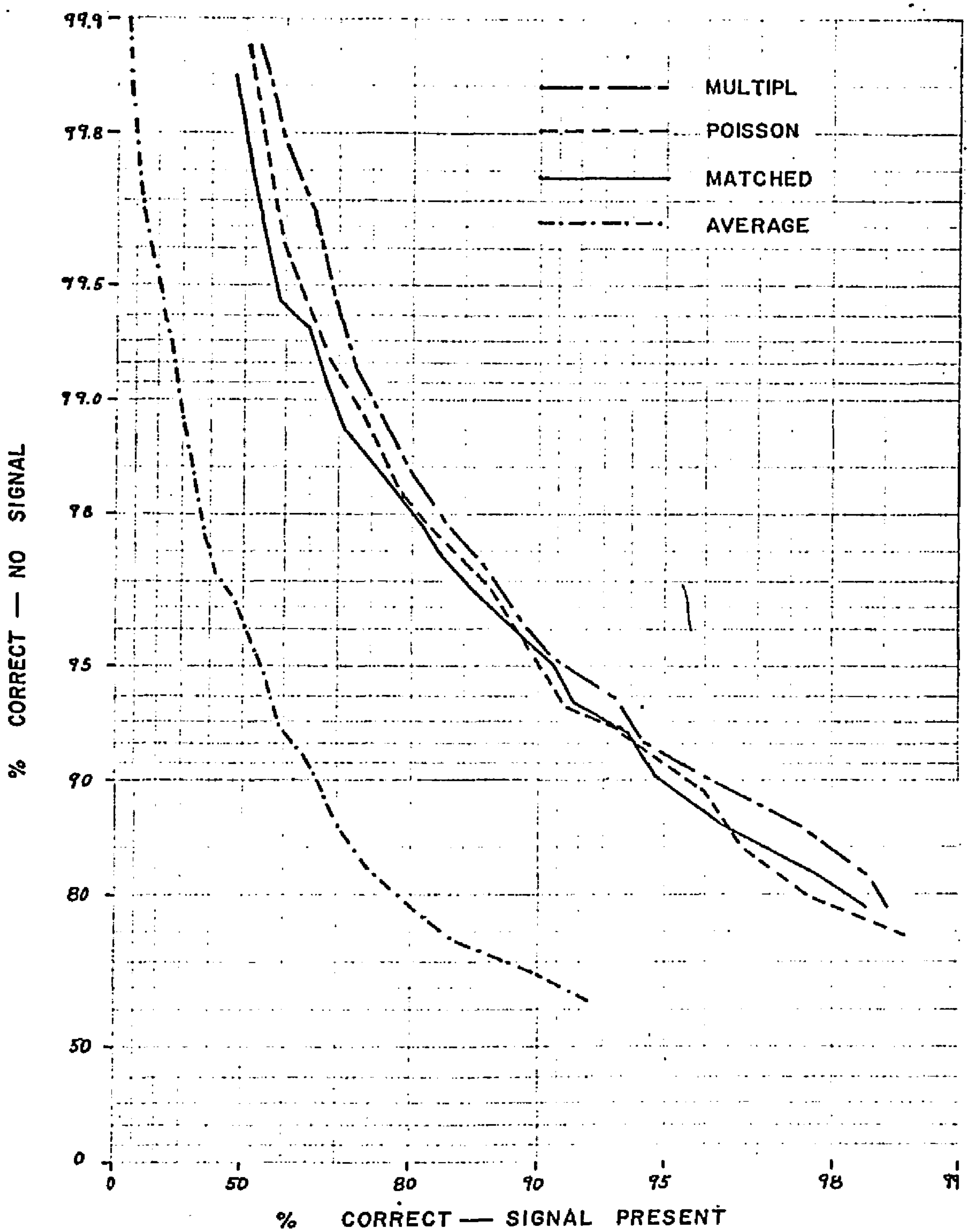


Fig. 19. Results of Processing Message with Varying Background Level.

Curves are based on 300 3 x 3 bandlimited square signals. Three different backgrounds were used, u = 0.5, 0.75, 1.0. The noise variance was given by $\sigma_N^2(x) = 0.231\,\bar{u}(x)$.

to the mean message level. The most frequently used distribution is illustrated in Fig. 20. While this is a reasonable model to apply to many physical processes, it is of interest to investigate the dependence of the processing methods proposed here to other noise distributions.

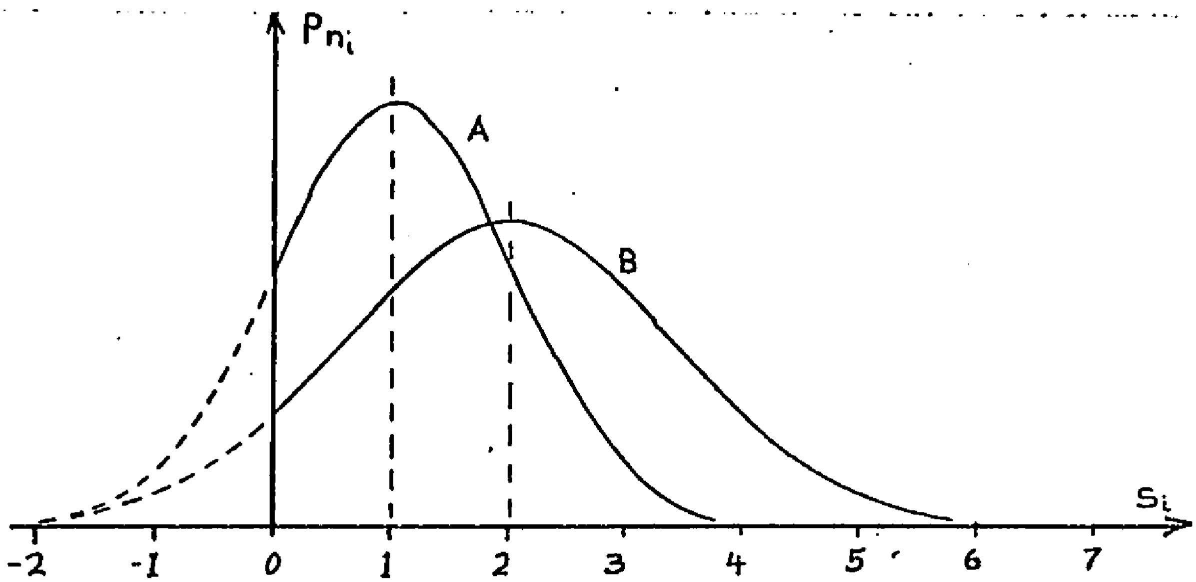


Fig. 20. Typical Gaussian Distributions of Noise Values.

The two curves are for (A) $p(n_i/s_i = 1.0)$ and (B) $p(n_i/s_i = 2.0)$ The variance is equal to s_i.

It would have been desirable to test the detection methods in the presence of pure Poisson noise. Because of the large number of points needed (approximately 100,000 each with different mean and variance) generating true Poisson noise would have been too costly. Instead, it was decided to use log normal distributed noise which is easier to generate and bears a close resemblance to the Poisson distribution.

A message containing 50 3 x 3 bandlimited square signals was recorded with a background equal to 1.0. The noise was log normal distributed with a variance equal to the mean signal level. The noise distribution for two points, $s_i = 1$ and $s_i = 2$, is shown in Fig. 21 and should be compared with the truncated Gaussian distribution shown in Fig. 20.

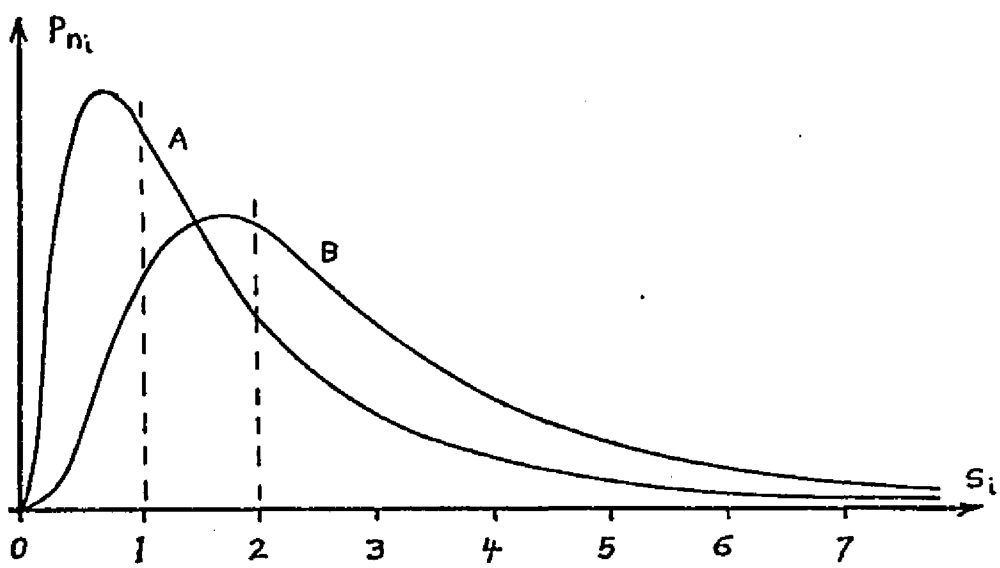


Fig. 21. Typical Log Normal Distributions of Noise Values.

