

NON-NUL INTERFEROMETER FOR TESTING OF ASPHERIC SURFACES

by

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TABLE OF CONTENTS

LIST OF FIGURES	10
LIST OF TABLES.....	35
ABSTRACT.....	40
1 INTRODUCTION	42
1.1 Interferometry	44
1.2 Interferometric Testing of Spherical Surfaces	46
1.3 Interferometric Testing of Aspheric Surfaces.....	50
1.4 Null Interferometric Testing of Aspheres	52
1.4.1 Stigmatic Imaging.....	52
1.4.2 Aberration Matching.....	54
1.4.3 Aberration Compensating	55
1.4.4 Sub-aperture Stitching	57
1.4.5 Annular Zonal Stitching.....	58
1.5 Non-Null Interferometric Testing of Aspheres.....	59
1.5.1 Collection: Vignetting/Ray Blocking	60
1.5.2 Detection.....	61
1.5.3 Calibration	62
1.6 Non-Null Sub-Nyquist Interferometer.....	63
2 REVIEW OF PHASE SHIFTING AND SUB-NYQUIST INTERFEROMETRY.....	64
2.1 Phase-Shifting Interferometry.....	64

2.1.1 Phase-Stepping vs. Phase-Ramping (Integrating Bucket)	66
2.1.2 Schwider-Hariharan Algorithm	68
2.1.3 Phase Unwrapping	69
2.1.4 Modulation.....	73
2.2 Sampling	74
2.3 Aliasing	77
2.4 Sub-Nyquist Interferometry	80
2.4.1 SNI Phase Unwrapping.....	83
2.4.2 Previous SNI Research	87
3 RAY TRACING SOFTWARE FOR MODLING A NON-NULL INTERFEROMETER	
.....	91
3.1 Ray Tracing a Conventional Imaging System	91
3.2 Ray Tracing a Non-Null Interferometer	95
3.2.1 Reference OPD	96
3.2.2 Normalized Field & Pupil Coordinates	100
3.2.3 Aperture Stop	102
3.2.4 Imaging in an Interferometer	103
3.2.5 Interferometer Errors	108
3.2.6 Ray Aiming.....	111
3.3 Zemax User Defined Programs.....	112
3.3.1 Wavefront Slope Calculations	113
3.3.2 Pupil Aberration Calculations.....	120

3.3.3 Caustic Calculations	127
4 DESIGN OF THE SUB-NYQUIST INTERFEROMETER.....	134
4.1 Contact Lens Inserts.....	134
4.2 Sub-Nyquist / Sparse Array Sensor	138
4.2.1 Measuring Sparse Array Sensor MTF	143
4.2.2 MTF Measurement Results.....	150
4.2.3 Measuring Sparse Array Sensor MTF Utilizing PSI	153
4.3 Interferometer Type	159
4.4 Light Source.....	166
4.5 Diverger Design.....	168
4.5.1 Transmission Sphere.....	172
4.5.2 Mirror.....	173
4.5.3 Multiple Element Diverger Lens	175
4.5.4 Single Element Diverger With an Aspheric Surface	178
4.5.5 Two Element Diverger With an Aspheric Surface	180
4.5.6 Comparing Diverger Designs	181
4.6 Imaging Lens	192
4.6.1 Paraxial Imaging Lens Design.....	192
4.6.2 Imaging Lens Induced Errors	199
4.6.3 Comparing Imaging Lens Designs	213
4.7 Beam Splitter	230
4.8 Reference Surface and Phase Shifter	236

4.9 Collimating Optics	238
4.10 Spurious Fringes	239
5 SNI SOFTWARE, RAY TRACING MODELS & MEASUREMENT PROCEDURE	
.....	249
5.1 Overview of the Measurement Process.....	249
5.2 SNI Software GUI.....	252
5.2.1 GUI Menu Bar	253
5.2.2 GUI Side Panel	257
5.2.3 GUI Acquire Data Tab.....	259
5.2.4 GUI Zernike Tab.....	286
5.2.5 GUI Math Tab.....	295
5.3 Simple Ray Trace Model	298
5.4 Reverse Optimization and Reverse Ray Tracing Model.....	309
5.4.1 Characterization and Modeling of the Collimated Input Beam	322
5.4.2 Characterization and Modeling of the Beam Splitter and Reference Surface	340
5.4.3 Characterization and Modeling of the Imaging Lens	358
5.4.4 Characterization and Modeling of the Diverger Lenses	366
5.5 Data Collection Process	377
6 MEASUREMENTS.....	384
6.1 Measurement of Cylindrical Surfaces.....	385
6.2 Measurement of a Conic Aspheric Surface Containing a Designed Defect	405
6.3 Measurement of a Toroidal Surface Containing a Designed Defect	416

6.4 Measurement of Two Aspheric Contact Lens Tooling Inserts	419
7 DISCUSSION & FUTURE WORK	430
7.1 Cylinder Lens Testing	430
7.2 Calibration with a Spherical Standard	434
7.3 Singlet Aspheric Diverger Lens	439
7.4 Doublet Diverger Lens	443
7.5 Performance Summary	448
7.6 Improvements	450
7.7 Conclusion	456
REFERENCES	458

LIST OF FIGURES

FIGURE 1.1 Laser-based Fizeau interferometer test of a spherical surface.....	46
FIGURE 1.2 Common Path Errors and Non-Common Path Errors	48
FIGURE 1.3 A wide range of spherical surfaces can be tested with the same transmission sphere.	49
FIGURE 1.4 The two null testing positions used to measure the radius of curvature of a spherical surface.....	50
FIGURE 1.5 An aspheric surface in a laser-based Fizeau interferometer.	51
FIGURE 1.6 Example of a null test for a parabolic mirror.	54
FIGURE 1.7 Refractive null lens used to produce a null interferometric test of an aspheric surface.....	56
FIGURE 1.8 CGH used to produce a null interferometric test of an aspheric surface.....	57
FIGURE 2.1 Phase-Stepping vs. Phase-Ramping	66
FIGURE 2.2 One Dimensional Phase Unwrapping.....	70
FIGURE 2.3 One dimensional phase unwrapping on sampled wavefront	71
FIGURE 2.4 Two dimensional phase unwrapping; A single interferogram (Left), the wrapped phase (Center) and the unwrapped phase (Right)	72
FIGURE 2.5 Pixelated Sensor Geometry	75
FIGURE 2.6 MTF of sensors with different G factors.	76
FIGURE 2.7 Three fringe frequencies which alias to the same recorded frequency when sampled.	78
FIGURE 2.8 Aliasing in Frequency Domain.....	79

LIST OF FIGURES-*Continued*

FIGURE 2.9 Mapping of frequencies above the Nyquist frequency back into the region below the Nyquist Frequency.	80
FIGURE 2.10 Aliasing causes multiple frequencies to be recorded as the same measured frequency ξ_m	81
FIGURE 2.11 One dimensional SNI phase unwrapping.	84
FIGURE 2.12 An aliased interferogram (Left), the PSI phase reconstruction (Center) and the SNI reconstruction (Right).....	86
FIGURE 3.1 Pupils are defined by the chief and marginal rays.....	93
FIGURE 3.2 Definition of transverse ray aberration, ε_y , longitudinal ray aberration ε_z , and the wavefront error, $W(\vec{H}, \vec{\rho})$	94
FIGURE 3.3 The default OPD_z calculation for a plane wave with the stop shifted between surfaces a, b, c, and d.	98
FIGURE 3.4 (a) Rays traced from a point source to a plane. (b) The OPD_z calculated with the reference set to Absolute.....	99
FIGURE 3.5 Pupil Imaging (Top) vs Conventional Imaging (Bottom).....	104
FIGURE 3.6 For a given interferogram two rays exist for each point on the detector plane, one from the reference wavefront (red) and one from the test wavefront (blue).	105
FIGURE 3.7 The image of the test part, which is the aperture stop of the interferometer, created by the diverger is the intermediate pupil of the system. Its image onto the detector by the imaging lens is the exit pupil of the interferometer.	107

LIST OF FIGURES-Continued

FIGURE 3.8 In the presence of pupil aberration rays that interfere at the detector from the test wavefront (blue) and reference wavefront (red) do not originate from the same point in the stop of the imaging system.	110
FIGURE 3.9 Direction Cosines	114
FIGURE 3.10 Example of a call to program ZPL23 from the Zemax merit function....	117
FIGURE 3.11 A wavefront (Left) and its corresponding wavefront slope map (Right) calculated with the WavefrontSlopeMap.zpl macro.	119
FIGURE 3.12 Pupil aberration fans for the same interferometer model with ray aiming turned off (Left) and with ray aiming turned on (Right)	122
FIGURE 3.13 The Zemax built in pupil mapping function PUPIL_MAP.	123
FIGURE 3.14 Example of a call to program ZPL49 from the Zemax merit function....	125
FIGURE 3.15 Example of Normalized Pupil Error Maps: Normalized to paraxial exit pupil semi-diameter (Right) Normalized to the real exit pupil semi-diameter (Left).....	126
FIGURE 3.16 Caustic produced from spherical aberration.	127
FIGURE 3.17 An example of an aspheric wavefront (red) in which only a small region exists in which the wavefront is not in a caustic region.	128
FIGURE 3.18 Examples of imaging a test wavefront onto the detector using a plano convex lens; A plane wavefront (Top) , an aspheric wavefront where the mapping is distorted but is still monotonic (Middle), and an aspheric wavefront where the imaging is not monotonic and the detector is located inside a confused region (Bottom).	129

LIST OF FIGURES-Continued

FIGURE 3.19 Output of ZPL43 as a surface is passed through a caustic region (blue) and the original binary caustic flag which only indicated when the surface was in a confused region (red).....	132
FIGURE 3.20 Example of a call to program UDO43 from the Zemax merit function ..	133
FIGURE 4.1 The basic process steps involved in making soft contact lenses	136
FIGURE 4.2 Examples of metal contact lens inserts.....	137
FIGURE 4.3 SEM images of the modified sparse array sensor.	139
FIGURE 4.4 SEM image of a typical pixel pinhole in the sparse array (a), overlaid with a square pixel (b), overlaid with a circular pixel (c).....	140
FIGURE 4.5 Sparse array sensor with circular pixels.	140
FIGURE 4.6 Comparison of the pixel MTF for square and circular pixels, or width a , in a sparse array sensor.	142
FIGURE 4.7 A Mach-Zehnder interferometer was used for measuring the pixel MTF of the sparse array camera.	145
FIGURE 4.8 Fringes created by the interference of two plane wavefronts.....	146
FIGURE 4.9 Horizontal Pixel MTF.....	151
FIGURE 4.10 Vertical Pixel MTF.....	151
FIGURE 4.11 Comparison of the spatial frequency calculated using the autocollimator to the measured spatial frequency from PSI during a measurement of the vertical pixel MTF.	155

LIST OF FIGURES-Continued

FIGURE 4.12 Comparison of the spatial frequency calculated using the autocollimator to the measured spatial frequency from PSI after accounting for aliasing.	156
FIGURE 4.13 The difference between the spatial frequencies calculated using the two techniques.	157
FIGURE 4.14 The spatial frequency measured with the autocollimator versus the spatial frequency measured using PSI during a horizontal pixel MTF measurement (Top Left). The difference between the measured spatial frequencies of the two techniques after unwrapping (Top Right). The horizontal pixel MTF as measured using the autocollimator (Bottom Left). The horizontal pixel MTF as measured with PSI (Bottom Right)	158
FIGURE 4.15 (Left) Twyman-Green Interferometer; (Right) Laser-Based Fizeau Interferometer	160
FIGURE 4.16 In order to meet the $F/\#$ requirement of the diverger the beam could be expanded in the test arm of the interferometer (Left) or a larger diameter collimated wavefront could be generated in the input arm of the interferometer (Right).	171
FIGURE 4.17 The layout of an off-axis parabolic mirror used as a diverger.	174
FIGURE 4.18 The pupil aberration at the intermediate pupil for a spherical surface measured using an off axis parabolic mirror is visible in the normalized pupil error map (Left) and the spot diagram (Right).	175
FIGURE 4.19 Three Element Spherical Diverger Lens Layout	176
FIGURE 4.20 Single Element Aspheric Diverger Lens Layout.....	179
FIGURE 4.21 Two Element Aspheric Diverger Lens Layout.....	181

LIST OF FIGURES-Continued

FIGURE 4.22 The distribution of the maximum aspheric departure for convex conic surfaces versus the BFS radius of curvature.	184
FIGURE 4.23 The distribution of the maximum aspheric departure for convex aspheric surfaces where multiple aspheric coefficients and the conic constant were allowed to vary versus the BFS radius of curvature.	186
FIGURE 4.24 The maximum departure of each convex surface generated versus the BFS radius of curvature.	187
FIGURE 4.25 The radius of curvature in the Y-Z plane versus the radius of curvature X-Z for the convex torodial surfaces generated.	188
FIGURE 4.26 The paraxial imaging of intermediate pupil onto the detector, where the blue ray represents a generic reference ray and the red rays represent the possible angular spread of the test rays bound by the fringe frequency limits of the detector.	194
FIGURE 4.27 The intermediate pupil is imaged onto the detector by the interferometer's imaging lens. In this example the magnification is -1 in order to make the rays visible. The reference rays are shown in blue.	202
FIGURE 4.28 The spread of the possible test rays at the edge of the exit pupil which all originated from the same point on the edge of the intermediate pupil. In a given interferogram there is only one test ray present for any point in the pupil. Therefore the size of the test wavefront at the exit pupil depends on the slope of the test ray in the intermediate pupil and the transverse ray error of the imaging lens.	203

LIST OF FIGURES-Continued

FIGURE 4.29 Interferograms, modeled (Left) and real (Right), where the induced mapping errors of the interferometer distort the circular stop into an elongated exit pupil when testing a torodial surface.	204
FIGURE 4.30 The induced OPD_E in the interferometers imaging optics, using a 200m plano-convex lens, from the center and the edge of an 18mm intermediate pupil (Red), along with the OPD_Z of the test arm (Blue).	209
FIGURE 4.31 The induced OPD_E in the interferometers imaging optics, using a 200m the three element lens, from the center and the edge of an 18mm intermediate pupil (Red), along with the OPD_Z of the test arm (Blue).	210
FIGURE 4.32 Induced Mapping Error: Plano Convex Lens ($F = 100\text{mm}$).....	216
FIGURE 4.33 Induced Mapping Error: Plano Convex Lens ($F = 200\text{mm}$).....	217
FIGURE 4.34 Induced OPD Error: Plano Convex Lens ($F = 100\text{mm}$).....	217
FIGURE 4.35 Induced OPD Error: Plano Convex Lens ($F = 200\text{mm}$).....	217
FIGURE 4.36 Induced Mapping Error: Cemented Doublet Lens	222
FIGURE 4.37 Induced OPD Error: Cemented Doublet Lens.....	222
FIGURE 4.38 Induced Mapping Error: Air Spaced Doublet Lens.....	223
FIGURE 4.39 Induced OPD Error: Air Spaced Doublet Lens	223
FIGURE 4.40 Induced Mapping Error: Custom Three-Element Lens (Small Aperture)	224
FIGURE 4.41 Induced OPD Error: Custom Three-Element Lens (Small Aperture)	224
FIGURE 4.42 Induced Mapping Error: Custom Three-Element Lens (Full Aperture)..	225

LIST OF FIGURES-Continued

FIGURE 4.43 Induced OPD Error: Custom Three-Element Lens (Full Aperture)	225
FIGURE 4.44 Twyman-Green Interferometer using a cube beam splitter.	230
FIGURE 4.45 Twyman-Green Interferometer using a plate beam splitter and a compensating plate in order to balance the OPL of the two arms.	231
FIGURE 4.46 The beam splitter used in the sub-Nyquist Interferometer.	233
FIGURE 4.47 Partially reflective beam splitter surface measured on WYKO 6000 laser- based Fizeau interferometer.	235
FIGURE 4.48 The AR coated beam splitter surface measured on WYKO 6000 laser- based Fizeau interferometer.	235
FIGURE 4.49 Reference Mirror Measured on WYKO 6000 laser-based Fizeau scaled to 532nm wavelength light.	236
FIGURE 4.50 Piezoelectric Optical mount used to phase shift the reference mirror.	237
FIGURE 4.51 Theoretical Transmitted Wavefront from Collimating Lens.	239
FIGURE 4.52 Diffraction of the test and reference wavefront off the sparse array sensor.	241
FIGURE 4.53 This is an example of a diffraction pattern present in the test arm of the interferometer which is generated by light from the reference arm diffracting off the sparse array sensor and brought to focus during its return trip though the interferometer by the imaging lens. The image appears skewed due to the angle at which the image was captured.	242
FIGURE 4.54 Diffraction pattern present on the sparse array sensor.	243

LIST OF FIGURES-Continued

FIGURE 4.55 Spurious fringes are visible in this interferogram captured without the absorbing pellicle in the imaging arm of the sub-Nyquist interferometer.	244
FIGURE 4.56 Spurious fringes are suppressed in this interferogram captured with the absorbing pellicle in the imaging arm of the sub-Nyquist interferometer.	244
FIGURE 4.57 The unwrapped wavefront from the interferogram recorded without the pellicle shows missing data where the SNI unwrapping algorithm failed (Left). The addition of the pellicle clearly improves the result of the SNI phase unwrapping algorithm (Right).	246
FIGURE 4.58 The transmitted wavefront error of the pellicle over a 50mm diameter (Left) and over a 38.5mm diameter (Right).....	247
FIGURE 4.59 Possible locations for the pellicle in the interferometer imaging arm.....	248
FIGURE 5.1 Flow chart of the SNI measurement process.....	251
FIGURE 5.2 Image of the Graphical User Interface (GUI).....	253
FIGURE 5.3 GUI Menu Bar.....	253
FIGURE 5.4 One phase shifted interferogram (Left) and the calculated phase shift at each pixel (Right).....	255
FIGURE 5.5 Histogram of number of pixels at each phase shift in degrees.....	255
FIGURE 5.6 An example of a sub Nyquist Interferogram and the Modulation Map calculated from 5 phase shifted interferograms.	256
FIGURE 5.7 GUI used to export Zernike Phase or Sag data into Zemax.	257
FIGURE 5.8 GUI Side Panel.....	258

LIST OF FIGURES-Continued

FIGURE 5.9 GUI Mask Panel	262
FIGURE 5.10 An interferogram generated by a cylindrical surface (Left) The user selected edge pixels shown in red (Right)	262
FIGURE 5.11 Ellipse calculated by least square fit of selected pixels (Left) Elliptical mask applied to interferogram (Right).....	263
FIGURE 5.12 An example of a sub-Nyquist interferogram (Left), an image of test wavefront with the reference arm blocked (Right)	264
FIGURE 5.13 Histogram of the intensity of the test wavefront.	265
FIGURE 5.14 Sobel Horizontal (Left) and Vertical (Right) Convolution Kernels.....	266
FIGURE 5.15 Image after the threshold (Left), Edges highlighted by the Sobel filter (Right)	267
FIGURE 5.16 Initial least squares fit (Left), and after a few iterations (Right).....	268
FIGURE 5.17 Final mask applied to the interferogram.....	268
FIGURE 5.18 Wrapped phase produced from a sub-Nyquist sampled interferogram (Left) After PSI unwrapping process (Right).....	269
FIGURE 5.19 Unwrapped three central columns (Left), next unwrap all rows to the right (Right)	272
FIGURE 5.20 Unwrap all rows to the left (Left), Output of the path dependent SNI unwrapping procedure (Right).....	273
FIGURE 5.21 Bad pixels binary array (Left), The unwrapped phase with bad pixels removed (Right)	275

LIST OF FIGURES-Continued

FIGURE 5.22 Border pixels to be unwrapped (Left). After a few iterations of the path independent SNI unwrapping algorithm (Right).	276
FIGURE 5.23 Unwrapped Wavefront	276
FIGURE 5.24 Calculated interferogram from the OPD generated by ray tracing the Zemax model (Left) and from the interferometer (Right).	277
FIGURE 5.25 A cartoon of the flat magnification target as viewed from the front and in cross-section (Left), and an image of the actual magnification target (Right).	277
FIGURE 5.26 A single interferogram produced by the magnification target (Left) and the modulation image (Right).....	279
FIGURE 5.27 Binary array of the edges detected using the Sobel filter (Left) and the largest connected region of the binary array overlaid on the modulation map (Right)..	279
FIGURE 5.28 Histogram of the radius of the edge pixels. Only edge points which correspond to peaks above the dashed threshold curve are kept. The small peaks represent light reflecting off the center of the concave rings, which are ignored.....	280
FIGURE 5.29 The final detected rings color coated in order of size and overlaid on top of the modulation data (Left), and an image generated by the Zemax model after optimization (Right).....	281
FIGURE 5.30 Error in the recovering the lens to detector distance from modeled data.	283
FIGURE 5.31 Sensitivity to error in placement of the magnification target.	284
FIGURE 5.32 Measured shift introduced between the lens and the detector.	285
FIGURE 5.33 Plotting Options Commands.....	285

LIST OF FIGURES-*Continued*

FIGURE 5.34 Examples of figures that can be generated using the plotting procedure	286
FIGURE 5.35 GUI Zernike Fitting Tab.....	287
FIGURE 5.36 OPD (Top Left), Zernike Polynomial Fit (Top Right), Difference (Bottom)	292
FIGURE 5.37 Zemax Zernike Fit (Top Left), IDL GUI Zernike Fit (Top Right), Difference (Bottom), all plots are in units of waves at 532nm.	293
FIGURE 5.38 GUI Math Tab	295
FIGURE 5.39 The three configurations that make up the simple Zemax model of the non- null interferometer. The image size and imaging distances in this figure are not to scale.	300
FIGURE 5.40 The merit function for the simple interferometer model	301
FIGURE 5.41 The OPD_z and interferogram of the wavefront at the intermediate pupil plane, when the distance between the diverger focus and test surface is equal to the negative of the base radius of curvature (Top), the negative of radius of curvature of the BFS (Middle) and distance that minimizes the maximum wavefront slope (Bottom). ..	304

LIST OF FIGURES-Continued

FIGURE 5.42 A Shack-Hartmann wavefront sensor measuring a plane-wave incident normal to the sensor top and an aberrated wavefront bottom. The side view of the SHWFS is shown on the left. Center view is looking along the optical axis, the outline of the lenslets are represented by the dark lines, the gray lines illustrate the individual pixels of the detector, and the dots represent the focal spots. A blown-up view of the focal spots produced by a single lenslet, for both a plane wavefront and an aberrated wavefront, is shown on the right.....	323
FIGURE 5.43 Procedure for measuring collimated wavefront with a Shack-Hartman Wavefront Sensor and Keplerian telescope.	325
FIGURE 5.44 The average of ten measurements of the error in the collimated wavefront over the full 47.1mm diameter beam (Left) and over the smaller 28.9mm beam needed to test the average aspheric insert as discussed in Chapter 4.6.3 (Right)	330
FIGURE 5.45 Difference between modeled wavefront error and measured wavefront error introduced by shifting the position of the collimating lens.....	332
FIGURE 5.46 Zemax grid phase surface representing a simulated measurement	335
FIGURE 5.47 Wavefront obtained by tracing forward through the test arm and backwards through the reference arm, without the Zemax grid phase surface inserted at the detector.....	335

LIST OF FIGURES-Continued

FIGURE 5.48 Wavefront obtained by tracing forward through the test arm and backwards through the reference arm, with the Zemax grid phase surface inserted at the detector. A large peak to valley error is encountered at the edge of the pupil (Left), which removed by stopping the aperture down by 1% (Right).	336
FIGURE 5.49 Error introduced into the simulated measurement by the presence of the errors in the collimated input wavefront.	336
FIGURE 5.50 The cumulative percentage of the aspheric surfaces for which the wavefront error, induced by the error in the collimated wavefront, is less than the magnitude displayed on the x axis.	338
FIGURE 5.51 The wavefront error introduced by the test beam's initial transmission through the beam splitter.	342
FIGURE 5.52 The wavefront error introduced into the test arm over a 48mm diameter collimated wavefront.	347
FIGURE 5.53 The interaction of the reference beam with the beam splitter and reference mirror.	349
FIGURE 5.54 The wavefront error introduced into the reference arm by the reference surface over the full 48mm diameter aperture.	351
FIGURE 5.55 The wavefront error introduced into the reference arm by all of its interactions with the beam splitter over the full 48mm diameter aperture.	352

LIST OF FIGURES-*Continued*

FIGURE 5.56 The wavefront error introduced into the reference arm by the combination of the beam splitter and reference surface (Left). The same error calculated after replacing two of the sag surface interactions with a phase surface representing the measured error in the collimated wavefront. (Right).....	354
FIGURE 5.57 The difference between the original model and partial phase model, shown in shown in FIGURE 5.56.	354
FIGURE 5.58 The difference between the original model and phase only model of the reference arm using a Zernike phase surface (Left) and a grid phase surface (Right). ..	356
FIGURE 5.59 The locations at which light from the PSM is focused back on itself from the first three lens surfaces.....	361
FIGURE 5.60 Example of a greatly exaggerated surface decenter in the lens causing a lateral shift in the location returned focal spot.....	361
FIGURE 5.61 Measured surface error the spherical surface of Edmund Optics plano-convex lens 63-496.	365
FIGURE 5.62 Measured surface error the planar surface of Edmund Optics plano-convex lens 63-49 (Left) and with 1.7λ of power removed (Right).....	365
FIGURE 5.63 Measured surface error the spherical surface of Newport Optics plano-convex lens KPX232 (Left) and of the planar surface (Right).	366
FIGURE 5.64 Measurements of the aspheric surface of the singlet diverger lens made by a stylus profiler (Left) and the Zygo Verifire Asphere (Right)	368

LIST OF FIGURES-*Continued*

FIGURE 5.65 The “null” fringe pattern generated by testing a spherical surface (Left) and the resulting OPD produced by the aspheric surface in double pass (Right).....	369
FIGURE 5.66 The difference between the aspheric surface of the singlet diverger lens measurement and the Zernike fit using Fringe terms (Left) and Standard terms (Right).....	370
FIGURE 5.67 Error in the first surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right).....	372
FIGURE 5.68 Error in the second surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right).....	372
FIGURE 5.69 Error in the third surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right).....	372
FIGURE 5.70 Error in the fourth surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right).....	373
FIGURE 5.71 The locations at which light from the PSM is focused back on itself from the four surfaces of the aspheric doublet diverger lens.....	374
FIGURE 5.72 Image of the sub-Nyquist interferometer taken from above the reference surface (not pictured). The collimated beam comes in from the right, the test rail containing the diverger and test part is shown in the left foreground, and the imaging rail containing the imaging lens and detector is shown in the background.....	378

LIST OF FIGURES-*Continued*

FIGURE 6.1 The interferograms produced by the spherical mirror (Left) and the -0.75diopter cylindrical lens surface (Right). The wavefront diameter produced by the spherical mirror is larger than the width of the detector and it is therefore cropped by the detector.....	387
FIGURE 6.2 The unwrapped OPD recorded at the detector plane produced by the spherical mirror (Left) and the -0.75 diopter cylindrical lens surface (Right).....	387
FIGURE 6.3 The Zernike polynomial fit to the OPD recorded from the cylindrical mirror (Left) and the difference between the OPD and the Zernike polynomial fit (Right).....	388
FIGURE 6.4 The wavefronts at the last surface of the RO model for two of the configurations just after the measured OPD data and surface properties are loaded. The wavefront for the spherical mirror is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.....	389
FIGURE 6.5 The wavefronts at the last surface of the RO model for two of the configurations after adjusting the tilt of the test parts. The wavefront for the spherical mirror is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.	392

LIST OF FIGURES-*Continued*

FIGURE 6.6 The wavefronts at the last surface of the RO model for two of the configurations after adjusting the distance between the test part and the imaging lens along with orientation of the imaging lens surfaces. The wavefront for the spherical mirror is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.	393
FIGURE 6.7 The wavefront at the last surface of the RO model for the -0.75 diopter cylindrical lens after allowing the radii of curvature to vary.....	394
FIGURE 6.8 The wavefronts at the last surface of the RO model for two of the configurations after completing the RO procedure. The wavefront for the spherical mirror, stopped down to the same diameter as the cylinder lens, is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.	394
FIGURE 6.9 The OPD introduced by the lens surface minus the OPD introduced by the cylindrical part shape, or twice the surface error. This is the Zernike fit of the OPD introduced by the surface cacluated by the RO procedure.	396
FIGURE 6.10 The residual error present in backwards ray trace through the model (Left), and the sum of the residual error from the forward and backwards ray trace (Right). ...	397
FIGURE 6.11 The OPD just after reflecting off the test surface for the forward propagating model (Left) and the backward propagating model (Right).	398

LIST OF FIGURES-Continued

FIGURE 6.12 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure and the Zernike representation of the measured OPD data. This is twice the surface error.	398
FIGURE 6.13 The fringes from a repeated measurement after rotating the test part 90° counter-clockwise (Left) and the recovered OPD introduced by the lens surface minus the cylindrical radii (Right). The measured OPD data is twice the surface error and has been rotated 180° to take into account inversion about the x and y axis introduce by the imaging lens.	399
FIGURE 6.14 The difference between the recovered wavefront error shown in FIGURE 6.12 and FIGURE 6.13, after aligning the axes of the cylinders.	400
FIGURE 6.15 The residual wavefront error present in the model immediately after loading the grid phase measured OPD data (Left), and the same data set with an aperture placed to remove the outside 5% of the wavefront error (Right).	401
FIGURE 6.16 The residual wavefront error present in the model after aligning the grid phase data to the model, utilizing the raw OPD data (Left) and data that has been processed through a low pass filter (Right).	402
FIGURE 6.17 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure and the full resolution OPD data from the sub-Nyquist sensor (Left). This is twice the surface error. The difference between this data and the Zernike fit of this data (Right).	403

LIST OF FIGURES-Continued

FIGURE 6.18 The interferogram (Left) and the unwrapped OPD (Right) for the cylindrical surface of the +0.75 diopter lens.....	404
FIGURE 6.19 The residual wavefront error at the last surface of the RO model for the cylindrical surface of the diopter +0.75 lens (Left). The OPD error introduced by the lens surface recovered by the RO process minus the cylindrical radii of curvature (Right). This is twice the surface error.	404
FIGURE 6.20 A second measurement of the OPD introduced by the cylindrical surface made at 90° with respect to the first measurement (Left). This is twice the surface error. The difference between this measurement and the previous measurement, after accounting for the rotation of the second measurement (Right).....	405
FIGURE 6.21 The designed surface error that was added to the two test parts.	406
FIGURE 6.22 Interferograms recorded for the convex conic aspheric test surface recorded in 0.25mm steps progressing away from the diverger from the top left to bottom right.	409
FIGURE 6.23 Unwrapped OPD for the first (Top Left), third (Top Right), and fifth (Bottom) interferograms shown in FIGURE 6.22.	410
FIGURE 6.24 The residual wavefront error in the model immediately after inserting the measured OPD data into model (Left) and the residual wavefront error after the first few steps of the RO procedure (Right).	413

LIST OF FIGURES-*Continued*

FIGURE 6.25 The final residual wavefront error in the model at the end of the reverse optimization procedure using the Zernike representation of the measured OPD (Left) and the same error present in the reverse ray trace of the model (Right).....	413
FIGURE 6.26 The final residual wavefront error in the model at the end of the reverse optimization procedure using grid phase surface representation of measured OPD (Left) and the same error present in the reverse ray trace of the model (Right).	414
FIGURE 6.27 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The OPD that should be introduced by the designed error (Right).....	414
FIGURE 6.28 A second measurement of the OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement is also shown (Right).	415
FIGURE 6.29 The “aligned” first (Left) and second (Right) measurements.	416
FIGURE 6.30 Interferograms recorded for the convex toroidal test surface recorded in 0.1mm steps progressing away from the diverger from the top left to bottom right.	417
FIGURE 6.31 The unwrapped OPD for the first (Left) and last (Right) interferograms shown in FIGURE 6.30.....	418

LIST OF FIGURES-Continued

FIGURE 6.32 The final residual wavefront error in the model at the end of the reverse optimization procedure using the grid phase representation of the measured OPD (Left) The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.	419
FIGURE 6.33 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure for the second measurement (Left) and the difference between the first and second measurement (Right).	419
FIGURE 6.34 Interferograms recorded for the aspheric surface with an 8mm radius of curvature and a 4 th order aspheric term equal to -6.0E-4. Fringes were recorded in 0.2mm steps progressing away from the diverger from the top left to bottom right.	420
FIGURE 6.35 Unwrapped OPD for the first (Top Left), third (Top Right), and fifth (Bottom) interferograms shown in FIGURE 6.34.	421
FIGURE 6.36 The residual wavefront error in the model at the end of the reverse optimization procedure using the Zernike representation of the measured OPD (Left). The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.	422
FIGURE 6.37 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).	423

LIST OF FIGURES-*Continued*

FIGURE 6.38 The final residual wavefront error in the model at the end of the reverse optimization procedure using the grid phase representation of the measured OPD (Left) The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.	423
FIGURE 6.39 The OPD introduced by the form error on the test surface from the second measurement calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).	424
FIGURE 6.40 Interferograms recorded for the aspheric surface with an 8mm radius of curvature and a conic constant equal to -0.8. Fringes were recorded in 0.2mm steps progressing away from the diverger from the top left to bottom right.	425
FIGURE 6.41 Unwrapped OPD for the first (Top Left), third (Top Right), and sixth (Bottom) interferograms shown in FIGURE 6.40.	426
FIGURE 6.42 The final residual wavefront error in the model at the end of the reverse optimization procedure using the Zernike representation of the measured OPD (Left). The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.	427
FIGURE 6.43 A second measurement of the OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).	428

LIST OF FIGURES-Continued

FIGURE 6.44 The final residual wavefront error in the model at the end of the reverse optimization procedure using the grid phase representation of the measured OPD (Left) The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.	428
FIGURE 6.45 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).	429
FIGURE 7.1 Surface errors present for a single measurement of the Grade 3 ball bearing as measured with the WYKO 6000 interferometer.....	435
FIGURE 7.2 The measured OPD with the ball bearing located at the “null” position made using the singlet diverger lens (Left) and the difference between the measured OPD and the Zernike Fit of the OPD (Right)	440
FIGURE 7.3 The residual OPD error left in the model after the RO procedure for the grade 3 ball bearing measured with the singlet diverger in which the measured OPD at the detector is modeled as a Zernike phase surface (Left) and as a grid phase surface (Right).	441
FIGURE 7.4 The OPD error incorrectly attributed to surface errors on the conic aspheric test part by the RO procedure (Left) and the same error minus the Zernike power term (Right) These errors are actually generated by the surface of the diverger lens and the method used to model the measured OPD at the detector.	442

LIST OF FIGURES-Continued

FIGURE 7.5 The OPD error attributed to surface errors on the conic aspheric test part tested using the singlet diverger lens and a grid phase representation of the measured OPD.....	443
FIGURE 7.6 The OPD introduced by the form error on the ball bearing surface calculated using the reverse raytracing procedure (Left) and the OPD introduced by the form error on the conic aspheric test part calculated using the reverse raytracing procedure (Right).	445
FIGURE 7.7 The final residual wavefront error in the model at the end of the reverse optimization procedure using grid phase surface representation of measured OPD (Left) and the OPD introduced by the form error on the conic asphere test surface calculated using the reverse raytracing procedure (Right).....	448
FIGURE 7.8 Example of a Zemax ray trace that stalled out for no apparent reason.	452
FIGURE 7.9 A random step change in the OPD calculated by Zemax (Left), a ray trace in which the results are unexplainable (Right).....	452

LIST OF TABLES

TABLE 1.1 The value of the conic constant for different types of conic sections.....	53
TABLE 3.1 Data returned by the programs ZPL23 and UDOP23	118
TABLE 3.2 Data returned by the programs ZPL29 and UDOP29	118
TABLE 3.3 Data returned by the program ZPL49	125
TABLE 3.4 The meaning of different ranges of c values.....	131
TABLE 3.5 Data returned by the programs ZPL43 and UDO43	133
TABLE 4.1 Aspheric Insert Properties	138
TABLE 4.2 Laser Properties (Lightwave Electronics Laser Manual).....	167
TABLE 4.3 Three Element Spherical Diverger Lens Prescription.....	176
TABLE 4.4 Single Element Aspheric Diverger Lens Prescription	180
TABLE 4.5 Two Element Aspheric Diverger Lens Prescription	181
TABLE 4.6 Examples of the aspheric test surface prescriptions for which only one aspheric coefficients or the conic constant was allowed to have a non-zero value.	183
TABLE 4.7 Examples of the aspheric test surface prescriptions for which multiple aspheric coefficients and the conic constant were allowed to have non-zero values.	185
TABLE 4.8 Examples of the Torodial Test Surface Prescriptions	187
TABLE 4.9 Results for the Rotationally Symmetric Test Surfaces	190
TABLE 4.10 Pupil Aberration of the Rotationally Symmetric Test Surfaces.....	190
TABLE 4.11 Results for the Toric Test Surfaces	191
TABLE 4.12 Pupil Aberration of the Toric Test Surfaces	191

LIST OF TABLES-Continued

TABLE 4.13 Paraxial imaging lens properties required to avoid vignetting and allow the imaging of fringes, up to the frequency limit of the detector, originating anywhere in the intermediate pupil.	197
TABLE 4.14 Paraxial imaging lens properties required to begin allowing for the imaging of fringes, corresponding to the frequency limit of the detector, in special cases.	199
TABLE 4.15 Plano-Convex Lens Prescription ($F = 100\text{mm}$)	214
TABLE 4.16 Plano-Convex Lens Prescription ($F = 200\text{mm}$)	215
TABLE 4.17 The percentage of the rotationally symmetric and toric test surfaces that could be properly imaged with the plano-convex lenses tested.....	218
TABLE 4.18 Summary of the Induced Errors for Rotationally Symmetric Test Parts ..	219
TABLE 4.19 Summary of the Induced Errors for Toric Test Parts.....	219
TABLE 4.20 Cemented Doublet Lens Prescription	222
TABLE 4.21 Air Spaced Doublet Lens Prescription.....	223
TABLE 4.22 Custom Three-Element Lens Prescription	224
TABLE 4.23 The percentage of the rotationally symmetric and toric test surfaces that could be properly imaged with the 200mm lenses tested.	227
TABLE 4.24 Summary of the induced errors for the rotationally symmetric test parts.	228
TABLE 4.25 Summary of the induced errors for the toric test parts.....	228
TABLE 4.26 Summary of the induced errors for the rotationally symmetric test parts that can be imaged by both the plano-convex lens and the custom triplet	229

LIST OF TABLES-Continued

TABLE 4.27 Summary of the induced errors for the toric test parts that can be imaged by both the plano-convex lens and the custom triplet.....	229
TABLE 4.28 Specifications of the PZT used for phase shifting the reference mirror. ..	237
TABLE 4.29 Collimating Lens Prescription	239
TABLE 5.1 Zemax Zernike Fringe Polynomials.....	290
TABLE 5.2 Reverse optimization and reverse raytracing model organized components into groups that can be turned on and off to enable forward and backward raytracing..	319
TABLE 5.3 Thorlabs Shack-Harmann Wavefront Sensor Specifications (Thorlabs)....	328
TABLE 5.4 The peak to valley of the measured wavefront for each of the ten measurements as well as the peak to valley and the rms of each wavefront minus the average wavefront.	329
TABLE 5.5 Pearson Product-Moment Correlation Coefficients.....	340
TABLE 5.6 The OPD _Z error introduced by the first pass of the test arm through the beam splitter, and the change in the OPD _Z error for perturbations of the various beam splitter properties.....	345
TABLE 5.7 The OPD _Z error introduced by the second interaction of the arm with the beam splitter, and the change in the OPD _Z error for perturbations of the various beam splitter properties.	348
TABLE 5.8 The OPD _Z error introduced by the reference surface, and the change in the OPD _Z error for perturbations of the reference surface orientation.	351

LIST OF TABLES-Continued

TABLE 5.9 The OPD _Z error introduced by the beam splitter into the reference arm and the change in the OPD _Z error for perturbations of the various beam splitter properties.	352
TABLE 5.10 The percentage out of 20,000 simulations in which the change in the OPD between the sag and phase models is less than the indicated value when the beam splitter and reference mirror properties are perturbed to simulate misalignments.	357
TABLE 5.11 The maximum tilt and decenter of each surface that could produce the measured $\pm 100\mu\text{m}$ shift in the spots, as well as the required	362
TABLE 5.12 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization for each imaging lens property varying over its listed range after testing the 3912 aspheric test surfaces previously described.	364
TABLE 5.13 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization when imaging lens properties are allowed to vary simultaneously.	364
TABLE 5.14 The decenter of the center of curvature (CoC) of each surface as measured with the PSM. The decenter and tilt of the surfaces that could cause the measured shifts.	374
TABLE 5.15 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization for each property of the diverger lens varying over its listed range after testing the 3912 aspheric test surfaces previously described.....	375

LIST OF TABLES-*Continued*

TABLE 5.16 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization when imaging lens properties are allowed to vary simultaneously.	376

ABSTRACT

The use of aspheric surfaces in optical designs can allow for improved performance with fewer optical elements. Their use has become common place due to advancements in optical manufacturing technologies. Standard interferometric testing of aspheric surfaces makes use of part specific null optics in order to match the test wavefront to the aspheric surface under test. Non-null interferometric testing offers the possibility to test a range of aspheric surfaces with a single interferometer design without the need for part specific null optics. However, non-null tests can generate interferograms with very high fringe frequencies that must be resolved and unwrapped, wavefronts with large slopes that must be imaged without vignetting, and induced aberrations which must be separated from the surface errors of the part.

The main goal of this project was the construction of a non-null interferometer capable of testing the aspheric tooling used in the manufacturing of soft contact lenses. Sub-Nyquist interferometry was used to allow for large wavefront departures which generate high fringe frequency interferograms to be both captured and unwrapped. The sparse array sensor at the heart of the Sub-Nyquist technique sets limits on both the range of the parts to be tested and the design of the interferometer. Characterization of the interferometer was achieved through the reverse optimization and reverse ray tracing of a model of the interferometer and was aided by multiple measurements of the test part at shifted positions.

The system was found to be capable of measuring parts with aspheric departure of over 60λ from the best fit sphere, which with introduced part shifts, generated over 300λ of OPD at the detector. The OPD introduced by the parts was measured to an accuracy of at least 0.76λ peak to valley and 0.12λ rms.

1 INTRODUCTION

In general an aspheric surface, or asphere, is by definition as any surface that is not spherical. Their use in optical systems can yield designs with less aberrations while simultaneously using fewer components than a system made up only spherical surfaces. The knowledge that aspheric surfaces are capable of outperforming their spherical counterparts has been known since approximately 200BC with the discovery of conic sections: parabolas, ellipses and hyperbolas. A Greek mathematician, Diocles, proved that parallel rays of light from the sun would be focused perfectly to a single point by use of a parabolic mirror. He also showed that this was not true of the spherical mirrors commonly used at the time, in his book “On Burning Mirrors.” (Pendergrast, 2003) In 1611, Johann Kepller suggested using conic surfaces for lenses as well as mirrors. However, because Snell’s Law of Refraction was not established until 1618, he was unable to prove their benefits for lenses. (Heynacher, 1979) In 1626, René Descartes proved that it was possible to design an aspheric lens with a plano-hyperbolic or an ellipso-spherical shape that completely eliminated the spherical aberration present in spherical lenses. (Burnett, 2005) In the latter half of 17th century, Newtonian, Gregorian and Cassegrain telescopes were designed utilizing conic surfaces. These telescopes were theoretically superior to contemporary refracting telescopes, which utilized spherical mirrors, because they didn’t suffer from spherical and chromatic aberration. However image quality at the time also suffered from poor craftsmanship and the general inability to produce high quality conic surfaces. It wasn’t until the 18th century, that progress was

made in the field of polishing aspheric surfaces. (Heynacher, 1979) Therefore, even though the advantages of aspheric surfaces have been known for some time, spherical surfaces have been much more commonly used because they are both easier and cheaper to produce. This is due to a unique property of spherical surfaces. When two spherical surfaces with the same radius, one convex and one concave, are brought together they will be in contact with each other at every point, regardless of the orientation and position of the surfaces. Therefore by placing an abrasive compound between two roughly spherical surfaces and randomly rubbing them together the high spots on both surfaces will wear down and both surfaces will become more spherical. (Hecht, 2002)

While conic surfaces were the first aspheres used in optical designs, they are far from the only type. Often, aspheric surfaces are described by using rotationally symmetric polynomial expansions. However, there is no requirement on rotational symmetry and cylindrical or toric surfaces or other non-rotationally symmetric functions, such as Zernike polynomials, can be used to define the surface. Regardless of the mathematical representation used to model the surface, aspheric surfaces provide more degrees of freedom than spherical surfaces making their use desirable. Historically since aspheric surfaces are more difficult to manufacture and test their use has been limited. However advances in manufacturing techniques such as single point diamond turning (SPDT) computer controlled polishing, magnetorheological finishing (MRF), conformal grinding and injection molding have made high quality aspheric surfaces more readily available.

As aspheric surfaces become more commonly used the need for aspheric metrology techniques increases. The goal of this research was to design and build an interferometer capable of measuring a range of aspheric surfaces without the need for custom null or auxiliary optical elements to enable the testing. This chapter will give a brief description of existing interferometric techniques for testing aspheres. It will also, introduce the differences between null and non-null interferometric testing and layout the general outline of the dissertation.

1.1 Interferometry

Interferometry is one of the most desirable methods of testing optical surfaces, because it captures information across the entire surface and is capable of measuring to sub-wavelength accuracies. (Mantravadi, 1992) In interferometric optical testing the phase difference, ϕ , between two overlapping wavefronts is measured by analysis of the generated interference pattern, or interferogram. The intensity distribution, I , of an interferogram is given in Equation 1.1, where I_1 and I_2 are the intensity distributions of the two wavefronts. (Goodwin & Wyant, 2006)

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi) \quad 1.1$$

Generally when performing an interferometric test on a surface, a reference wavefront is reflected off of a high quality reference surface and is compared to the wavefront reflected off of the test surface. A single wavefront represents a surface of uniform phase and are often represented as a bundle of light rays which propagate normal to the wavefront and indicate the direction of energy flow. The phase difference for a given

point in an interferogram is the difference in optical path length, OPL, of the two rays, one from each wavefront, which intersect at that point. OPL is defined as the physical distance, t , a ray travels through a medium multiplied by the refractive index, n , of the medium.

$$OPL = nt \quad 1.2$$

In the case of a ray traveling through an optical system, made up of several homogeneous materials with different refractive indices, the total OPL is simply the sum of the OPL through each medium.

$$OPL = \sum_i OPL_i = \sum_i n_i t_i \quad 1.3$$

The optical path difference, OPD, is defined as the OPL of a ray in the test arm minus the OPL of the corresponding ray from the reference arm at a given point in the interferometer.

$$OPD = OPL_{Test} - OPL_{Ref} \quad 1.4$$

The OPD is related to the phase difference by Equation 1.5, where λ , is the wavelength of the light source. The OPD and phase difference are computed at each point on the surface in order to produce a measurement of the surface shape.

$$OPD(x, y) = \frac{\lambda}{2\pi} \phi(x, y) \quad 1.5$$

Interferometers are traditionally used to perform null tests in which the system is designed such that a perfect test surface, when properly aligned, produces an interferogram consisting of a single “null” fringe. Thus, fringes that appear in the

measurement are a result of errors in the surface under test. Prior to the advent of phase shifting interferometry several straight tilt fringes were often introduced into the measurement to aid in analysis of the static interference pattern. Errors on the part would then show up as deviations in the straightness of the tilt fringes. Phase shifting interferometry will be reviewed in Chapter 2.1.

1.2 Interferometric Testing of Spherical Surfaces

A common interferometric test for a spherical surface using a laser-based Fizeau interferometer is shown in FIGURE 1.1. In which the spherical test surface is illuminated with a spherical wavefront originating from its center or curvature. The reference surface is the final element of the transmission sphere or converging lens.

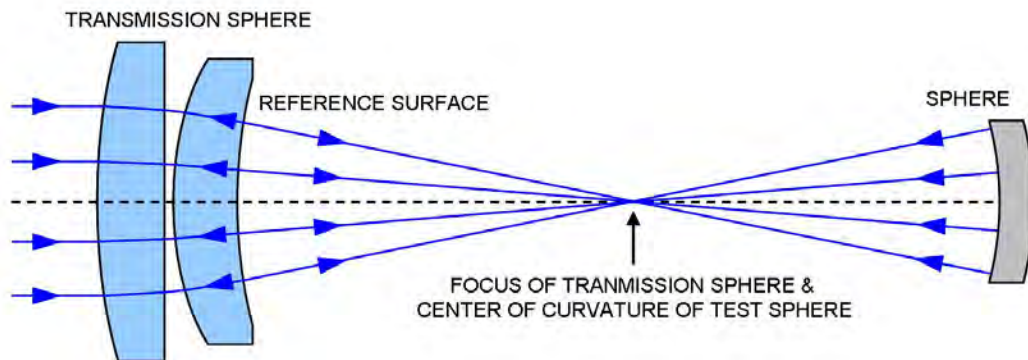


FIGURE 1.1 Laser-based Fizeau interferometer test of a spherical surface.

The rays follow the same path to the reference surface, where they split into reference rays which are reflected, and test rays which are transmitted. The OPL of the reference and test arms are the same, except the test arm contains the extra path length to and from

the test surface. Since the spherical test surface is placed so that its center of curvature is coincident with the transmission spheres focus, the wavefront at the test part is exactly matched to the test surface, and all the test rays approach and reflect perpendicular to the test surface. Therefore, the test rays all travel the same distance to and from the test surface; thus, the OPD for every ray is identical, leading to a uniform phase value across the pupil and a null fringe. Assuming the test surface is properly aligned, any deviation from the null condition can be attributed to deviations in the test surface from a perfect sphere. Thus, height errors, h , on the surface under test can simply be calculated by Equation 1.6.

$$h(x, y) = \frac{1}{2} OPD(x, y) \quad 1.6$$

This generally only holds when the height errors on the surface are small. Large height errors will tend to lead to changes to the localized surface slope. Slope errors will cause the rays to be deviated. This leads to an increase in the fringe frequency and at some point a violation of the null testing condition.

It is important to note that errors in system components that are common to both the reference and test arms in an interferometer will cancel, since the same error will be introduced into both wavefronts at the same pupil location in the wavefronts. These errors are known as common path errors. In order for a component to be considered common path it must add the same OPL to both wavefronts. Components that are not common path, like the reference surface, will introduce errors into the measurement that will be indistinguishable from errors in the test part. For example, consider a bump of

height, t , on the reference surface of the laser-based Fizeau interferometer. The rays of both the reference and test wavefronts which travel through this bump will pick up an extra OPL of $2nt$. However, the distance between the reference surface and test surface will also shrink by $2t$ for the corresponding test ray. The net effect is a decrease in the OPD between the two arms of the interferometer and an apparent bump on the surface under test. This type of error is a non-common path error, or system error. Examples of both common and non-common path errors are shown in FIGURE 1.2.

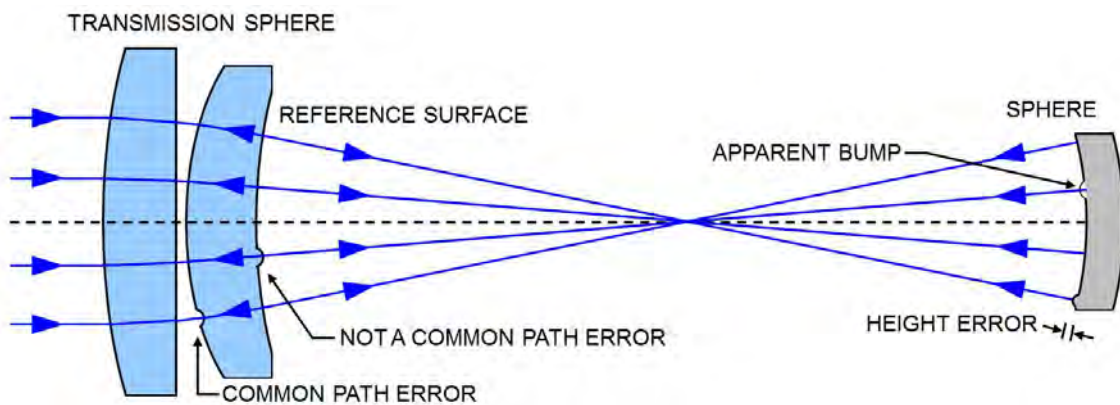


FIGURE 1.2 Common Path Errors and Non-Common Path Errors

While non-common path errors are initially indistinguishable from errors on the test surface, they can be measured. Procedures have been described by Creath and Wyant (1990), Evans (1993), Parks (Parks et al, 1998) and Griesmann (Griesmann et al, 2005) for measuring non common path errors in interferometric testing. These errors are not part specific, and always occur in the same pupil location with respect to the reference wavefront they can be accounted for in future measurements.

A laser-based Fizeau interferometer has the ability to test a wide range of spherical surfaces, due to the fact that a spherical wavefront remains spherical as it propagates. The range of testable surfaces depends on the F-number ($F/\#$) of the transmission sphere, see FIGURE 1.3. A concave spherical surface can be fully tested provided the $F/\#$ of the transmission sphere is less than or equal to the radius of curvature (R) of the surface divided by its diameter (D), or $R/\#$. If the $F/\#$ of the transmission sphere is greater than the $R/\#$ of the surface, then only a portion of the surface can be tested; which is the case for R_3/D_3 in FIGURE 1.3. The same holds true for convex surfaces, with the added requirement that the radius of curvature of the test surface must be shorter than the radius of the reference surface; otherwise, the test part would have to be positioned inside the transmission sphere.

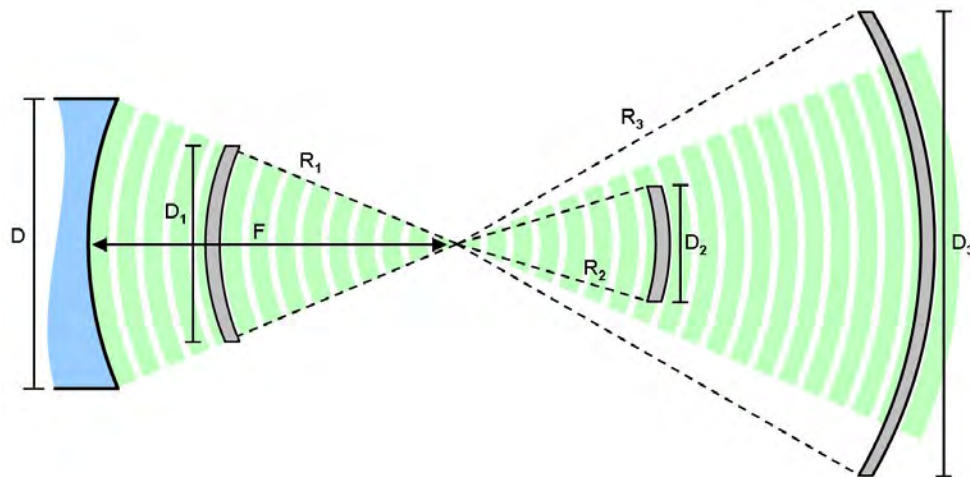


FIGURE 1.3 A wide range of spherical surfaces can be tested with the same transmission sphere.

In the test described above, the surface's departure from the spherical reference surface is measured; yet there is no information provided on the radius of curvature of the part.

However, the radius of curvature of the surface can be measured by making use of another position the test surface can be placed to generate a null fringe, commonly known as the cat's eye position. In the cat's eye position, the surface is placed at the focus of the transmission sphere. Measuring the axial distance between the cat's eye position and the confocal position yields the radius of curvature of the surface, FIGURE 1.4.

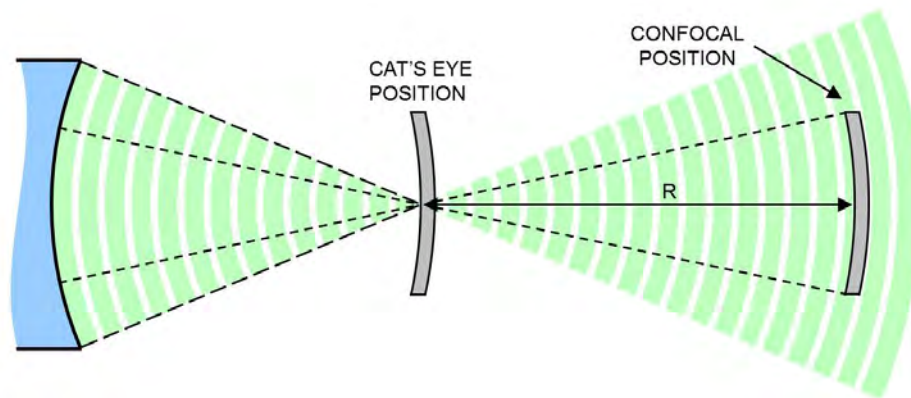


FIGURE 1.4 The two null testing positions used to measure the radius of curvature of a spherical surface.

1.3 Interferometric Testing of Aspheric Surfaces

Ideally an interferometer could be developed to test aspheric surfaces as easily and accurately as spherical surfaces can be tested. However, illuminating an aspheric surface with a spherical wavefront will not cause the rays to intersect the surface perpendicularly at every location across the wavefront; thus, the rays no longer follow the same path back through the system (Wyant 1988), as seen in FIGURE 1.5.

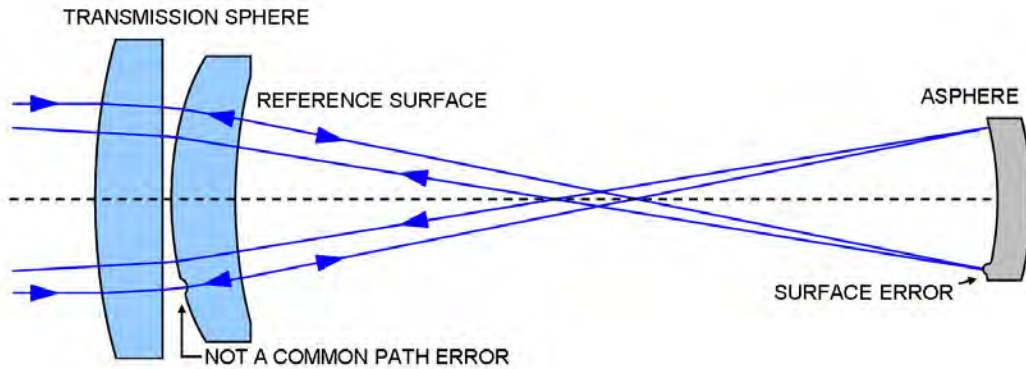


FIGURE 1.5 An aspheric surface in a laser-based Fizeau interferometer.

The result is a non-null interferogram, where the number and frequency of fringes depends on the surface's departure from the reference wavefront, or more precisely on the maximum slope present in the difference between the test and reference wavefronts. As the amount of asphericity increases, it can become difficult to accurately detect the fringe pattern; which will be discussed in detail in Chapter 2. Also, the OPD between the reference and test arms is not only the result of the tests surfaces departure from the incident spherical wavefront, but also includes the changes to the OPL of the test rays as they propagate though the rest of the interferometer. Errors in parts common to both arms may no longer cancel because they can occur at different pupil positions in the test and reference wavefronts, as seen in FIGURE 1.5. Additionally the added OPL introduced into the test arm depends on the wavefront generated by the aspheric surface under test. Therefore, the interferogram contains aberrations generated by the aspheric surface combined with aberrations introduced by the interaction of the aberrated wavefront with the interferometer optics. These aberrations are known as induced

aberrations or retrace error, which will be discussed in greater detail in Chapter 3. (Kurita et al, 1986) (Kurita, 1989) (Hoffman, 1993) Another error that is often present when testing aspheres is an error in the conversion between the measured wavefront to surface figure. (Kurita, 1989) Since the wavefront at the test part does not match the surface under test the simple scaling of the measured phase difference at the detector to surface height error shown in Equations 1.5 and 1.6 is no longer adequate. In order to remove these errors, the system has to be ray traced, which will be discussed in Chapter 3.2. However, in order to avoid these error sources, and the problems with detecting interferograms with high fringe frequencies, additional optics are generally used to create null tests for aspheric surfaces.

1.4 Null Interferometric Testing of Aspheres

A null test is any test which produces an interference pattern consisting of a single phase value. Since the final interferogram is the difference between the test and reference wavefronts there is more than one method of creating a null test for an aspheric surface. Stahl (Stahl,1991), divided null tests for aspheric surfaces into three categories: stigmatic imaging, aberration compensating, and aberration matching.

1.4.1 Stigmatic Imaging

Stigmatic imaging tests make use of the fact that conic aspheric surfaces have two conjugate foci that provide perfect imaging. The sag, z , of a conic surface is described by

Equation 1.7, where C represents the surface curvature, which is the inverse of the radius of curvature, R , and the conic constant, k .

$$z = \frac{Cr^2}{1 + \sqrt{1 - (1+k)C^2r^2}}; \quad C = 1/R; \quad r^2 = x^2 + y^2 \quad 1.7$$

Surface Type	Conic Constant
Hyperboloid	$k < -1$
Paraboloid	$k = -1$
Prolate Ellipsoid	$-1 < k < 0$
Sphere	$k = 0$
Oblate Ellipsoid	$k > 0$

TABLE 1.1 The value of the conic constant for different types of conic sections.

By illuminating a conic surface with a spherical wavefront from one focus an aberration free spherical wavefront centered at the other focus is produced. A sphere has both foci located at its center of curvature; thus, a sphere images its center of curvature back onto itself. Since the foci of other conics, such as parabolas, ellipses, and hyperbolas, are located at two distinct points; additional optics can be used to image one focus onto the other. One example of such a test is the auto collimation test of a parabola. Here a parabola is illuminated by a spherical wavefront centered at the prime focal position, creating an aberration free image at infinity. Then a high quality flat mirror is used to return the collimated beam and re-image the wavefront at the prime focus position. See FIGURE 1.6.

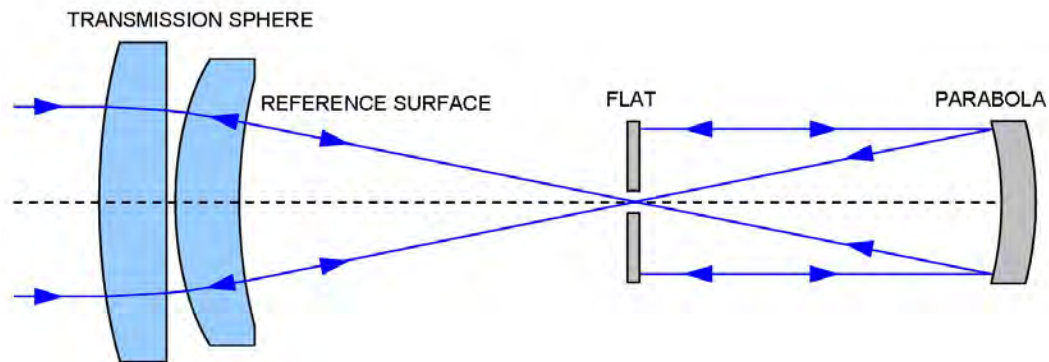


FIGURE 1.6 Example of a null test for a parabolic mirror.

Similar tests exist for testing other types of conic surfaces such as those proposed by Hindle, Silvertooth, Parks & Shao, and Meinel & Meinel. (Offner and Malacara 1992) These tests are generally performed with the asphere in double pass. Many, but not all, require either an aperture at the center of the aspheric surface or will produce an obscuration at the center of the aspheric surface. Most of these tests require flat or spherical surfaces, as large as or larger than the conic surface under test, which must be higher quality than the desired measurement accuracy. The alignment of the spherical reference wavefront to the foci of the conical surface is critical. Additionally they can suffer from errors in the conversion from measured wavefront to surface figure since the wavefront at the test part doesn't match the test surface.

1.4.2 Aberration Matching

In an aberration matching null test the wavefront at the aspheric surface does not match the surface under test; rather, the reference and test wavefronts are made to match, in order to create a null interference pattern. This could be accomplished using a Twyman-

Green interfereometer in which the reference is replaced by a perfect master asphere.

This arrangement is referred to as a Willimans interferometer and is often noted as a way of testing large spherical mirrors, but it could be extended to testing an asphere against a master asphere (Malacara, 2007b). Therefore both the test and reference arms contain matched aberrated wavefronts such that a null interference pattern is generated.

The major disadvantage to this type of setup is the requirement of a master asphere to test against. Another method of accomplishing aberration matching is to use a real or computer generated hologram (CGH) in the imaging arm of an interferometer. A real hologram can be recorded in the imaging arm with the use of a master aspheric part, or the interference pattern could be predicted, by ray tracing the system, and encoded onto a CGH. (Creath & Wyant, 1992) Since the wavefront does not follow the same path to and from the aspheric part, reverse raytracing may be required. (Goodwin & Wyant, 2006)

1.4.3 Aberration Compensating

In an aberration compensating test, an additional null optic or null compensator is used to generate a wavefront that exactly matches the perfect test surface causing the wavefront to retro reflect at every point along the surface. These tests are ideal because they do not suffer from system induced aberrations or errors in the wavefront to surface figure conversion. Null compensators can be reflective, refractive or diffractive components or any combination thereof. Reflective and refractive null correctors such as those

designed by Couder, Burch, Holleran, Ross, Shaffer, Dall and Offner, have been used to test conic aspheric surfaces for a long time. (Offner and Malacara 1992)

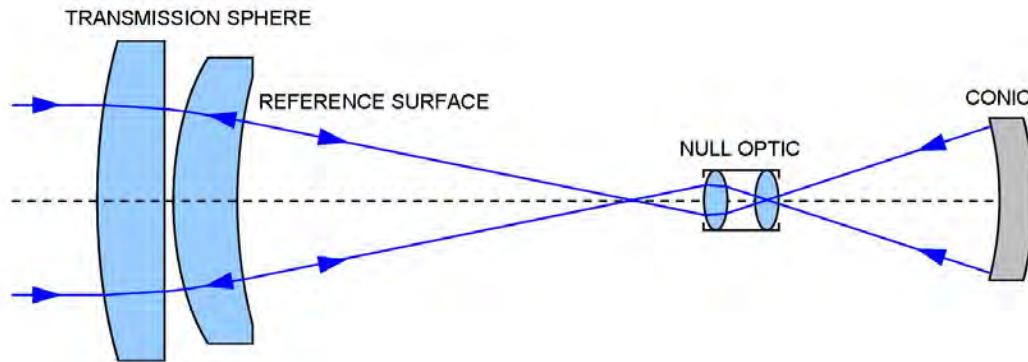


FIGURE 1.7 Refractive null lens used to produce a null interferometric test of an aspheric surface.

The advantage of using null lenses are that they are often much smaller than the surface under test and often easier to align than tests based on conic properties (chapter 1.4.1) (Stahl 1991). However, since refractive nulls often require multiple elements they can be difficult to accurately characterize. Characterization and alignment are crucial, since any error the wavefront produced by the null optic will appear as an error in the aspheric surface. Instead of using a multiple element refractive or reflective nulls, it is often preferable to use a single element diffractive null, such as a computer generated hologram (CGH), as shown in FIGURE 1.8. The CGH is designed to produce a wavefront identical to the surface under test, with the addition of a tilt carrier frequency needed to separate the various diffraction orders.

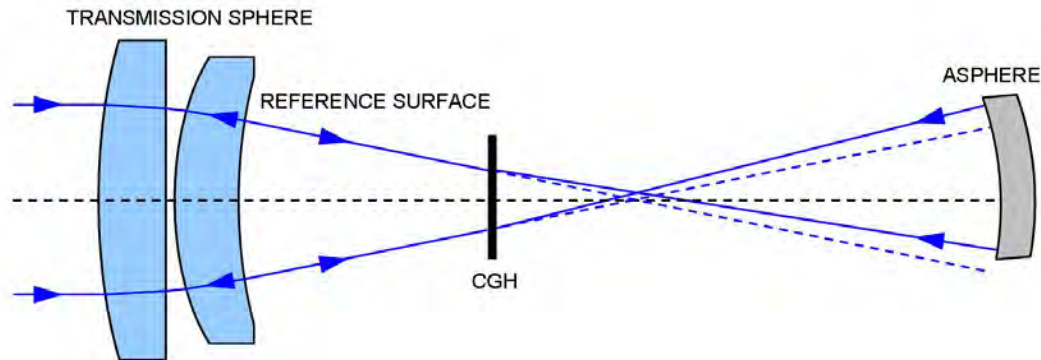


FIGURE 1.8 CGH used to produce a null interferometric test of an aspheric surface.

Using a CGH in between the test arm has several advantages over using a CGH in the imaging arm as discussed in Chapter 1.4.2. First the test can be accomplished by using a commercially available laser-based Fizeau interferometer. Additionally because the CGH is placed in between the transmission sphere and the aspheric test surface the exact prescription of the interferometer isn't needed for the design of the CGH. (Wyant, 2006) However, because the CGH is used in double pass it must have higher diffraction efficiency than a CGH designed to be used in the imaging arm. Additionally the substrate must have less thickness variation in order to avoid adding error into the OPL of the test arm. (Wyant, 2006) Regardless of the type of null optic used, a separate null must be designed and manufactured to compensate for every aspheric surface tested, leading to higher costs for the productions of aspheres.

1.4.4 Sub-aperture Stitching

With null optics being expensive and time consuming to produce, there is a great desire to find an interferometer capable of testing aspheres without utilizing custom null optics.

One possible technique is sub-aperture stitching, in which several interferometric measurements are made over a grid or lattice of overlapping regions of the test surface and then combined into a single surface map. (Fleig et al, 2003) In testing an aspheric surface, rather than matching the asphericity of the entire surface, the test part is moved to produce a null fringe pattern generated over the smaller sub-aperture. However, in order to accurately reconstruct the entire surface, the part's location during each measurement must be known very accurately. As the aspheric shape becomes more steeply sloped more measurements must be taken, since the subapertures over which the wavefront is sufficiently nulled becomes smaller. If the wavefront over the apertures are only nulled enough to record the fringes and not to meet the null fringe requirement then retrace errors will be present in the individual measurements, which must be corrected. (Murphy et al, 2006) Additionally it has been shown that a variable null lens can be used to reduce the fringe densities present in the measurements of each sub-aperture which reduces the number of sub-apertures needed and decreases measurement time. (Murphy et al, 2009) (Tricard et al, 2010)

1.4.5 Annular Zonal Stitching

Annular zonal stitching is another form of sub-aperture testing, except rather than utilizing a grid of distinct measurements it utilizes several measurements of null rings of increasing diameter. (Liu et al, 1988) It works by using an interferometer such as a laser-based Fizeau to produce a null fringe at the center of the interferogram. Then, by scanning the test part along the optical axis, the location of the null fringe moves outward

from the center. By tracking the lateral position of the null fringe, with phase shifting interferometry, while recording axial position of the part, the surface height is calculated. (Kuechel, 2006) Since only the position of the null fringe is used, there are no retracing or system errors introduced into the measurement. (Küchel, 2009) This technique works for rotationally symmetric surfaces where the null fringe will take the shape of an expanding ring allowing the entire surface to be measured. However non-rotationally symmetric parts cannot be measured since they will not produce a null ring. The Zygo Verifire Asphere is a commercially available instrument that makes use of this technique. (Zygo Corporation Middlefield, CT)

1.5 Non-Null Interferometric Testing of Aspheres

A different approach to testing aspheric surfaces is non-null interferometric testing. In a non-null test the wavefront at the aspheric surface is not matched to the aspheric surface. Likewise the test and reference wavefronts are not made to match at the detector. Rather the OPD is only reduced to the point where the fringe frequencies present at the detector are within the measurement range of the sensor, which allows for a wide range of parts to be tested. However, there are three requirements that must be satisfied by the interferometer in order to successfully test in a non-null configuration. (Greivenkamp & Gappinger, 2004)

Collection: The interferometer must not allow rays, especially those associated with the high-slope portions of the test wavefront or surface, to vignette.

Detection: The sensor must be able to record the fringes produced by the interference of the test and reference wavefronts with sufficient dynamic range and precision.

Calibration: The interferometer must be calibrated in order to account for the errors which result from the violation of the null condition.