The two curves are for (A) $p(n_i/s_i = 1.0)$ and (B) $p(n_i/s_i = 2.0)$ The variance is equal to s_i.

This field was processed and the resulting detection curves are shown in Fig. 22. The obvious conclusions are (1) the MULTIPL processing is significantly inferior to the MATCHED and POISSON programs

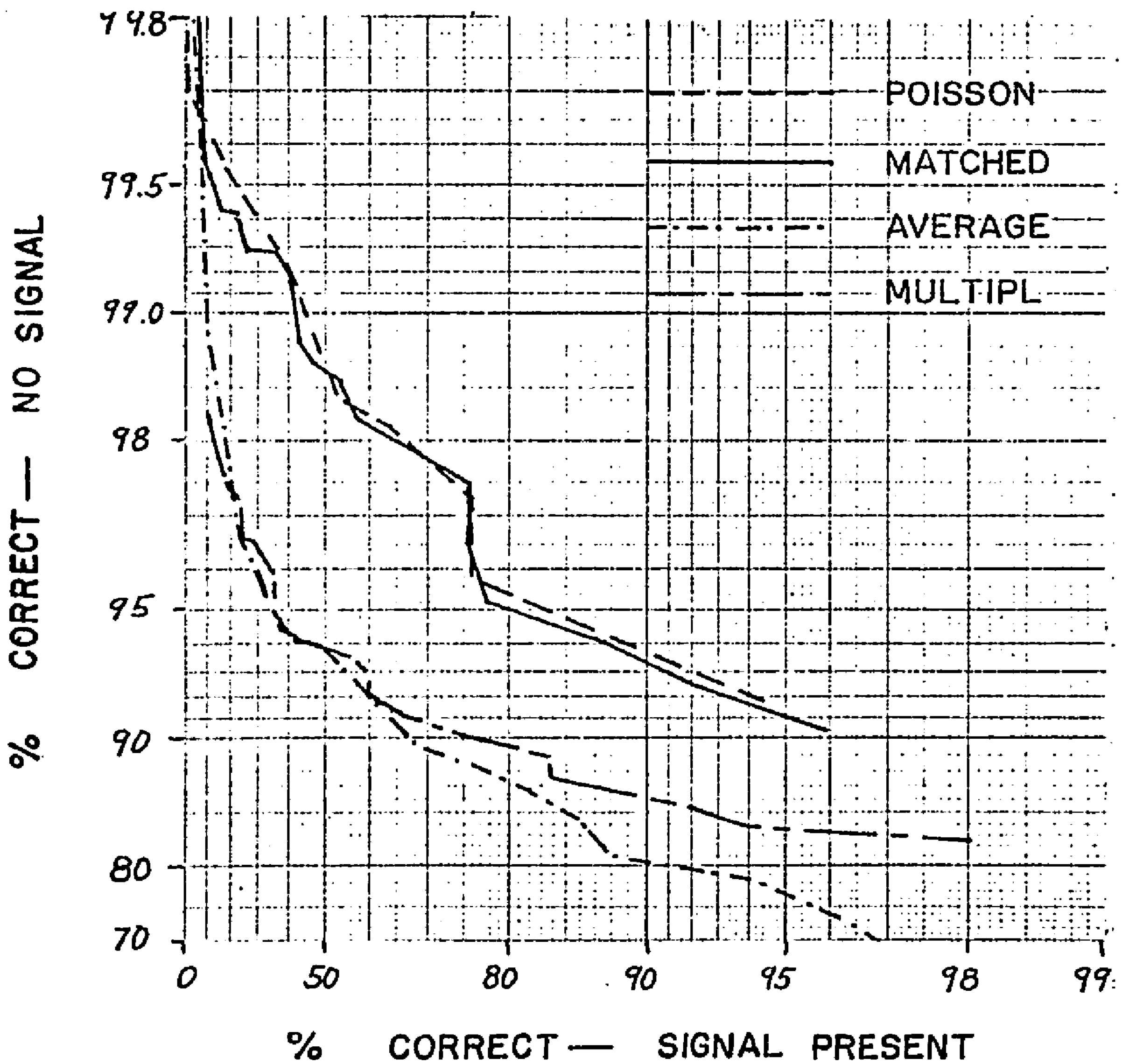


Fig. 22. Detection Curves for Signals in the Presence of Log Normal Noise.

Results are based on the processing of 50 bandlimited Gaussian signals of amplitude equal to 3.0 in noise with unit variance.

and (2) the MULTIPL performance is relatively worse in the region where a low false alarm rate is used. Both of these conclusions can be seen intuitively by a careful examination of Fig. 21. The probability of receiving a noise value when no signal is present that is many times larger than the average level when the signal is present is quite high. Squaring these message values accentuate this problem and makes the elimination of all false alarms very difficult.

These results emphasize the importance of accurately knowing the noise statistics in a message before attempting to process.

Signal Discrimination

The theory of optimum signal discrimination is discussed in Chapter 3. It is mentioned there that the problem of discriminating between the occurrence of two (or more) known signals in the presence of noise can be considered as an extension of the general signal detection problem. If brief, a field containing only two signals of known shape should be processed by convolving the message with a signal that is related to the difference of the two signals. More specifically, the optimum processing signal for Gaussian, stationary noise is

$$sp(m,n) = \frac{1}{\sigma_o^2}\,[u1(m,n) - u2(m,n)]$$

and for multiplicative, Gaussian noise is

$$sp(m,n) = \frac{u_o}{2\sigma_o^2}\,\left[\frac{1}{u2(m,n)} - \frac{1}{u1(m,n)}\right]$$

where $u1(m,n)$ and $u2(m,n)$ are descriptions of the known signals.

An experiment was conducted using 100 bandlimited circles of a radius giving both signals an integrated area of 25. The signals had unit amplitude and were recorded in the presence of Gaussian, multiplicative noise with unit variance and background level. Cross sections of two signals are shown in Fig. 23 along with the optimum processing signals described by the equations above.

The MATCHED and MULTIPL programs were revised to handle the signal discrimination problem. They were applied to the combined field of bandlimited circles and squares and the resulting discrimination curves are shown in Fig. 24.

Fig. 24 shows that processing on the assumption of multiplicative noise does give fewer errors than the independent noise model. The improvement is on the order of 10% fewer wrong decisions. This appears to break down in the regions above 90% correct decisions for either of the signal types but this is probably due to the small number of samples (less than 10) involved in establishing these points.

Simple Filtering

The general topic of filtering as a method of processing is much too broad to be covered comprehensively in a study of this type. The purpose of including this topic here is to present some simple examples of signal processing in the presence of signal-dependent noise by operating in the frequency domain.

Continuous Detection Filtering

In the section, Signal Detection, message fields are processed to determine if a known signal is present at some specific location.