1.5.1 Collection: Vignetting/Ray Blocking

Collecting the light that is reflected off of an aspheric surface and relaying it to the detector is a major concern in designing a non-null interferometer to measure aspheric surfaces. Since the incoming wavefront will not match the aspheric test surface the rays will not retrace their path on reflection of the test surface. As the difference between the incoming wavefront and aspheric test surface increases, the slope errors will cause the maximum angular deviation of the rays on reflection to increase leading to the possibility that some rays will vignette as they travel through the rest of the interferometer. The Vignetting of rays associated with higher slope portions of the aspheric wavefront is a serious problem with non-null interferometry. Although the term *vignette* as used here may be misleading as vignetting is usually considered to be the loss of irradiance in an imaging system for an off-axis image point due to the partial or total blocking of the ray bundle from the corresponding object point. Yet an interferometer that uses a point source is a zero field system and therefore there is only one ray corresponding to each point on test surface. Therefore, it is more appropriate to say that rays are simply blocked by an aperture in the system, rather than vignetted. If a ray from a portion of the test

surface is blocked there will be no information from that point relayed to the detector. Additionally due to the high slopes associated with non-null testing, ray blocking is not restricted to the rays from the edge of the test part. (Greivenkamp et al, 1996)

Ray blocking becomes very difficult to control in a non-null system designed to measure a range of aspheric surfaces. Since every aspheric surface is different and will produce a different distribution of ray angles on reflection and therefore different wavefront diameters at each system aperture. Two possible solutions to this problem are to oversize the interferometer optics or to reduce the diameter of the aspheric wavefront. Large optics are generally more expensive and harder to fabricate than their smaller counterparts. The diameter of the test wavefront can usually be reduced at a given element in the system by shifting the test part axially from the ideal testing location. However this will only help if the new maximum fringe frequency created by the shift is still with-in the dynamic range of the detector. Also shifting the test part to control the beam diameter at one system aperture may cause the wavefront diameter to increase at a different aperture. In the end, the possible size of the test wavefront at each component over the range of possible test parts should be considered in the initial design of the system. This will be discussed with the design of the system in Chapter 4.

1.5.2 Detection

The major advantage of a non-null interferometry test is that the test wavefront is free to depart from the reference wavefront. The departure of the test wavefront from the

reference wavefront at the detector is limited by the maximum fringe frequency which can be resolved by the detector which is proportional to the maximum slope difference between the wavefronts. (Greivenkamp et al, 1996) Most interferometers use standard cameras and phase shifting interferometry (PSI) to record the inference pattern and recover the wavefront at the detector. Chapter 2.1, will discuss the limit PSI places on the measureable wavefront slope and how the use of a sparse array camera and sub-Nyquist phase unwrapping can greatly increase this range. The specifics of the sparse array detector used for this research will be discussed in Chapter 4.2 and the unwrapping algorithm in Chapters 2.4 and 5.2. In addition to the fringe frequency requirement, the test surface should be imaged onto the detector which will be discussed in Chapters 3.2.4 and 4.6.

1.5.3 Calibration

The major obstacle to non-null optical testing is the need for accurate system calibration in order to remove aberrations introduced into the measurement from the violation of the null condition. There have been several papers which discuss removal of these errors by utilizing reverse ray tracing and reverse optimization. (Lowman and Greivenkamp 1995) (Greivenkamp et al, 1996) (Gappinger and Greivenkamp, 2003) (Gappinger and Greivenkamp, 2004) (Greivenkamp, 2006) In order to accurately predict and remove the aberrations introduced by the system, including the test part, the interferometer components must be accurately characterized and modeled. Properties for each optic in the system such as curvatures, index of refraction and center thickness must be measured.

While well corrected optics may be able to reduce the overall aberrations of the system, using a minimal number of components in the interferometer will help to simplify the model and calibration process. (Gappinger and Greivenkamp, 2003) The use of cemented surfaces should be avoided due to the inability to accurately measure the buried surface. (Gappinger and Greivenkamp, 2004) Finally, it may be necessary to make several measurements of the same part with known perturbations of the system in order for the reverse optimization routine to be successful. (Gappinger and Greivenkamp, 2003). The reverse optimization process used will be discussed in Chapters 5.4 and 6.

1.6 Non-Null Sub-Nyquist Interferometer

The main goal of this project was the construction of a non-null sub-Nyquist interferometer capable of testing the aspheric tooling used in the manufacturing of soft contact lenses. In Chapter 2 the theory behind phase shifting interferometry and the extension to sub-Nyquist interferometry will be reviewed. Chapter 3 will discuss the basics of modeling a non-null interferometer with sequential ray tracing software along with a description of a few programs written in the ray tracing software. Chapter 4 will discuss the design of the non-null interferometer built for this research. Chapter 5 will cover the process for making measurements with the non-null interferometer including a description of the ray trace models used for the calibration and the software that was written to accomplish the task. Chapter 6 will present the measurement results and Chapter 7 will contain the conclusion and suggestion of future work.

2 REVIEW OF PHASE SHIFTING AND SUB-NYQUIST INTERFEROMETRY

This chapter will cover the theory behind sub-Nyquist interferometry (SNI), starting with a review of phase shifting interferometry, of which SNI is an extension. PSI and SNI data collection techniques and algorithms used to recover the unwrapped wavefront that are used for the sub-Nyquist interferometer built for this research will be discussed, as will a basic explanation of phase unwrapping procedures for both PSI and SNI. A brief description of aliasing and sampling will be given, as they are important concepts to the foundation of SNI. Most of the topics discussed in this chapter are explained in greater detail in the first paper on SNI by Greivenkamp (1987) and by Greivenkamp and Bruning in Chapter 14 of “Optical Shop Testing” by Malacara (Greivenkamp & Brunning 1992). This chapter will conclude with a brief review of previous SNI work.

2.1 Phase-Shifting Interferometry

In order to solve for the relative phase difference between the two interferometer arms, start with the general expression for the reference and test wavefronts.

$$W_{ref}(x, y, \delta) = A_{ref}(x, y) e^{i[\phi_{ref}(x, y) - \delta]} \quad 2.1$$

$$W_{test}(x, y) = A_{test}(x, y) e^{i\phi_{test}(x, y)} \quad 2.2$$

Where A_{ref} and A_{test} are the amplitudes, ϕ_{ref} and ϕ_{test} are the phases of the reference and test wavefronts, and δ is the phase shift between the two beams. When the two

wavefronts are interfered, the resulting intensity pattern or interferogram is then given by, Equation 2.3, which reduces to the fundamental equation for PSI, Equation 2.4 (Greivenkamp & Brunning, 1992).

$$I(x, y, \delta) = |W_{test}(x, y) + W_{ref}(x, y, \delta)|^2 \quad 2.3$$

$$I(x, y, \delta) = I'(x, y) + I''(x, y) \cos[\phi(x, y) + \delta] \quad 2.4$$

Where the average intensity or the intensity bias is given by Equation 2.5, the fringe modulation or half of the peak to valley intensity modulation is given by Equation 2.6, and the phase difference is Equation 2.7.

$$I'(x, y) = A_{test}^2(x, y) + A_{ref}^2(x, y) \quad 2.5$$

$$I''(x, y) = 2A_{test}(x, y)A_{ref}(x, y) \quad 2.6$$

$$\phi(x, y) = \phi_{test}(x, y) - \phi_{ref}(x, y) \quad 2.7$$

Since Equation 2.4 contains three unknown terms, at least three interferograms with unique phase shifts are required to solve for the phase difference. A common method of introducing a phase shift into the reference arm of a Twyman-Green interferometer, and the one employed in this research, is a movable reference mirror. The OPL of the reference arm, and thus the OPD, is changed by twice the distance the reference mirror is translated along the optical axis by the use of a piezo-electric transducer (PZT). The hardware used to perform the phase shifting will be discussed in Chapter 4.8.

2.1.1 Phase-Stepping vs. Phase-Ramping (Integrating Bucket)

In order to control the data acquisition process, the movement of the mirror must be synchronized to the camera. There are two common data acquisition techniques used for PSI; phase-stepping and phase-ramping. The differences between these two approaches are highlighted in FIGURE 2.1.

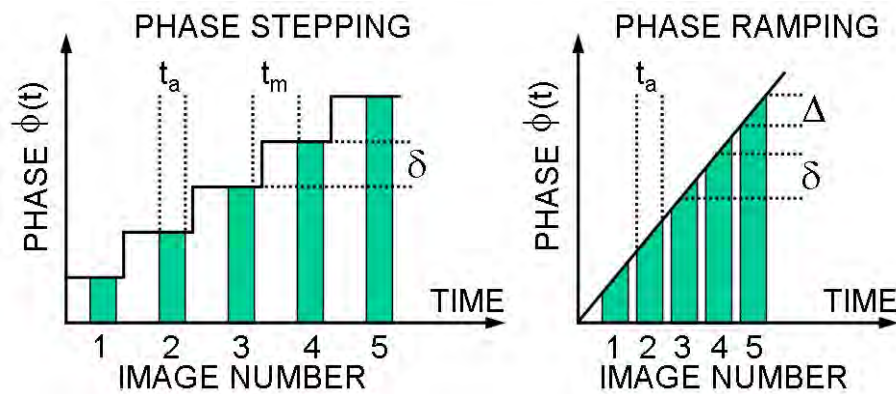


FIGURE 2.1 Phase-Stepping vs. Phase-Ramping

In phase-stepping the mirror is moved to several discrete positions, or “steps”, in-between image captures and held stationary during the acquisition time, t_a , of a given interferogram. In phase stepping it is often necessary to wait for the mirror to stabilize at the desired position, due to oscillations in the PZT, before the next interferogram can be recorded. The added time required to move and wait for the reference mirror to come to rest, t_m , can greatly increase the length of time required to record all the necessary interferograms. As the overall measurement time is increased the system becomes more susceptible to time dependent errors, such as vibration and air turbulence.

In phase-ramping, first proposed by Wyant (1975) the mirror is moved at a constant rate allowing the capturing of successive frames to complete the measurement. This shortens the overall measurement time and therefore decreases the interferometers' sensitivity to vibration and air turbulence. However, there is a trade off when using phase-ramping: since the mirror is always in motion, the interferograms are no longer static during the acquisition time. The effect the phase change during acquisition has on recorded interferograms can be calculated by integrating Equation 2.4 over the phase change during a given phase shift, Equation 2.8 (Greivenkamp, 1984).

$$I_i(x, y) = \frac{1}{\Delta} \int_{\delta_i - \frac{\Delta}{2}}^{\delta_i + \frac{\Delta}{2}} I(x, y, \delta) d\delta \quad 2.8$$

Where, Δ is the change in the phase change during the acquisition of interferogram I_i , and δ_i is the corresponding average phase shift. The result of this integration is given in Equation 2.9, where the sinc function is the normalized sine cardinal function given by Equation 2.10.

$$I_i(x, y) = I'(x, y) + I''(x, y) \text{sinc} \left[\frac{\Delta}{2\pi} \right] \cos [\phi(x, y) + \delta_i] \quad 2.9$$

$$\text{sinc}(x) \equiv \begin{cases} 1 & \text{if } a = 0 \\ \frac{\sin(x\pi)}{x\pi} & \text{else} \end{cases} \quad 2.10$$

Thus, the penalty for changing the phase during the acquisition of an interferogram is a reduction in the modulation by the sinc function. Note that if the change in the phase

shift during the capture of an interferogram, Δ , goes to zero, Equation 2.9 reduces to Equation 2.4, the phase-stepping solution.

2.1.2 Schwider-Hariharan Algorithm

As previously stated, Equation 2.4, and now Equation 2.9, contain three unknowns and therefore at least three interferograms with unique phase shifts are required to solve for the phase difference. Several algorithms utilizing different numbers of phase shifted interferograms have been developed to solve for the relative phase. Each has slightly different sensitivities to known error sources, such as vibration, inaccurate phase shifts, non-linear phase shifts, detector non-linearity, and harmonic sensitivities (Schreiber & Bruning 2006). One of the more commonly used algorithms, and the one employed by this system, is the Schwinder-Hariharan algorithm which offers a good compromise between the number of frames required and the sensitivity to errors (Schwider et al, 1983) (Hariharan et al, 1987). The algorithm requires five interferograms separated by a phase shift, α .

$$i = 1, 2, 3, 4, 5 \quad \delta_i = -2\alpha, -\alpha, 0, \alpha, 2\alpha \quad 2.11$$

The five interferograms required can be derived from Equation 2.9 and are given by Equations 2.12-2.16.

$$I_1(x, y) = I'(x, y) + I''(x, y) \operatorname{sinc}\left[\frac{\Delta}{2\pi}\right] \cos[\phi(x, y) - 2\alpha] \quad 2.12$$

$$I_2(x, y) = I'(x, y) + I''(x, y) \operatorname{sinc}\left[\frac{\Delta}{2\pi}\right] \cos[\phi(x, y) - \alpha] \quad 2.13$$

$$I_3(x, y) = I'(x, y) + I''(x, y) \operatorname{sinc}\left[\frac{\Delta}{2\pi}\right] \cos[\phi(x, y)] \quad 2.14$$

$$I_4(x, y) = I'(x, y) + I''(x, y) \operatorname{sinc}\left[\frac{\Delta}{2\pi}\right] \cos[\phi(x, y) + \alpha] \quad 2.15$$

$$I_5(x, y) = I'(x, y) + I''(x, y) \operatorname{sinc}\left[\frac{\Delta}{2\pi}\right] \cos[\phi(x, y) + 2\alpha] \quad 2.16$$

Equations 2.12-2.16 can be combined and simplified by the applying the appropriate trigonometric identities to Equation 2.17.

$$\frac{\tan[\phi(x, y)]}{2 \sin \alpha} = \frac{I_2(x, y) - I_4(x, y)}{2I_3(x, y) - I_5(x, y) - I_1(x, y)} \quad 2.17$$

The phase shift, α , is then picked to minimize the effect of phase shift errors by minimizing Equation 2.17. This occurs when $\sin(\alpha)$ is maximized at α equals $\pi/2$. Phase shifts of $\pi/2$ can be created by moving the reference mirror a distance of $\lambda/8$.

Substituting $\pi/2$ into Equation 2.17 for α and solving for ϕ yields Equation 2.18.

$$\phi(x, y) = \tan^{-1} \left[\frac{2(I_2(x, y) - I_4(x, y))}{2I_3(x, y) - I_5(x, y) - I_1(x, y)} \right] \quad 2.18$$

2.1.3 Phase Unwrapping

Unfortunately the result of Equation 2.18 is phase wrapped by modulo of π due to the inverse tangent function having a range of π from $-\pi/2$ to $\pi/2$. The range can be extended by considering the signs of the numerator and denominator of Equation 2.18

independently, yielding wrapped phase modulo 2π . Assuming that the original phase surface is continuous, any discontinuities in measured phase must be a result of wrapping due to the inverse tangent. Therefore, by adding or subtracting the appropriate multiple of 2π to the wrapped portion of the wavefront, the original wavefront can be recovered. FIGURE 2.2 shows an example of phase unwrapping in one dimension. The wrapped phase is shown in the grey box under 2π , and the unwrapped phase is the continuous line shown on top.

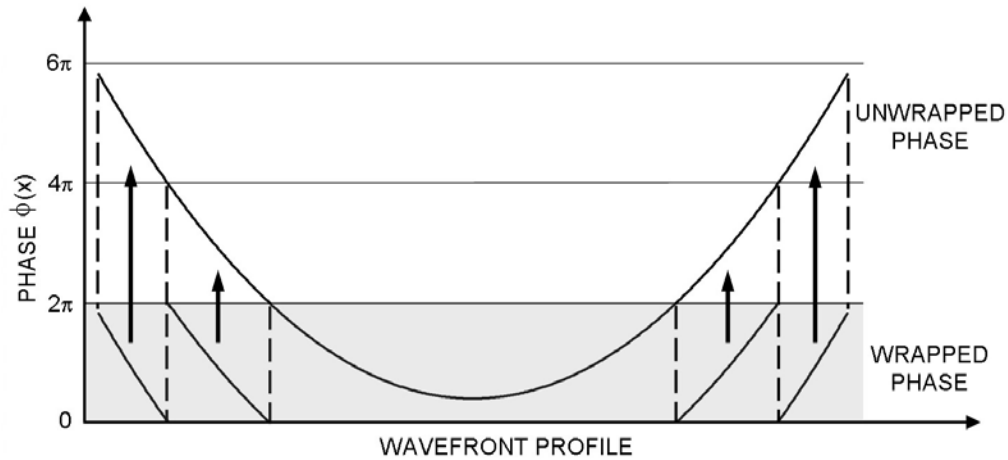


FIGURE 2.2 One Dimensional Phase Unwrapping

In FIGURE 2.2, the discontinuities are obvious because the phase profile hasn't been sampled along the x-axis, so the wrapping locations are where the phase changes between 0 and 2π instantaneously. When the wavefront is sampled there must be a threshold placed on the phase change between adjacent pixels, that when exceeded, the wavefront is assumed to have wrapped. PSI assumes that the phase changes by less than π between adjacent sampled points or pixels. The effects of spatial sampling on interferograms will be discussed further in Chapter 2.2. The phase unwrapping algorithm selects solutions to

Equation 2.18, such that the phase at every pixel is within π of the adjacent pixels. This can be seen graphically in FIGURE 2.3.

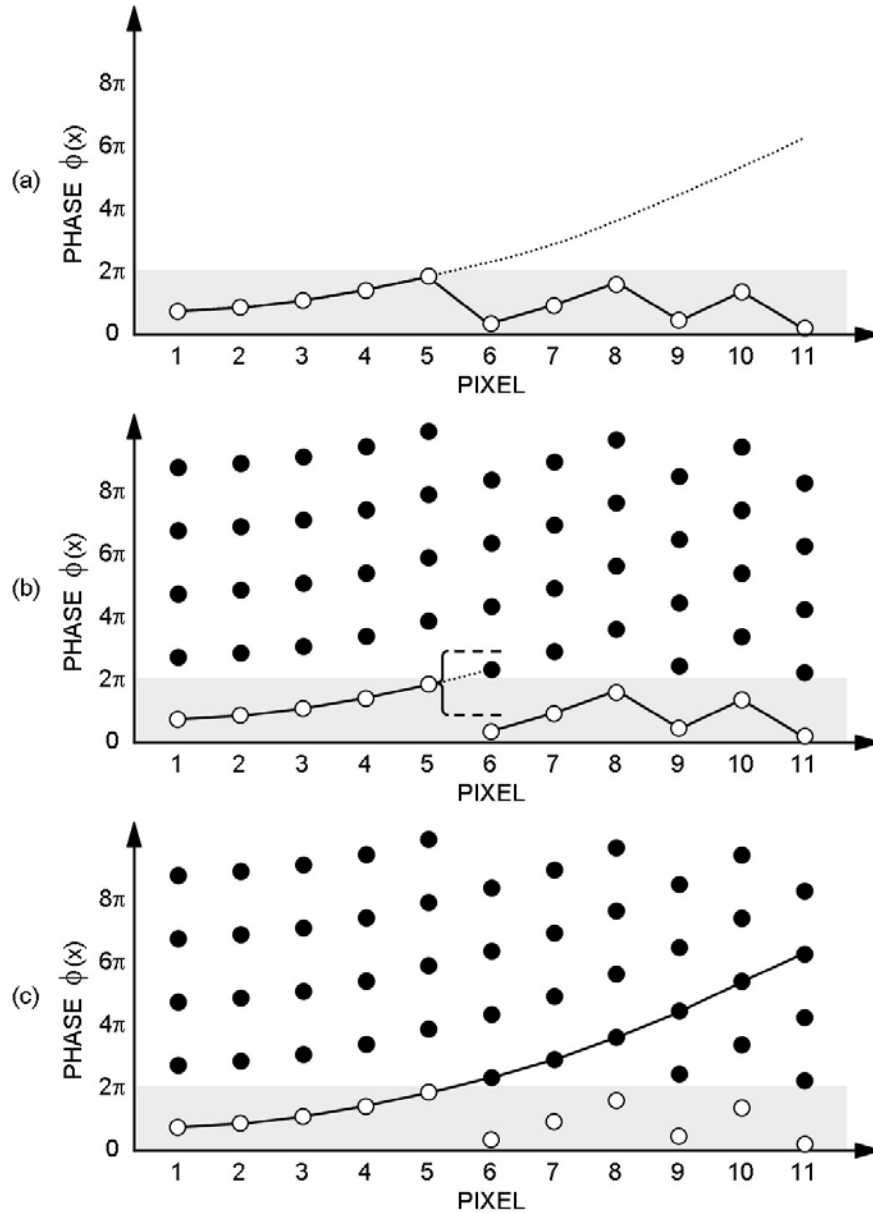


FIGURE 2.3 One dimensional phase unwrapping on sampled wavefront

FIGURE 2.3(a) shows the original phase profile as the dotted line, as well as the measured, modulo 2π , phase data represented by the open circles. FIGURE 2.3(b) shows

some of the possible solutions to the inverse tangent of Equation 2.18 represented by the closed circles. The phase unwrapping procedure starts at the first pixel and moves outward selecting the phase value at the next pixel that is within $\pm\pi$. The dashed lines indicate the $\pm\pi$ range from the fifth to sixth pixel. It is clear that the correct solution at the sixth pixel location is the one just outside the 2π range. Thus the measured phase profile wrapped at this location and the unwrapping algorithm needs to add 2π to phase value of at the sixth pixel. This process is repeated across the entire profile, until the original phase has been restored, as shown in FIGURE 2.3(c). Also, note that if a different pixel was chosen as the starting point, the same phase profile would be recovered, with the exception of a possible vertical shift, keeping in mind that there are also negative solutions to the inverse tangent. Phase unwrapping can be extended to 2 dimensions by unwrapping a profile in one dimension and then using the values along this profile as the starting points for an unwrapping procedure in the orthogonal direction, FIGURE 2.4

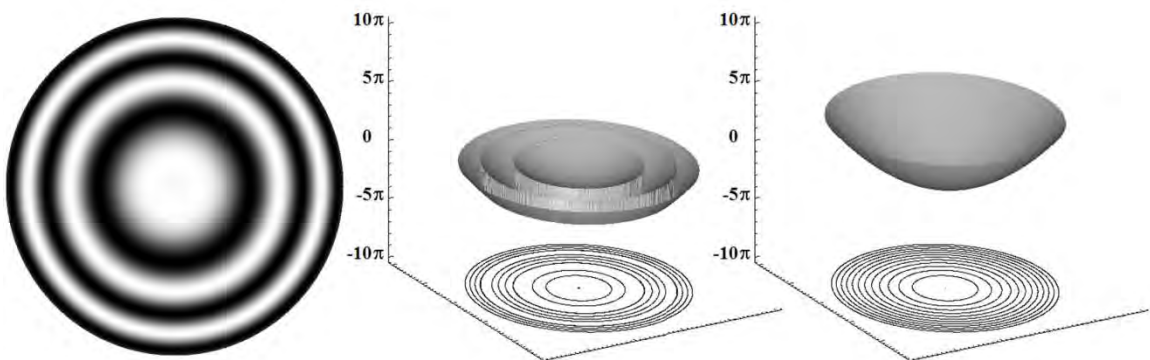


FIGURE 2.4 Two dimensional phase unwrapping; A single interferogram (Left), the wrapped phase (Center) and the unwrapped phase (Right)

While this process is conceptually simple because it is path dependent it can fail due to localized errors, such as areas of low modulation, or missing data due to obstructions or bad pixels. In order to isolate these areas, modifications can be made to the algorithm, such as following the path of high modulation or using several well established path independent algorithms. “Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software” by Dennis C. Ghiglia, Mark D. Pritt, is a good starting point for more information (Ghiglia & Pritt, 1998).

2.1.4 Modulation

In addition to solving for the phase the average intensity, $I'(x, y)$, and the fringe modulation, $I''(x, y)$, could also be solved from Equations 2.12 - 2.16. Generally of more interest is the data modulation, $\gamma(x, y)$, which is related to the fringe modulation and average intensity by, Equation 2.19.

$$\gamma(x, y) = \frac{I''(x, y) \operatorname{sinc} \left[\frac{\Delta}{2\pi} \right]}{I'(x, y)} \quad 2.19$$

In terms of the five interferograms the solution is given in Equation 2.20. Note that the $I_i(x, y)$ notation for the interferograms has been simplified to, I_i , to save space.

$$\gamma(x, y) = \frac{3\sqrt{4(I_4 - I_2)^2 + (I_1 + I_5 - 2I_3)^2}}{2(I_1 + I_2 + 2I_3 + I_4 + I_5)} \quad 2.20$$

Now the change in the modulation for phase-stepping and phase ramping using the Schwider-Hariharan algorithm can be calculated. Assuming five consecutive frames are

used for phase-ramping, the change in the phase shift, Δ , over the course of a frame should be approximately equal to δ or $\pi/2$. Since Δ is zero for phase-stepping, the modulations are related by,

$$\gamma_{ramp}(x, y) = \gamma_{step}(x, y) \text{sinc}\left[\frac{\Delta}{2\pi}\right] \approx \gamma_{step} \text{sinc}\left[\frac{1}{4}\right] \approx 0.9\gamma_{step} \quad 2.21$$

Therefore theoretically phase ramping causes a 10% degradation in the modulation compared to phase-stepping. The actual reduction is smaller, around 4%, for several reasons. One being that the acquisition time, t_a , for a single frame is shorter than the length of time between frames and therefore the change in the phase shift, Δ , is less than $\pi/2$. Both techniques were implemented in the hardware built for this research. Usually phase ramping was used because of the increase in speed of data collection and reduction is sensitivity to air turbulence and vibration. However if the loss in modulation couldn't be tolerated phase stepping could be used with the instrument isolated to minimize the effects of air turbulence and vibration.

2.2 Sampling

In addition to the loss of modulation due to the interferogram varying over the acquisition time, there will also be a reduction in the modulation due to averaging over the active area of the pixels in the sensor used to record the interferogram.

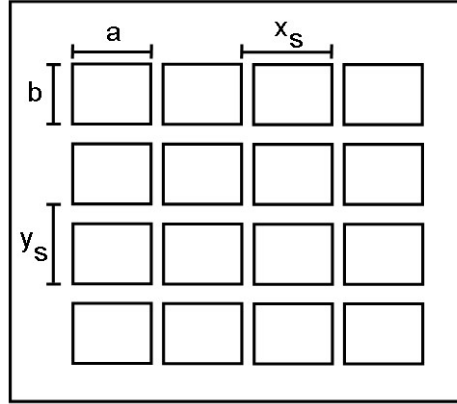


FIGURE 2.5 Pixelated Sensor Geometry

If a sensor geometry shown in FIGURE 2.5 is assumed, with rectangular pixels of width (a) and height (b), spaced in the horizontal and vertical dimensions by x_s and y_s respectively, then the sampled interferogram is given by Equation 2.22, (Greivenkamp & Brunning, 1992)

$$I_i^s(x, y) = \left[I_i(x, y) ** \text{rect}\left(\frac{x}{a}, \frac{y}{b}\right) \right] \text{comb}\left(\frac{x}{x_s}, \frac{y}{y_s}\right) \quad 2.22$$

Where I_i is given by Equation 2.9, which is convolved (**) with the 2 dimensional rect function to represent the average intensity over the rectangular active area of a pixel, and multiplied by the comb function, which generates one value of this average at every pixel location in the two dimensional array. The frequency-spaced representation of Equation 2.22, in terms of the spatial frequency coordinates ξ and η , can then be found by taking the Fourier transform.

$$\tilde{I}_i^s(\xi, \eta) = \left[\tilde{I}_i(\xi, \eta) \text{sinc}(a\xi, b\eta) \right] ** \text{comb}(x_s\xi, y_s\eta) \quad 2.23$$

Where,

$$\text{sinc}(a\xi, b\eta) = \text{sinc}(a\pi\xi) \text{sinc}(b\pi\eta) \quad 2.24$$

The result of the interferogram being averaged over the size of the pixel is a reduction in the contrast by the value of the sinc function dependent on the fringe frequency present at the pixel. This result is similar to the reduction in modulation due to time varying phase shift as shown in Equation 2.19. The absolute value of the $\text{sinc}(a\xi, b\eta)$ term is known as a pixel MTF, and its first zero is the pixel MTF cutoff frequency. The width-to-pitch ratio, G , is a useful parameter for comparing the pixel MTF of different sensors. Where $G_x = a/x_s$ for the horizontal axis and $G_y = b/y_s$ for the vertical axis. (Grievenkamp, 1987) When $G=1$ the pixels are contiguous; as G decreases, the sensor becomes more sparsely populated with pixels. The pixel MTF in one dimension is shown in FIGURE 2.6 for $G=1$, $G=1/2$, and $G=1/4$. By halving the G factor, the cutoff frequency is doubled.

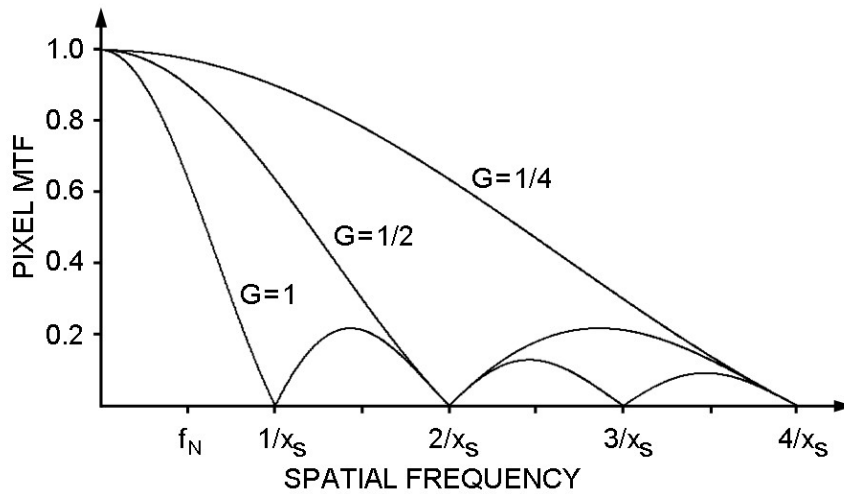


FIGURE 2.6 MTF of sensors with different G factors.

The quantity f_N is the Nyquist frequency of the sensor and is defined to be half the sampling frequency, Equation 2.25, in the horizontal and vertical direction respectively.

$$f_{N_x} \equiv \frac{1}{2x_s}, \quad f_{N_y} \equiv \frac{1}{2y_s} \quad 2.25$$

The Nyquist frequency is the limiting resolution of the sampled system. If a signal above the Nyquist frequency is present on the sensor it will alias to a lower frequency, which will be discussed in the next section. In order to avoid aliasing most sensors are not designed to respond to frequencies above the Nyquist frequency. Sensors are typically designed to maximize the active area of the pixel to improve light collection, resulting in pixels size approximately equal to the pixel spacing and a G factor of approximately 1. When the sensor is illuminated with fringes at the Nyquist frequency one fringe covers two pixels. The modulation is reduced because a single pixel will record the average intensity over half of the fringe. At the pixel cutoff frequency, the width of the fringe is equal to the width of a pixel. Therefore every pixel will record the average intensity over the entire fringe, regardless of the phase shift, resulting in zero modulation.

2.3 Aliasing

Aliasing is the property of sampling systems to display high frequency signals, those above the Nyquist frequency, as low frequency signals. This can be seen in FIGURE 2.7, which shows three input signals which increase in frequency. The vertical lines represent pixels which sample the signals and the dots represent the sampled values at each pixel. The first signal has a frequency below the Nyquist frequency, at two-thirds the Nyquist frequency. While the second and third have frequencies above the Nyquist frequency, at four-thirds and eight-thirds respectfully. However, when each of the input signals is sampled the same values are recorded at each sampled location, represented by the dots. Thus, each signal is recorded as the low frequency, two thirds the Nyquist frequency.

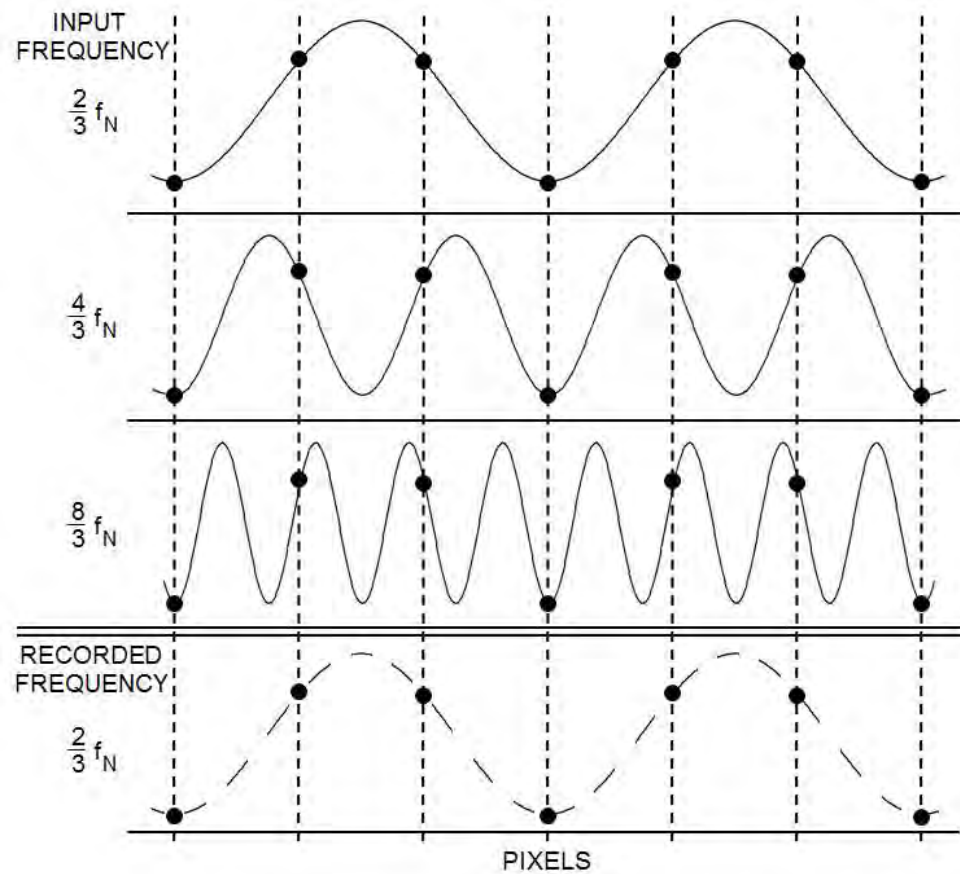


FIGURE 2.7 Three fringe frequencies which alias to the same recorded frequency when sampled.

The origin of aliasing can be seen by graphing Equation 2.23 in one dimension, shown in FIGURE 2.8. For fringes to be sufficiently sampled, the bandwidth, $1/a$, must be less than the Nyquist frequency of the sensor, FIGURE 2.8(a). In this case there is no confusion about the recorded fringe frequencies, because the replicated frequency spectra do not overlap. Therefore, the recorded fringe frequency is always the same as the input frequency. However, if the fringe frequency bandwidth is greater than the Nyquist frequency, FIGURE 2.8(b), then the replicated fringe frequency spectra overlap creating aliasing. There is no longer a one-to-one relationship between the input and recorded frequencies.

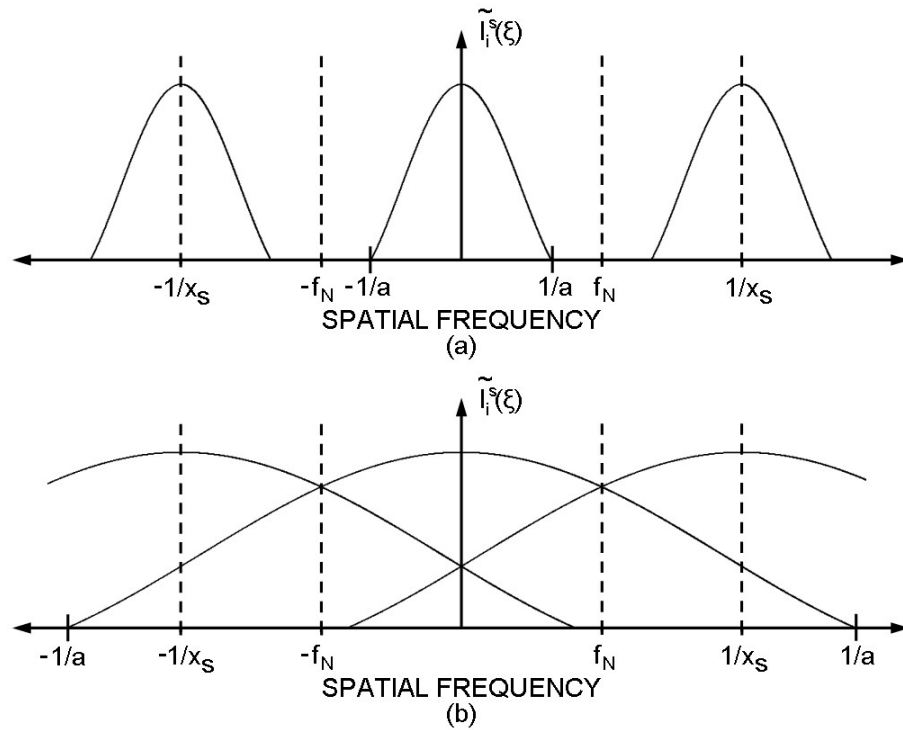


FIGURE 2.8 Aliasing in Frequency Domain

A frequency higher than the Nyquist frequency will be displayed as a lower frequency inside the sensor's baseband; which is the region between 0 and the Nyquist Frequency. The mapping of the frequencies above the Nyquist frequency back into the baseband is shown in FIGURE 2.9, using the same frequencies discussed for FIGURE 2.9.

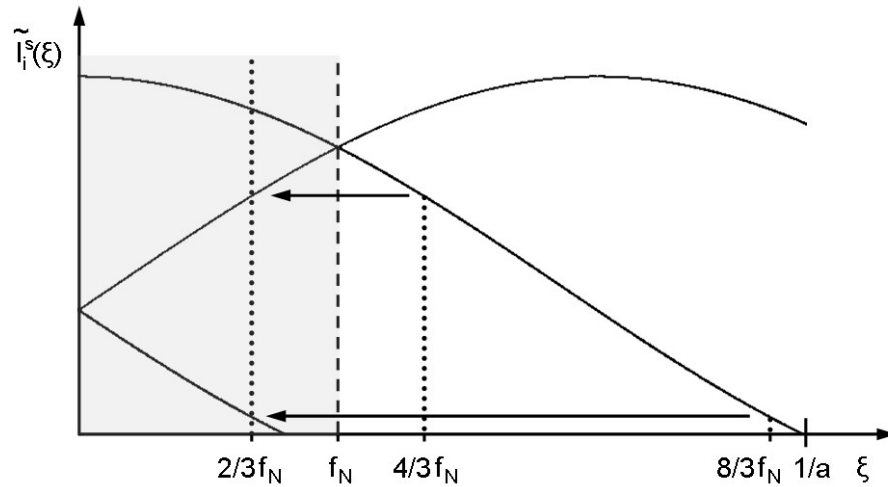


FIGURE 2.9 Mapping of frequencies above the Nyquist frequency back into the region below the Nyquist Frequency.

2.4 Sub-Nyquist Interferometry

Sub-Nyquist Interferometry is an extension of PSI that can recover the original phase information from aliased fringe patterns by using *a priori* information about the wavefront (Greivenkamp 1987). The Whittaker-Shannon sampling theorem states that a signal can be recovered without error from its sampled values provided the signal is band-limited to less than the Nyquist frequency of the sensor used to sample the signal (Gaskill 1978), (Bracewell 2000). However, it does not claim that the inverse is true. In fact a signal that is not band-limited to the Nyquist frequency of the sensor can in some instances be recovered without error provided additional information is known about the signal. One method of recovering aliased scene without error is Sub-Nyquist sampling. (Barratt & Lucas 1979) The recorded frequency present at a given pixel of a sub-Nyquist sampled image, such as an interferogram, will be recorded as a frequency within the baseband of the sensor due to aliasing. However, if additional information is known

about the frequency content at that pixel it may be possible to remap the recorded frequency back to its original frequency. For instance, looking back at FIGURE 2.9, if a frequency of $2/3F_N$ is recorded for some portion of a scene, the original frequency could be $2/3F_N$, $4/3F_N$ or $8/3F_N$. If it is known that the area of the scene from which the frequency was observed does not contain a frequency less than F_N or greater than $2F_N$, then the original input frequency must have been $4/3F_N$.

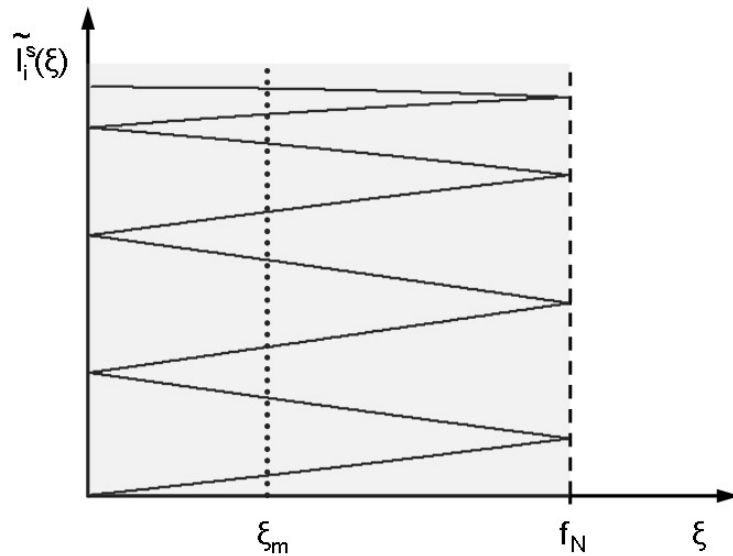


FIGURE 2.10 Aliasing causes multiple frequencies to be recorded as the same measured frequency ξ_m .

A more general example is shown in FIGURE 2.10 in which the frequency response of a detector has been folded back and forth, between 0 and the Nyquist frequency, until the entire curve is within the base band of the sensor. This wrapping of high frequency signals back into the baseband of the sensor causes ambiguity into which original frequency, ξ_o , was recorded as the measured frequency ξ_m . There are several possible

frequencies ξ_o given by, Equation 2.26, where the total number of possible values of ξ_o is limited by the cutoff frequency of the sensor.

$$\xi_o = |\xi_m \pm 2nf_N|, \quad n = 0, 1, 2, 3... \quad 2.26$$

In order to remap a frequency from ξ_m back to ξ_o , the intersection of the vertical line at ξ_m with the correct branch of the folded response curve shown in FIGURE 2.10 must be known. Interferograms are ideal candidates for sub-Nyquist sampling because in an interferogram the aliasing is a localized phenomenon, meaning aliasing will occur in areas of the interferogram where high frequency fringes are present, but will not produce artifacts in other areas of the image. The assumption used, to provide the *a priori* information in sub-Nyquist interferometry, is that the derivative of the wavefront under test is continuous. This means that the change in the fringe frequency across the interferogram must be continuous, since the wavefront slope is directly related to the fringe frequency.

The PSI unwrapping procedure assumes that the original fringe frequency is the measured fringe frequency; therefore fringe frequencies are bound to the baseband of the sensor or the top branch of FIGURE 2.10. When ξ_o increases past the Nyquist frequency ξ_m begins to decrease causing the PSI unwrapping procedure to incorrectly assume that ξ_o is decreasing. This leads to the slope of the wavefront suddenly changing signs whenever the fringe frequency is equal to an odd multiple of the Nyquist frequency.

In SNI unwrapping, the assumption of slope continuity means that while the slope of the wavefront is increasing, the fringe frequency must be increasing, and while the slope is decreasing, the fringe frequency must also be decreasing. The SNI unwrapping procedure should start in a location containing frequencies in the baseband of the sensor, along the top branch of FIGURE 2.10. As ξ_0 increases past the Nyquist frequency, the SNI unwrapping procedure moves on to the next branch. As the slope of the wavefront increases, the fringe frequency is assumed to be “walking down” the graph from branch to branch. As the slope of the wavefront decreases, the fringe frequency is assumed to be walking back up the graph; thus, the SNI unwrapping is able to predict the correct original fringe frequency from the measured fringe frequency. SNI will run into a limit when the slope of the wavefront changes by more than π /pixel/pixel. At this point the slope between neighboring pixels is changing so rapidly that an entire branch is being skipped. There is the possibility of using the assumption that the 2nd derivative is also continuous in order to slightly increase the range even further as described by Greivenkamp (1987).

2.4.1 SNI Phase Unwrapping

The slope continuity requirement allows wrapped phase generated from aliased interferograms to be interpreted. The wrapped phase is calculated in the exact same manner as PSI, using Equation 2.18. The phase unwrapping procedure, however, must be modified to ignore the π per pixel height constraint of PSI and implement the slope

continuity constraint of SNI. This can be seen graphically in FIGURE 2.11

(Greivenkamp 1987).

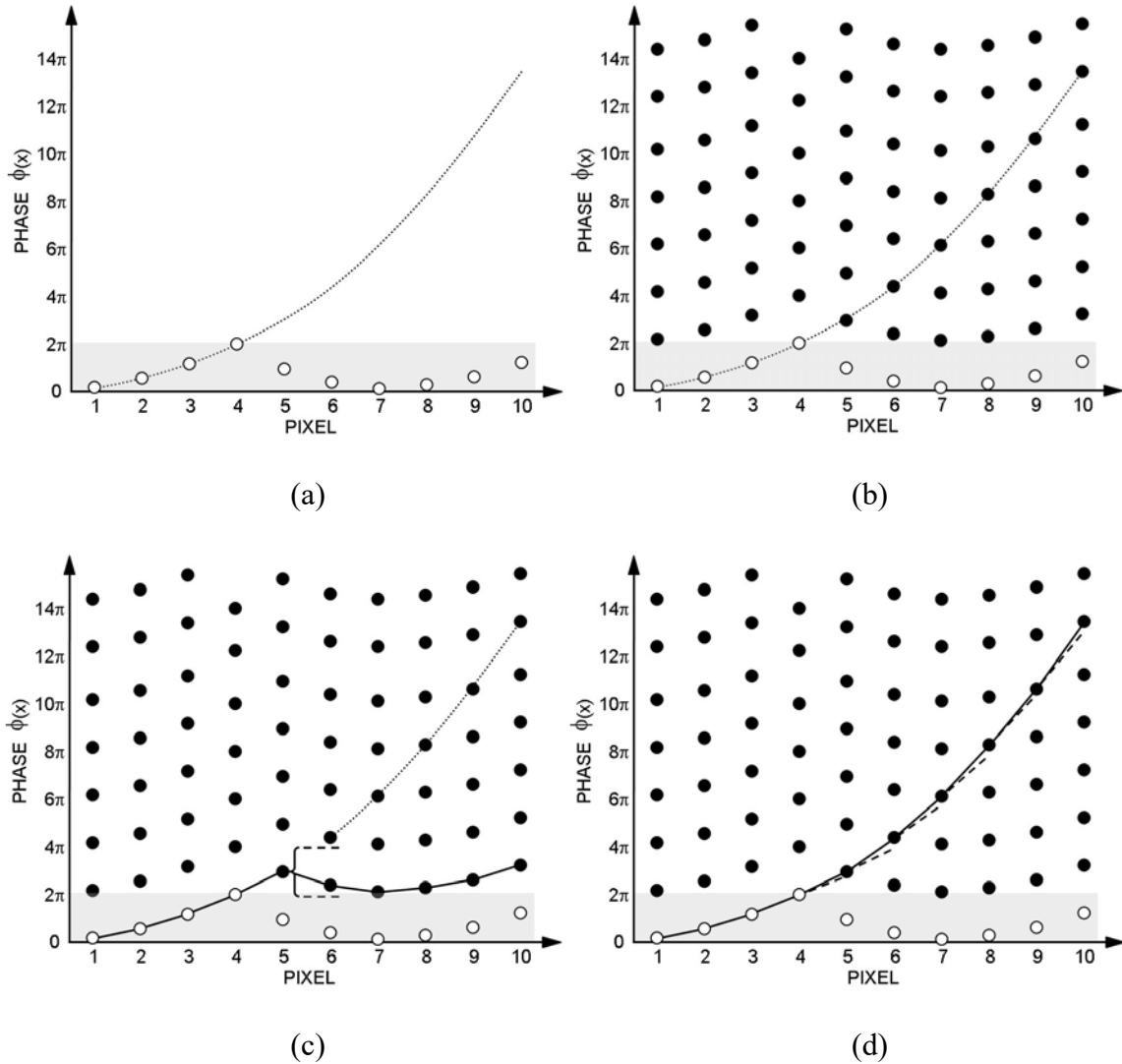


FIGURE 2.11 One dimensional SNI phase unwrapping.

In FIGURE 2.11 (a) the original phase profile is shown as the dotted line and the measured modulo 2π phase data is represented by the open circles. The first step in the unwrapping process is to calculate the possible solutions to the inverse tangent of Equation 2.18 as shown in FIGURE 2.11(b) closed circles. The PSI reconstruction is

shown FIGURE 2.11(c) which fails between the 5th and 6th pixel because the phase change is greater than π . This is the location where the fringe frequency exceeds the Nyquist frequency, and the slope of the reconstructed wavefront suddenly changes signs. The SNI unwrapping procedure is illustrated in FIGURE 2.11(d). The slope of the wavefront between pixels, shown as the dashed line, is used to predict the phase at the next pixel. The closest solution to the projected value is then selected as the correct solution. This procedure is continued outwards towards the edge of the interferogram. Note that if a pixel after the 4th pixel was chosen as a starting point of the SNI unwrapping procedure, a significant amount of tilt would be introduced into the wavefront. Therefore, it is important to start the unwrapping procedure at a pixel where the fringes are not aliased. There are several methods for determining which fringe pattern is the result of non-aliased fringes. First, the region with the highest modulation, as calculated by Equation 2.20, is likely the non-aliased region of the interferogram. Another method is to introduce a slight vibration into the sensor, by lightly tapping on the camera housing and observing live video of the fringe pattern. High fringe frequencies are more sensitive to the vibration and will grey out before lower frequency fringes. Furthermore the case may arise in which the interferogram does not contain the null fringe. One method that guarantees the identification of the null fringe is visual inspection of the fringe pattern. This works because the human eye will not perceive the aliasing of the sensor so only the true low frequency fringes will be visible.

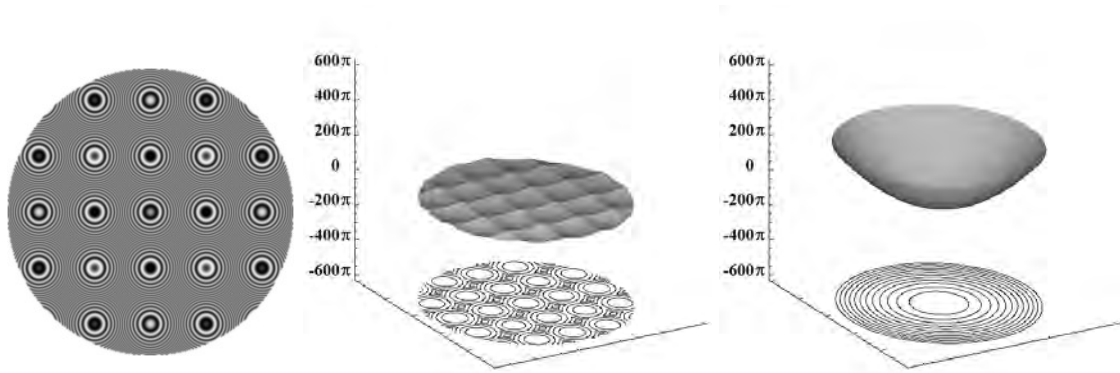


FIGURE 2.12 An aliased interferogram (Left), the PSI phase reconstruction (Center) and the SNI reconstruction (Right).

Examples of sub-Nyquist unwrapping of an aliased interferogram are shown in FIGURE 2.12. Where FIGURE 2.12(a) is one of five aliased interferograms required to calculate the wrapped phase surface using the Schwider-Hariharan algorithm. The center ring pattern contains the non-aliased fringes, while the multiple surrounding patterns are the result of higher frequencies aliasing into the base band. The incorrect PSI reconstruction is shown in FIGURE 2.12(b), while the correct SNI reconstruction is illustrated in FIGURE 2.12(c). The SNI reconstruction can be performed directly on the wrapped phase; however, it requires a starting point that is free of 2π ambiguities to calculate the correct slope of the wavefront. For wavefronts with large departures, the area that is free of any wrapping artifacts can be very small. Generally, first the PSI unwrapping algorithm is applied to the wrapped phase, which correctly unwraps the wavefront until the fringe frequency exceeds the Nyquist frequency of the sensor. This widens the area that is free of 2π ambiguities around the null fringe of the interferogram. The SNI reconstruction is then started within this region. The actual unwrapping procedure

written for this research and the method of correcting errors caused by path dependent unwrapping will be discussed in more detail in Chapter 5.2.

2.4.2 Previous SNI Research

As previously stated, the idea of sub-Nyquist interferometry was first described by Greivenkamp (1987). In this paper the theory behind sub-Nyquist sampling of interferograms were first discussed. Computer simulations demonstrating the performance of SNI unwrapping procedure were presented as well as a comparison of SNI to two-wavelength phase shifting interferometry as methods of extending the range of PSI. Palum and Greivenkamp (1990) built a sub-Nyquist interferometer by modifying a commercially available laser-based Fizeau interferometer. They demonstrated the ability to record and unwrap highly aliased fringe patterns generated from by a defocused spherical surface as well as an aspheric surface with 42 waves of departure. They also highlighted the need for calibration of the interferometer in order to account for the errors introduced by the violation of the null condition.

Additionally several problems were discovered in converting the interferometer to handle the interferograms produced from aspheric test parts, leading to several design considerations for future non-null interferometers. For example, in many commercial phase shifting interferometers the interferogram is imaged onto a rotating ground glass plate and then reimaged onto a detector by a zoom lens. The ground glass changes the interferogram from a coherent image into and incoherent object for the zoom lens,

removing the need to consider the OPD generated by the zoom lens. This design allows the interferometer to accommodate a larger range in test part diameters since the image of the interferogram at the ground glass can be scaled by the zoom lens to fill the detector. Palum and Greivenkamp discovered that the grain size of the ground glass obscured the high frequency fringes associated with non-null testing of aspheric surfaces and thus it needed to be removed from the system. However once it was removed the multiple surfaces of the zoom lens created many spurious fringe patterns on the detector. Also, without the ground glass in the system, the zoom lens produces a coherent image of the interferogram at the detector and thus errors resulting from the non-common path of the reference and test wavefronts through the zoom lens must be considered. Thus the zoom lens was removed from the system and the detector was placed at the image plane previously occupied by the ground glass. In order to block spurious light from reaching the detector a pinhole is often placed at the focus of the imaging lens in an interferometer in order to act as a spatial filter. However, Palum and Greivenkamp noticed that the pinhole also blocked light from the high slope regions of the aspheric test part and thus it too needed to be removed. Finally, problems were encountered with the A to D converter bandwidth and the A to D converter synchronization. Fringes recorded at the Nyquist frequency cause the signal at neighboring pixels to alternate between bright and dark. The camera electronics and the electronics used to digitize the signal must have the bandwidth to record such a signal with good modulation. The timing sampling of the video read out signal by the digitization electronics is important to maintain good modulation since fringes recorded at the Nyquist frequency will cause the video signal to

oscillate at half the clock pulse frequency. If the synchronization is off by half the pixel clock period the modulation will go to zero for fringes at the Nyquist frequency.

Lowman and Grievenkamp designed and built a Sub-Nyquist Twyman-Green interferometer to test aspheric surfaces (Lowman & Grievenkamp 1994), (Lowman, 1995), (Grievenkamp et al, 1996). They highlight the need for the interferometer to be designed to account for the vignetting that can occur when highly aberrated wavefronts propagate through an interferometer. They developed a test method to measure the MTF of a sparse-array sensor at multiples of the Nyquist frequency using a self-calibrating fringe pattern (Lowman & Grievenkamp, 1994). This test allows problems with the A/D bandwidth to be detected and the A/D synchronization to be optimized. (Gappinger et al, 2004) improved the process to allow data from non-multiples of the Nyquist frequency to be measured. This process will be discussed in more detail in Chapter 4.2. Additionally a reverse optimization procedure was developed which used a ray tracing program and a model of the interferometer in order to calibrate the interferometer and account for the retrace errors introduced by the interferometer (Lowman 1995). Measurements of a defocused spherical surface, generating 100λ of surface departure, were made and calibrated to better than a quarter wave peak-to-valley. However the interferometer could not be characterized sufficiently to calibrate measurements of aspheric surfaces.

Gappinger and Grievenkamp built a Mach-Zehnder sub-Nyquist interferometer to measure the aspheric transmitted wavefronts of progressive bifocal lenses (Gappinger,

2002), (Gappinger & Greivenkamp 2003), (Greivenkamp & Gappinger 2004). This interferometer employed an iterative reverse optimization process in order to successfully remove up to 25λ of interferometer induced aberrations on a wavefront of more than 240λ of aspheric departure. Calibration of $\lambda/6$ PV was demonstrated for wavefronts with departure of 200λ .

3 RAY TRACING SOFTWARE FOR MODLING A NON-NULL INTERFEROMETER

The ray tracing software used for this project was Zemax produced by Zemax LLC. (Kirkland, WA) The primary function of a sequential ray tracing software like Zemax is the design of imaging systems. While its capacity for modeling other types of optical systems is constantly being expanded many of its definitions and functions are focused on imaging optics. This chapter will highlight some of the definitions, settings and user written programs used to design and model a non-null interferometer which will be referenced in future chapters. While this discussion will be centered on the properties of Zemax and the steps that are nessecary to model a non-null interferometer in Zemax, much of this discussion may be applicable to other commercial ray tracing programs. Zemax, like any commercial software is constantly being updated, so spefic properties of the software are subject to change. The version of Zemax used in this research range from the 2003 release through the July 2011 release.

3.1 Ray Tracing a Conventional Imaging System

The underlying principles and the techniques for designing well corrected lenses will not be discussed here as they are the subject of countless papers and books, such as “Lens Design Fundamentals” by Rudolf Kingslake (1978) and “The Art and Science of Optical Design” by R. R. Shannon (1997). However, a very basic description of the process and definitions needed for to describe the modeling a non-null interferometer will be discussed. Lens design software, such as Zemax, models the performance of optical

systems by tracing bundles of rays originating at several points on the object plane sequentially, from surface to surface, through the optical system and onto the image plane. The angular spread of the bundle from the axial object point is limited by a physical aperture known as the aperture stop (Greivenkamp 2004). The images of the aperture stop into object and image space by the system are defined as the entrance and exit pupil of the system. Rays are defined by two vectors, the normalized field vector, $\vec{H}$, and their normalized aperture or pupil vector, $\vec{\rho}$. The normalized field vector is defined as the vector pointing from the optical axis to the rays starting location in the object plane. The normalized pupil vector yields the initial angle at which the ray is launched by defining the vector pointing from optical axis to the rays intersection with the entrance pupil. There are two special rays known as the marginal ray and the chief ray which define the paraxial locations of the pupils and the image plane. By definition the marginal ray starts at the axial position in the object plane, $H = 0$, and passes through the edge of the entrance pupil, $\rho = 1$. The chief ray starts at the edge of the object, $H = 1$, and passes through the center of the entrance pupil $\rho = 0$. An image is formed whenever the marginal ray crosses the axis and the size is determined by the height of the chief ray at that point. Likewise a pupil is defined whenever the chief ray crosses the axis and its size is determined by the height of the marginal ray, FIGURE 3.1.

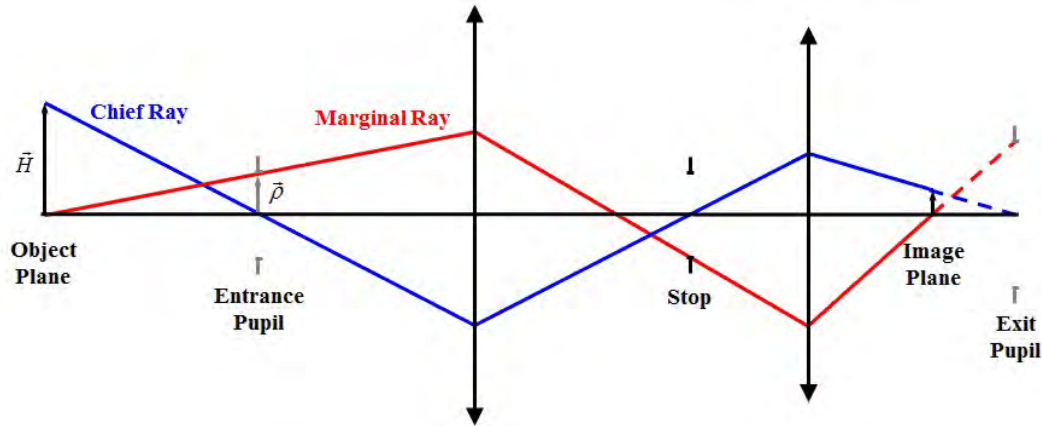


FIGURE 3.1 Pupils are defined by the chief and marginal rays.

The quality of the image, and thus the optical system, is determined by the errors in the location of the rays arriving at the image plane, known as ray aberrations, and the error in the phase of the wavefront, known as wavefront aberrations, relative to a perfect spherical wavefront (Shannon 1997). This perfect spherical wavefront is known as the reference sphere and is centered at the paraxial image location. Wavefronts are calculated from the traced rays, whereby a wavefront is defined as a surface over which rays have a constant OPL. The direction of ray propagation defines the normal vector to the wavefront. Wavefront aberrations or wavefront error, $W(\vec{H}, \vec{\rho})$, is the difference between wavefront calculated from the rays and the reference sphere in the exit pupil of the system FIGURE 3.2. Transverse ray errors, ϵ_x and ϵ_y , and longitudinal ray errors ϵ_z are measured with respect to the paraxial image point. The transverse ray aberrations, ϵ_x and ϵ_y , are related to the slope of the wavefront aberrations by Equations 3.1 and 3.2 (Greivenkamp 2004). Where x_p and y_p are the components of the pupil vector, R is the radius of the reference sphere and r_p is the radius of the exit pupil, FIGURE 3.2.

$$\varepsilon_x(\vec{H}, x_p, y_p) = -\frac{R}{r_p} \frac{\partial W(\vec{H}, x_p, y_p)}{\partial x_p} \quad 3.1$$

$$\varepsilon_y(\vec{H}, x_p, y_p) = -\frac{R}{r_p} \frac{\partial W(\vec{H}, x_p, y_p)}{\partial y_p} \quad 3.2$$

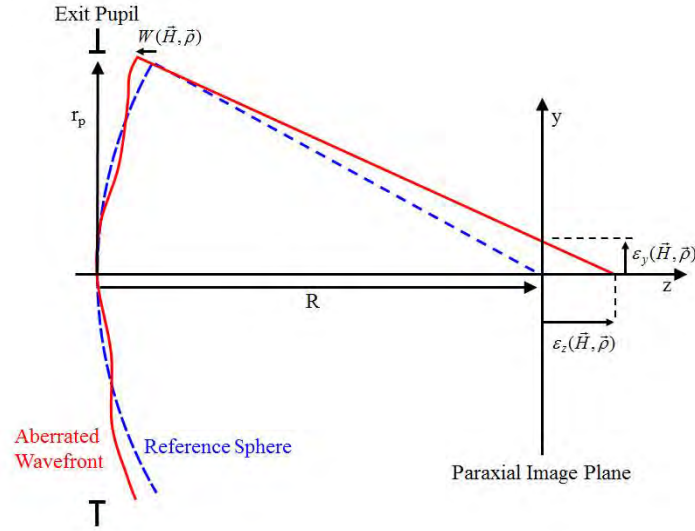


FIGURE 3.2 Definition of transverse ray aberration, ε_y , longitudinal ray aberration ε_z , and the wavefront error, $W(\vec{H}, \vec{\rho})$.

The goal of the designer and the software is to produce a lens which minimizes these errors while maintaining specific lens properties, such as the field of view, focal length and numerical aperture. This is accomplished by using a merit function in which target values and weights are assigned to various system properties and errors. The designer comes up with a reasonable starting design and then selects properties of the lens, such as surface curvatures, surface separations and indices of refraction to be variables. Then through a combination of insights by the lens designer and optimization of the merit function by the software a solution to the variables is found which minimizes the merit function, and thus improves the lens design.

3.2 Ray Tracing a Non-Null Interferometer

Zemax ray tracing software was used to model many aspects of the non-null interferometer. In addition to aiding in the design of the system, it was used to simulate measured data, to determine the optimal system set up for testing a specific aspheric surface, and finally for the reverse optimization and reverse ray tracing process. Each of these aspects place slightly different requirements on the software, and each will be covered more in depth in subsequent chapters. However, the goal of most of these processes is to predict the OPD between the test and reference arms of the interferometer and the resulting interference pattern or a property of the interference pattern, such as the maximum fringe frequency. In the interferometer design process, the interferograms for a range of test parts are generated in order to understand and improve the dynamic range of the system. Once the system is built, the predicted interference pattern is used to determine if a specific test part falls within the testable range of the interferometer and to determine the optimal testing configuration. During reverse optimization and reverse ray tracing the recorded interference pattern, or more precisely the measured wavefront from the actual system, is compared to the wavefronts generated by the model in order to quantify and separate the errors associated with the interferometer and the test part. While Zemax is capable of making these predictions; the implications of its default settings need to be considered and in some cases the settings must be changed. Also while Zemax has a large number of built in programs used for analysis and optimization

of a conventional lens designs it does not provide many options for analyzing interferograms. Therefore, several programs were written to extend its capability.

3.2.1 Reference OPD

One method of modeling the interference between the test and reference arm of an interferometer in Zemax is through the use of multiple configurations. Typically multiple configurations are used for non-static optical systems, such as zoom lenses in which the locations of the optical elements are varied between configurations to produce a lens with a changeable focal length. Two configurations can also be used to independently model the test and reference arms of an interferometer. Ideally the Zemax ray tracing engine could be used to trace rays through both configurations and record the OPL of each ray. The OPL of the reference rays could then be subtracted from their corresponding test ray to produce the OPD between the test and reference wavefronts. Unfortunately, Zemax does not always keep track of the OPL of every ray it traces especially when a large number of rays are traced simultaneously. Zemax does however keep track of the wavefront error or the optical path difference (OPD_Z) across a single wavefront. The word “difference” here refers to the difference between the traced wavefront and a perfect spherical wavefront at the exit pupil. It is important to note that this optical path difference, denoted OPD_Z , is not the same as the OPD defined in Chapter 1.1, which is the difference in the OPL of the test and reference rays. The method used to calculate OPD_Z is defined in the Zemax manual (Zemax LLC, 2011).

“Zemax by default uses the exit pupil as a reference for OPD_z computations. Therefore, when the OPD_z is computed for a given ray, the ray is traced through the optical system, all the way to the image surface, and then is traced backward to the "reference sphere" which lies in the exit pupil. The OPD_z as measured back on this surface is the physically significant phase error important to diffraction computations, such as MTF, PSF, and encircled energy. The additional path length due to the tracing of the ray backwards to the exit pupil, subtracted from the radius of the reference sphere, yields a slight adjustment of the OPD_z called the "correction term".” (Zemax LLC, 2011)

Calculating the optical path difference at the exit pupil relative to a spherical wavefront centered at the paraxial image is the standard definition when modeling an imaging system (Shannon 1997). Since the spherical wavefront will collapse to form a perfect image point, any non-zero OPD_z represents a departure from the diffraction limited image. In modeling an interferometer the OPD_z of the reference and test wavefronts can be used to calculate the OPD between them provided the same reference sphere is used for both OPD_z calculations. However, this can get complicated since the OPD_z will change if the stop, and therefore the exit pupil, is moved even if the move causes no additional OPL to be added to a given ray. This is shown in FIGURE 3.3 where a plane wavefront is traced from surface (a) to surface (d). The OPD_z is calculated at surface d, for four different locations of the stop. The reference sphere is centered on the image surface (d). Since stop position (a) produces the longest radius of curvature reference

sphere, it shows the least OPD_z . Decreasing the radius of curvature of the reference increases the calculated OPD_z across a plane wavefront relative to the reference sphere.

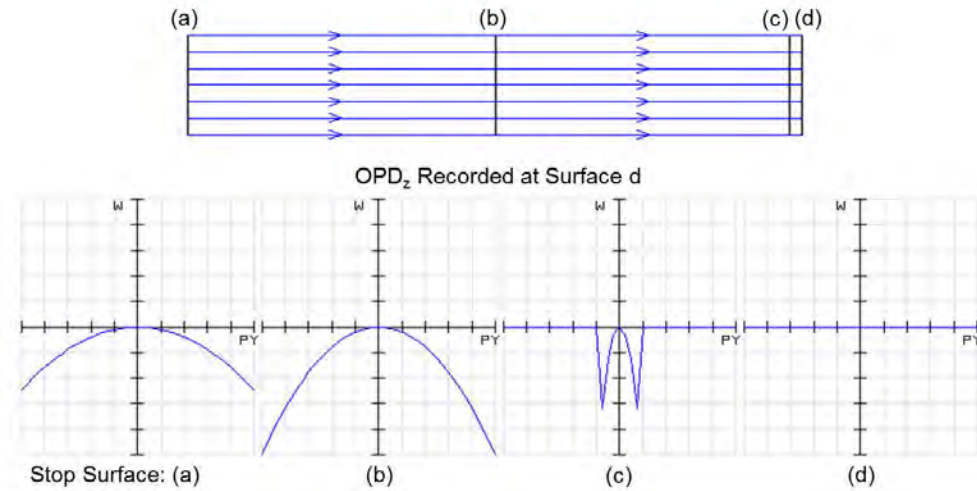


FIGURE 3.3 The default OPD_z calculation for a plane wave with the stop shifted between surfaces a, b, c, and d.

In FIGURE 3.3(c) the radius of the reference sphere is less than the semi-diameter of the surface; therefore the correction term is only applied over the portion of the aperture that is less than this radius. Note that having the stop at the last surface (d) does produce a uniform OPD_z across the pupil. This was not always the case as some versions of Zemax would attempt to use a reference sphere with a radius of curvature at or near zero, causing erroneous results.

Fortunately the method that Zemax uses to calculate the OPD can be modified by changing the Reference OPD_z setting to “Absolute”. In this mode OPD_z is defined as follows,

“The reference to "Absolute" means that Zemax does not add any correction term at all to the OPD_Z computation, but adds up the total optical path length of the ray and subtracts it from the chief ray.” (Zemax LLC, 2011)

Essentially this is the same as simply subtracting a piston term equal to the OPL of the chief ray from the OPL of the test and reference arms which allow the Zemax OPD_Z to be used as a substitute for the OPL. However there is one important distinction: rather than subtracting the OPL of the chief ray from every ray other ray traced, which would be the same as setting the OPL of the chief ray to zero, Zemax subtracts the length of every other ray from the chief ray, Equation 3.3.

$$OPD_Z = OPL_{ChiefRay} - OPL \quad 3.3$$

The effect of this definition can be seen in FIGURE 3.4 where light from a point source is traced to a plane. The rays on the outside of the ray bundle have a longer OPL than the center rays; however the OPD_Z for these rays is negative. In working within Zemax the negative sign has no impact; however this sign convention must be accounted for when importing real measurement data or exporting simulated data to an external program.

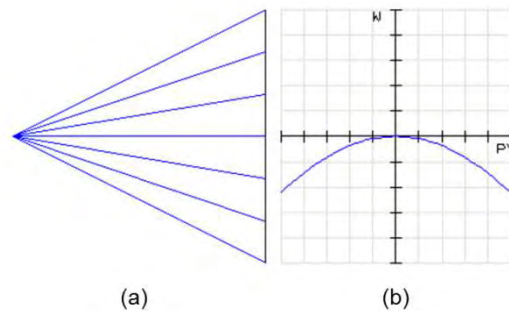


FIGURE 3.4 (a) Rays traced from a point source to a plane. (b) The OPD_Z calculated with the reference set to Absolute.

3.2.2 Normalized Field & Pupil Coordinates

Zemax defines rays by the x and y components of their normalized field, $\vec{H}$, and pupil, $\vec{\rho}$, vectors. By default Zemax uses the size and location of the paraxial entrance pupil to define the normalized pupil coordinates. Therefore Zemax traces rays for each field coordinate by first launching them at an even grid of points spread across the entrance pupil of the system. The Zemax manual provides a good explanation as to why rays are traced by normalized coordinates (Zemax LLC, 2011).

“By using normalized coordinates, the same ray set will work unaltered if the entrance pupil size or position or object size or position is changed later, or perhaps even during the optimization procedure.” (Zemax LLC, 2011)

However, there are a few reasons why using normalized field and pupil coordinates are problematic when modeling an interferometer. The interferometer used in this research is a Twyman-Green interferometer, which will be discussed in more detail in Chapter 4.3. The Twyman-Green interferometer, like many interferometer designs, makes use of a point light source. Therefore the field of view of the interferometer has no extent, or the x and y components of the field vector, $\vec{H}_x$ and $\vec{H}_y$, are zero for all rays. This also means that the chief ray is poorly defined. In the case where the field vector is zero for every ray in the model, Zemax uses the ray that propagates on the optical axis through the entrance pupil as the chief ray. However since pupils are located where the chief ray crosses the optical axis and this ray lies along the optical axis the locations of the pupils

are also poorly defined. In this situation Zemax traces several rays about the chief ray with small non-zero pupil vector to calculate the location of the pupils.

When using normalized pupil coordinates to trace rays in a model of an interferometer a problem arises when trying to compare the reference and test wavefronts generated by using two different configurations, or when comparing a simulated wavefront from the Zemax modeled to real data captured by the space array sensor. These comparisons need to be made at the same coordinates in real space. In order to ensure that rays with the proper real coordinates are being compared either the pupil coordinates and real coordinates must be forced to overlap at the surface of interest or calculations have to be done so that rays are launched at pupil coordinates which correspond to the correct real coordinates at the surface of interest. However the relationship between real coordinates and normalized pupil coordinates at a given surface in the Zemax model is complicated and it depends on several Zemax settings, as well as the pupil aberrations of the interferometer, which will be discussed later in this chapter. The Zemax settings which affect the relationship are the method chosen to define the aperture stop, as well as its size and location, and the use of ray aiming. Additionally the effect of ray aiming, which will be discussed in Chapter 3.2.6, depend on the interferometer errors and the role of the imaging lens in the interferometer.