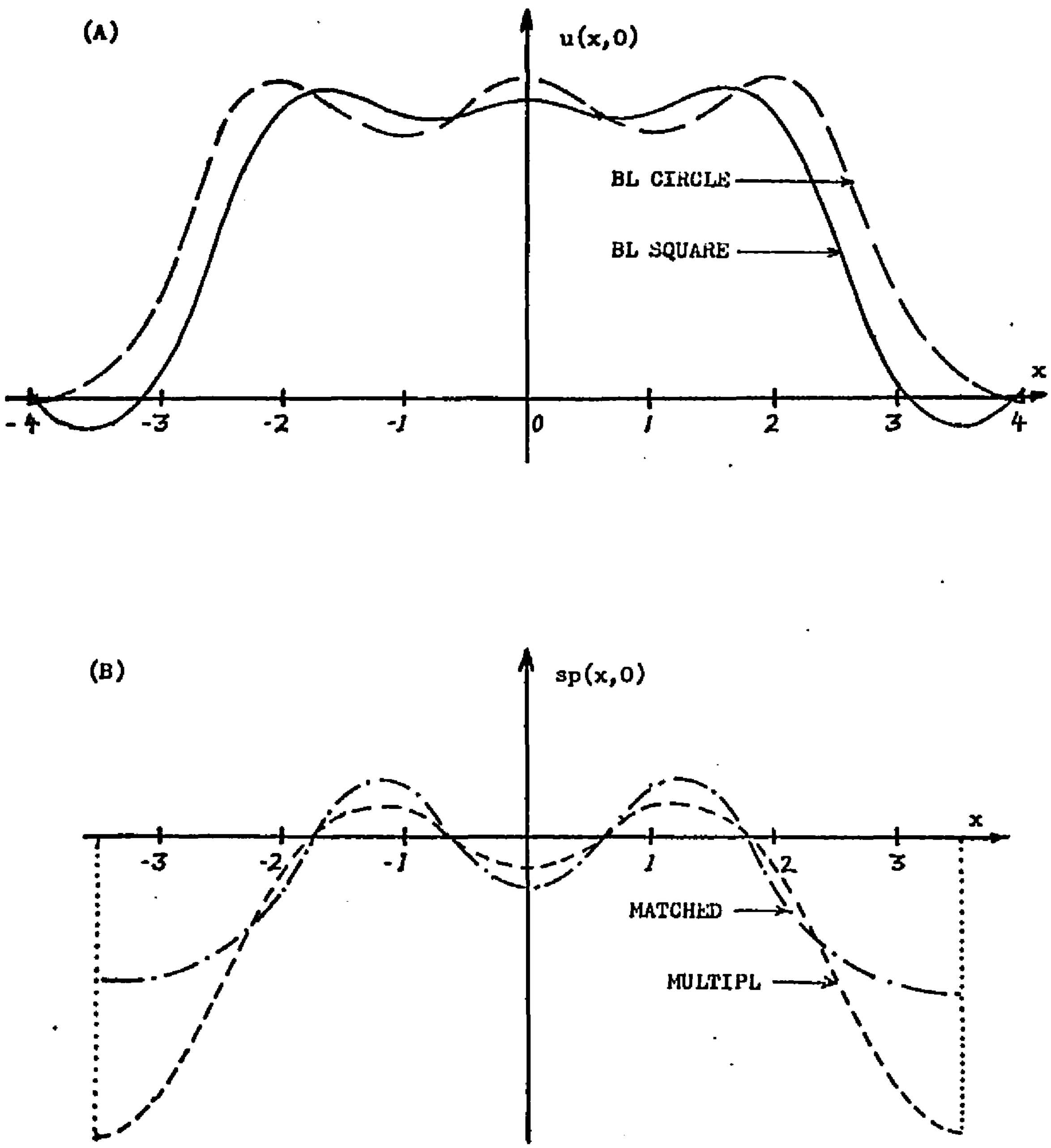


Fig. 23. Optimum Signal Discrimination--Bandlimited Circles and Squares.

Part A shows cross sections of the reconstruction of the bandlimited, sampled signals. Part B shows similar reconstructions of the optimum processing signals for stationary and multiplicative, Gaussian noise.

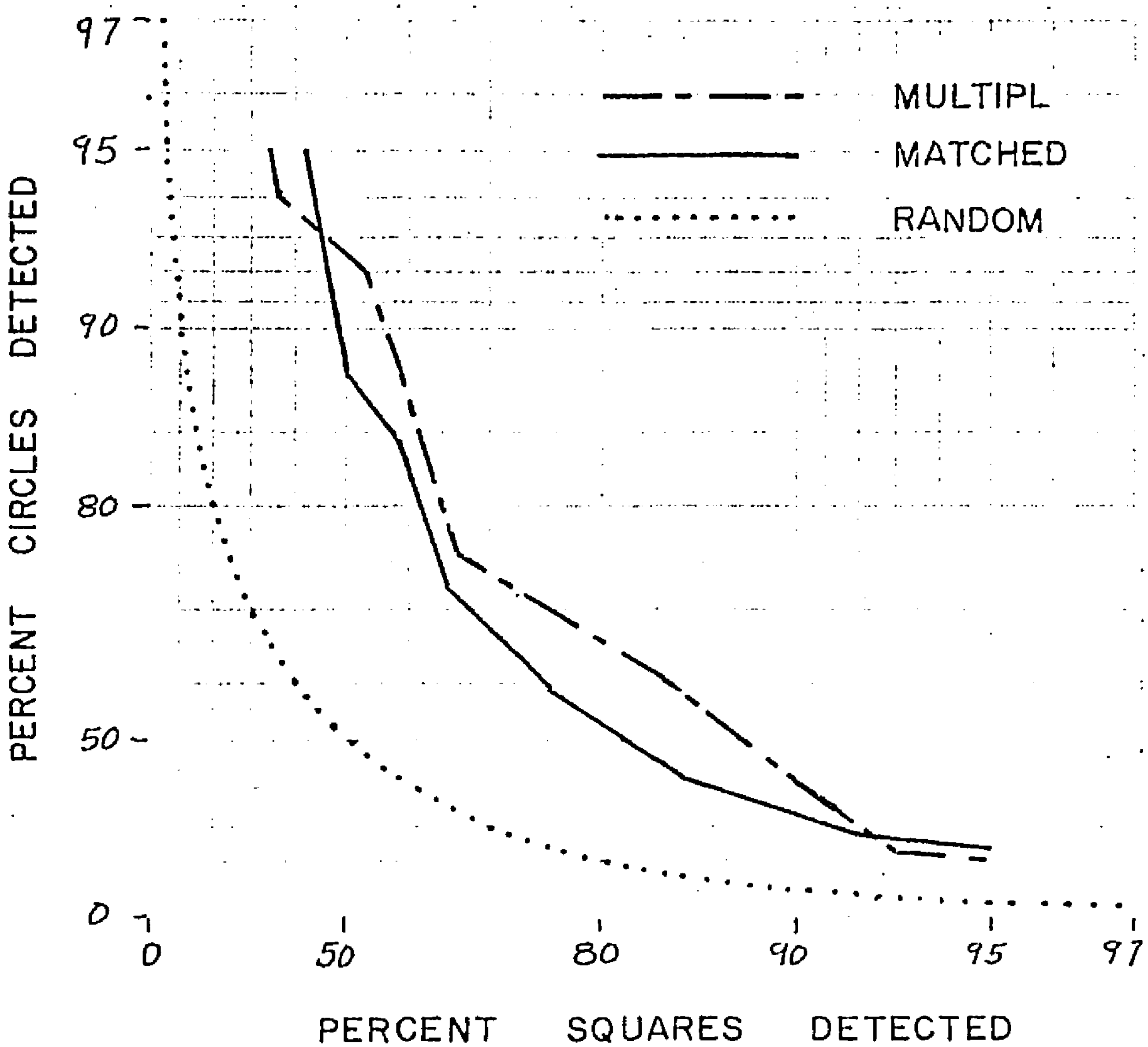


Fig. 24. Discrimination Curves for Bandlimited Circles and Squares.

Results of process 100 bandlimited circles and 100 bandlimited squares--all of equal signal energy--in the presence of multiplicative, Gaussian noise. Approximate signal-to-noise ratio is one.

This is done by a convolution type of process where the convolution is evaluated only at the locations of interest. If the location of the known signal is not known it becomes necessary to evaluate the convolution process for every point in the field. If the system under consideration is defined in a way which preserves linearity (note that the multiplicative, Gaussian noise model requires squaring the message field) then this processing may be performed by a simple filtering operation in the frequency domain. The filter is described by the Fourier transform of the convolving processing signal.

In the case of signal-independent noise, the matched filter is just the complex conjugate of the Fourier transform of the signal itself. When the noise is multiplicative, Gaussian, or Poisson distributed, the filter becomes the complex conjugate of the Fourier transform of the optimum processing signals derived in Chapter 3. The interpretation of this filter is not so simple. Not only are the Fourier transforms of the processing signals difficult to find in general, but they vary as a function of the noise level. To illustrate this effect, a one dimensional cosine wave was used as the object and the matched filter was calculated for varying levels of multiplicative, Gaussian noise. The results are shown in Fig. 25.

In the limiting case of the background, u_o, being much larger than the signal, $u(x)$, it can be seen that the filter becomes identical to that used for signal-independent noise. As u_o becomes small (for this example it cannot be less than $u'(0)$ because of the positive