3.2.3 Aperture Stop

There are several methods of either directly or indirectly defining the size and the location of the aperture stop in Zemax, such as by directly specifying the entrance pupil diameter or the object space NA. These and similar methods are useful when designing a system where the physical size of the stop is not specified by the design criteria since these methods allow the stop location and size to vary as the system is optimized. In an interferometer if the entire test surface is to be measured then there should not be another aperture which limits the light that reflects off the test surface and is relayed to the detector. Therefore the test surface should serve as the aperture stop of the interferometer. If the size of the test part is known, the stop diameter and location can be defined in Zemax by using the “Float by Stop Size” aperture setting. This ensures that test part remains fully illuminated as changes are made to the interferometer either in the design stage or during reverse optimization. However there are situations where it is advantageous to move the aperture stop away from the test part in the Zemax model, these situations will be discussed as they arise.

Since the test surface is the aperture stop of the interferometer then it is obviously also the stop of the test arm when the two interferometer arms are modeled as separate configurations. However in this situation the stop of the reference arm is not well defined. The true aperture stop of the reference arm is the aperture that physically limits the light in the reference arm. Its location depends on the design of the interferometer but likely candidates are the collimating lens, the reference surface or the detector. In

practice the reference wavefront needs to be the same size or larger than the test wavefront at the detector so that an interference pattern is observed for the entire test wavefront. Therefore the test wavefront at the detector acts as the stop for the model of the reference arm. The exact method used to set the stop size of the reference arm changes depending on how the interferometer model is being used and will be discussing in the applicable sections.

3.2.4 Imaging in an Interferometer

The role of an imaging lens in an interferometer and the relationship between aberrations in an interferometer and a conventional imaging system are described in depth by Murphy, Brown and Moore (Murphy et al, 2000a). The basic function of an imaging lens in an interferometer is not to form an image of the object, which in a Twyman-Green interferometer is a point source; rather it is to image the wavefront at the test surface onto the detector. As previously described the test part serves as the aperture stop of the system which means its image on the detector is the exit pupil of the system. However, when modeling the interferometer imaging optics independently of the rest of the interferometer, the aperture stop of the interferometer serves as both the entrance pupil and aperture stop of the interferometer's imaging optics. This leads to the roles of the marginal and chief rays being reversed.

As previously discussed the interferometer has only a single on axis field point, however in considering the interferometer imaging optics as an independent system the multitude

of possible test ray angles can be modeled as different field vectors. FIGURE 3.5 shows the difference between pupil imaging and a conventional imaging system.

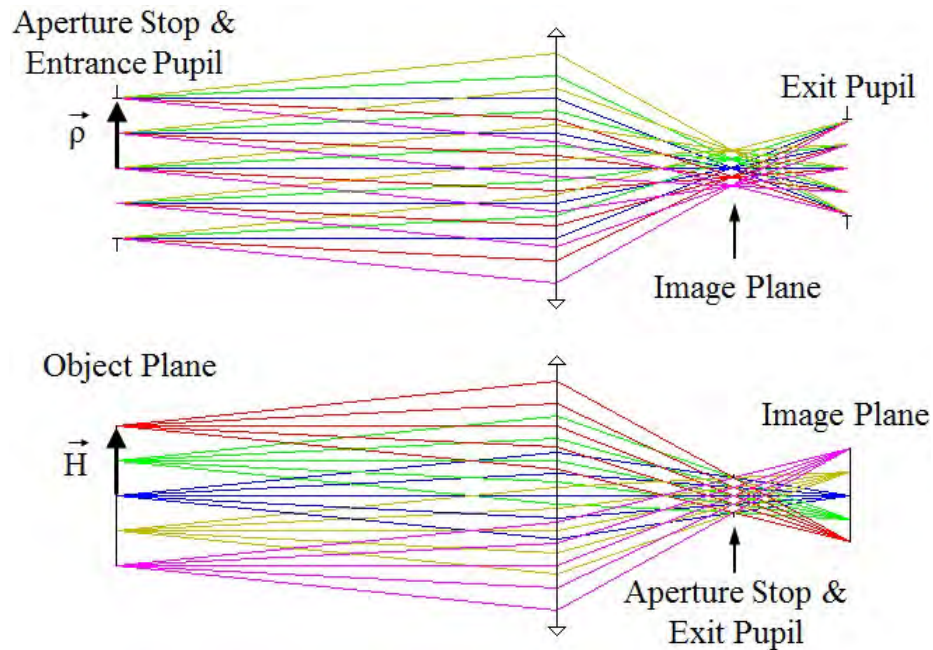


FIGURE 3.5 Pupil Imaging (Top) vs Conventional Imaging (Bottom)

In the pupil imaging case, the object plane is located at negative infinity and the aperture stop represents the plane of the wavefront under test. In the corresponding conventional imaging case, the aperture stop, and exit pupil, of the system are located at the rear focal point of the lens, making the system object space telocentric. The ray sets in both systems are identical however they are colored by field coordinates to highlight the difference in the definitions $\vec{H}$ and $\vec{\rho}$ for the same rays in the two types of imaging.

There is an important distinction between the two types of imaging when used to model an interferometer in Zemax. The OPD_z for identical rays in the two models can be drastically different. This is because the chief rays for each field coordinate, to which the OPD_z for all other rays with the same field coordinate are referenced, are different in the

two models. In both cases the chief rays for each field of view travel through the center of the entrance pupil and aperture stop. In the pupil imaging case, FIGURE 3.5 (Top), this means that all rays leaving the center of the first surface of the model, which represents the wavefront under test, will have an OPD_z of zero. However in conventional imaging case, FIGURE 3.5 (Bottom), all rays that leave the first surface parallel to the optical axis will have an OPD_z of zero. As discussed in Chapter 3.2.1, OPD_z is used as substitute for the OPL of the rays in calculating the OPD between the interferometer arms. While it is useful to look at the spread of possible test rays, in the actual interferometer there are only two rays for each point on the interferogram, one from the test wavefront and one from the reference wavefront (Murphy et al, 2000a), FIGURE 3.6. This means that when calculating the OPD or modeling phase errors in an interferometer the pupil imaging case will provide more meaningful results where the rays across both the test and reference wavefronts are referenced to the ray at the center of the aperture stop.

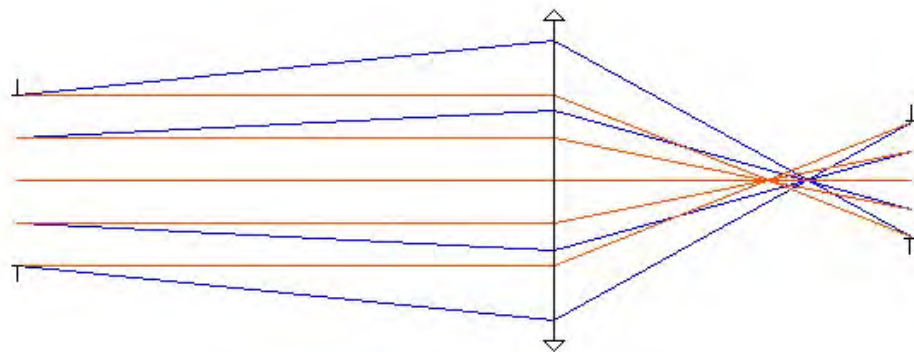


FIGURE 3.6 For a given interferogram two rays exist for each point on the detector plane, one from the reference wavefront (red) and one from the test wavefront (blue).

In considering only the interferometers imaging optics, as a pupil imaging system, the location of each ray at the test plane defines its pupil vector and its field vector is determined by the normal to the test wavefront at the test plane or the angle at which the test ray reflects off the test surface. The corresponding reference ray is the reference ray which interferes with the test ray at the detector or exit pupil. If a flat reference wavefront is used, as shown in FIGURE 3.6, then every reference ray has a null field vector. In modeling the imaging of the reference arm in Zemax it is convenient to use an aperture stop located in the same optical plane as the test part or wavefront, even if there is not a physical stop at this location. This will force the exit pupil to be located the same distance from the imaging lens for both interferometer arms in the model.

If additional optics are used in the test arm of the interferometer the image of the test plane at detector is the combination of the image produced by the additional optics and the imaging lens. In any optical system the image of the stop through each surface can be thought of as an intermediate pupil of the system (Hoffman 1993). In modeling an interferometer, in which a diverger is used to collect light off the test part, it is often useful to analyze the wavefront at the image of the test part through the diverger. This plane will be referred to as the intermediate pupil in this research, even though the system contains many intermediate pupils, FIGURE 3.7.

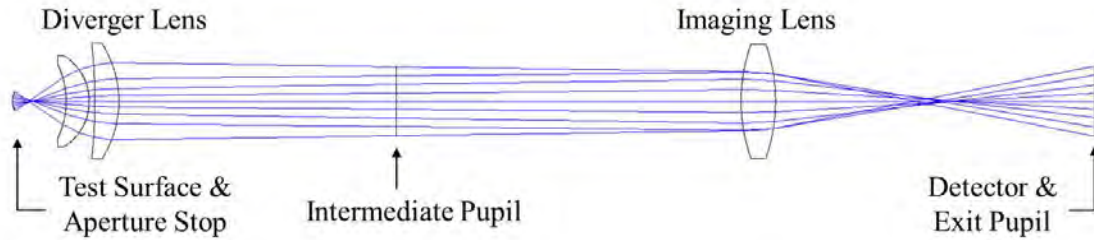


FIGURE 3.7 The image of the test part, which is the aperture stop of the interferometer, created by the diverger is the intermediate pupil of the system. Its image onto the detector by the imaging lens is the exit pupil of the interferometer.

Since the intermediate pupil is conjugate to the detector and the reference wavefront is a plane wave, in the same optical space, the test wavefront or the OPD_z of the test arm at the intermediate pupil is very similar to the measured OPD at the detector. In Zemax the location of the intermediate pupil and the exit pupil of the system can be found by using a pupil position solve on a prior surface thickness. A Zemax “solve” is a function in Zemax which actively adjusts a parameter in the lens design, such as a thickness, a curvature or an index of refraction, in order to maintain a specific condition as the lens design is changed. In this case, a pupil position solve which normally calculates the thickness a surface needs to be so that the chief ray crosses the optical axis at the next surface. However in this case the chief ray is collinear with the optical axis so Zemax uses a slightly different method described below.

“The pupil position is determined by tracing real, differential rays about the central field chief ray.” (Zemax LLC, 2011)

3.2.5 Interferometer Errors

The aberration in an interferometer can be separated into two categories: phase errors and mapping errors (Murphy et al, 2000a). Phase errors are simply the OPD between the test and reference rays in an interferometer as a result of their non-common path through the interferometer. Mapping errors are the nonlinear relationship between a uniform grid of rays at the test plane, and corresponding reference plane, and the resulting grid of rays at the detector. These mapping errors are analogous to transverse ray aberrations in a conventional imaging system.

In a null interferometer the phase error is generally small and is attributed entirely to the error in the sag or alignment of the test surface. This is due to null interferometers being designed so that the test and reference arms are either common path, meaning the same wavefront aberrations are introduced into both arms, or well corrected, so that no additional OPL is introduced in the non-common path section of the interferometer.

Mapping errors, in a null interferometer, simply result in confusion between the measured OPD at a given detector pixel and its corresponding location on the test surface.

Therefore in a null interferometer the imaging lens should produce a distortion free image of the test part on the detector. (Malacara et al, 2007) Additionally since the test surface is the stop of the system and the image plane is at the exit pupil, mapping errors are the result of pupil aberration.

Aberration in an optical system can also be divided into two categories based on their source; intrinsic and induced aberrations (Hofmann 1993). Intrinsic aberrations are the

aberrations introduced into an optical system when the incoming wavefront is perfect. While induced aberrations are the additional aberrations introduced when the incoming wavefront is aberrated. In a non-null interferometer phase and mapping errors become intertwined and difficult to separate due to induced aberrations. Since the test and reference wavefronts do not follow the same path through a non-null interferometer the induced aberrations of each wavefront are dissimilar leading to different pupil aberrations in the two wavefronts. Murphy, Brown and Moore (Murphy et al, 2000a) show that the mapping functions for the test and reference rays, in one direction, can be defined in terms of transverse ray aberration and take the form given in Equations 3.4 and 3.5. In which h' represents the location of the ray at the detector as a function of its location at the test part. The mh term represents the linear mapping and the ε_y term represents the transverse ray aberration, which depends on the location of the ray at the test surface and the ray's pupil vector, which is the angle at which the ray leaves the test surface. The height and angle of reflection are obviously dependent on the shape of the test surface.

$$h'_{\text{test}}(h) = mh + \varepsilon_y(\rho_{\text{test}}(h), h) \quad 3.4$$

$$h'_{\text{ref}}(h) = mh + \varepsilon_y(0, h) \quad 3.5$$

Calculating the mapping errors or pupil aberrations in the Zemax model of the non-null interferometer will be discussed in Chapter 3.3.2 and 4.6.2. One important impact that the mapping errors have on the modeling of a non-null interferometer is that, because induced aberrations in each arm of the interferometer are different, the diameter of the test and reference arms entrance pupils, which correspond to the same size exit pupil, are

not necessarily equal, FIGURE 3.8. This adds complexity to the retracing model because Zemax traces rays by normalized pupil coordinates and these coordinates are no longer the same for the two arms of the interferometer.

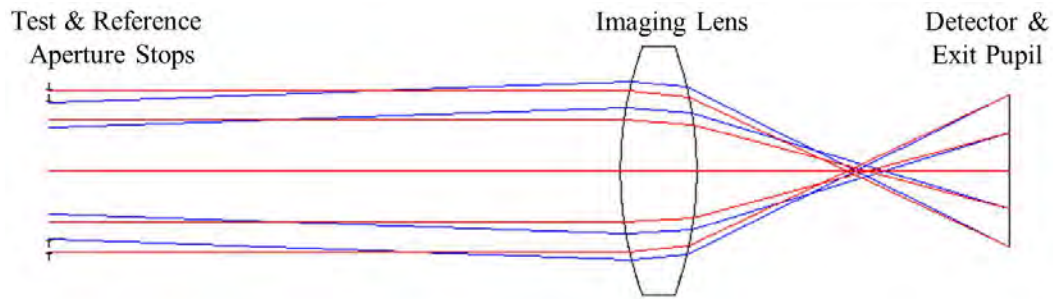


FIGURE 3.8 In the presence of pupil aberration rays that interfere at the detector from the test wavefront (blue) and reference wavefront (red) do not originate from the same point in the stop of the imaging system.

The phase error in a non-null interferometer is still the OPD between the test and reference wavefront at the detector, however it cannot be attributed entirely to the error in the sag and alignment of the test surface. Rather the phase error is the result of the aspheric wavefront, generated by the mismatch of the test surface to the incoming wavefront, and the induced aberration the wavefront generates as it propagates through the interferometer to the detector. Additionally because the mapping errors of the reference and test wavefront are different the phase error experienced at each point in the wavefront depends on the test and reference rays that actually interfere at the detector. Therefore it is difficult to separate the induced phase errors which result from aspheric departure of the test surface from the phase errors introduced by the different mapping errors of the test and reference arms of the interferometer. In a non-null interferometer reverse ray tracing is used to separate the induced phase errors of the interferometer from the phase errors produced by the form errors of the aspheric test part. Additionally the

phase errors introduced by the interferometers imaging optics and a method of calculating their shape and magnitude will be discussed in Chapter 4.6.2.

3.2.6 Ray Aiming

As stated previously, by default Zemax initially launches rays over an equally spaced grid at the paraxial entrance pupil. However due to pupil aberrations this will not produce an equally spaced grid of rays of the stop or exit pupil. This can also lead to the stop surface, the test part in the interferometer, not being fully illuminated. Additionally in modeling a non-null interferometer the reference and test arms will have different pupil aberrations.

The effects of pupil aberration can be accounted for by the use of ray aiming. With ray aiming turned on, pupil coordinates are defined at the stop surface and thus the pupil vector is normalized to the size of stop, rather than the paraxial entrance pupil. This is useful when calculating the interference between two configurations representing the two arms of an interferometer. If the surface at which the interference is to be calculated can be set to the stop in both configurations and ray aiming is turned on then the pupil coordinates for both configurations are guaranteed to overlap. However care must be taken to ensure that the size of the new stop corresponds to the size of the test wavefront at this surface. Zemax doesn't have an option to define the pupil vector at the exit pupil, so in order to accomplish this, the location of the exit pupil must first be found, and then the corresponding surface can be set to be the aperture stop. Moving the aperture stop to

the detector surface is useful when trying to calculate the interference at the detector surface over a uniform grid of points corresponding to the physical location of the detector's pixels.

Two important things to note on ray aiming are first ray aiming only insures that a grid of uniform spaced pupil coordinates corresponds to a uniform grid of real coordinates at the defined stop surface and doesn't actually correct for the mapping errors present in the interferometer. Second, in order to generate the uniformly distributed ray set at the stop surface Zemax must trace rays from the object surface to the stop surface and iteratively adjust the angle at which the ray is launched until it crosses the aperture stop at the correct location. This can significantly increase the length of time required to perform ray tracing. In the Zemax manual it is stated that this can increase the ray tracing time by a factor of two to eight (Zemax LLC, 2011). However in practice if the stop is placed on the last surface of a lens file that includes many surfaces or complex surfaces types, such as grid phase or grid sag, the length of time can be increased by more than a factor of 100 and in some instances cause the Zemax ray tracing engine to crash.

3.3 Zemax User Defined Programs

Zemax has a large number of built in programs for performing analysis on lens designs. Many of these programs are designed to produce graphs and numbers to be analyzed by the designer such as calculating the modular transfer function, point spread function or aberration coefficients. Other programs perform calculations on the lens design and

supply the results to the Zemax merit function so that they can be used in the optimization process. In addition to the built in programs, Zemax allows users to write their own programs using two different methods. The first is using a native programming language similar to BASIC called Zemax Programming Language (ZPL). Programs written in this language are generally referred to as ZPL macros. The second method is to write a Zemax Extension using third party software such as C. The primary difference between the two options is that ZPL macros are simple to write and are executed entirely within Zemax while Extensions are more complicated and make use of the Dynamic Data Exchange protocol defined within Microsoft Windows operating system. Some of the advantages of Extensions are that they can trace large number of rays simultaneously, perform calculations faster than comparable ZPL macros, and allow communication with external programs. The user defined functions written to aid in the design and analysis of the non-null interferometer are described in this section.

3.3.1 Wavefront Slope Calculations

The range of the wavefronts that can be measured by the sparse array sensor is limited by the maximum fringe frequency of the interference pattern at the detector. Therefore a critical calculation that needs to be made in the model of a non-null interferometer is the wavefront slope or fringe frequency present in the interference of the test and reference wavefront. Several programs were written to calculate the wavefront slope of each ray traced over a single wavefront or the slope of the difference between two wavefronts. More specifically these programs return the maximum wavefront slope (MWS) of a

wavefront modeled within a single configuration or the maximum wavefront slope difference (MWSD) between the wavefronts modeled in two configurations at a user specified surface in Zemax. In the case of a single configuration the absolute wavefront slope is calculated, meaning the slope of the reference sphere is not considered in the calculation or rather a plane wavefront is always used as the reference. In the case of comparing the wavefront between two configurations the second configuration is used as the reference for the first configuration. When modeling an interferometer this allows the fringe frequencies to be calculated from the interference of the test and reference wavefronts.

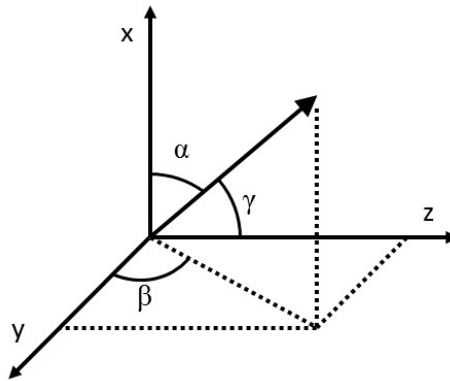


FIGURE 3.9 Direction Cosines

Making use of the fact that the rays propagate along the normal vector of the wavefront the programs calculate the wavefront slope of each ray from the direction cosines, l , m and n utilized by the Zemax ray tracing engine as defined by Equation 3.6 - 3.8. Since the direction cosines of a ray at any surface can be retrieved from the Zemax ray trace data, the wavefront slope can be calculated at any surface in the model.

$$l = \cos(\alpha) \quad 3.6$$

$$m = \cos(\beta) \quad 3.7$$

$$n = \cos(\gamma) \quad 3.8$$

The direction cosines also satisfy the relationship given in Equation 3.9.

$$l^2 + m^2 + n^2 = 1 \quad 3.9$$

The x and y components of the wavefront slope for each ray are calculated using Equation 3.10 and 3.11.

$$u_x = \frac{l}{n} \quad 3.10$$

$$u_y = \frac{m}{n} \quad 3.11$$

Finally the magnitude of the slope is calculated by Equation 3.12.

$$|u| = \sqrt{(u_x - u'_x)^2 + (u_y - u'_y)^2} \quad 3.12$$

Where u and u' are the ray slopes from the first and second configuration respectfully.

If only one configuration is traced the slope u'_x and u'_y are equal to zero. In the case where the wavefronts from two configurations are compared it is important that the rays from the two configurations intersect at the surface of interest. One way to accomplish this is to make the surface at which the difference is to be calculated the stop in both configurations. Then by using the Zemax ray aiming feature the rays from the two configurations will be forced to overlap. However when calculating the MWSD at the intermediate pupil or at the detector plane the stop would have to be moved from its ideal location, at the test surface, making this approach not very useful when optimizing. To get around this a simple ray aiming procedure was written in which the positions of each

ray from the first configuration are saved and then rays from the second configuration are iteratively traced until they overlap these points within a user defined distance.

Additionally these programs keep track of the radial distance of each ray, in the xy plane, from the chief ray. The magnitude of the slope is then scaled to waves/radius by multiplying by the maximum radial distance and dividing by the wavelength.

Two programs were written to calculate the MWS or MWSD in the Zemax merit function to be used during optimization, and one program was written to produce a map of the wavefront slope. Two copies of each of the merit function programs were written, one in the native Zemax Programming Language (ZPL) designated by the ZPLM merit function line and one written in C, designated by the UDOP (User Defined Operand) merit function line. The ZPL and C versions of the code are nearly identical. However the ZPL only allows one ray to be traced at a time, whereas the C version can trace a large number of rays simultaneously. This makes the UDOP version of the code much faster especially as the number of rays to be traced becomes large. In Zemax user defined merit functions data is passed to the merit function by using the columns labeled Hx, Hy, Px and Py. For native Zemax merit function operands these are used to specify the field and pupil coordinates of a ray. In a user defined operand these columns can be used to pass in different types of data but the column headings do not change, FIGURE 3.10.

Oper #	Type	Prog#	Data	Hx	Hy	Px	Py		Target	Weight	Value	% Contrib
1: CONF	CONF	2										
2: ZPLM	ZPLM	23	0	51.0000	-7.0000	3.00000	27.0000		0.00000	1.00000	563.398	100.00000
3: ZPLM	ZPLM	23	1						0.00000	0.00000	0.47059	0.0000000
4: ZPLM	ZPLM	23	2						0.00000	0.00000	0.00000	0.0000000
5: ZPLM	ZPLM	23	3						0.00000	0.00000	3.45622	0.0000000
6: ZPLM	ZPLM	23	4						0.00000	0.00000	163.010	0.0000000
7: ZPLM	ZPLM	23	5						0.00000	0.00000	4.95637	0.0000000

FIGURE 3.10 Example of a call to program ZPL23 from the Zemax merit function

In ZPL23 and UDO23 the user defines the number of equal spaced rays to trace between the chief ray and the edge of the pupil in the Hx column. The value in the Hy column, if greater than or equal to zero, is used to specify a specific polar angle of the rays in the xy plane or, if the number is less than zero, its absolute value defines the number of equal spaced polar angles to trace rays starting with rays aligned to the +x axis. If the Px column contains a valid configuration number then the wavefront from that configuration is used as the reference for calculating the MWSD. If the Px column is zero then the MWS of the current configuration is calculated. Finally, the Py column specifies the surface number at which the slope calculation is to be performed. In addition to returning the maximum wavefront slope in waves/radius to the merit function, these programs return the maximum slope in both cycles/mm and degrees, the pupil coordinates where the maximum slope occurs, and the maximum radius of the wavefront with respect to the chief ray. The value returned by the programs depends on the value specified in the data column and is outlined in TABLE 3.1.

Data #	Returned Value
0	MWS or MWSD [Waves / Radius]
1	Normalized pupil radius at which the MWS or MWSD occurred
2	Polar angle at which the MWS or MWSD occurred [Degrees]
3	Maximum radius of the wavefront with respect to the chief ray [mm]
4	MWS or MWSD [Cycles / mm]
5	MWS or MWSD [Degrees]

TABLE 3.1 Data returned by the programs ZPL23 and UDOP23

Programs ZPL23 and UDOP23 are useful when the system is either radially symmetric or has symmetry about a few axes such as when a toric surface is used in the lens design model. Alternatively, two programs ZPL29 and UDOP29 trace a uniform grid rays across the pupil, where the ray density is set by the user. These are useful when the wavefront has no rotational symmetry such as when a free form optical surface is tested. In these programs the Hx column is used to pass in the number of rays, n_{rays} , to trace across the semi diameter of the beam. The programs sets up a grid of $(2n_{\text{rays}} - 1) \times (2n_{\text{rays}} - 1)$ rays across the wavefront, however rays that would fall outside the aperture stop, $\rho < 1$, are not traced. Also in these programs the Hy column is not used and the returned values are slightly different, TABLE 3.2.

Data #	Returned Value
0	MWS or MWSD [Waves / Radius]
1	Normalized pupil x coordinate at which the MWS or MWSD occurred
2	Normalized pupil y coordinate at which the MWS or MWSD occurred
3	Maximum radius of the wavefront with respect to the chief ray [mm]
4	MWS or MWSD [Cycles / mm]
5	MWS or MWSD [Degrees]

TABLE 3.2 Data returned by the programs ZPL29 and UDOP29

Finally a third program, WavefrontSlopeMap.zpl, was written to create a map of the wavefront slope or the wavefront slope difference, as shown in FIGURE 3.11 (Right). It also uses a uniform square grid of rays where the density is specified by the user.

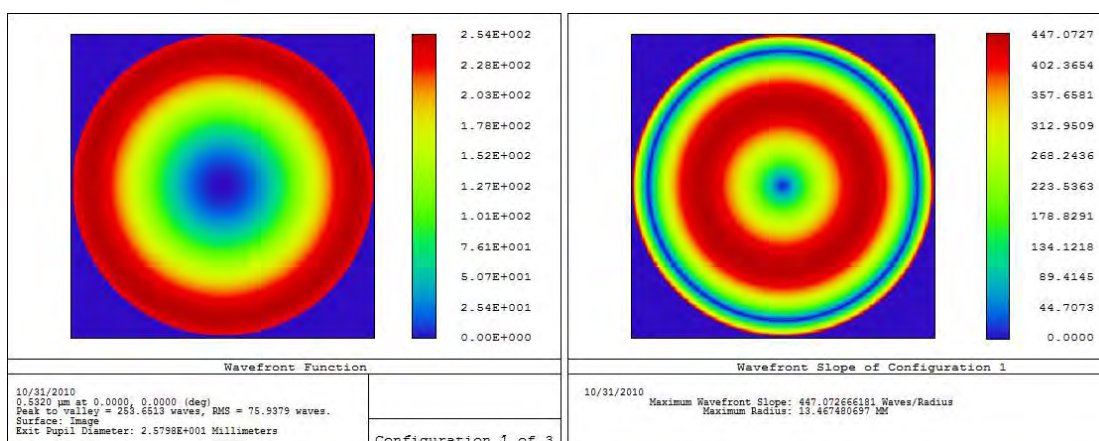


FIGURE 3.11 A wavefront (Left) and its corresponding wavefront slope map (Right) calculated with the WavefrontSlopeMap.zpl macro.

In these programs the MWS or MWSD values are simply the maximum values out of all the rays traced, therefore the density of rays traced will affect the calculated value. The error between the actual MWS or MWSD and their calculated values is the result of a ray not being traced at the exact location of maximum slope. In general the error in measurement will decrease as the density of rays traced is increased. Since the density of rays traced by these programs is specified by the user, in practice the density should be increased until the change in the calculated MWS or MWSD is less than the desired tolerance.

3.3.2 Pupil Aberration Calculations

In modeling a non-null interferometer it is often beneficial to look at the pupil aberration present in the mapping of the test surface onto the detector. In a conventional imaging system pupil aberrations can be used to determine how the image aberrations change as the object surface is shifted (Sasián 2010). This is based on the idea that an object shift in a conventional imaging system is analogous to a stop shift in a pupil imaging system. Wynne demonstrated that pupil aberrations can be defined in terms of the 3rd order wavefront aberrations or Seidel sums (Wynne 1952), which Sasián then extended to 6th order (Sasián 2010). However another interpretation of pupil aberrations discussed by Hoffman (1993) and Sasián (2006) is that pupil aberrations are simply distortions, or errors, in the mapping of coordinates between the entrance and exit pupil of an optical system. This interpretation was used in the modeling of the non-null interferometer where the value of the individual pupil aberration coefficients is less important than the overall magnitude and shape of the mapping error. Additionally in modeling a non-null interferometer the mapping between the entrance pupil and the exit pupil is not as much of a concern as the mapping between the stop and the exit pupil, which correspond to the test part and detector respectively.

Two different situations in which calculating the pupil aberration is important in the modeling of a non-null interferometer are when comparing the pupil aberrations experienced for a single wavefront and when calculating the range of aberrations that can be expected over the dynamic range of the interferometer. Considering the latter case

first, the Zemax spot diagram can be used to calculate the magnitude and shape of the mapping error in the exit pupil for a given point in the entrance pupil, stop or intermediate pupil over the range expected ray slopes. However this requires that the model is setup properly so that the rays traced for the spot diagram match the rays that may be generated in the interferometer. This process will be discussed in greater detail with the imaging lens design considerations in Chapter 4.6. In the case of calculating the pupil aberrations for a specific wavefront the native pupil aberration calculations built into Zemax are inadequate for use in modeling a non-null interferometer and the wavefronts they produce. The first built in procedure is a pupil aberration fan which shows the entrance pupil distortion as a function of the pupil coordinates. It calculates the difference between the real ray intercept on the stop surface and the paraxial ray intercept as a percentage of the paraxial stop radius (Zemax LLC, 2011). It was designed to be used as a method of determining if ray aiming is needed in a traditional optical system. Even when there is pupil aberration present between the entrance pupil and the aperture stop as soon as ray aiming is turned on the pupil aberration fan will always show zero aberration, FIGURE 3.12.

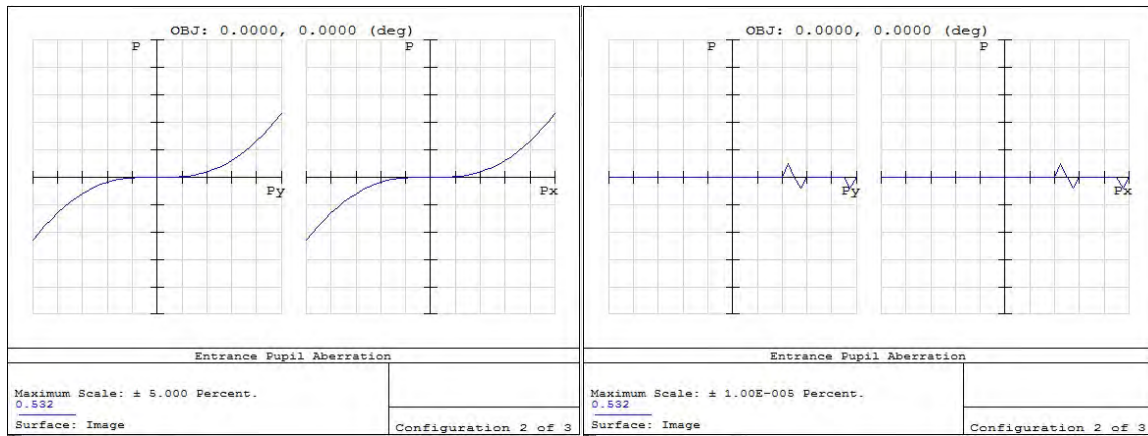


FIGURE 3.12 Pupil aberration fans for the same interferometer model with ray aiming turned off (Left) and with ray aiming turned on (Right)

The second built in function, called PUPIL_MAP, calculates the maximum percent distortion between rays at a specified surface and the paraxial pupil size. It outputs a figure similar to a spot diagram where the real x and y locations of the rays on the surface of interest are drawn relative to a perfect grid representing the size of the paraxial exit pupil. The PUPIL_MAP program is also insufficient for use in modeling the non-null interferometer for three reasons. First, since the image of the test part, or stop, must be imaged onto the detector the real size of the exit pupil is more of a concern than the paraxial size of the exit pupil. Second the map itself is difficult to interpret, especially with a dense grid of rays, FIGURE 3.13. Finally the program is not written to allow the calculated percent distortion to be used by the Zemax merit function which means it cannot be used during the optimization process.

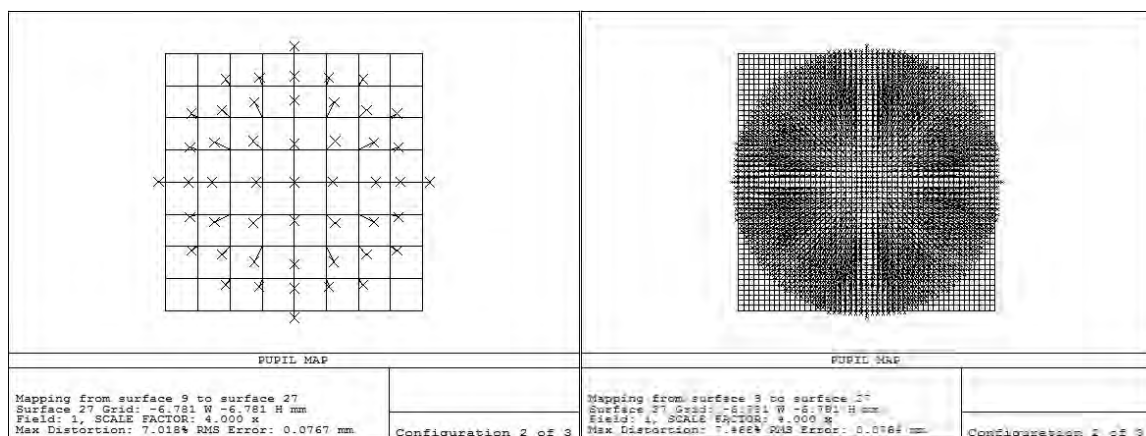


FIGURE 3.13 The Zemax built in pupil mapping function PUPIL_MAP.

Therefore two new programs were written to calculate the shape and magnitude of the pupil aberration present in a non-null interferometer, Normalized_Pupil_Error_Map and ZPL49. While they essentially perform the same calculations, the first is used to produce a map of the aberration while the other is used in the Zemax merit function. In order to make the programs more flexible, they allow the distortion of the wavefront between either the stop or paraxial entrance pupil and any surface in the Zemax model to be calculated. It is important to note that performing the calculation at a plane that is not a pupil may produce meaningless results. To this end the designer should place a pupil solve on the thickness prior to the surface of interest to force it to be located at a pupil. The program Normalized_Pupil_Error_Map allows the pupil aberration to be calculated relative to the semi-diameter of the paraxial exit pupil or to the real semi-diameter of the pupil. The program ZPL49 always calculates the aberration relative to the real pupil size. The pupil aberration is calculated by tracing a uniform grid of rays, by pupil coordinates, through the system. The density of this grid and the surface to trace rays to are specified in the same manner used in ZPL29. If ray aiming is turned off then the rays represent a

uniform grid at the paraxial entrance pupil. If ray aiming is on then the rays represent a uniform grid at the stop. The program `Normalized_Pupil_Error_Map` allows the field coordinates of the rays can be specified by the user; however ZPL49 assumes the field coordinate of every ray is zero as is the case for modeling the non-null interferometer.