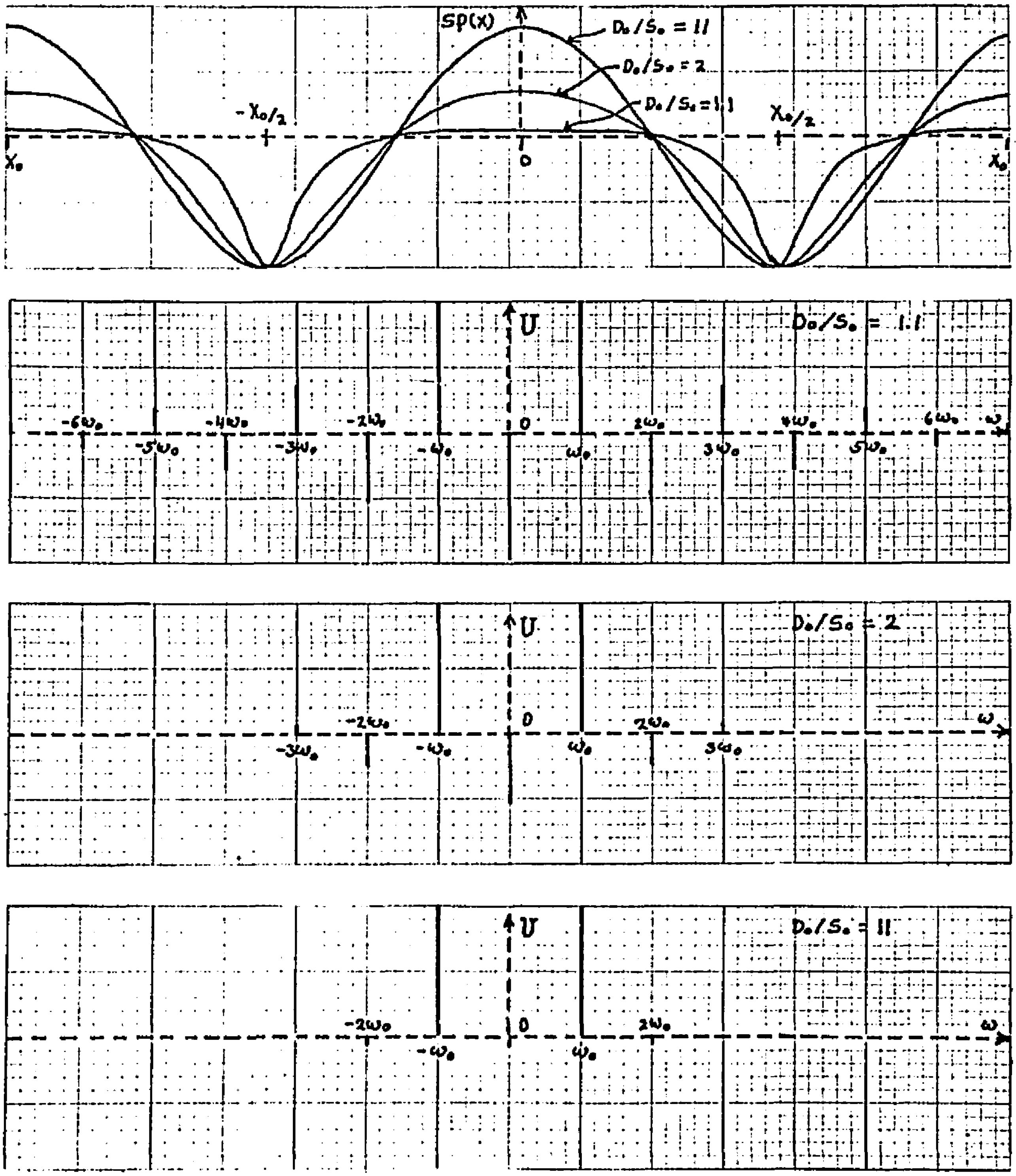


Fig. 25. Matched Filter for Single Frequency in Gaussian Multiplicative Noise.

U_o is the background level. The signal is given by $u'(x) = s_o \cos(2\pi f_o x)$.

message restriction) the processing signal and its Fourier transform depart markedly from the independent noise case. It should also be remembered that the message being filtered in this example is $s^2(x)$ where $s(x) = u'(x) + u_o + n(x)$.

The principles of continuous filtering for optimum signal detection have been applied to a real problem.[14] A section of the sky that contained several weak star images was photographed on a Kodak 103a-D photographic plate. The density profile from a portion of the plate was sampled and recorded on magnetic tape (approximately 20,000 readings). The field was then processed using the CDC 6400 computer by passing it through a matched filter for a weak star image in the presence of multiplicative, Gaussian noise. The output at each point in the field was compared to ten different threshold values and an appropriate number was assigned to each. Contour lines of equal probability of occurrence were then drawn on the digitized output. The results are shown in Fig. 26.

Figures 4A and 4B are both processed fields. Fig. 4A has one additional lower contour level. Fig. 4C is a contour plot of the original field before processing with threshold levels which can be compared to those in 4B. Note that some of the weaker spots in 4C disappear in 4B whereas others are enhanced. This action is presumably the discrimination of weak signals from noise. Unfortunately, due to the nature of the original field it is impossible to verify the results.

(A)

(B)

(C)

Fig. 26. Continuous Optimum Signal Detection Filtering of a Star Field.

(A) Contours of equal probability that a star was present centered at that point, (B) same output with lowest contour eliminated, and (C) isodensity contours of original field.

Bandpass Filtering

Filtering for purposes of detecting the presence of a known signal is a relatively specialized problem. More commonly, the precise shape of the signal is unknown and the problem is to filter out as much of the noise as possible while leaving the signal spectrum relatively undistorted. When the noise is stationary, the filtering procedure is well established and, with the aid of the Fourier transform, is easily conceptualized. When the noise is signal dependent the situation is less clear.

In Chapter 3 a model was presented to aid in visualizing the effects of operations in the frequency domain of a message containing signal-dependent noise. To test the validity and usefulness of this model, a simple message was recorded on Kodak 35 mm Tri-X film and placed in a coherent optical filtering system similar to the one illustrated in Fig. 6, Chapter 3.[15] The object transparency, photographed in coherent light, is shown in Fig. 27-A. When this transparency is placed in a coherent beam (He-Ne laser) the transmitted amplitude can be written as

$$s_t(x,y) = u_t(x,y) + n_t(x,y)$$

where u_t is the ideal two-level signal. The statistics of the transmission noise, n_t, are dependent on the signal level. Specifically, if the film density is assumed to have a noise distribution which is Gaussian and a variance which is proportional to the mean density level, then it can be shown that if the transmitted field, u_t, is described

(A)

(B)

(C)

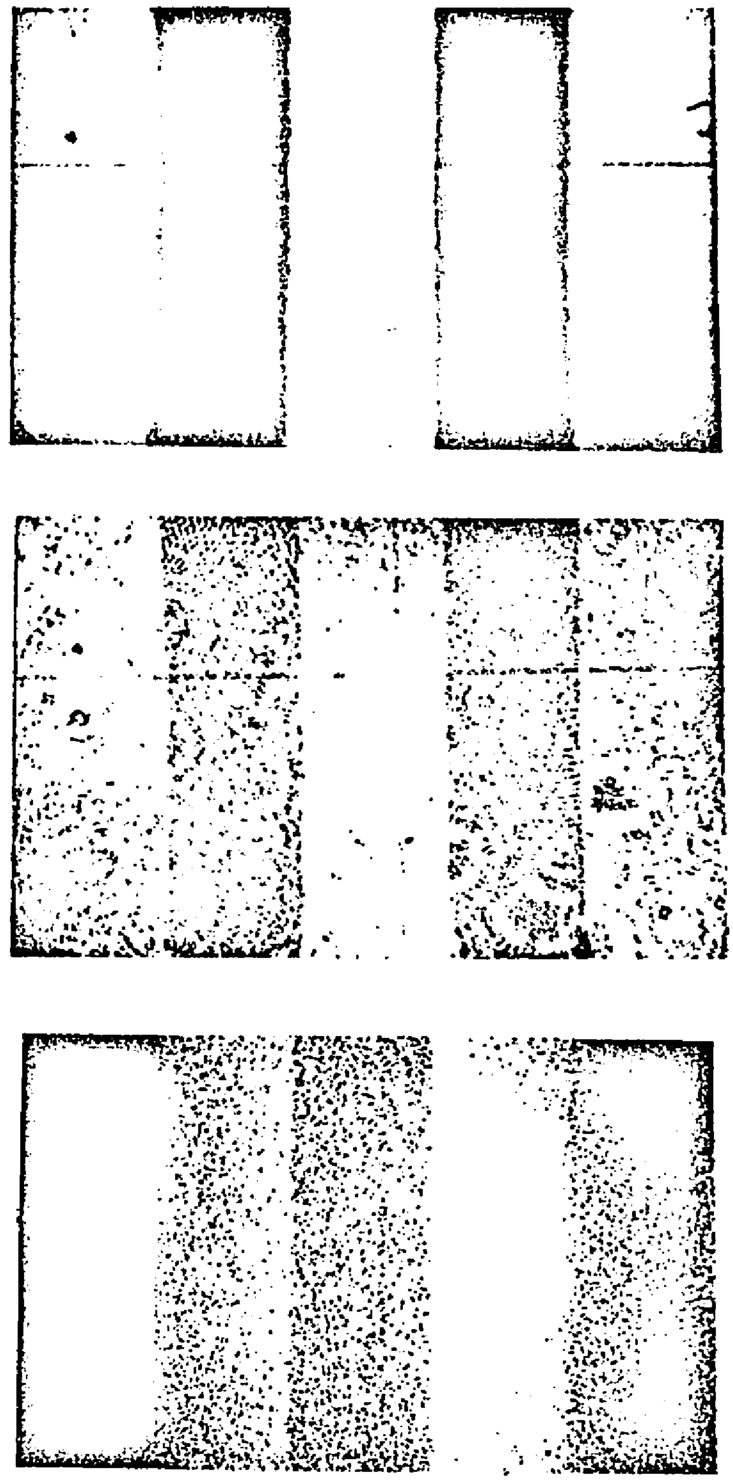


Fig. 27. Comparison of Images in Signal-Dependent Noise.

All images are approximately 15X magnification. Conditions are (A) incoherent illumination, (B) coherent illumination, and (C) coherent illumination using noise spectrum only.

by

$$u_t = u_{tmax} e^{-ku_D}$$

then the variance of the field is given by

$$\sigma_t^2(x,y) \propto u_{tmax}^2 - \bar{u}_t^2(x,y)$$

where u_{tmax} is the transmitted field at zero density and $\bar{u}_t$ is the expected transmission. With this noise distribution and the aid of the Multiplicative Noise Model presented in Chapter 3, it is possible to consider the transmitted field as the independent sum of the ideal coherent, transmitted signal field and an incoherent field of intensity given by σ_t^2 (x,y). In the transform plane of the coherent optical filtering system the signal transform is superimposed on the partially coherent field from the noise term.

Figures 27-B and 27-C are recordings of the image plane for two different pupil (filter plane) configurations. Figure 27-B is the ordinary coherent image obtained by passing all spatial frequencies in the f/10 system as shown in Fig. 28-A. Fig. 27-C is the image obtained when a portion of the frequency spectrum containing no information from the signal term is used. This filter condition is shown in Fig. 28-B.

In comparing the photographs in Fig. 27, it is significant to notice that the contrast has reversed in Fig. 27-C. This is due to the use of light from the signal-dependent noise term only which is described by $\sigma_t^2(x,y)$.

This contrast reversal was not obtained for all pupils which excluded the signal spectrum. For lower noise frequencies this simple model typically breaks down due, apparently, to the exclusion of the effects of phase noise in expressing the transmitted field in terms of film density. A more thorough study of these effects would be desirable.

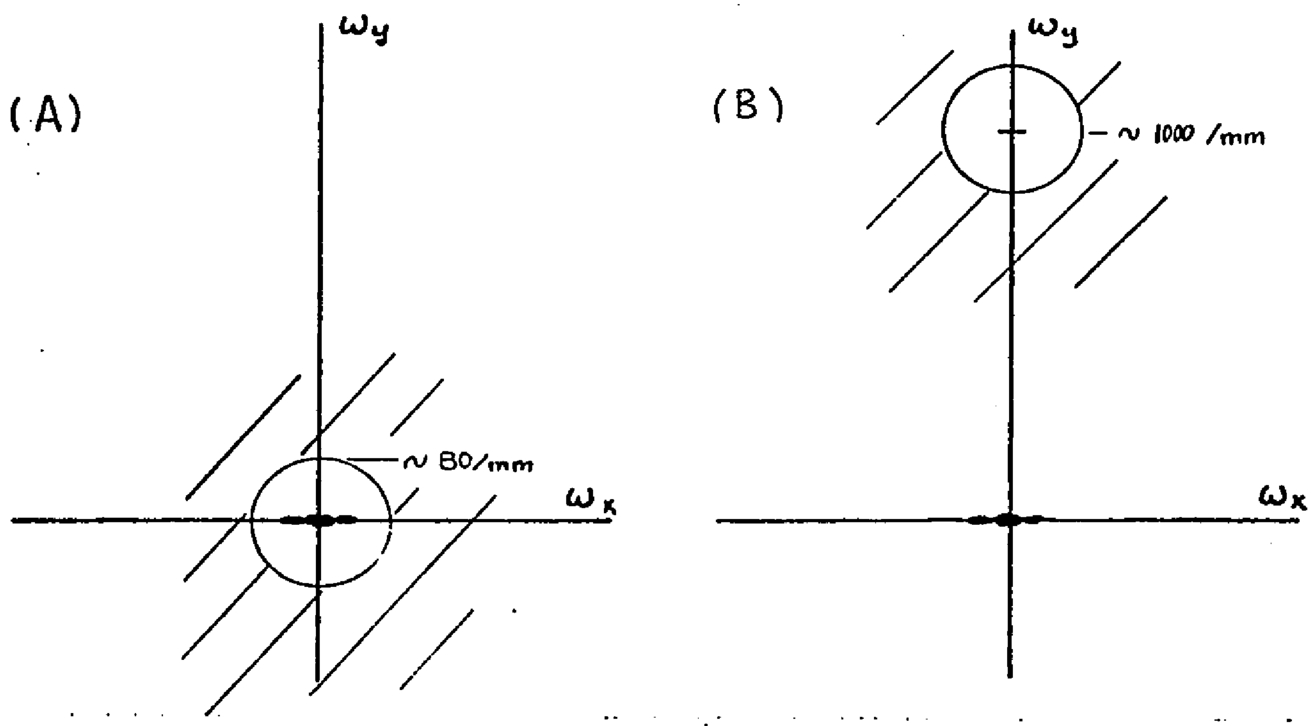


Fig. 28. Illustration of Filter Plane Conditions.

(A) is the configuration used in obtaining the photograph in Fig. 27-B (B) was used for Fig. 27-C.

CHAPTER 5

CONCLUSIONS

The first objective of this study was to develop a better understanding of the significance of signal-dependent noise. While all of the results presented here help to achieve this goal, the section "Filter Theory" (Chapter 3) and the experiment described in "Bandpass Filtering" (Chapter 4) are of particular interest.

In Chapter 3, a method for simulating a message recorded in noise of any arbitrary signal dependence by using a coherent optical imaging system was introduced. This method follows from the observation (first suggested by A. Lohmann in 1965) that the expression for the mutual intensity of a partially coherent field is mathematically equivalent to a noise autocorrelation function.[13] Thus, white noise becomes equivalent to an incoherent field with an intensity everywhere proportional to the noise variance. The Fourier Transform of the incoherent field describes the noise spectrum. The power spectrum of the message can be observed as the irradiance distribution in the back focal plane of the first lens in the system.

The experiment reported in Chapter 4 illustrates both the signal-dependent nature of film grain noise and the utility of the above model in predicting the effects of signal-dependent noise in a simple bandpass filtering system. It shows that even though there is useful information in the noise spectrum about the signal, care must be taken in filtering the spectral components because the reverse contrast of the "noise" image would normally subtract from the contrast of the "signal" image.

It would be of interest to explore this model further by using it to predict or measure the nature of the signal-dependence of grain noise in other types of emulsions.

The second objective of this study is to derive statistical tests for the optimum detection of signals recorded on photographic film. The sections "Signal Detection" and "Signal Discrimination" in Chapter 3 are addressed to this problem.

It was found that for the detection of a known signal in the presence of multiplicative Gaussian noise, the optimum processing of a sampled message is obtained by generating the test statistic given by

$$\Lambda = \sum_{i=1}^{N} s_i^2 \cdot \frac{u_i - u_o}{u_i}$$

Where the known signal is described by the N values $\{u_i\}$, the sampled message is described by $\{s_i\}$ and u_o is the background level when no signal is present. When the multiplicative noise is described by Poisson statistics, the optimum test statistic is found to be

$$\Lambda = \sum_{i=1}^{N} s_i \cdot \log_e(u_i/u_o)$$

When discriminating between two signals, $\{s_{1i}\}$ and $\{s_{2i}\}$, the optimum test statistics become

$$\Lambda = \sum_{i=1}^{N} s_i^2 \cdot \left(\frac{1}{u_{2i}} - \frac{1}{u_{1i}} \right)$$

for multiplicative, Gaussian statistics and

$$\Lambda = \sum_{i=1}^{N} s_i \cdot \log_e(u_{1i}/u_{2i})$$

for Poisson statistics.

The work done in simulating signal detection problems indicates that these tests do indeed yield improvements in the detection rate when applied to systems with noise distributions of the type typically found in photographic emulsions. It is possible to extend this study to cover such problems as the optimum detection of signals with unknown phase, the detection of signals of unknown location and the estimation of signal parameters. Although solutions to these problems for the case of multiplicative noise are not presently available in published form, they can be obtained by using the results of Chapter 3 and paralleling the solutions already developed for additive, signal-independent noise.

The last objective of this study is to explore the practical limitations of these new tests. The computer-simulated experiments in Chapter 4 were performed to determine under what conditions, if any, the new processing methods would provide an advantage over the more commonly used methods which are based on additive, stationary noise.

When a message is recorded in the presence of Gaussian, multiplicative noise, several conclusions can be reached. First, a definite advantage over processing based on the signal-independent noise assumption (on the order of 20-30% increase in the detection rate) is observed for signals processed according to the Gaussian, multiplicative noise model. This advantage was observed using signals recorded at a signal-to-noise ratio of approximately one. As the signal-to-noise ratio is increased, the advantage increases slightly but the need for sophisticated processing techniques is usually decreased. As the signal-to-noise ratio is decreased, all processing methods tend to become equivalent. Little effect is observed as the structure of the known signal is changed. For the case of Gaussian, multiplicative noise, it appears that the primary processing advantage stems from the squaring of the received message rather than from the differences in the shape of the filters. This is an important observation as it suggests that very nearly optimum performance can be obtained by squaring the message and then using a simple average over the signal as the test statistic.

When the tests are made on messages recorded in multiplicative noise distributed according to log normal statistics, the results change dramatically. Processing these data according to the multiplicative, Gaussian distribution assumption yields clearly inferior results. The Gaussian, stationary processing and the Poisson, multiplicative processing gave essentially identical results. This presents a paradox. Intuitively, if the Gaussian, multiplicative processing is used on a message recorded with multiplicative noise, then it should give better results than the Gaussian, stationary processing. This argument is apparently false, however, as it cannot be supported theoretically and is not observed experimentally.

These results provide evidence of the importance of knowing the correct noise statistics before attempting to process. This is particularly important when working with photographic film since the associated grain noise statistics vary greatly as a function of film type, exposure, development, method of measurement, and many other parameters.