The programs then trace rays to the surface specified by the user and record the real x and y location of each ray. Additionally the program calculates the ideal x and y locations for a uniform square grid of points, with a width equal to $2R$. In these calculations, the normalization radius, R , is either the semi-diameter of the paraxial exit pupil, or the real semi-diameter of the exit pupil determined by the maximum distance from the chief ray of all the rays traced. Next the program calculates the distance between the actual and ideal location for each ray relative the chief ray, similar to the transverse ray aberration in conventional imaging, and normalizes the distances to the size of the pupil, Equation 3.13 and 3.14. The magnitude of the aberration for each ray is also calculated by Equation 3.15.

$$\vec{\epsilon}_x = \frac{x_{Actual} - x_{Ideal}}{R} \hat{x} \quad 3.13$$

$$\vec{\epsilon}_y = \frac{y_{Actual} - y_{Ideal}}{R} \hat{y} \quad 3.14$$

$$\epsilon_{Mag} = \sqrt{\epsilon_x^2 + \epsilon_y^2} \quad 3.15$$

Additionally the programs keep track of the minimum and maximum magnitude of the aberration as well as the minimum and maximum of the x and y components. These values are returned to the merit function along their corresponding peak to valley value, the normalization radius, and the real x and y intercept of the chief ray, TABLE 3.3. This

allows the calculations to be used in the Zemax optimization process. The program Normalized_Pupil_Error_Map produces false color maps of the x component, y component and magnitude of the pupil aberrations, FIGURE 3.15.

Oper #	Type	Prog#	Data	Hx	Hy	Px	Py		Target	Weight	Value	% Contrib
16: CONF	CONF	2										
17: ZPLM	ZPLM	49	0	111.00000	0.0000000	0.0000000	26.000000		0.0000000	0.0000000	0.0274719	0.0000000
18: ZPLM	ZPLM	49	1						0.0000000	0.0000000	0.0541247	0.0000000
19: ZPLM	ZPLM	49	2						0.0000000	0.0000000	0.0544896	0.0000000
20: ZPLM	ZPLM	49	3						0.0000000	0.0000000	3.5013029	0.0000000
21: ZPLM	ZPLM	49	4						0.0000000	0.0000000	-3.4E-004	0.0000000
22: ZPLM	ZPLM	49	5						0.0000000	0.0000000	-1.6E-004	0.0000000
23: ZPLM	ZPLM	49	6						0.0000000	0.0000000	0.0272361	0.0000000
24: ZPLM	ZPLM	49	7						0.0000000	0.0000000	0.0000000	0.0000000
25: ZPLM	ZPLM	49	8						0.0000000	0.0000000	0.0272361	0.0000000
26: ZPLM	ZPLM	49	9						0.0000000	0.0000000	-0.026889	0.0000000
27: ZPLM	ZPLM	49	10						0.0000000	0.0000000	0.0272960	0.0000000
28: ZPLM	ZPLM	49	11						0.0000000	0.0000000	-0.027194	0.0000000

FIGURE 3.14 Example of a call to program ZPL49 from the Zemax merit function

Data #	Returned Value
0	Peak to valley ε_{Mag}
1	Peak to valley ε_x
2	Peak to valley ε_y
3	Normalization radius, R [mm]
4	Real x coordinate of the chief ray [mm]
5	Real y coordinate of the chief ray [mm]
6	Maximum ε_{Mag}
7	Minimum ε_{Mag}
8	Maximum ε_x
9	Minimum ε_x
10	Maximum ε_y
11	Minimum ε_y

TABLE 3.3 Data returned by the program ZPL49

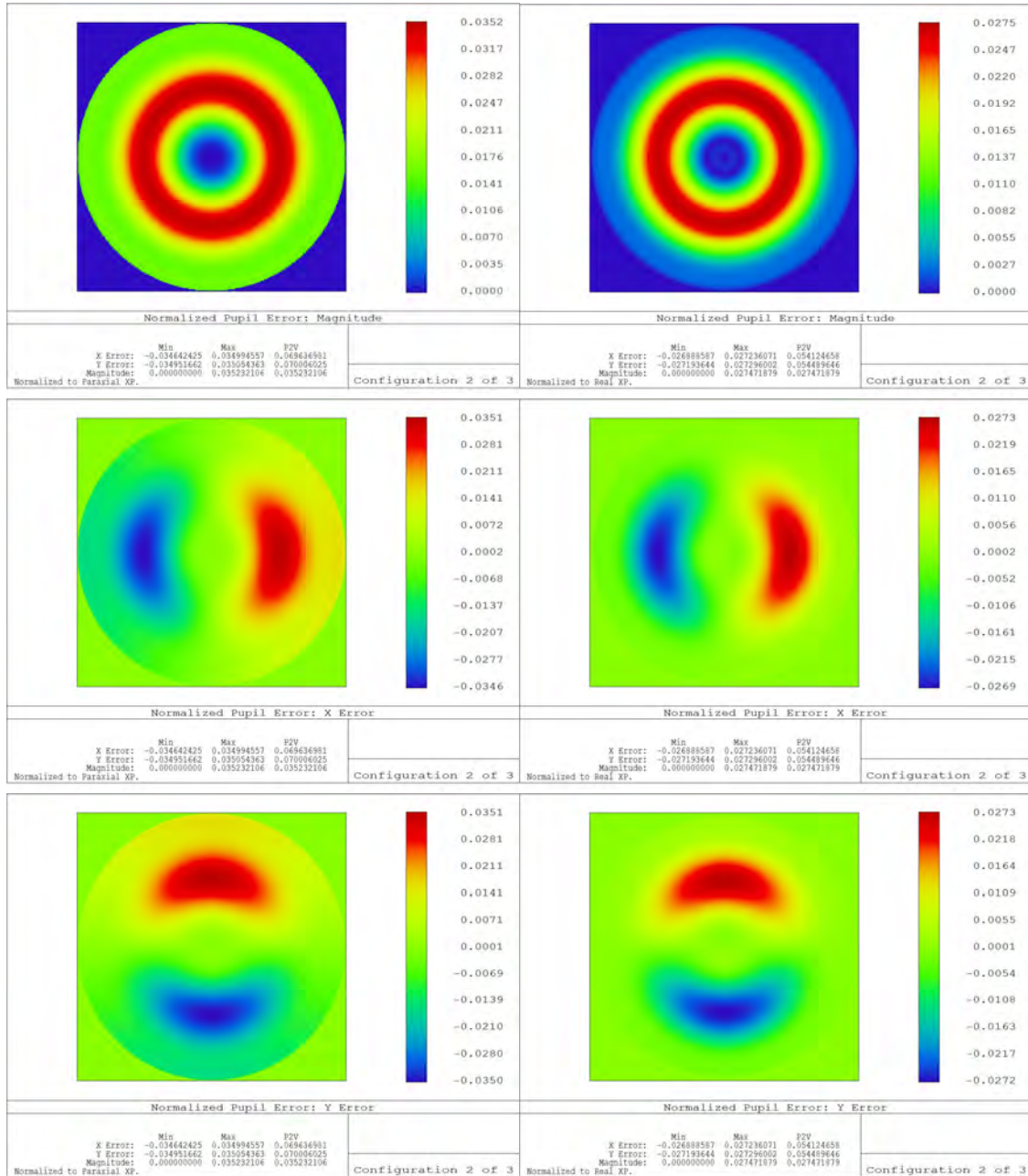


FIGURE 3.15 Example of Normalized Pupil Error Maps: Normalized to paraxial exit pupil semi-diameter (Right) Normalized to the real exit pupil semi-diameter (Left)

3.3.3 Caustic Calculations

The shape of a non-planar wavefront constantly changes as it propagates. The previous calculation of pupil aberration gives the magnitude of the distortion a uniformly spaced set of rays across a wavefront will experience after the wavefront has propagated some distance. However the calculation doesn't indicate if the mapping of the rays between the two planes is no longer monotonic. The loss of one to one mapping occurs when the wavefront folds over onto itself producing a caustic surface. Two principle curvatures can be defined for every point on a wavefront. The locus of these principal centers of curvature is the caustic surface. (Stavroudis 1972) For this analysis the shape of the caustic surface is not as important as the region along the optical axis over which the caustic surface exists. If the image of the interferometer stop falls into this confused region the mapping of points across the stop into the exit pupil will not be monotonic. This would lead to the wavefront interfering with itself and an inability to reconstruct the wavefront at the stop from the recorded interferogram at the detector.

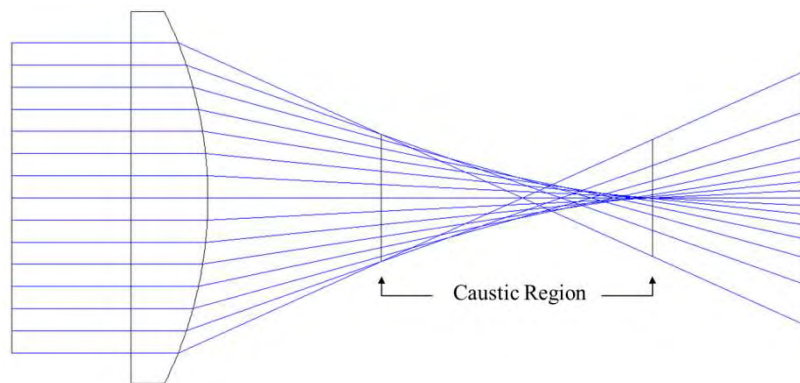


FIGURE 3.16 Caustic produced from spherical aberration.

A classic example of a caustic surface is that of a collapsing near spherical wavefront containing spherical aberration, such as the wavefront produced by a plano-spherical lens

focusing an incoming plane wave, FIGURE 3.16. In terms of pupil imaging in an interferometer this is not the best example since a virtual intermediate pupil would be required in order for the exit pupil to fall into the confused region. However it does show the distortion experienced by the ray set as the wavefront propagates away from the lens as well as the folding of the wavefront onto itself inside the confused region. In this case the caustic extends from this point to the paraxial focus of the lens. After the wavefront emerges from paraxial focus the rays are inverted over the optical axis, yet are again in the original order based on the radial distance from the optical axis. The size and shape of a caustic created by third and fifth order spherical aberration is well documented in several source such as Modern Optical Engineering (Smith 2000). Additionally, Malacara derives the dimensions for a caustic produced from reflection of a point source at the center of curvature for aspheric surfaces of the form given in Equation 3.16.

(Malacara 2007a)

$$z = \frac{Cr^2}{1 + \sqrt{1 - (1+k)C^2r^2}} + A_1r^4 + A_2r^6 + A_3r^8 + A_4r^{10} \quad 3.16$$

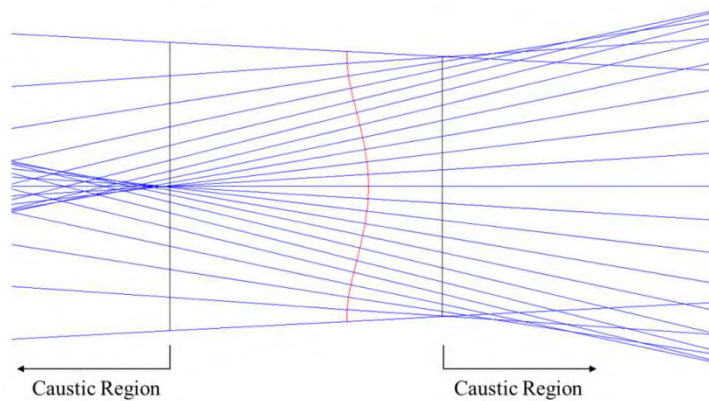


FIGURE 3.17 An example of an aspheric wavefront (red) in which only a small region exists in which the wavefront is not in a caustic region.

While the previous examples the confused region is localized near the focus, an aspheric wavefront can produce a confused region which extends to infinity, FIGURE 3.17. In order to test an aspheric wavefront, such as the one shown in FIGURE 3.17, the wavefront is imaged onto the detector. If the imaging is free of pupil aberration then each ray will be mapped to the appropriate image point regardless of the rays angle in the wavefront under test. However if there is aberration in the imaging lens then the point at which each ray is mapped will depend on its location in the wavefront, its angle of propagation and the shape and magnitude of the pupil aberration of the imaging lens. If the induced aberrations by the imaging lens resulting from the steep wavefront slopes present in a non-null test of an aspheric wavefront are large enough it may not be possible to monotonically map the test wavefront onto a detector at a given magnification, as shown in FIGURE 3.18.

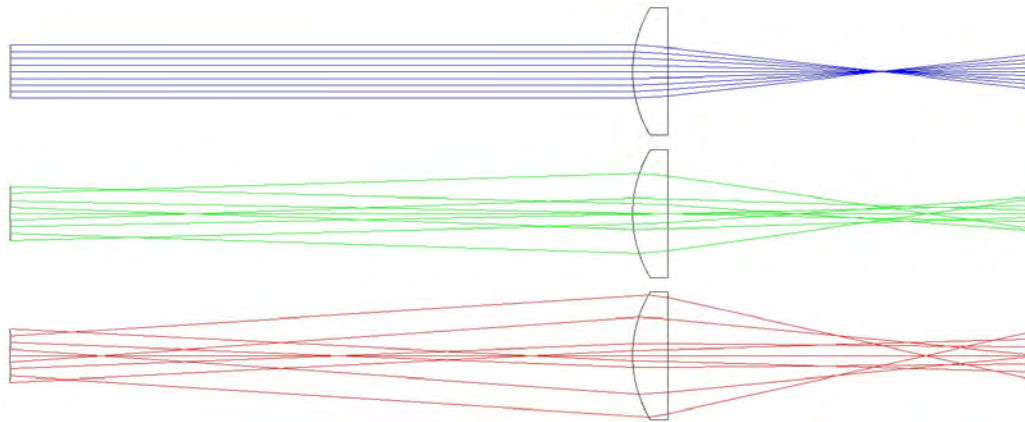


FIGURE 3.18 Examples of imaging a test wavefront onto the detector using a plano convex lens; A plane wavefront (Top) , an aspheric wavefront where the mapping is distorted but is still monotonic (Middle), and an aspheric wavefront where the imaging is not monotonic and the detector is located inside a confused region (Bottom).

In the final interferometer used to test the aspheric surfaces the pupil aberrations which can preclude the monotonic mapping of the test surface onto the detector are the result of both the imaging lens, the diverger optic and the aspheric surface itself. Therefore a program capable of determining if a surface is located in a confused region was needed in order to avoid placing the detector in a position where the mapping of the test wavefront is not monotonic. Zemax does not have a built in method of performing this task so two user defined macro programs, ZPL43 or UDO43, were written. Originally these programs would return a binary flag if the surface was located in a confused region. This was accomplished by tracing a uniform spaced line of rays in either the entrance pupil or aperture stop along a user defined polar angle, through the lens design model to the surface of interest. The program would then check if the rays were in the original order by radial distance from the chief ray. If the rays were in the original order then a confused region was not detected and the program would return a 0. If they were not in the original order then a confused region was detected and the program would return a 1. However this approach has some major drawbacks. First since the program looks for when neighboring rays cross to determine if the surface is in a confused region the result is very dependent on the density or spacing of the rays traced. Second the binary output doesn't work well as a merit function operand. Unless the surface is on the edge of a confused region; the merit function operand will not change value as the Zemax optimization procedure introduces small perturbations into the optical model. Therefore there is no feedback to indicate when a surface is approaching an edge of a confused region, only once it has been crossed.

The program was modified to trace rays from the entrance pupil to the stop in order of increasing radial pupil coordinate. The ray trace data is used to calculate the distance of each ray from the chief ray at the surface of interest, r_i , and the change in distance from the previous ray traced, dr_i , Equation 3.17.

$$dr_i = r_i - r_{i-1} \quad 3.17$$

After all the rays have been traced the program returns the minimum distance between rays at the surface of interest divided by the average spacing of the rays at the surface of interest, Equation 3.18.

$$c = \frac{\min(dr_i)}{\frac{1}{n_{Rays}} \sum_{i=1}^{n_{Rays}} dr_i} \quad 3.18$$

If there is no distortion in the wavefront the minimum change in distance between rays is equal to the average ray spacing and c will equal one. If there is distortion then the minimum change in distance will be less than the average spacing and c will be less than one. When surface is located in a caustic region the distortion is so great that the rays cross and the minimum change distance will be negative and c will be less than zero,

TABLE 3.4.

Value	Interpretation
$c = 1$	Wavefront is not distorted
$0 < c \leq 1$	Wavefront is distorted but not in the confused region
$c = 0$	Wavefront is located at the start or end of the confused region
$c \leq 0$	Wavefront is inside the confused region

TABLE 3.4 The meaning of different ranges of c values.

Since the value c constantly changes as the surface is moved the merit function can determine when the surface is approaching the edge of a confused region and the direction the surface needs to move in order to avoid it. FIGURE 3.19 shows the output of ZPL43 calculated at different distances along the optical axis through a confused region. Finding the exact start and end of the confused region is still dependent on the density of rays traced. However the main use of these programs is to allow the Zemax merit function the ability to avoid the confused region, not to find their exact locations. This can be accomplished by setting the target on c to be 0.3 or larger and then increasing the number of rays traced to until the change in the calculated value of c within a tolerance determined by the user. The exact number of rays needed depends on the shape of the wavefront under test, but in this research 50 to 150 rays were generally sufficient.

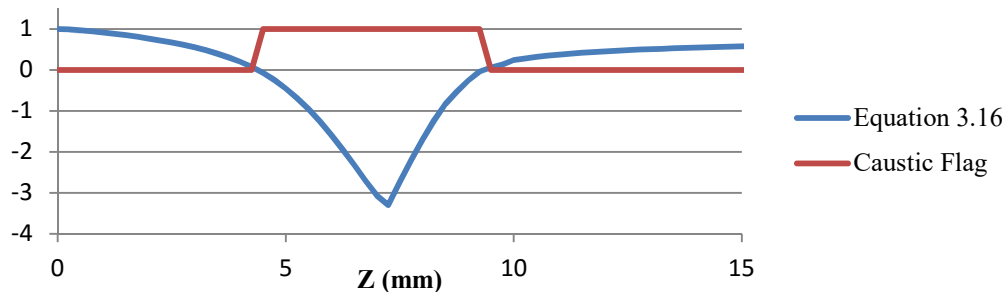


FIGURE 3.19 Output of ZPL43 as a surface is passed through a caustic region (blue) and the original binary caustic flag which only indicated when the surface was in a confused region (red).

User input into the ZPL43 and UDO43 is the same as was described for ZPL23, except only one configuration is used. The normal method that Zemax uses to determine the semi-diameter of a surface fails inside a caustic (Zemax LLC, 2011). The semi-diameter of the beam at the surface of interest is returned to the merit function by keeping track of the ray with the largest displacement from the chief ray. The programs also report the

location of the first ray crossing and whether any rays have vignetted before reaching the surface of interest. The full output of a merit function call is shown in FIGURE 3.20 and listed in TABLE 3.5. One of the main limitations of these programs is they only look for ray crossings in the radial direction along the polar angle chosen by the user. Since the programs were normally used for wavefronts where the majority of the change in the wavefront slope occurred in the radial direction, as is the case near the focus of the imaging lens, this wasn't an issue. However if wavefronts were to be tested where the slope changed rapidly as a function of the polar angle, then this program would have to be modified to either look for crossings in both polar coordinates, or to look for crossings in both Cartesian coordinates for a grid of rays.

Oper #	Type	Prog#	Data	Hx	Hy	Fx	Fy		Target	Weight	Value	% Contrib
19: UDOP	UDOP	43	0	91.000000	90.000000	0.000000	28.000000		1.000000	1.000000	-0.9579974	100.000000
20: UDOP	UDOP	43	1						1.000000	1.000000	3.2994716	0.000000
21: UDOP	UDOP	43	2						0.000000	0.000000	90.000000	0.000000
22: UDOP	UDOP	43	3						0.000000	0.000000	1.000000	0.000000
23: UDOP	UDOP	43	4						0.000000	0.000000	1.000000	0.000000
24: UDOP	UDOP	43	5						0.000000	0.000000	0.000000	0.000000

FIGURE 3.20 Example of a call to program UDO43 from the Zemax merit function

Data #	Returned Value
0	Equation 3.18
1	Semi-diameter of the beam [mm]
2	Radial pupil coordinate of first ray crossing
3	Angular pupil coordinate of first ray crossing [Degrees]
4	Caustic Flag
5	Vignetting Flag

TABLE 3.5 Data returned by the programs ZPL43 and UDO43

4 DESIGN OF THE SUB-NYQUIST INTERFEROMETER

The main goal of this project was the construction of a non-null sub-Nyquist interferometer capable of testing the aspheric tooling used in the manufacturing of soft contact lenses. In this chapter a brief description of the function of these tools will be given along with the information that was known about their aspheric departure. The design of the non-null interferometer will then be discussed, starting with an overview of the sub-Nyquist sensor around which the system was designed. This will be followed by a discussion of the type of interferometer and the design of each component, such as the diverger, imaging lens along with important concepts regarding light collection and imaging in a non-null interferometer.

4.1 Contact Lens Inserts

Ideally a single interferometer could be constructed to test any aspheric surface. Yet by definition an aspheric surface can take on almost any shape or size, from a small cell phone lens to a large primary telescope mirror. However it might be possible to test a range of similar aspheres with a single machine and contact lenses seem to offer one such set. Since contact lenses must be designed to fit on the human eye, they all have to be approximately the same size and shape. Manufacturers of contact lenses would like the ability to use aspheric surfaces in their designs, since rotationally symmetric aspheres could offer better correction than spherical surfaces. A toroidal, or toric, surface can be

used to correct astigmatism of the eye. Generalized non-rotationally symmetric aspheric surfaces could also be used to provide custom correction for an individual patient.

However performing interferometric measurements on the surfaces of contact lenses presents several problems. Since most contact lenses are soft they deform easily and change shape. They also need to be kept hydrated or they shrink, curl up and dry out. Additionally they are thin so separating the interference pattern from each surface could be difficult. Therefore, instead of testing the contact lenses surfaces directly, the tooling from which they are made will be tested. These tools are also called inserts. The manufacturing of soft contact lenses is a multiple step process shown in FIGURE 4.1. First four brass inserts are manufactured utilizing single point diamond turning (a). One insert has basically the same shape as the front, convex, surface of the contact lens and one matches the back, concave, surface of the contact lens. These inserts are then used in combination with two other generic inserts to make two pieces of plastic, called molds, by injection molding (b). These molds are then brought together and filled with liquid contact lens material (c). The liquid is then cured by exposure to ultraviolet light (d). Finally, the contact lens is removed from the molds, hydrated and packaged in a saline solution (e).

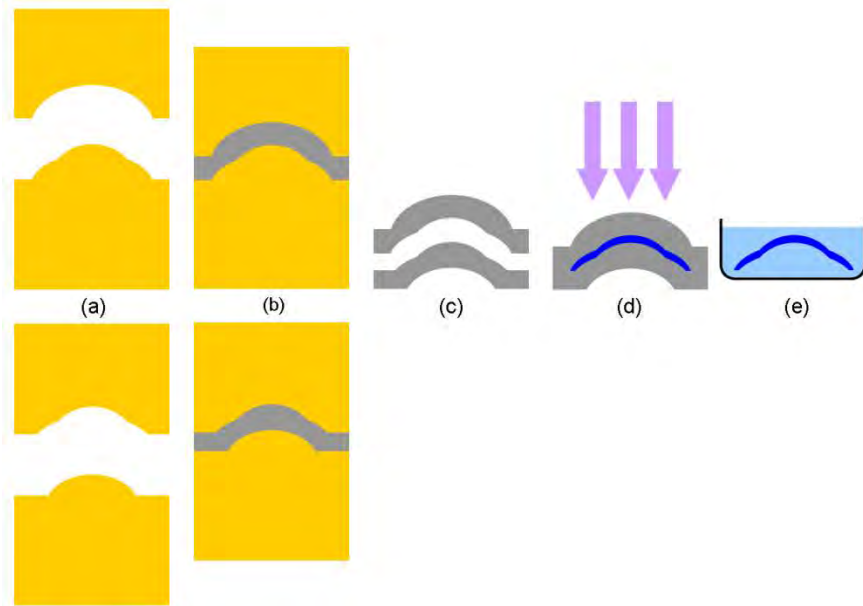


FIGURE 4.1 The basic process steps involved in making soft contact lenses

The contact lens surfaces, and thus the inserts used to make them, consist of two regions, the optical zone and the periphery. The optical zone rests over the pupil of the eye and is responsible for the optical correction. The periphery is the area around the optical zone that is used to hold the lens on the eye. This research was to test the optical zones of contact lenses only. The size of the optical zone depends on the design of the contact lens, but typically it is around 8-10mm in diameter, however it was decided for this project to only test over an 8mm diameter.



FIGURE 4.2 Examples of metal contact lens inserts.

One major obstacle to the design and construction of this system is that the designs of the parts to be tested were not provided at the beginning of this project. Ultimately the surface shape is needed to determine the maximum wavefront slope and fringe frequency that will be generated in a non-null test. Since this information was also unknown it was not possible to predict the range of fringe frequencies that would be present. Information that was known is shown in TABLE 4.1. Since contact lenses are meniscus shaped the surfaces to be tested could be either convex or concave. The best fit sphere radius of curvature would be between 6 to 10mm. The maximum departure from best fit sphere would be $50\mu\text{m}$ for rotationally symmetric parts and $100\mu\text{m}$ for toric parts. The sign of the aspheric departure from best fit sphere was unknown and thus it had to be assumed that both positive and negative aspheric departures would be utilized.

Specification	Value
Concavity	Convex & Concave
Best Fit Sphere Radius	6mm - 10mm
Optical Zone Diameter	8-10mm
Rotationally Symmetric Aspheric Departure	<50 μ m
Toric Departure	<100 μ m

TABLE 4.1 Aspheric Insert Properties

The sub-Nyquist sensor will set the limit on the wavefront slope that can be detected. Since new sparse array cameras were purchased before this research began, the design process was started by reviewing the specifications of these cameras. The MTF of the cameras was measured to determine the maximum fringe frequency that could be measured. This information was then used to generate surfaces that approximated contact lens surfaces. These surfaces were then used for the rest of the interferometer design process which will be discussed in this chapter.

4.2 Sub-Nyquist / Sparse Array Sensor

The sparse array sensor is what allows fringe frequencies higher than the Nyquist frequency to be detected and what makes sub-Nyquist interferometry possible, as discussed in Chapters 2.2-2.4. The sparse array sensor used in this research is a modified charge injection device (CID) manufactured by Thermo CIDTEC. (Liverpool, NY) The actual detector is identical to the one used by Gappinger (2002). However, the supporting electronics were updated by Thermo CIDTEC. The unmodified sensor is a 512 x 512 grid of 15 μ m square nearly contiguous pixels. The Nyquist frequency of the

sensor is 33.33cycles/mm with a cutoff frequency of 66.67cycles/mm. The camera outputs a non-interlaced analog video with a frame rate of 30 frames per second. In order to create a sparse array sensor an aluminum mask consisting 2.35 μm holes on a 15 μm square grid was placed directly over the sensor. The pixel width to pitch, or G factor, of the sparse array sensor is 0.157 in both the horizontal and vertical directions. The Nyquist frequency in both directions is unchanged at 33.33 cycles/mm. FIGURE 4.3, shows magnified images from a scanning electron microscope of the modified sparse array sensor. The CID sensor contains raised horizontal and vertical electrodes for reading the recorded electric signals that run through the center of each pixel. In order to avoid these features the pinholes were placed off center in the pixels. Although these features are covered by the aluminum mask, the deformation they cause in the aluminum layer makes them visible in the scanning electron microscope (SEM) images. The high reflectance of the aluminum mask and the print through of these electrodes causes stray light issues that will be discussed in Chapter 4.10.

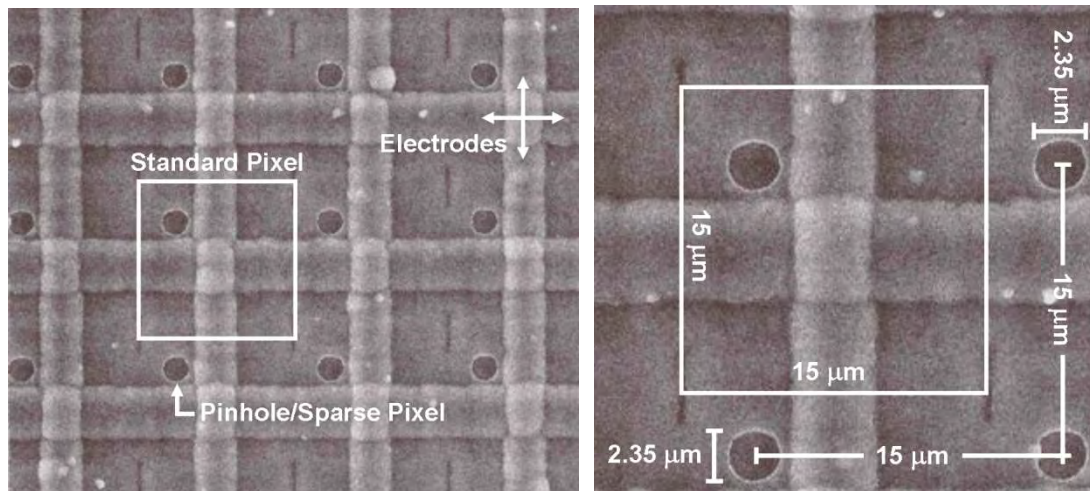


FIGURE 4.3 SEM images of the modified sparse array sensor.

The cutoff frequency of the sparse array sensor can be determined from the pixel MTF calculation discussed in Chapter 2.2. However, in Chapter 2.2 rectangular pixels were assumed. While the pixels from the sub-Nyquist sensor are fairly anamorphic, they are probably better modeled as circular apertures than rectangular pixels as shown in FIGURE 4.4.

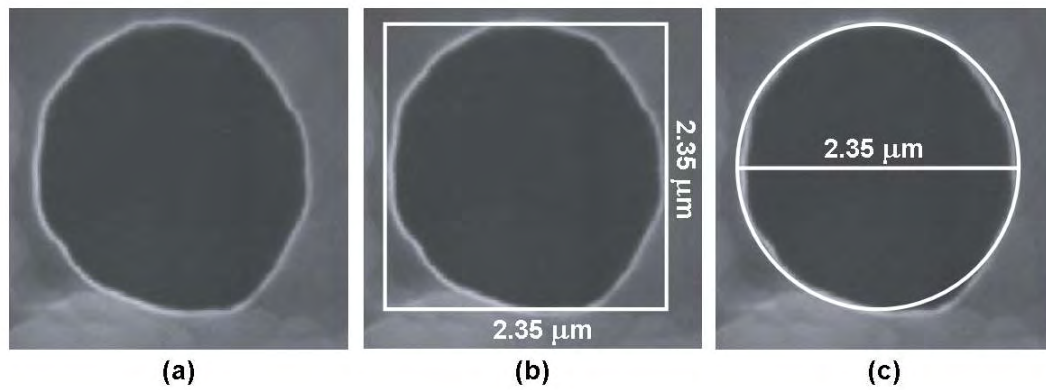


FIGURE 4.4 SEM image of a typical pixel pinhole in the sparse array (a), overlaid with a square pixel (b), overlaid with a circular pixel (c).

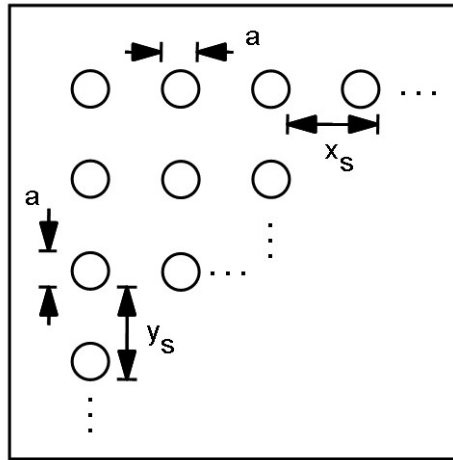


FIGURE 4.5 Sparse array sensor with circular pixels.

The cutoff frequency for a sparse array with circular pixels, as shown in FIGURE 4.5 can be found by rewriting Equation 2.22 as Equation 4.1 by replacing the $\text{rect}()$ function with the $\text{circ}()$ function defined in Equation 4.2.

$$I_i^s(x, y) = \left[I_i(x, y) ** \text{circ}\left(\frac{\sqrt{x^2 + y^2}}{a}\right) \right] \text{comb}\left(\frac{x}{x_s}, \frac{y}{y_s}\right) \quad 4.1$$

$$\text{circ}(r) = \begin{cases} 1 & 0 \leq r \leq 1/2 \\ 0 & \text{else} \end{cases} \quad 4.2$$

Taking the Fourier transform to find the frequency-spaced representation of Equation 4.1 yields,

$$\tilde{I}_i^s(\xi, \eta) = \left[\tilde{I}_i(\xi, \eta) \text{somb}\left(a\sqrt{\xi^2 + \eta^2}\right) \right] ** \text{comb}(x_s\xi, y_s\eta) \quad 4.3$$

The Fourier transform of the circ() function is the Sombrero function, defined in Equation 4.4, where $J_1()$ is the first order Bessel function of the first kind (Gaskill 1978).

$$\text{somb}(r) = \frac{2J_1(\pi r)}{\pi r} \quad 4.4$$

Thus for circular pixels the pixel MTF is the absolute value of the Sombrero function and its first zero is the pixel MTF cutoff frequency. The cut off frequency occurs at $1.22/a$, compared to a frequency of $1/a$ for square pixels.

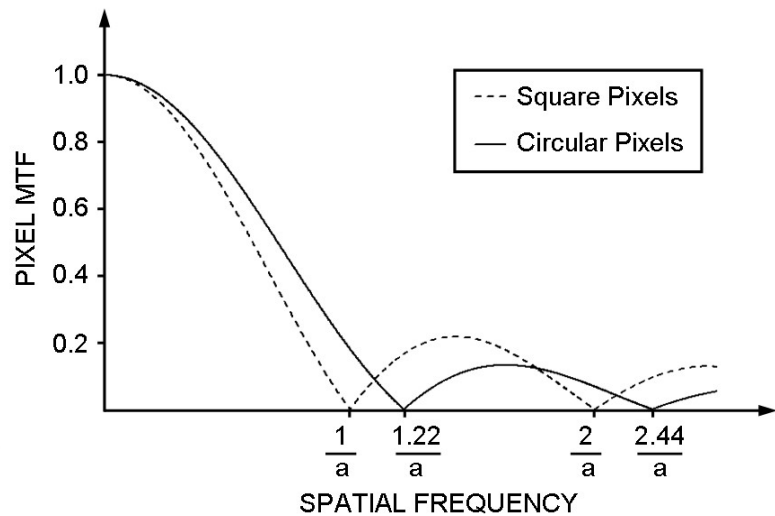


FIGURE 4.6 Comparison of the pixel MTF for square and circular pixels, or width a , in a sparse array sensor.

Therefore the theoretical cutoff frequency of the sensor used in this project would be 425 cycles/mm if square pixels are assumed or 519 cycles/mm if circular pixels are assumed. In reality because of the irregular shape of the pinhole pixels the true cutoff frequency is probably somewhere in between these values. Note that these are improvements of 6.4 to 7.8 times the unmodified sensor cutoff frequency. As discussed in Chapter 2.4.1, PSI is limited by the Nyquist frequency of the sensor, in this case 33.33 cycles/mm, while SNI is not. Therefore using the sparse array sensor with SNI increases the theoretical maximum measurable fringe frequency by a factor of 12.7 to 15.6 over the limit of using the unmodified sensor with PSI.

Unfortunately, in addition to the sensor geometry, the supporting electronics will affect the MTF of the sensor and the maximum measurable fringe frequency. In order to avoid aliasing, camera electronics are designed to limit the frequency response to well below the Nyquist frequency by the use of low pass filters. In order for the sparse array camera to support frequencies near the Nyquist frequency these filters were either removed or modified by Thermo CIDTEC. Additionally an internal pixel clock signal is generated by the camera and used to synchronize the readout of the voltage from each pixel. When a signal is sampled at the Nyquist frequency adjacent pixels record high and low voltages. Thus the maximum video signal frequency will be equal to half the pixel clock frequency. The camera electronics also need to prevent the clock signal from interfering

with or bleeding into the video signal. The new cameras electronics are a modified CIDTEC model 8723 camera controller unit (CCU) which has better clock noise cancellation than the older 2250D CCU, upon which the old camera was based. (Tony Chapman) The frame grabber used to capture video signal from the camera and convert the frames to digital images will also affect the system's MTF. In order to ensure the analog video signal is sampled at the appropriate time the pixel clock of the camera must be used to control the frame grabber acquisition timing. The frame grabber used in this system was the BitFlow™ Raven frame grabber, which was the updated model of the frame grabber used in Gappinger's work. Ideally the same frame grabber would have been used to compare the new and old cameras performance, however, the old frame grabber, a BitFlow™ Raptor, was no longer in working condition and the product line had subsequently been retired. In order to fairly compare the frequency response of the new and old cameras new data was taken with both cameras using the new frame grabber, results of which was discussed in Chapter 4.2.2.