The brief experiment on signal discrimination served to illustrate the technique. The conclusions derived from the signal detection work are also applicable to the discrimination problem.

The results of Chapter 4 can now be applied to any of a large class of signal detection problems involving photographic film. It is now clear that the noise statistics of any photographically recorded message must be carefully measured. If the grain noise is multiplicative and Gaussian distributed, the processing methods studied here might (depending on other parameters in the specific problem) be of significant value. If, on the other hand, the noise statistics are found to be multiplicative and Poisson distributed it appears that retention of the more frequently used, additive, signal-independent noise assumption is likely to be justified.

The objectives set forth in the Introduction of this report have been met. A better understanding of the significance of signal-dependent noise has been developed, new optimum statistical tests have been proposed for use on photographic films, and their limitations have been explored. One specific application has been included as an example. Studies of additional specific applications are recommended as the next area of activity.

APPENDIX I

SIMULAT LIST

```
PROGRAM    SIMULAT  FORTRAN EXTENDED VERSION 2.0/C        01/28/70              10

              PROGRAM SIMULAT (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
              DIMENSION  DBLSM(21,21),CM(10),CN(10),DM(21),S(22,22)
        C              READ AND WRITE PARAMETERS FOR UNIT SIGNAL CELL
              READ(5,10) NUMBER,NSUBX,NSUBY
        10    FORMAT (3I3)
              WRITE(6,15) NUMBER,NSUBX,NSUBY
        15    FORMAT(1H1,40X,* SIMULATION NUMBER *,I3////20X,*DESCRIPTION OF UNI
             1T SIGNAL CELL*//* NUMBER OF ROWS  NSUBX =*,I3/* NUMBER OF COLUMNS
             2 NSUBY =*,I3//)
        C                DESCRIBE NON-BANDLIMITED SIGNAL
              READ(5,20) SIG1,SIG2,XO,YO
        20    FORMAT(2A10,2F10.0)
              WRITE(6,25)SIG1,SIG2,XO,YO
        25    FORMAT(1H0,20X,*DESCRIPTION OF SIGNAL BEFORE BANDLIMITING*//
             2* TYPE OF SIGNAL*,5X,2A10/* SIGNAL CENTER AT XO =*,F5.1,5X,
             3*YO =*,F5.1//)
              DO 30 M=1,NSUBX
              DO 30 N=1,NSUBY
              RM=M
              RN=N
              S(M,N)=0.
        30    IF(ABS(RM-XO).LT.1.5.AND.ABS(RN-YO).LT.1.5) S(M,N)=1.
              WRITE(6,34)
        34    FORMAT(1H0,* SAMPLED DESCRIPTION OF S(X,Y)*//)
              DO 36 M=1,NSUBX
              WRITE(6,35)(S(M,N),N=1,NSUBY)
        35    FORMAT(/5X,21F6.2)
        36    CONTINUE
        C                DESCRIBE COEFFICIENTS OF SPECTRUM OF S(M,N)
              READ(5,40) LIMX,LIMY
        40    FORMAT(2I3)
              WRITE(6,45) LIMX,LIMY
        45    FORMAT(//1H0,20X,*SIGNAL SPECTRUM PARAMETERS*//
             1* HIGHEST HARMONIC IN COLUMN DIRECTION   LIMX=*,I3/
             2* HIGHEST HARMONIC IN ROW DIRECTION   LIMY=*,I3//)
              RSUBX=NSUBX
              DO 50 M=1,LIMX
              RM=M
        50    CM(M)=3./20.*SIN(3.*3.1416*RM/RSUBX)/(3.*3.1416*RM/RSUBX)
              CMO=3./20.
              WRITE(6,55) CMO,CM
        55    FORMAT(* HARMONIC COEFFICIENTS IN COLUMN DIRECTION*//(F8.4))
              RSUBY=NSUBY
              DO 60 N=1,LIMY
              RN=N
        60    CN(N)=CM(N)
              CNO=CMO
              WRITE(6,65) CNO,CN
        65    FORMAT(//* HARMONIC COEFFICIENTS IN ROW DIRECTION*//(F8.4))
        C            CALCULATE UNIT BANDLIMITED SIGNAL MATRIX
        C            SIGNAL IS REAL, EVEN, AND SEPARABLE
              FACTRX=6.2832/NSUBX
              FACTRY=6.2832/NSUBY
              WRITE(6,69)
        69    FORMAT(///20X,*DESCRIPTION OF UNIT BANDLIMITED SIGNAL MATRIX*/)
              DO 86 M=1,NSUBX
              RM=M
              DO 80 N=1,NSUBY
```

```
          RSUBY=NSUBY
          DO 60 N=1,LIMY
          RN=N
   60     CN(N)=CM(N)
          CNO=CMO
          WRITE(6,65) CNO,CN
   65     FORMAT(//* HARMONIC COEFFICIENTS IN ROW DIRECTION*//(F8.4))
   C               CALCULATE UNIT BANDLIMITED SIGNAL MATRIX
   C               SIGNAL IS REAL, EVEN, AND SEPARABLE
          FACTRX=6.2832/NSUBX
          FACTRY=6.2832/NSUBY
          WRITE(6,69)
   69     FORMAT(///20X,*DESCRIPTION OF UNIT BANDLIMITED SIGNAL MATRIX*/)
          DO 86 M=1,NSUBX
          RM=M
          DO 80 N=1,NSUBY
```



```
          RN=N
          SUMX=0.
          DO 70 I=1,LIMX
          RI=I
   70     SUMX=SUMX+CM(I)*COS(FACTRX*(RM-XO)*RI)
   C               IF S(X,Y) IS SYMMETRIC, DO LOOP 71 CAN BE SKIPPED
          SUMY=0.
          DO 71 J=1,LIMY
          RJ=J
   71     SUMY=SUMY+CM(J)*COS(FACTRY*(RN-YO)*RJ)
   80     UBLSM(M,N)=(CMO+2.*SUMX)*(CNO+2.*SUMY)
          WRITE(6,85) (UBLSM(M,N),N=1,NSUBY)
   85     FORMAT(/5X,21F6.2)
   86     CONTINUE
   C               ENTER PARAMETERS OF COMPOSITE MATRIX
          READ(5,110) NSIGX,NSIGY
  110     FORMAT(2I3)
          WRITE(6,115) NSIGX,NSIGY
  115     FORMAT(///20X,*PARAMETERS OF COMPOSITE MATRIX*//
         2* NUMBER OF ROWS OF SIGNALS     NSIGX=*,I3/
         3* NUMBER OF COLUMNS OF SIGNALS     NSIGY=*,I3//)
          READ(5,120)AMP
  120     FORMAT(F10.0)
          WRITE(6,125) AMP
  125     FORMAT(//* SIGNAL AMPLITUDE     AMP = *,F5.2/)
   C               ENTER NOISE PARAMETERS
          READ(5,130) SIGMAO,DZERO,TYPE1,TYPE2
  130     FORMAT(2F10.0/2A10)
          WRITE(6,135) TYPE1,TYPE2,DZERO,SIGMAO
  135     FORMAT(///20X,*NOISE PARAMETERS*//
         2* TYPE OF NOISE DISTRIBUTION*,5X,2A10/
         3* MEAN BACKGROUND LEVEL     DZERO=*,F8.4/
         4* STANDARD DEVIATION AT DZERO     SIGMAO=*,F8.4///)
          NFLAG=0
   C               CALCULATE NOISEY MEMBERS OF COMPOSITE MATRIX
          READ(5,136) NTAPE,NREC
  136     FORMAT(I5,I3)
          WRITE(6,137) NTAPE,NREC
  137     FORMAT(1H0,20X,*DESCRIPTION OF COMPOSITE MATRIX*//
         2* RECORDED ON TAPE NUMBER  *,I5,10X,*RECORD NUMBER  *,I3/)
          CALL RANSET(2)
          DO 139 I=1,NSUBX
          DO 139 J=1,NSUBY
  139     UBLSM(I,J)=AMP*UBLSM(I,J)+DZERO
          CONST=(SIGMAO**2)/DZERO
          NROW=0
          DO 150 M=1,NSIGX
```

```
          DO 150 I=1,NSUBX
          NROW=NROW+1
          DO 150 N=1,NSIGY
          IF(UNIT(1)) 140,196,198
   140    DO 144 J=1,NSUBY
          SIGMAS=CONST*UBLSM(I,J)
          IF(NFLAG.EQ.1) GO TO 142
   141    X1=RANF(1)
          X2=RANF(2)
          RNOISE=-2.*ALOG(X1)
          X2=6.832*X2
```

PROGRAM SIMULAT FORTRAN EXTENDED VERSION 2.0/C 01/28/70 1

```
          DM(J)=UBLSM(I,J)+SQRT(SIGMAS*RNOISE)*COS(X2)
          NFLAG=1
          IF(DM(J)) 142,144,144
   142    DM(J)=UBLSM(I,J)+SQRT(SIGMAS*RNOISE)*SIN(X2)
          NFLAG=0
          IF(DM(J)) 141,144,144
   144    CONTINUE
          BUFFER OUT(1,1)(DM(1),DM(NSUBY))
   150    CONTINUE
          GO TO 200
   196    WRITE(6,197)
   197    FORMAT(* END OF FILE DETECTED ON BUFFEO*)
          GO TO 200
   198    WRITE(6,199)
   199    FORMAT(* PARITY ERROR DETECTED ON BUFFEO*)
   200    STOP
          END
```