4.2.1 Measuring Sparse Array Sensor MTF

Since the actual limit on fringe density is a function of the sensor and the supporting electronics, those of the camera and frame grabber. The pixel MTF of the sub-Nyquist camera system, the sensor and electronics, was measured using a procedure outlined by Gappinger (Gappinger et al, 2004) in order to determine the actual maximum measurable fringe frequency. This procedure is based on earlier work by Marchywka and Socker (1992) and Greivenkamp and Lowman (1994), in which an interferometer is used to

generate sinusoidal straight line tilt fringes directly onto the sub-Nyquist sensor. The advantage of this technique is that any spatial frequency can be created by simply adjusting the angle between the two beams of the interferometer. The pixel modulation and fringe frequency can be calculated using Fourier analysis as described by Marchywka and Socker (1992). However if the fringe pattern is phase shifted then the modulation can also be calculated using Equation 2.20, from the Hariharan-Schwider algorithm, as discussed previously in Chapter 2.1.4. Thus by collecting phase shifted interferograms for multiple tilts of the beam splitter the MTF of the system can be mapped out. As the angle between the interfering beams increases, the generated fringe frequency will also increase while the recorded fringe frequency oscillates between zero and the Nyquist frequency of the sensor due to aliasing.

In the procedure outlined by Greivenkamp and Lowman (1994) a Twyman-Green interferometer is used and measurements are only made at multiples of the Nyquist frequency. Even multiples of the Nyquist frequency can be found wherever aliasing produces a null fringe pattern and odd multiples can be found by observing the Moiré beat pattern between the fringes and the sensor pixels. Provided the angle between the two beams is constantly increased consecutive multiples of the Nyquist frequency can be mapped out. Measurements made using this technique are self-calibrated to the Nyquist frequency of the detector. However no information is recorded on frequencies between multiples of the Nyquist frequency. Also, knowledge of the pixel pitch is required in order to convert the spatial frequency into units of cycles/mm.

In Gappinger's procedure a Mach-Zehnder interferometer is used to generate the tilt fringes, FIGURE 4.7. The frequency of the tilt fringes can easily be varied by tilting the second beam splitter. The sensor is placed as close as possible to the second beam splitter to maximize the range of fringe frequencies that can be generated before the first beam walks off the sensor. Additionally an autocollimator can be used to track the tilt of the second beam splitter allowing frequencies other than multiples of the Nyquist frequency to be measured.

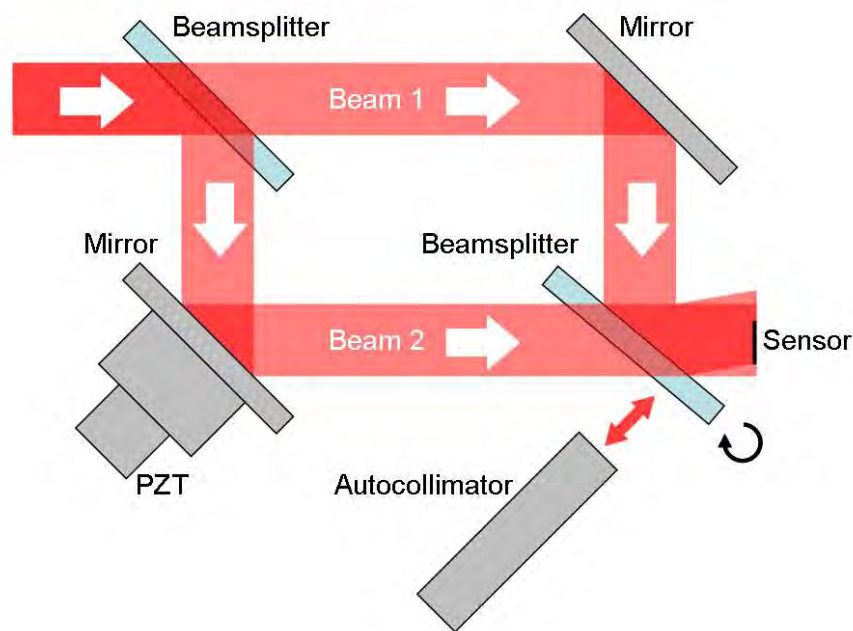


FIGURE 4.7 A Mach-Zehnder interferometer was used for measuring the pixel MTF of the sparse array camera.

Gappinger used a model of the system in lens design software to calculate the peak to valley OPD of the wavefront across the detector for a given angle of the second beam

splitter. The frequency of the fringes can be calculated from the OPD, by the Equation 4.5, where the OPD is in waves and D is diameter of the detector in millimeters.

$$\xi = \frac{2OPD}{D} \quad 4.5$$

However the fringe frequency can also be calculated mathematically from the angle between the two interfering plane waves, which in turn can be calculated from the tilt of the beam splitter, eliminating the need for modeling the system in lens design software. The derivation of the fringe frequency from the rotation angle of the beam splitter starting with the interference of two plane waves is outlined below FIGURE 4.8. The MTF measurements made for this research made use of these equations rather than relying on a lens design model.

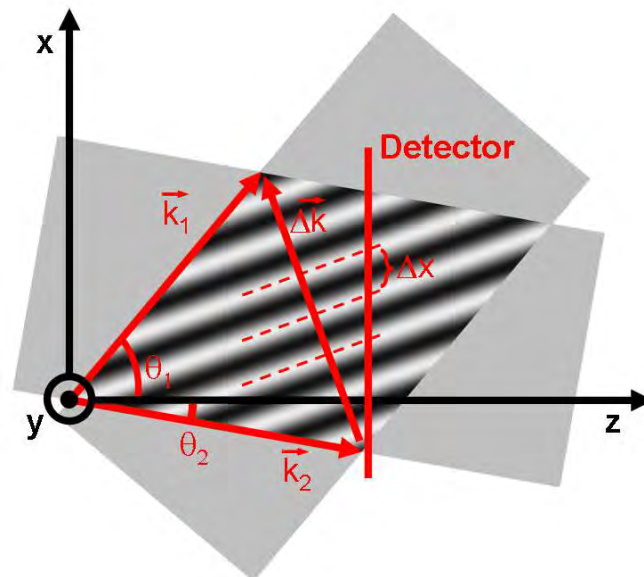


FIGURE 4.8 Fringes created by the interference of two plane wavefronts.

The equation for two plane waves polarized along the y axis and propagating in the x-z plane with the same wavelength are represented by Equations 4.6 and 4.7. The irradiance resulting for the interference of these two plane waves is then given by Equation 4.8.

$$\vec{E}_1 = A_1 e^{i(\vec{k}_1 \cdot \vec{r} - \omega t + \delta_1)} \hat{y} \quad 4.6$$

$$\vec{E}_2 = A_2 e^{i(\vec{k}_2 \cdot \vec{r} - \omega t + \delta_2)} \hat{y} \quad 4.7$$

$$I = |\vec{E}_1 + \vec{E}_2|^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r} + \delta_1 - \delta_2) \quad 4.8$$

In these equations the constant phase terms δ_1 and δ_2 shift the fringe pattern but do not change the frequency of the fringes, so their contribution can be ignored yielding Equation 4.9.

$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos(\Delta \vec{k} \cdot \vec{r}) \quad 4.9$$

$$\Delta \vec{k} = (\vec{k}_1 - \vec{k}_2) \quad 4.10$$

Since the wavefronts are propagating in the x z plane the vector terms of Equations 4.9 and 4.10 can be written as Equations 4.11-4.14.

$$\vec{r} = x \hat{x} + z \hat{z} \quad 4.11$$

$$\vec{k}_1 = \frac{2\pi}{\lambda} [\sin \theta_1 \hat{x} + \cos \theta_1 \hat{z}] \quad 4.12$$

$$\vec{k}_2 = \frac{2\pi}{\lambda} [\sin \theta_2 \hat{x} + \cos \theta_2 \hat{z}] \quad 4.13$$

$$\Delta \vec{k} = \frac{2\pi}{\lambda} [(\sin \theta_1 - \sin \theta_2) \hat{x} + (\cos \theta_1 - \cos \theta_2) \hat{z}] \quad 4.14$$

From Equation 4.9 a bright fringes occurs whenever the cosine term is equal to one, Equations 4.15 and 4.16.

$$\Delta \vec{k} \cdot \vec{r} = 2\pi m \quad m = 0, \pm 1, \pm 2 \dots \quad 4.15$$

$$\Delta \vec{k} \cdot \vec{r} = \frac{2\pi}{\lambda} [x(\sin \theta_1 - \sin \theta_2) + z(\cos \theta_1 - \cos \theta_2)] = 2\pi m \quad 4.16$$

If the assumption is made that the detector is perpendicular to the z axis and is located at the point $z = 0$, then the locations of bright fringes along the x axis are given by Equation 4.17.

$$x = \frac{m\lambda}{\sin \theta_1 - \sin \theta_2} \quad 4.17$$

The spacing of bright fringes along the detector plane and the corresponding fringe frequency are then given by Equations 4.18 and 4.19 respectively.

$$\Delta x = x_{m+1} - x_m = \frac{(m+1)\lambda}{\sin \theta_1 - \sin \theta_2} - \frac{m\lambda}{\sin \theta_1 - \sin \theta_2} = \frac{\lambda}{\sin \theta_1 - \sin \theta_2} \quad 4.18$$

$$\xi = \frac{1}{\Delta x} = \frac{(\sin \theta_1 - \sin \theta_2)}{\lambda} \quad 4.19$$

Equation 4.19 can be rewritten in terms of the rotation of the second beam splitter, θ_r , by assuming that the interferometer is set up such that when θ_r is equal to zero both the beams propagate along the z axis. Referring back to FIGURE 4.7, the interferometer is setup such that beam 1 reflects off the back surface of the beam splitter, without transmitting through the glass, thus beam 1 is simply deviated by twice the rotation of the beam splitter.

$$\theta_1 = 2\theta_r \quad 4.20$$

If the beam splitter is a perfect plane parallel plate then beam 2 is displaced but not deviated as the beam splitter is rotated. In this case θ_2 is always equal to zero. However, if there is wedge in the beam splitter then there is a slight deviation of beam 2 with rotation of the beam splitter. In the setup used for this experiment the wedge of the beam splitter is aligned parallel to the optical table so that the deviation is also in the x-z plane. Therefore the deviation of beam 2 is the same as the deviation of a prism as a function of angle of incidence, Equation 4.21 (Hecht 2002).

$$\delta(\theta_i, n, \alpha) = \theta_i + \sin^{-1} \left[\sin \alpha \sqrt{n^2 - \sin^2 \theta_i} - \sin \theta_i \cos \alpha \right] - \alpha \quad 4.21$$

Where n is the index of refraction of the beam splitter and α is the wedge in the beam splitter. Assuming that beam 2 propagates along the z axis when θ_r is equal to zero then the angle at which the light exits the beam splitter is given by Equation 4.22.

$$\theta_2 = \delta(\theta_i + \theta_r, n, \alpha) - \delta(\theta_i, n, \alpha) \quad 4.22$$

The initial angle of incidence, θ_i , is the angle at which light enters the second beam splitter when θ_r is equal to zero. Assuming the reflective surface of the beam splitter is set at an angle of 45° with respect to the z axis the angle of incidence can be found from Snell's law.

$$n_{\text{exit}} \sin \theta_{\text{exit}} = n \sin \theta_{\text{internal}} \quad 4.23$$

$$n \sin(\theta_{\text{internal}} + \alpha) = n_i \sin \theta_i \quad 4.24$$

Since the initial angle at which light exits the beam splitter, θ_{exit} , should also be equal to 45° and the beam splitter is in air; solving Equations 4.23 and 4.24 for the angle of incidence yields Equation 4.25.

$$\theta_i = \arcsin \left[n \sin \left(\alpha + \arcsin \left[\frac{1}{n\sqrt{2}} \right] \right) \right] \quad 4.25$$

The resulting fringe frequency as a function of the rotation angle of the beam splitter can be found by plugging Equations 4.20 and 4.22 into Equation 4.19.

$$\xi = \frac{\sin 2\theta_r - \sin [\delta(\theta_i, n, \alpha) - \delta(\theta_i + \theta_r, n, \alpha)]}{\lambda} \quad 4.26$$

The beam splitters used in this experiment were made from BK7 with an index of refraction of 1.519 at 532nm and had a wedge angle of 0.5° between the two surfaces. The initial angle on incidence, from Equation 4.25 is 45.957°. The change in beam 2 as a result of the wedge in the beam splitter is very small, approximately 6 arc minutes over the entire measurement range. This translates to a difference in the calculated fringe frequency of approximately 3 cycles/mm at the end of the measurement range. It was included in the calculations for completeness but, depending on the tolerance at which the MTF is to be mapped out, it could be ignored.

4.2.2 MTF Measurement Results

In order to test the horizontal and vertical MTFs independently of each other the MTF measurement was performed twice with the camera being rotated 90° between measurements. The measurement for one of the new cameras is shown in FIGURE 4.9 and FIGURE 4.10, alongside a measurement of the old camera, using the new frame grabber. Also plotted is the theoretical modulation for the sensor assuming both square

and circular fringes, as well as for a standard sensor with G factor equal to one.

Measurements were taken out to 433 cycle/mm or $13f_N$.

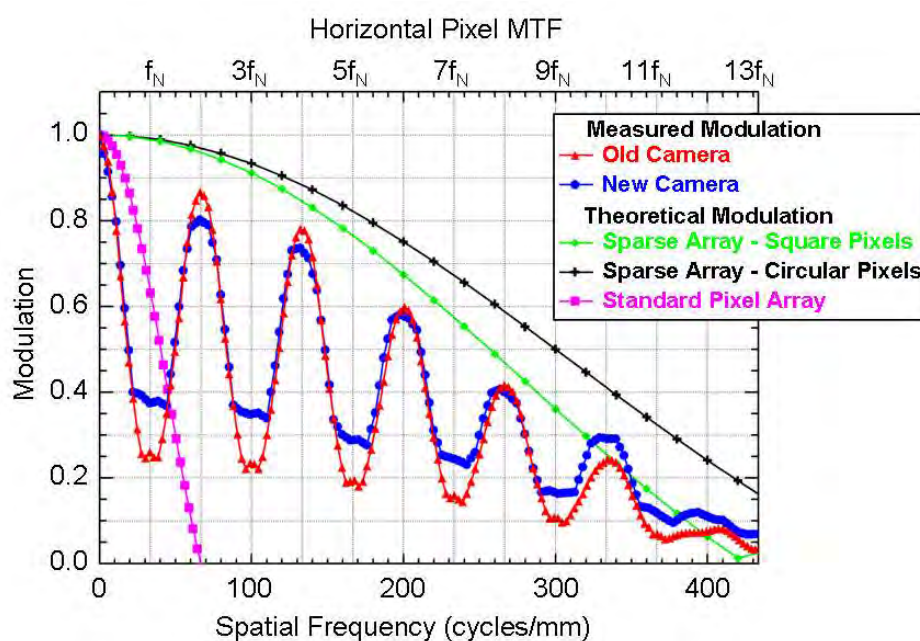


FIGURE 4.9 Horizontal Pixel MTF

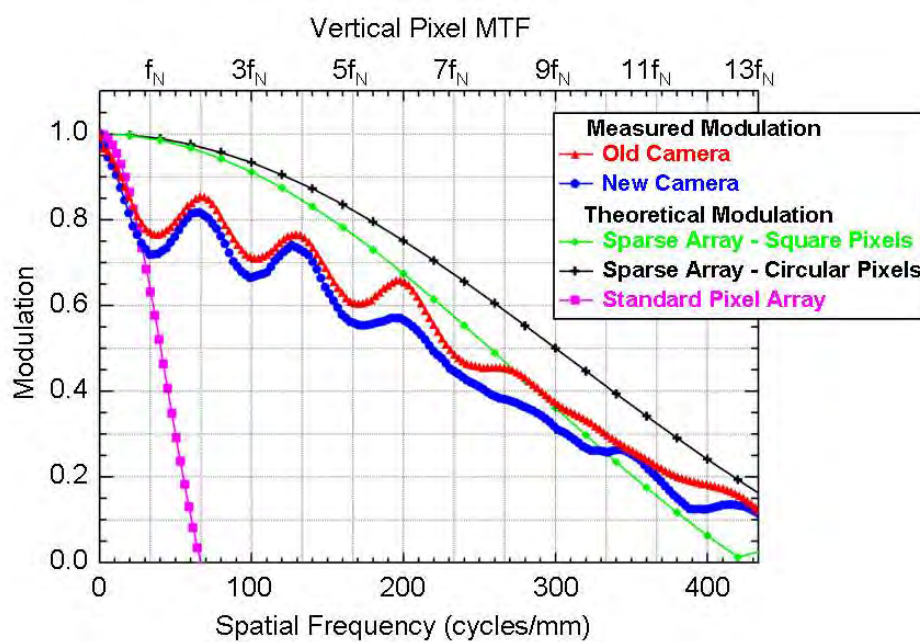


FIGURE 4.10 Vertical Pixel MTF

From these measurements of the camera several things become apparent. First the horizontal MTF of both cameras is much lower, especially at odd multiples of the Nyquist frequency, than the vertical MTF. This is due the read out process of the sensor, which reads out pixel values row by row. In the horizontal MTF measurement adjacent pixel's values alternate between high and low voltages across a row and are uniform by column. Therefore the read out signal is oscillating at half the pixel clock frequency. In the vertical measurement adjacent pixels produce uniform voltages across a row and alternating by column. Since the read out occurs across the rows the video signal oscillates at the half line rate, which is approximately $1/512$ the pixel clock frequency.

This means that the horizontal MTF determines the limit on the fringe frequency which can be measured with this system. Also note that the new camera has a slightly lower vertical MTF than the old camera and has a comparable horizontal MTF across most spatial frequencies. However there is a noticeable improvement at the odd multiples of the Nyquist frequency in the horizontal MTF. The ability to detect and unwrap fringes at these frequencies is what ultimately limits the dynamic range of the system. Therefore the improved modulation at these frequencies in the horizontal direction is worth the tradeoff of slightly lower modulation in the vertical direction. Gappinger stated that “a fringe modulation of 10% is sufficient to obtain good interferometric data” (Gappinger et al, 2004). The horizontal MTF of the new sensor does not cross the 10% modulation threshold until after 366 cycles/mm. The vertical MTF would be sufficient out to 400 - 433 cycles/mm. However this test represents the MTF under almost ideal conditions,

since the intensity of the two arms is almost perfectly matched across the entire sensor. When testing an aspheric wavefront particularly one with large changes in wavefront slope the intensity across the wavefront can vary due to pupil aberrations decreasing the modulation further. In order to allow for some degradation in the modulation it was assumed that the maximum fringe frequency would be 300 cycles/mm. While the fringe frequency in cycles/mm is useful when discussing the difference between the test and reference wavefronts at the detector it is not very useful when analyzing the wavefronts elsewhere in the interferometer. For a given interferogram the fringe frequency in cycles/mm will change as diameter of the interferogram changes. A more useful unit is waves/radius which will remain unchanged as the interferogram is scaled. The largest circle that could be inscribed in the square detector has a radius of 3.8325mm. Therefore the maximum slope difference between the test wavefront and the reference wavefront is 1150 waves/radius.

4.2.3 Measuring Sparse Array Sensor MTF Utilizing PSI

Measuring the MTF with this method is a rather tedious and time consuming process, generally taking more than 2 hours per direction. This is because the beam splitter was turned by hand while visually monitoring the tilt through the autocollimator. Since the autocollimator could only measure a range of 25 arc minutes, and 400 arc minutes of tilt is required for the full measurement the autocollimator had to be carefully repositioned 16 times. At the beginning of this research there were eight cameras that needed to be tested and compared, many multiple times after requiring repairs, in order to find the

camera with the best MTF. Additional measurements were required to find optimal settings for the frame grabber and therefore a faster method of measuring the MTF was desirable.

Since PSI is already being used to calculate the modulation at each pixel by Equation 2.20, the wrapped phase can be calculated from the same interferograms with Equation 2.18. The phase can then be unwrapped and converted to OPD from which the fringe frequency can be calculated using Equation 4.5. Knowledge of the pixel pitch is required to calculate the frequency in cycles/mm. The problem with this method is that as the fringe frequency increases past the Nyquist frequency the fringes will alias. Thus the measured frequency will always be in the base band of the sensor as discussed in Chapter 2.3. Sub-Nyquist unwrapping cannot solve this problem since there is only one fringe frequency across the entire sensor. Therefore there is not a zero order fringe to serve as the starting point for the sub-Nyquist unwrapping procedure. However, the aliasing problem can be solved by always increasing the fringe frequency between measurements, and keeping track of when a multiple of the Nyquist frequency has been crossed by visual observation. Then the measured frequencies, ξ_m , can be remapped to the actual frequency, ξ_o , utilizing, Equation 2.26, which can be broken down into two cases, Equation 4.27 and 4.28, based on the last multiple of the Nyquist frequency encountered.

$$\xi_o = (n+1)f_N - \xi_m, \quad n = 1, 3, 5 \dots \quad 4.27$$

$$\xi_o = nf_N + \xi_m, \quad n = 0, 2, 4 \dots \quad 4.28$$

In order to test the accuracy of this approach a MTF measurement was performed where the spatial frequency was calculated by monitoring the tilt of the beam splitter with an autocollimator, as described in Chapters 4.2.1 and 4.2.2, and from the OPD recovered with PSI, shown in FIGURE 4.11 and FIGURE 4.12.

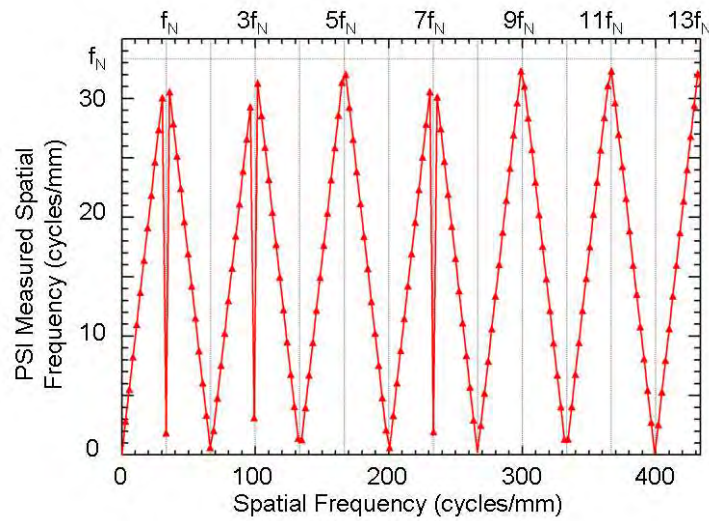


FIGURE 4.11 Comparison of the spatial frequency calculated using the autocollimator to the measured spatial frequency from PSI during a measurement of the vertical pixel MTF.

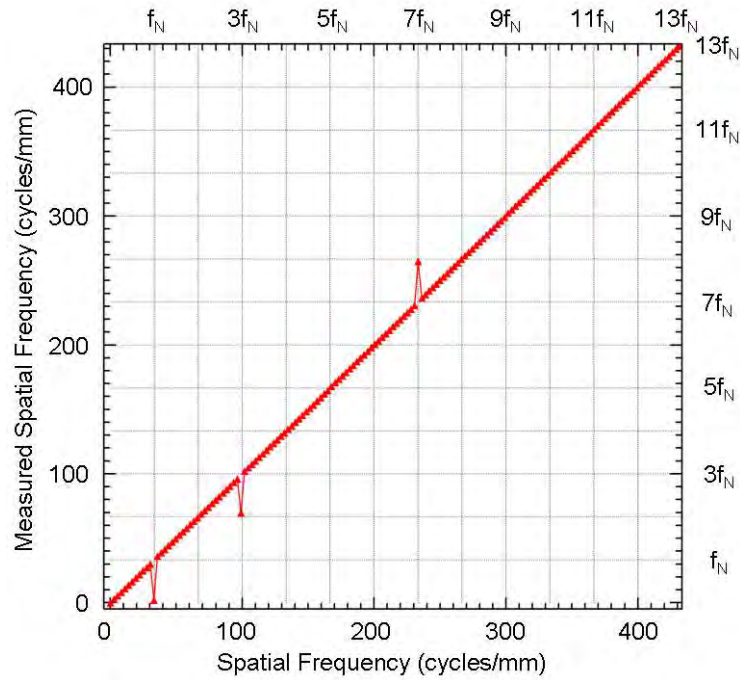


FIGURE 4.12 Comparison of the spatial frequency calculated using the autocollimator to the measured spatial frequency from PSI after accounting for aliasing.

The outliers in FIGURE 4.12 occur at or very near odd multiples of the Nyquist frequency, which are caused by the failure in the PSI unwrapping. This issue can be avoided by reverting back to the method specified by Greivenkamp and Lowman (1994) at these frequencies; that is rotate the beam splitter until a null Moiré beat pattern is observed so that the measurement will be self-calibrated to the Nyquist frequency of the detector.

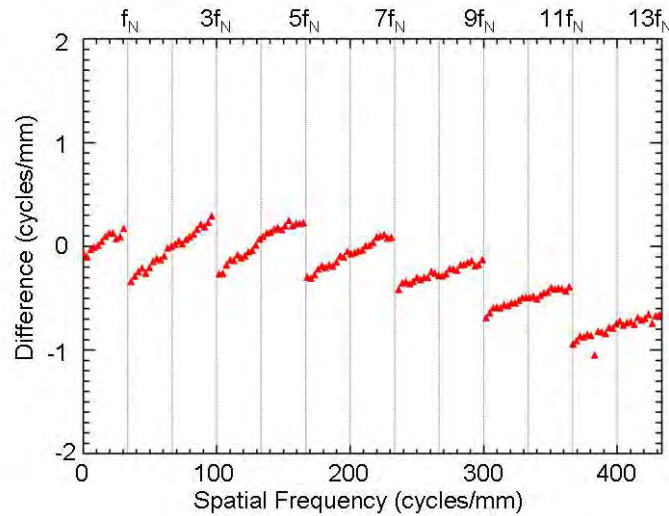


FIGURE 4.13 The difference between the spatial frequencies calculated using the two techniques.

The difference between the two methods of measuring the spatial frequency is shown in FIGURE 4.13, where the outlying points have been thrown out. The peak to valley difference is 1.33 cycles/mm with an RMS 0.32 cycles/mm. The sawtooth pattern in the difference plot is caused by the OPD switching signs as odd multiples of the Nyquist frequency are crossed due to aliasing. The magnitude of the difference is most likely caused by noise in the unwrapped wavefront and tilt of the detector with respect to the incoming wavefront. The general downward slope of the graph is likely due to a systematic error in the repositioning of the autocollimator every 25 arc minutes.

The main drawback to this method is that if the modulation drops too low the PSI measurement of the OPD will fail. This problem is exacerbated around the odd multiples of the Nyquist frequency during measurements of the horizontal pixel MTF. This means this technique can fall apart around the areas of most interest. However this in and of

itself can be useful information, since the desired result of these tests is to find the maximum fringe frequency that will be able to be recorded and unwrapped. With the exception of at odd multiples of the Nyquist frequency, as long as the modulation stays above 20% the recovered spatial frequency is within 1 cycles/mm of the value calculated from the autocollimator measurement.

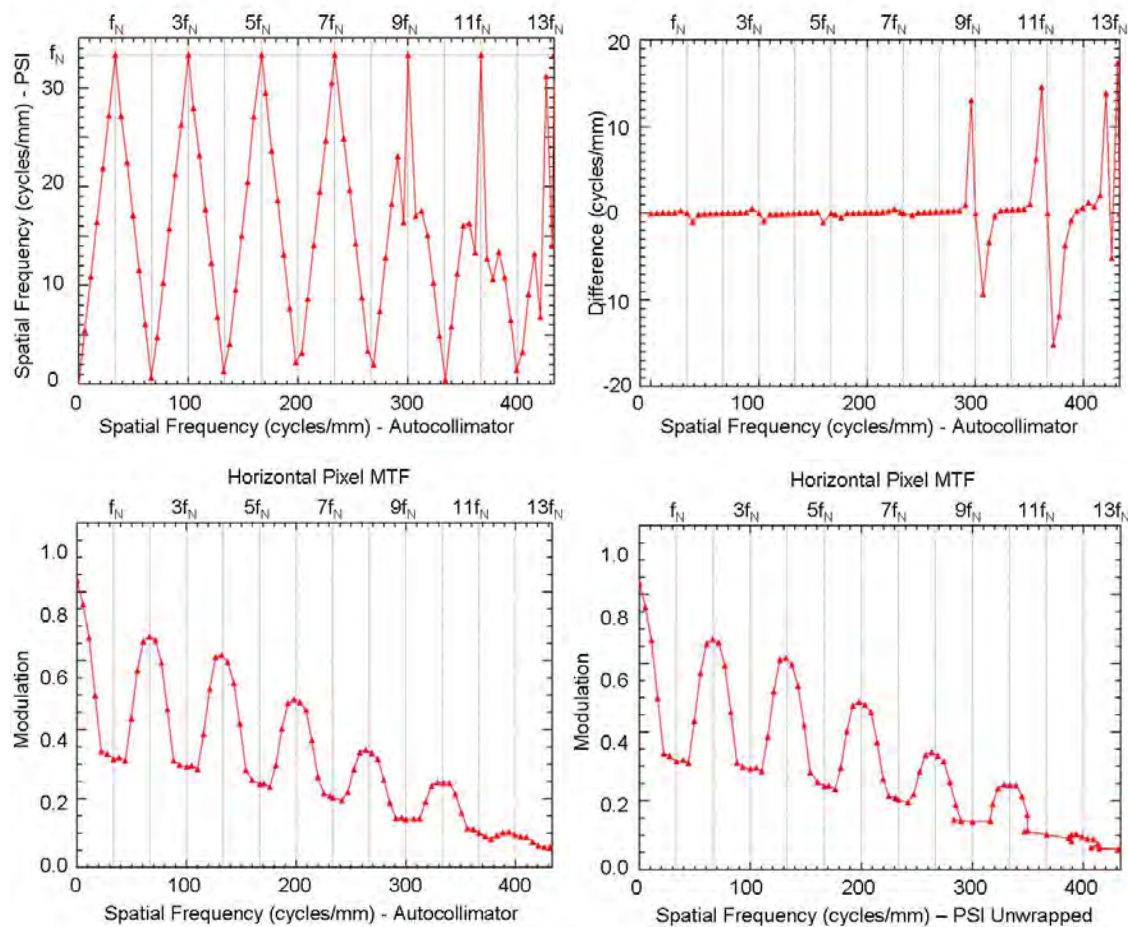


FIGURE 4.14 The spatial frequency measured with the autocollimator versus the spatial frequency measured using PSI during a horizontal pixel MTF measurement (Top Left). The difference between the measured spatial frequencies of the two techniques after unwrapping (Top Right). The horizontal pixel MTF as measured using the autocollimator (Bottom Left). The horizontal pixel MTF as measured with PSI (Bottom Right)

FIGURE 4.14 shows the results of a measurement performed utilizing both techniques. The modulation drops below the 20% threshold around $9F_n$ or 300cycles/mm. Notice the error in the recovered spatial frequency when the modulation dips below 20%, FIGURE 4.14 (Top Left), (Top Right) and (Bottom Right). Frequencies at odd multiples of the Nyquist frequency were recorded by observing the null Moiré beat pattern and assigning the appropriate spatial frequency without using PSI to recover the phase. The main benefit of this approach is that a measurement could be completed in approximately 20 minutes and does not require the use of the autocollimator. It was used often to make quick measurements of the MTF when knowledge the exact spatial frequency at each point was not required.

4.3 Interferometer Type

The next step in the design process was to decide on the basic type of interferometer. Two types of unequal-path phase shifting interferometers that are commonly used for surface testing are the laser-based Fizeau and Twyman-Green. The long coherence length of the laser allows them to be non-equal path. In the initial design phase of this system both interferometers types were considered as a starting point. The basic layout of each interferometer is shown in FIGURE 4.15.

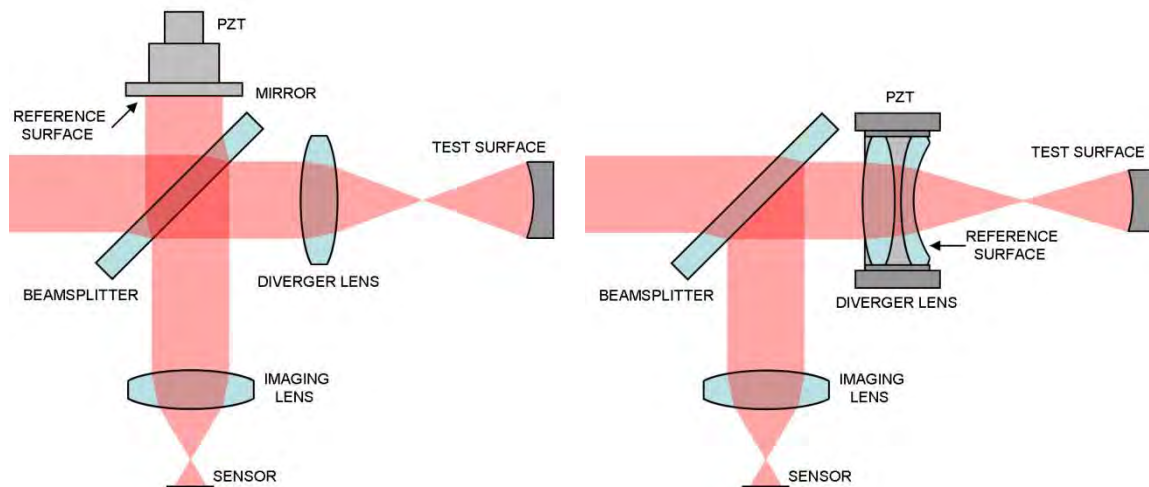


FIGURE 4.15 (Left) Twyman-Green Interferometer; (Right) Laser-Based Fizeau Interferometer

Both interferometers are made up of the same basic components. In the case of spherical surface testing both utilize a lens, which will be referred to as a diverger lens, to illuminate the test surface with a spherical wavefront where the center of the wavefront and test part are coincident. The main difference between the two is the location of the reference surface. In a Twyman-Green the beam splitter divides the incoming wavefront between the two arms of the interferometer; the reference arm and the test arm. The reference wavefront is reflected off a reference surface and sent back through the beam splitter into the imaging arm. The test wavefront travels through the diverger, onto the test part and back again until it is also sent into the imaging arm by the beam splitter. Then both arms propagate through the imaging arm and onto the sensor. In the Twyman Green interferometer the reference and test wavefronts are physically separated thus the optics in each arm will contribute to the OPL of that arm alone. Therefore when used for a null test any optic in the test or reference arm must be well corrected so that errors introduced into the measured wavefront by the interferometer are minimized.

In a laser-based Fizeau both the test and reference arm travel through the beam splitter and the diverger lens until they reach the reference surface. In null testing of spherical surfaces with a laser-based Fizeau the diverger lens is referred to as a transmission sphere, in which the last surface of the transmission sphere is the reference surface of the interferometer. The only difference between the reference and test arm of a laser-based Fizeau interferometer is that the reference arm reflects off the reference sphere, while the test arm transmits through the reference surface, reflects off the test part, and finally returns back through the reference surface. As discussed in Chapter 1.2, the light from both arms travel the same path through the system contributing the same OPL to both arms, thus the OPD is made up of the contributions of only the reference and test surfaces. In a null test the errors introduced by the interferometer are minimized provided a high quality reference surface is used, because all other surfaces are common path to both arms. Therefore the rest of the system components do not have to be as high of quality as their counter parts in a Twyman-Green interferometer. However, when testing an aspheric surface without a null optic the rays do not retrace their same path after reflecting off the test surface. Therefore one of the major advantages of the laser-based Fizeau, not requiring as high quality of optics as the Twyman-Green, is diminished when testing in a non-null configuration.

Another similarity between the two interferometers is that they both typically use a piezoelectric transducer (PZT) in order to move the reference surface $\lambda/8$ steps and

introduce the phase shifts of $\pi/2$ required for PSI and SNI as discussed in Chapter 2.1.2. However because the location of the reference surface is different the shift has a slightly different impact on each system. In the Tywman-Green interferometer the reference wavefront and surface are both flat. Therefore by moving the reference surface parallel to the incident light a constant phase shift is introduced across the wavefront. The Fizeau interferometer is typically phase shifted by moving the entire diverger optic in $\lambda/8$ steps parallel to the optical axis. This motion changes the distance between the reference and test surface by $\lambda/4$. However since both of these surfaces are concentric to the spherical waverfront produced by the transmission sphere the linear shift is not parallel to all the rays and thus the phase shift is not uniform over the wavefront (Moore and Slaymaker 1980). The effect is greater when testing fast optical surfaces with high NA transmission spheres. This problem has been solved in the case of PSI by utilizing an algorithm that makes use of the calculated phase shift at each pixel (Creath and Hariharan 1994) (de Groot 1995). However, these types of algorithms were developed for PSI and Creath and Hariharan specifically warns that difficulties can arise when the phase difference is close to an integer multiples of π . In a non-null interferogram it is possible that the phase difference will contain several multiples of π . If a laser-based Fizeau is to be used it would have to be verified that the solutions used for PSI could also be used for non-null measurements performed with SNI.