APPENDIX II

MULTIPL LIST

```
PROGRAM       MULTIPL  FORTRAN EXTENDED VERSION 2.0/C          01/28/70           1

                  PROGRAM MULTIPL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
                  DIMENSION COMPAR(50),THRESH(50),ID(50),NFA(50),NHITS(50)
                  DIMENSION S(5,5),SP(5,5),DM(5,130),DP(130)
                  INTEGER FLAG
                  LOGICAL LOGICA,LOGICB
                  COMMON/READER/KWORKT,DENS,NSUBY,NFA,NHITS,ID,NLEVEL,COMPAR,THRESH
                  COMMON/ROWOUT/NPTSY,DP,NMAX,INTERY,NCENTY,LOGICA,LOGICB
            C        READ AND WRITE RUN NUMBERS
                  READ(5,10) NRUN,NSIM
            10    FORMAT(2I3)
                  WRITE(6,15) NRUN,NSIM
            15    FORMAT(1H1,30X,*PROCESSING ASSUMING MULTIPLICATIVE NOISE*///
                 1 * RUN NUMBER *,I3/* INPUT DATA FROM SIMULATION NUMBER *,I3//)
            C        READ AND WRITE INPUT DATA DESCRIPTION
                  READ(5,20) SIG1,SIG2,TYPE1,TYPE2,DZER,SIGMA
            20    FORMAT(4A10/2F10.0)
                  WRITE(6,25) SIG1,SIG2,TYPE1,TYPE2,DZER,SIGMA
            25    FORMAT(20X,*DESCRIPTION OF INPUT DATA*//
                 1 * TYPE OF SIGNAL=     *,2A10/* TYPE OF NOISE=     *,2A10/
                 2 * BACKGROUND DENSITY=     DZERO = *,F6.3/
                 3 * STANDARD DEVIATION OF NOISE=     SIGMAO = *,F6.3//)
            C        READ AND WRITE PROCESSING INFORMATION
                  READ(5,30) NSUBX,NSAMPX,NSUBY,NSAMPY,NMAX,MMAX
            30    FORMAT(6I3)
                  WRITE(6,35) NSUBX,NSAMPX,NSUBY,NSAMPY,MMAX,NMAX
            35    FORMAT(20X,*PROCESSING INFORMATION*//
                 1 * NUMBER OF ROWS IN UNIT SIGNAL MATRIX=     NSUBX = *,I3/
                 2 * NUMBER OF ROWS IN DATA MATRIX=     NSAMPX = *,I3/
                 3 * NUMBER OF COLUMNS IN UNIT SIGNAL MATRIX=     NSUBY = *,I3/
                 4 * NUMBER OF COLUMNS IN DATA MATRIX=     NSAMPY = *,I3/
                 5 * NUMBER OF ROWS IN PROCESSING SIGNAL=     MMAX = *,I3/
                 6 * NUMBER OF COLUMNS IN PROCESSING SIGNAL=     NMAX = *,I3//)
            C        READ AND WRITE NOISE ASSUMPTIONS
                  READ(5,40) TYPE3,TYPE4,DZERO,SIGMAO
            40    FORMAT(2A10/2F10.0)
                  WRITE(6,45) TYPE3,TYPE4,DZERO,SIGMAO
            45    FORMAT(20X,*NOISE ASSUMPTIONS*//
                 1 * TYPE OF NOISE DISTRIBUTION ASSUMED=     *,2A10/
                 2 * BACKGROUND OF NOISE ASSUMED=     DZERO = *,F6.3/
                 3 * STANDARD DEVIATION OF NOISE ASSUMED=     SIGMAO = *,F6.3//)
            C         READ AND WRITE SIGNAL PARAMETERS
                  WRITE(6,50)
            50    FORMAT(20X,*SIGNAL DESCRIPTION*//)
                  DO 56 M=1,MMAX
                  READ(5,51) (S(M,N),N=1,NMAX)
            51    FORMAT(10F10.0)
                  WRITE(6,55) (S(M,N),N=1,NMAX)
            55    FORMAT(10X,10F8.3)
            56    CONTINUE
            C              READ AND WRITE SIGNAL AMPLITUDE
                  READ(5,62) AMP
            62    FORMAT(F10.0)
                  WRITE(6,65) AMP
            65    FORMAT(///* AMPLITUDE OF SIGNALS = *,F5.2/)
            C          OUTPUT DISPLAY PARAMETERS
                  READ(5,75) NLEVEL
            75    FORMAT(I3)
                  READ(5,76) (THRESH(I),I=1,NLEVEL)
```

```
76      FORMAT(8F10.0)
        N1=NLEVEL+1
        DO 77 I=1,N1
77      ID(I)=I
C       CALCULATE BIAS AND SP(M,N) FOR MULTIPLICATIVE PROCESSING
        BIAS1=0.
        BIAS2=0.
        DO 80 M=1,MMAX
        DO 80 N=1,NMAX
        SP(M,N)=AMP*S(M,N)/(AMP*S(M,N)+DZERO)
        BIAS1=BIAS1+ALOG(1.+AMP*S(M,N)/DZERO)
80      BIAS2=BIAS2+S(M,N)*AMP
        BIAS=BIAS1*SIGMA0**2+BIAS2*DZERO
        DO 85 I=1,NLEVEL
85      COMPAR(I)=2.*(SIGMA0**2)*ALOG(THRESH(I))+BIAS
        WRITE(6,87) BIAS
87      FORMAT(* BIAS =*,F8.4///20X,
       2 *DESCRIPTION OF PROCESSING SIGNAL, SP(M,N)*//)
        DO 89 M=1,MMAX
        WRITE(6,88)(SP(M,N),N=1,NMAX)
88      FORMAT(10X,10(F10.4)/)
89      CONTINUE
C               DEFINE TARGET LOCATIONS
        READ(5,90) INTERX,INTERY,NCENTX,NCENTY
90      FORMAT(4I3)
        WRITE(6,95) INTERX,INTERY,NCENTX,NCENTY
95      FORMAT(//* ROW SPACING OF HYPOTHETICAL SIGNAL LOCATIONS    INTERX=
       2 *,I3/* COLUMN SPACING OF HYPOTHETICAL SIGNAL LOCATIONS   INTERY =
       3 *,I3/* ROW SPACING OF ACTUAL SIGNALS    NCENTX = *,I3/
       4 * COLUMN SPACING OF ACTUAL SIGNALS    NCENTY = *,I3////)
        DO 100 I=1,N1
        NFA(I)=0
100     NHITS(I)=0
C           CALCULATE LIMITS FOR INTEGER VARIABLES
        IMIN=1
        JMIN=1
        IMAX=NSAMPX-MMAX+1
        JMAX=NSAMPY-NMAX+1
        NPTSY=NSAMPY-NMAX+1
        M1=1
C                SET UP INITIAL DENSITY MATRIX
        K=DKT=61
        DO 330 K=1,MMAX
        DO 330 I=1,NSAMPY
        CALL READEN
330     DM(K,I)=DENS
        NROW=MMAX
        NRO=NROW+1-(MMAX+NCENTX)
        LOGICB=.FALSE.
C               LOGICA IS TRUE IF ROW CONTAINS ANY HYPOTHETICAL SIGNALS
        LOGICA=(NRO/INTERX*INTERX.EQ.NRO)
        IF(.NOT.LOGICA) GO TO 420
C               LOGICB IS TRUE IF ROW CONTAINS ANY ACTUAL SIGNALS
        LOGICB=(NRO/NSUBX*NSUBX.EQ.NRO)
C           FORM LOOP TO PROCESS EACH COLUMN
370     DO 405 J=JMIN,JMAX
        ICOL=J-NCENTY
        IF(.NOT.(ICOL/INTERY*INTERY.EQ.ICOL)) GO TO 405
        K1=J-JMIN+1
        K2=K1+NMAX-1
C           PERFORM CONVOLUTION OPERATION ON EACH SAMPLE POINT
        DP(K1)=0.
        LS=1
        L1=M1
        L2=MMAX
        FLAG=0
390     DO 400 L=L1,L2
        KS=1
        DO 395 K=K1,K2
        DPSUM=SP(KS,LS)*DM(L,K)**2
        DP(K1)=DP(K1)+DPSUM
```

```
      394 KS=KS+1
      400 LS=LS+1
          IF (M1.EQ.1.OR.FLAG.EQ.1) GO TO 405
          FLAG=1
          L1=1
          L2=M1-1
          GO TO 390
      405 CONTINUE
      410 CALL ROWOUT
  420     DO 440 I=1,NSAMPY
          CALL READEN
  440     DM(M1,I)=DENS
          NROW=NROW+1
  500     M1=M1+1-M1/MMAX*MMAX
          NRO=NROW+1-(MMAX+NCENTX)
          LOGICR=.FALSE.
  C               LOGICA IS TRUE IF ROW CONTAINS ANY HYPOTHETICAL SIGNALS
          LOGICA=(NRO/INTERX*INTERX.EQ.NRO)
          IF(.NOT.LOGICA) GO TO 420
  C               LOGICB IS TRUE IF ROW CONTAINS ANY ACTUAL SIGNALS
          LOGICR=(NRO/NSUBX*NSUBX.EQ.NRO)
          GO TO 370
  1000    STOP 1
          END
```

SUBROUTINE READEN   FORTRAN EXTENDED VERSION 2.0/C   01/28/70   12.

```
          SUBROUTINE READEN
          COMMON/READEN/KWDKT,DENS,NSUBY,NFA,NHITS,ID,NLEVEL,COMPAR,THRESH
          DIMENSION COMPAR(50),THRESH(50),ID(50),NFA(50),NHITS(50)
          DIMENSION D(50),PRCNFA(50)
          IF(KWDKT-NSUBY) 40,40,10
  10      BUFFER IN (1,1) (D(1),D(NSUBY))
          IF(UNIT(1)) 30,20,50
  20      WRITE(6,21)
  21      FORMAT(/////7X,1HN,8X,*LAMDA C*,7X,*COMPAR*,8X,*NHITS*,6X,*NFA*,
         2 8X,*PERCENT MISSES*,3X,*PERCENT FALSE ALARMS*/5X,5H*****,5X,
         3 9H*********,4X,9H*********,7X,5H*****,5X,5H*****,7X,14H**********
         4****,6X,14H**************//)
          N1=NLEVEL+1
          DO 22 I=1,N1
          D(I)=NHITS(I)
  22      PRCNFA(I)=NFA(I)
          DO 23 I=1,NLEVEL
          J=NLEVEL+1-I
          D(J)=D(J)+D(J+1)
  23      PRCNFA(J)=PRCNFA(J)+PRCNFA(J+1)
          DNOT=D(1)
          PRCNOT=PRCNFA(1)
          DO 24 I=1,N1
          D(I)=(1.-D(I)/DNOT)*100.
  24      PRCNFA(I)=PRCNFA(I)/PRCNOT*100.
          WRITE(6,25) (ID(I),NHITS(I),NFA(I),D(I),PRCNFA(I),THRESH(I),
         2 COMPAR(I),I=1,N1)
  25      FORMAT(5X,I3,37X,I3,7X,I3,11X,F7.2,14X,F7.2/15X,1PE9.2,5X,0PF7.2)
          STOP 2
       30 KWDKT=1
       40 DENS=D(KWDKT)
          GO TO 90
       50 PRINT 51
  51      FORMAT (* PARITY ERROR IN TAPE OF DENSITY READINGS *)
          GO TO 20
       90 KWDKT=KWDKT+1
          RETURN
          END
```

```
      SUBROUTINE ROWOUT
      COMMON/READEN/KWORT,DENS,NSUBY,NFA,NHITS,ID,NLEVEL,COMPAR,THRESH
      COMMON/ROWOUT/NPTSY,DP,NMAX,INTERY,NCENTY,LOGICA,LOGICB
      DIMENSION DP(130),NFA(50),NHITS(50),ID(50),COMPAR(50),THRESH(50)
      LOGICAL LOGICA,LOGICB
      DO 100 I=1,NPTSY
      IF(.NOT.LOGICA) GO TO 100
      ICOL=I-NCENTY
      IF(.NOT.(ICOL/INTERY*INTERY.EQ.ICOL)) GO TO 100
      IF(LOGICB.AND.ICOL/NSUBY*NSUBY.EQ.ICOL) GO TO 65
      KLUE=-1
      GO TO 75
   65 KLUE=1
   75 DO 80 J=1,NLEVEL
      IF(DP(I)-COMPAR(J)) 95,80,80
   80 CONTINUE
   95 IF(KLUE) 98,100,99
   98 NFA(J)=NFA(J)+1
      GO TO 100
   99 NHITS(J)=NHITS(J)+1
  100 CONTINUE
      RETURN
      END
```

APPENDIX III

MULTIPL OUTPUT

PROCESSING ASSUMING MULTIPLICATIVE NOISE

RUN NUMBER 19
INPUT DATA FROM SIMULATION NUMBER 15

DESCRIPTION OF INPUT DATA

TYPE OF SIGNAL- 3X3 RANDLIM SQUARE
TYPE OF NOISE- MULTIPLICATIVE
BACKGROUND DENSITY- DZERO = .500
STANDARD DEVIATION OF NOISE- SIGMAO = .163

PROCESSING INFORMATION

NUMBER OF ROWS IN UNIT SIGNAL MATRIX- NSUBX = 20
NUMBER OF ROWS IN DATA MATRIX- NSAMPX = 400
NUMBER OF COLUMNS IN UNIT SIGNAL MATRIX- NSUBY = 20
NUMBER OF COLUMNS IN DATA MATRIX- NSAMPY = 100
NUMBER OF ROWS IN PROCESSING SIGNAL- MMAX = 5
NUMBER OF COLUMNS IN PROCESSING SIGNAL- NMAX = 5

NOISE ASSUMPTIONS

TYPE OF NOISE DISTRIBUTION ASSUMED- MULTIPLICATIVE
BACKGROUND OF NOISE ASSUMED- DZERO = .500
STANDARD DEVIATION OF NOISE ASSUMED- SIGMAO = .163

SIGNAL DESCRIPTION

0.000	.030	.030	.030	0.000
.030	.930	.960	.930	.030
.030	.960	.990	.960	.030
.030	.930	.960	.930	.030
0.000	.030	.030	.030	0.000

AMPLITUDE OF SIGNALS = .20

BIAS = .9721

DESCRIPTION OF PROCESSING SIGNAL, SP(M,N)

0.0000	.0119	.0119	.0119	0.0000
.0119	.2711	.2775	.2711	.0119
.0119	.2775	.2837	.2775	.0119
.0119	.2711	.2775	.2711	.0119
0.0000	.0119	.0119	.0119	0.0000

ROW SPACING OF HYPOTHETICAL SIGNAL LOCATIONS INTERX= 5
COLUMN SPACING OF HYPOTHETICAL SIGNAL LOCATIONS INTERY = 5
ROW SPACING OF ACTUAL SIGNALS NCENTX = 8
COLUMN SPACING OF ACTUAL SIGNALS NCENTY = 8

N	LAMDA C	COMPAR	NHITS	NFA	PERCENT MISSES	PERCENT FALSE ALARMS
1			0	*84	0.00	100.00
	1.00E-01	.85				
2			0	57	0.00	22.63
	1.50E-01	.87				
3			1	56	0.00	18.56
	2.50E-01	.90				
4			0	47	1.00	14.56
	4.00E-01	.92				
5			1	30	1.00	11.21
	6.50E-01	.95				
6			0	40	2.00	9.06
	1.00E+00	.97				
7			1	12	2.00	6.21
	1.45E+00	.99				
8			0	11	3.00	5.35
	2.00E+00	1.01				
9			0	11	3.00	4.57
	2.80E+00	1.03				
10			1	9	3.00	3.78
	3.90E+00	1.04				
11			0	16	4.00	3.14
	5.50E+00	1.06				
12			2	5	4.00	2.00
	7.50E+00	1.08				
13			3	5	6.00	1.64
	1.00E+01	1.09				
14			3	7	9.00	1.28
	1.45E+01	1.11				
15			0	2	12.00	.79
	2.00E+01	1.13				
16			1	2	12.00	.64
	2.80E+01	1.15				
17			3	4	13.00	.50
	3.90E+01	1.17				
18			6	1	16.00	.21
	5.50E+01	1.19				
19			2	1	22.00	.14
	7.50E+01	1.20				
20			2	0	24.00	.07
	1.00E+02	1.22				
21			2	1	26.00	.07
	1.45E+02	1.24				
22			0	0	28.00	0.00
	2.00E+02	1.25				
23			2	0	28.00	0.00
	2.80E+02	1.27				
24			0	0	30.00	0.00
	3.90E+02	1.29				
25			4	0	30.00	0.00
	5.50E+02	1.31				
26			4	0	34.00	0.00
	7.50E+02	1.33				
27			6	0	38.00	0.00
	1.00E+03	1.34				
28			7	0	44.00	0.00
	1.45E+03	1.36				
29			2	0	51.00	0.00
	2.00E+03	1.38				
30			4	0	53.00	0.00
	2.80E+03	1.40				
31			4	0	57.00	0.00
	3.90E+03	1.41				
32			4	0	61.00	0.00
	5.50E+03	1.43				
33			3	0	65.00	0.00
	7.50E+03	1.45				
34			4	0	68.00	0.00
	1.00E+04	1.46				
35			5	0	72.00	0.00
	1.50E+04	1.48				
36			2	0	77.00	0.00
	2.50E+04	1.51				
37			5	0	79.00	0.00
	4.00E+04	1.54				
38			2	0	84.00	0.00
	6.50E+04	1.56				
39			3	0	86.00	0.00
	1.00E+05	1.59				
40			11	0	89.00	0.00

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