Another difference between the two interferometers is that the reference surface of the Fizeau must be partially transparent. It must reflect some portion of the incident light to

create the reference wavefront while also allowing the test wavefront to pass through twice. In Chapter 4.2, the MTF testing was performed with two beams of equal intensity; if the intensity of the two beams is not equal the modulation will be reduced. From Equation 2.5, 2.6, and 2.19, the data modulation depends on the fringe modulation, I'' , and the average intensity, I' , where

$$\gamma = \frac{I''}{I'} = \frac{2A_{test}A_{ref}}{A_{test}^2 + A_{ref}^2} = \frac{2\sqrt{I_{test}I_{ref}}}{I_{test} + I_{ref}} \quad 4.29$$

In a laser-based Fizeau, if I is the intensity of the light incident on the reference surface the intensity of the reference beam after reflecting off the reference surface would be

$$I_{ref} = R_{ref}I \quad 4.30$$

where R_{ref} , is the reflectivity of the reference surface. The test beam will transmit though the references surface twice and reflect off the test surface once, thus the intensity of the test beam is,

$$I_{test} = (1 - R_{ref})^2 R_{test}I \quad 4.31$$

where $(1 - R_{ref})$ is the transmittance of the reference surface and R_{test} is the reflectivity of the test surface. Typically the reference surface of a commercially available transmission sphere for a laser-based Fizeau is uncoated glass with a reflectance of about 4%. If the test surface is also uncoated glass then the intensity of the reference beam and test beam will be $0.04I$ and $0.037I$, with a resulting modulation of 0.999. However if the test surface is a mirror with R_{test} approximately equal to one, the reference intensity remains

unchanged but the test beam intensity increases to $0.92I$, resulting in a modulation of only 0.399.

Normally when testing a highly reflective surface on a commercial laser-based Fizeau interferometer an absorbing or reflecting pellicle is placed between the test and reference surfaces in order to reduce the intensity of the test beam. Since the pellicle is only in the test arm of the interferometer it will impart OPD into the measurement and therefore must be of high quality. Additionally if a fast convex surface is to be tested the gap between the reference surface and test surface can be very small making using a pellicle difficult or impossible if the test surface must be nested inside the outer edge of the reference surface or its mount.

Alternatively, the intensities of the two arms could be altered by adding a coating to the reference surface. Solving Equations 4.30 and 4.31 for R_{ref} when R_{test} is equal to 1 yield that a reference surface reflectance of approximately 0.382 would result in equal reference and test intensities and a modulation of 1.0. However this could introduce a problem with spurious fringes since light making two round trips between the test and reference surface would still have an intensity of $0.15I$ and a modulation of 0.9 with either the test or reference beams.

The Twyman-Green interferometer allows for greater flexibility in adjusting the intensity of each arm independently, because unlike the laser-based Fizeau, the intensity of the test

arm does not depend on the reflectance of the reference surface. Since the two arms are physically separated the percentage of light entering into each arms depends on the beam splitter coating. Typically a coating is used that reflects 50% and transmits 50% of the input light in order to keep the light in both arms equal. Then a reference surface is selected that matches the reflectivity of the test surface in order to maintain a high modulation. Additionally if an absorbing pellicle is needed it does not have to be sandwiched in-between the last surface of the diverger and the test surface, it could simply be placed prior to the diverger lens. The pellicle would still have to be high quality in order to avoid adding OPD into the measurement.

In the end, while either interferometer type could be used as the base of a sub-Nyquist interferometer, the Twyman-Green interferometer was selected as the base of the design because:

- 1) The major advantage of the Fizeau interferometer, the fact that the reference and test wavefronts are common path, is not maintained in a non-null test.
- 2) The method employed to phase shift a laser-based Fizeau interferometer introduces non uniform phase steps across the pupil.
- 3) It is easier to maintain equal intensities of the test and reference wavefront, which is important to avoid further degrading the MTF of the system, in a Twyman-Green interferometer.

Now that the type of interferometer has been selected, the individual elements must be specified. The required list of parts include a light source, optics to produce a collimated beam, a beam splitter, a reference mirror along with a phase shifter, a lens to collect the light off the test parts, an imaging lens and a sensor.

4.4 Light Source

The light source used for the sub-Nyquist interferometer was a Lightwave Electronics (Mountainview, CA) frequency doubled 532nm Nd:YAG laser, model 142H-532-200. The light source was chosen early on in the design process so that its properties such as wavelength, power output and coherence length were known for the rest of the system design. The primary reason this particular laser was chosen is because it was already on hand and had been used for the sensor MTF measurements. The properties that made it appealing for use in sub-Nyquist interferometry are its relatively high power and long coherence length. The power of the light source is a concern in SNI because of the reduced sensitivity of the sparse array camera due to the reduction of the active area of each pixel by the pinhole array. The area of a $2.35\mu\text{m}$ diameter circular pixel, $4.3\mu\text{m}^2$, is over fifty times smaller than that of a $15\mu\text{m}$ square pixel, $225\mu\text{m}^2$. The high power of the laser allows for glass or plastic surfaces to be measured which reflect only 4% of the incident light. The diamond turned brass inserts however have a much higher reflectivity than glass so the laser power is typically cut to approximately 20mW using a variable neutral density filter. The long coherence length of this laser prevents the fringe visibility from decreasing as a result of the non-equal path length of the reference and test arms.

Any decrease in the fringe visibility would mean a further reduction in the modulation beyond that inherent to the sub-Nyquist sensor presented in Chapter 4.2.2, which could possibly limit the interferometer to a lower maximum fringe frequency. However, the laser used has a theoretical coherence length of greater than a kilometer, TABLE 4.2. The extremely long coherence length of this laser allows the length of the test arm to be varied greatly, in order to accommodate measuring different aspheric surfaces, yet still produce fringes with high visibility. However, in hindsight this laser may have too long of a coherence length since stray light reaching the sensor will interfere with the test and reference wavefronts to produce high visibility spurious fringes. Using a laser with a coherence length on the order of a few meters or less may have been beneficial even if it required changing the length of the reference arm to maintain high visibility fringes when the length of the test arm was changed.

Specification	Value
Wavelength	532nm
CW Power	200 mW
Spatial Mode	TEM ₀₀
Longitudinal Mode	Single Frequency
Linewidth	<10 kHz/ms
Calculated Coherence Length	>1000m
Frequency Drift	<10MHz/min
Linear Polarization	1000:1

TABLE 4.2 Laser Properties (Lightwave Electronics Laser Manual)

4.5 Diverger Design

As discussed in Chapter 1.4.3 when an aspheric surface is tested in a null configuration the diverger serves as a null optic which creates a wavefront that exactly matches the form of the test surface. In a non-null test the diverger will not produce a wavefront that matches the test surface, however it still needs to be able to fully illuminate the test surface and capture the reflected wavefront. Since the Twyman-Green interferometer makes use of a flat reference wavefront the diverger will subtract a nominally spherical wavefront from the test wavefront at the test part. However, as discussed in Chapter 3.2.5, phase errors are introduced as a result of the null condition being violated. Additionally, the diverger is involved in the imaging of the test part onto the detector. The diverger images the test part, which serves as the aperture stop of the system, to the intermediate pupil of the interferometer. If this imaging is not free of pupil aberrations then the diverger induces mapping errors into the measurement.

Two different design strategies that could be pursued for the diverger design are to either minimize the number of optical components to aid the reverse optimization procedure as discussed by Lowman (Lowman 1995), or to increase the design complexity in order to minimize the induced aberrations. Considering the latter, if the interferometer is designed around testing a single aspheric surface, or a range of surfaces with similar aspheric departures, then the diverger optic could be designed to generate a wavefront that approximately matches the test surface. This type of diverger optic would be considered a partial null since it does not completely null out the aberrations from the

aspheric surface. Rather a partial null is used to minimize the induced phase aberrations in the test arm and as a result reduce either, or both, the number of fringes or the maximum fringe frequency produced in the interferometric test. An attempt could also be made to design the diverger to reduce the induced mapping errors or pupil aberrations. Again this would be easier if only one aspheric surface is to be tested since there would be only two conjugate planes and one set of rays for which the imaging correction would have to be designed. If multiple surfaces are to be tested, and the distance between the diverger and test surface is changed from part to part, then the correction will have to be made over a range of conjugate planes. Additionally if both convex and concave parts are to be tested with the same diverger then the test surfaces will have to be situated inside and outside of the diverger focal point. This would lead to the imaging correction needing to be made for both real and virtual intermediate pupils. The main problem with pursuing an aberration correction strategy during the design of this SNI system was that the part prescriptions were not known. Therefore it was unknown if the parts would all have a similar aspheric departure or even if the sign of the departure would be consistent. This made it impractical to design a single diverger which would correct for the induced phase and mapping aberrations of the unknown test parts, or to determine the number of divergers/partial nulls that would be required to cover the unknown range of aspheric surfaces. Therefore the strategy of designing a simple system in order to aid the reverse optimization and reverse ray tracing was employed. The goal of the design was then to find a single diverger that was capable of capturing the light off both convex and concave

parts with both positive and negative aspheric departures, the specifications of which are given in TABLE 4.1.

The diverger must emit a collapsing wavefront so that convex and concave parts can be tested. The input wavefront into the diverger will be a plane wavefront so that a flat reference surface can be utilized. The range of image space $F/\#$'s required to fill the test surface can be calculated from test part diameter, 8mm, and the range of BFS radii of 6mm to 10mm. This leads to an $F/\#$ range from $F/0.75$ to $F/1.25$ or an approximate NA range of 0.67 to 0.4, Equation 4.32.

$$F/\# \approx \frac{1}{2NA} \quad 4.32$$

This range serves as a good starting point for the design. However, in addition to illuminating the test part the diverger needs to be able to collect the reflected light. The NA to collect the reflected light of the test surface may be larger than the NA required to illuminate the surface depending on the slope difference between the aspheric test surface's and the illuminating wavefront. The last surface of the diverger should be concave in order aid in the collection of the light which reflects off the test part. Since, in comparison to a convex surface, a concave surface will reduce the angle from the surface normal at which reflected rays strike the surface. The back focal distance, BFD, of the lens has to be large enough to accommodate a convex surface with a best fit sphere radius of curvature of 10mm, TABLE 4.1. In order to aid system alignment it is beneficial for the BFD of the diverger to be long enough to prevent a convex test surface from having to be nested inside the last surface of the diverger. Therefore, a target of 15mm was used

for the diverger designs. A decision must be made on the diameter of the input wavefront into the test arm in order to meet the BFD and $F/\#$ requirements. Either a smaller diameter plane wavefront can be expanded and collapsed in the test arm, FIGURE 4.16 (Left) or a larger plane wavefront can be generated before the beam splitter and used in both arms of the interferometer, FIGURE 4.16 (Right). The expansion of the beam could be accomplished by the addition of an afocal system, such as a Galilean telescope, or it could be incorporated into the diverger design or collimating optics.

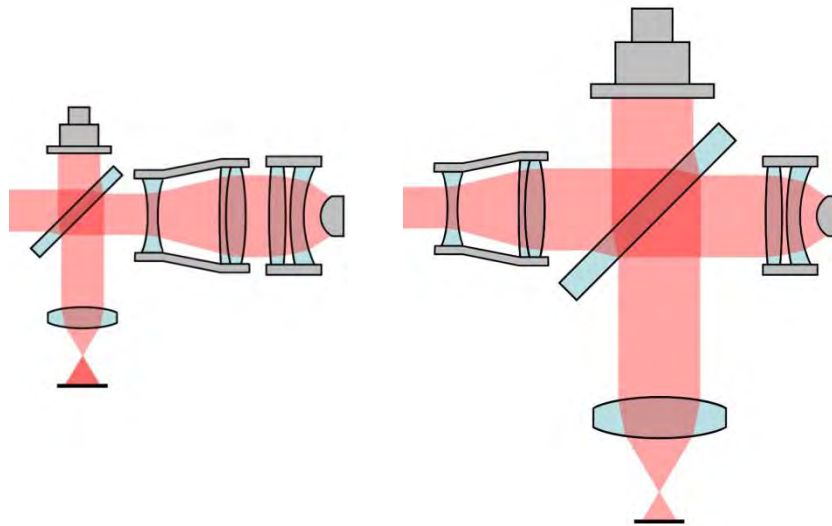


FIGURE 4.16 In order to meet the $F/\#$ requirement of the diverger the beam could be expanded in the test arm of the interferometer (Left) or a larger diameter collimated wavefront could be generated in the input arm of the interferometer (Right).

The advantage of using a smaller input wavefront and expanding the beam in the test arm is that the rest of the interferometer components, such as the beam splitter, reference surface and PZT mount, can be relatively small. The advantage of using a larger input wavefront is that the number of components in the test arm of the interferometer is minimized (Lowman 1995) (Gappinger 2002). This means fewer surfaces that the aspheric wavefront will interact with, thus simplifying the reverse optimization and ray

tracing procedures. Therefore the latter design strategy was implemented. The diameter of the input wavefront was limited by the collimating lens and PZT, which were selected from parts that were already on hand at the time of the design. Both components have a nominal 50mm diameter, but will be discussed in more detail in Chapter 4.8 and 4.9.

Finally, it can aid in the alignment of the interferometer and finding the starting point for the reverse optimization process if the diverger allows for null tests to be performed on a spherical test surface and a surface located at the cat's eye position. These procedures will be discussed in more detail in Chapter 5 and 6, however in order to implement them the diverger must produce a nearly spherical wavefront. If the OPD_z of the light focused by the diverger is under half a wave then the resulting null test using a spherical test surface will be under one wave. In Chapters 4.5.1 - 4.5.5 several different diverger options will be presented. The performance of a few of the designs will be compared in Chapter 4.5.6.

4.5.1 Transmission Sphere

One of the first options considered for the diverger lens was a commercially available transmission sphere. Either utilizing it as intended, in a laser-based Fizeau, or in the test arm of a Twyman-Green interferometer. Since they are used in many commercial interferometers, and are generally interchangeable between interferometer manufactures, they are readily available from a number of manufactures and second hand sellers. As discussed previously, Chapter 4.3, matching the intensities of the reference and test wavefronts would be a challenge. If used in a Twyman-Green a separate reference flat

would be used allowing the intensities of the two arms to be matched for highly reflective test surfaces. However there would still be a 4% reflection off the last surface of the transmission sphere which is concentric to the focus of the lens. One possible solution would be to have the reference surface of the transmission sphere anti-reflection coated. The biggest obstacle to the use of a transmission sphere is that their designs are proprietary, faster transmission spheres appear to contain several elements and because they are designed for null testing only the reference surface needs to be extremely high quality, making reverse optimization difficult. However, if a transmission sphere manufacturer was willing to provide the design as well as manufacturing tolerances a transmission sphere could conceivably be used. Even better would be to obtain measured data from a transmission sphere as it is being assembled, such as index of refractions of the glasses, surface figure measurements and surface separations and decenters. Yet, without the cooperation of a manufacturer, a transmission sphere would have to be taken apart and the individual parameters measured in order to reverse engineer the design. The elements would then have to be precisely reassembled, which was impractical for the purpose of this research.

4.5.2 Mirror

Another option explored was the use of a mirror for the diverger. Obviously a mirror would be the solution with the fewest number of optical surfaces simplifying the model for reverse optimization. A parabolic mirror would seem to be an ideal candidate; as a parabolic mirror produces a spherical wavefront from the incoming plane wavefront.

Additionally there are well known interferometric testing setups that can be used to quantify the errors in the parabolic surface for inclusion into the optical model, as shown in Chapter 1.4.1. An on-axis parabolic mirror cannot be used to test the contact lens inserts since the insert would block the central portion of the test wavefront. The next logical solution would be an off-axis parabolic mirror. An off-axis parabolic mirror can still be tested with the previously discussed technique since the surface is simply a smaller section of the larger parent parabola. In order to avoid the test surface, the rest of the insert and the mounting hardware from blocking any portion of the beam the input test beam must be moved significantly off axis, as shown in FIGURE 4.17. In this example a parabolic mirror with a radius of curvature of 46.892mm was used. The optical axis of the input wavefront is 30mm from the center of rotation of the parabola. The chief ray is deviated by 66° upon reflecting off the parabolic mirror.

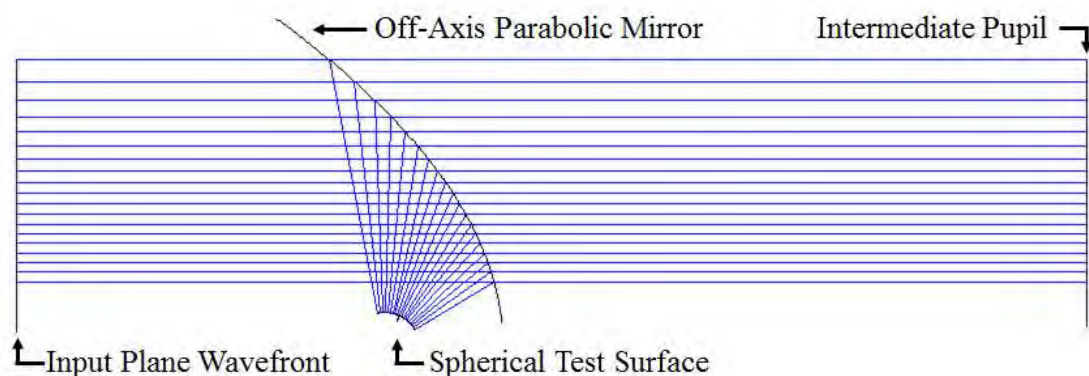


FIGURE 4.17 The layout of an off-axis parabolic mirror used as a diverger.

In FIGURE 4.17 a spherical test surface with a radius of 7mm is used as the aperture stop of the system. The problem with using an off-axis parabolic mirror can be seen in the uneven distribution of rays across the intermediate pupil. The off axis parabolic mirror introduces a large amount of non-rotationally symmetric pupil aberrations into the

measurement even when testing a rotationally symmetric surface, as shown in FIGURE 4.18. Therefore the use of an off-axis parabolic mirror was not further investigated as a potential diverger design.

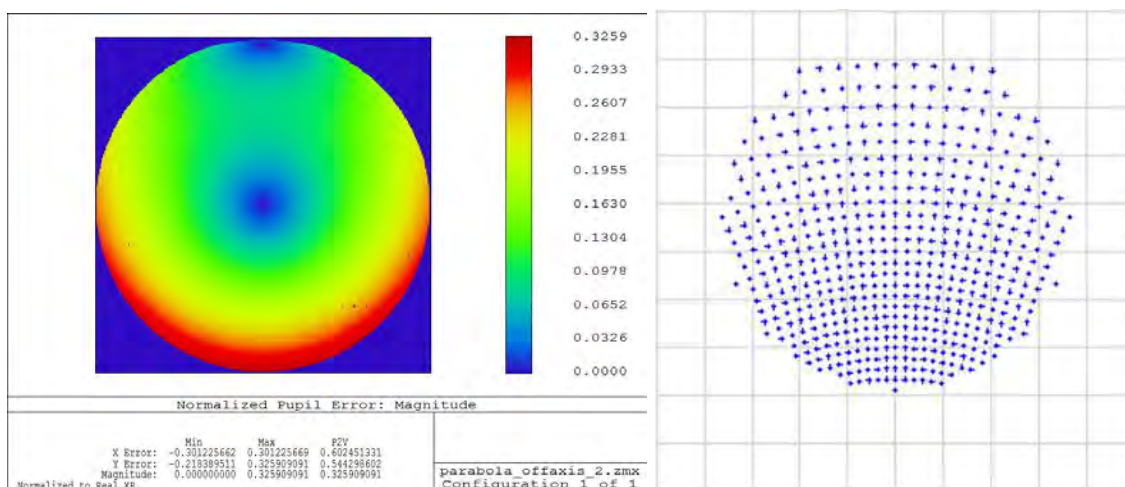


FIGURE 4.18 The pupil aberration at the intermediate pupil for a spherical surface measured using an off axis parabolic mirror is visible in the normalized pupil error map (Left) and the spot diagram (Right).

4.5.3 Multiple Element Diverger Lens

The next option for the diverger was to use a custom designed lens composed of spherical surfaces. It was found that in order to meet the $F/\#$ requirement while also reducing the OPD_z to under a half a wave a three lens solution was required, as shown in FIGURE 4.19. The design made use of the high index of refraction glass, S-NPH2, since it allows for better reduction of the OPD_z compared to a lens using a lower index of refraction glass with the same $F/\#$. This glass was chosen because it was the preferred high index glass of the lens manufacturer used for this research. The index of refraction of S-NPH2 at 532nm is 1.9389. The diverger was designed for a 36mm diameter plane input wavefront, which corresponds to a working $F/\#$ of 0.74 and an image space NA of 0.59.

It produces a spherical wavefront with a peak to valley OPD_z of 0.04 waves. However, the lens will accept a plane input wavefront with a diameter as large as 46mm which corresponds to a working $F/\#$ equal to 0.61 and an image space NA of 0.68, while maintaining a peak to valley OPD_z of 0.21 waves. The BFD of the design had to be reduced to only 12.9mm in order to accommodate the use of three lenses. This is slightly under the design target of 15mm, but is still long enough to allow convex aspheric surfaces with a BFS of 10mm to be tested.

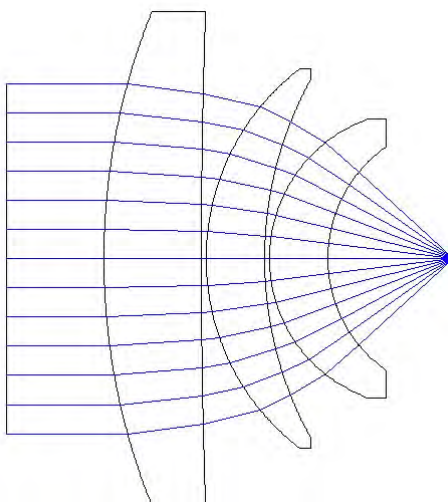


FIGURE 4.19 Three Element Spherical Diverger Lens Layout

Surf	Type	Radius	Thickness	Glass	Semi-Diameter	Conic
OBJ	Standard	Infinity	Infinity		0.00000	0.00000
STO	Standard	Infinity	0.00000		18.00000	0.00000
2*	Standard	67.64524	10.00000	S-NPH2	25.40000	0.00000
3*	Standard	738.45614	0.50000		25.40000	0.00000
4*	Standard	24.56624	6.00000	S-NPH2	19.50000	0.00000
5*	Standard	38.15276	0.50000		18.50000	0.00000
6*	Standard	15.19801	6.00000	S-NPH2	14.30000	0.00000
7*	Standard	13.99825	12.91791		11.50000	0.00000
IMA	Standard	Infinity	-		2.7389E-004	0.00000

TABLE 4.3 Three Element Spherical Diverger Lens Prescription

One drawback to this diverger lens is that the three element design leads to a large number of variables, or errors in the construction of the lens, that have to be taken into consideration for the reverse optimization and reverse retracing procedures. These variables include the shape of the six optical surfaces, the index or refraction of each element, the three glass thicknesses, the two air gaps, as well as the decenters and tilts of all six surfaces for a grand total of thirty-eight variables. Each of these lens properties is a potential source of error in the final reverse ray tracing if the physical system is different than the ray tracing model used. Therefore each variable must be dealt with in one of three ways. The first is to measure the lens property and determine if the difference between the nominal value and the measured value, combined with the uncertainty of the measurement, is large enough to induce significant error into the reverse ray tracing procedure. If the lens property is close enough to the nominal value that no significant error is introduced into the reverse ray tracing procedure then it can be ignored in the model of the system. The second method is to update the nominal value in the model to match the measured value if it is determined that significant error would be introduced. Finally, if the property cannot be measured, or the uncertainty in the measurement is large enough to introduce a significant error into the reverse ray tracing procedure, the property must be made a variable in the reverse optimization procedure. It is worth pointing out that the shape of the optical surface is actually quite a bit more complicated than just a single variable, since it is essentially the difference between the nominal sag and actually sag of each point across the surface. For a spherical surface the

surface error map can be determined using a laser-based Fizeau interferometer and a precision slide to measure the radius of curvature. This design appears to be a viable option and will be compared against other designs in Chapter 4.5.6.

4.5.4 Single Element Diverger With an Aspheric Surface

The diverger can be reduced to a single element, while still satisfying the $F/\#$ and OPD_z requirements, if an aspheric surface is used in the design. The advantage of such a design is that the number of lens properties that must be measured, or set as variables in the reverse optimization procedure, is greatly reduced from the three element design. The design only has two optical surfaces, their decenter and tilts, one glass index of refraction and one glass thickness for a total of twelve variables. The aspheric surface adds an element of complexity into the characterization of the lens. However since the lens is designed to produce a spherical wavefront the lens could be measured in double pass using a precision spherical surface. If the rest of the lens has been accurately characterized then it may be possible to attribute the measured error to the aspheric surface.

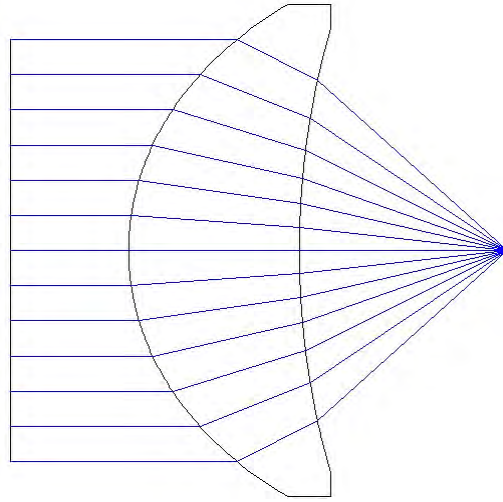


FIGURE 4.20 Single Element Aspheric Diverger Lens Layout

The single element aspheric diverger design also makes use of S-NPH2 glass. The first surface is a Zemax Even Asphere surface type as described by general Equation 4.33, where only the conic and α_1 term are non-zero for this design. The full prescription of the lens is given in

TABLE 4.4.

$$z = \frac{Cr^2}{1 + \sqrt{1 - (1+k)C^2r^2}} + \alpha_1 r^2 + \alpha_2 r^4 + \alpha_3 r^6 + \alpha_4 r^8 + \alpha_5 r^{10} + \alpha_6 r^{12} + \alpha_7 r^{14} + \alpha_8 r^{16} \quad 4.33$$

It has a BFD of 17.66mm which exceeds the target 15mm distance. It was also designed to for a 36mm diameter plane input wavefront at which it has a working F/# of 0.75, an image space NA of 0.56 and a peak to valley OPD_Z of 0.01 waves. The maximum diameter of the input wavefront is slightly smaller than the three element diverger at 40.5mm. This corresponds to a working F/# of 0.67, an image space NA of 0.60, and a quarter of a wave peak to valley OPD_Z. The minimal number of surfaces and reverse optimization variables of this design lead it to be selected as the original diverger for use

with the interferometer. Ultimately it failed due to a manufacturing defect, which will be discussed in Chapter 5.4.4, however this design will still be compared against other designs in Chapter 4.5.6.

	Surf:Type	Radius	Thickness	Glass	Semi-Diameter	Conic	2nd Order Term
OBJ	Standard	Infinity	Infinity		0.00000	0.00000	
STO	Standard	Infinity	0.00000		18.00000	U 0.00000	
2*	Even Asphere	16.69399	14.50000	S-NPH2	21.00000	U -0.51982	-2.2163E-003
3*	Standard	68.71527	17.66497		19.00000	U 0.00000	
IMA	Standard	Infinity	-		2.4832E-004	0.00000	

TABLE 4.4 Single Element Aspheric Diverger Lens Prescription

4.5.5 Two Element Diverger With an Aspheric Surface

A compromise between the two previous designs is a two element diverger that makes use of a single aspheric surface. As in the previous designs S-NPH2 glass is used for the elements. Only a conic term is used for the aspheric departure of the first surface. The total number of lens properties that must be considered for the reverse optimization procedure is twenty-five. Unlike the previous lenses which were designed by the author; this lens was designed by OPTICS 1, Inc. (Westlake Village, CA) It was based on a design created by the author but was modified to be used in a different sub-Nyquist interferometer. Their interferometer used a larger diameter collimated beam as the input wavefront which resulted in this lens being slightly slower than the previous lenses. At the 36mm diameter the working $F/\#$ of the lens is 0.81 and the image space NA is 0.55. The OPD_z at this diameter is 0.16 waves peak to valley. At the maximum input wavefront diameter of 50mm the $F/\#$ is 0.63 with a image space NA of 0.68 and a OPD_z

of 0.65 waves peak to valley. The layout of the lens is shown in FIGURE 4.21 and the prescription is given in TABLE 4.5.

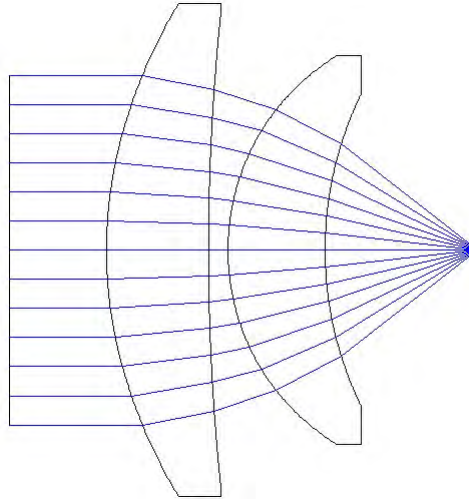


FIGURE 4.21 Two Element Aspheric Diverger Lens Layout

Surf:Type		Radius	Thickness	Glass	Semi-Diameter		Conic
OBJ	Standard	Infinity	Infinity		0.00000	0.00000	
STO	Standard	Infinity	0.00000		18.00000	U 0.00000	
2*	Standard	43.77000	10.50000	S-NPH2	25.40000	U -0.89291	
3*	Standard	234.85000	2.00000		24.00000	U 0.00000	
4*	Standard	23.48000	10.00000	S-NPH2	20.00000	U 0.00000	
5*	Standard	36.63900	15.71834		16.00000	U 0.00000	
IMA	Standard	Infinity	-		1.360E-003	0.00000	

TABLE 4.5 Two Element Aspheric Diverger Lens Prescription

4.5.6 Comparing Diverger Designs

In order to compare the performance of the different diverger lenses a series of aspheric test surfaces, meeting the specification given in TABLE 4.1, were generated. For the purpose of this test two types of aspheric surfaces were used, the Zemax Even Asphere surface which is described by Equation 4.33 and the Zemax Toroidal surface. The former

was used to produce rotationally symmetric surfaces and the latter was used to produce non-rotationally symmetric surfaces. The process for creating the test surfaces will be discussed first followed by the results of testing the surfaces with the three diverger lenses.

Two sets of rotationally symmetric aspheric surfaces were generated. In the first set only one of the aspheric coefficients, or the conic constant, was allowed to vary. In the second, multiple coefficients and the conic constant were varied. Both sets were generated by using the Zemax Best Fit Sphere Data, BFSD, merit function operand and simple macro programs. In order to generate the first set of aspheres a surface in a Zemax model was set to be an Even Asphere surface type, with a diameter of 8mm. The radius of curvature of the surface and one of the aspheric coefficients, or the conic constant, were set as variables. The BFS radius of curvature calculation returned by the BFSD operand was targeted to a value of 6mm while the sag difference between the BFS and the aspheric surface was targeted to 5 μ m. Next the radius of curvature of the aspheric surface was set to the targeted value of 6mm and the conic constant was set to be a small positive number. The Zemax optimization procedure was then run to solve for the two variables. The procedure was then repeated with the initial conic constant value set to be a small negative number. These steps were then repeated ten times with the target aspheric departure increasing by 5 μ m each time up to a total departure of 50 μ m. This in turn was repeated for targeted BFS radius of curvatures ranging from 6mm to 10mm and from -10mm to -6mm in 0.5mm steps. Finally the entire process was repeated

with each of the eight aspheric coefficients used to generate the aspheric departure. Additionally the spherical surfaces with radii of curvature ranging from -10mm to -6mm and 6mm to 10mm were added to the list of test surfaces. Examples of some of the surface prescriptions are given in TABLE 4.6 and a graph of the distribution of aspheric departure for convex conic surfaces versus the BFS radius of curvature is given in FIGURE 4.22. This graph represents only a small fraction of the surfaces since there is a complimentary distribution for concave conics as well as both concave and convex lists for each of the eight aspheric coefficients.

Radius [mm]	Conic	α_1	α_2	$\alpha_3 \dots \alpha_8$	BFS Radius [mm]	Difference [μm]
5.9318	-0.0893	0	0	0	6.0	5
6.0538	0.0756	0	0	0	6.0	5
5.8635	-0.1787	0	0	0	6.0	10
6.1057	0.1491	0	0	0	6.0	10
-9.872	-0.0193	0	0	0	-7.5	15
6.3431	0	-0.0101	0	0	7.0	15
-7.8419	0	0.0061	0	0	-8.5	5
7.9710	0	0	-9.43E-4	0	10	50

TABLE 4.6 Examples of the aspheric test surface prescriptions for which only one aspheric coefficients or the conic constant was allowed to have a non-zero value.

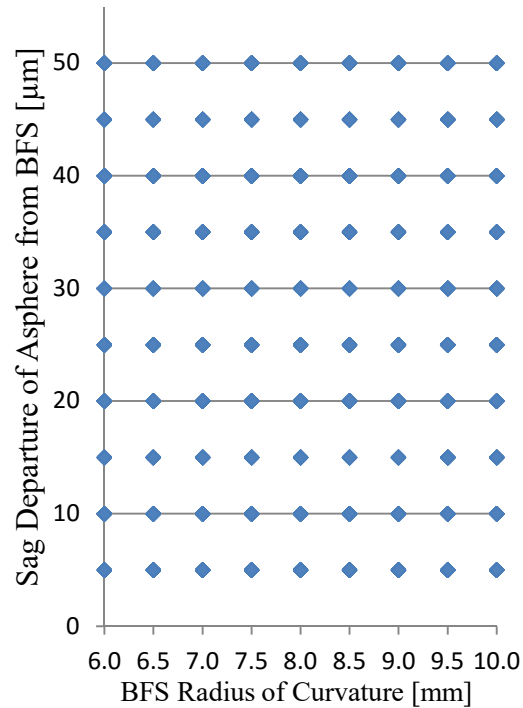


FIGURE 4.22 The distribution of the maximum aspheric departure for convex conic surfaces versus the BFS radius of curvature.

Next the second series of aspheric surfaces were generated using multiple aspheric terms and the conic constant. This was done by randomly assigning values to the radius of curvature and the aspheric terms to create potential test surfaces. The radius of curvature was chosen to be uniformly distributed random number between 5mm and 12mm for convex surfaces or between -12mm and -5mm for concave surfaces. Each of the aspheric terms and the conic constant was then set to a normally distributed random number. The ranges for these coefficients were set to be twice their range found in the previous procedure. The reason for the increase in range was to allow for cases where one term balances the aspheric departure of another. After a random prescription was generated it was loaded into Zemax and the BFS radius of curvature and aspheric departure were calculated using the BFSD operand. If the surface met the criteria of TABLE 4.1 its

prescription was saved to a text file along with the BFS D stats, if not it was discarded.

For half of the surfaces generated the α_1 term was not used. This process tended to skew the departure towards the higher end of the range. Therefore after 10,000 surfaces were generated they were sorted into bins of increasing aspheric departure spaced $5\mu\text{m}$ apart. Then an equal number of surfaces were selected from each bin for a total of 6000 test surfaces. Examples of these aspheric surfaces are shown in TABLE 4.7 and a graph of the aspheric departure versus the BFS radius of curvature is shown in FIGURE 4.23 for the convex surfaces only.

Radius [mm]	Conic	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	BFS Radius [mm]	Departure [μm]
-9.224	1.001	0.00E+0	-9.23E-4	4.17E-5	8.47E-7	-8.39E-8	-1.97E-09	-4.27E-11	3.33E-12	-7.837	35.4
-10.125	2.136	-2.77E-2	-6.18E-4	3.83E-5	8.36E-8	4.49E-9	-5.44E-10	5.95E-12	-1.09E-11	-6.391	8.6
-8.660	-1.441	0.00E+0	-1.65E-4	-3.50E-5	-8.39E-8	4.44E-9	-1.45E-10	-7.36E-12	9.62E-12	-8.122	23.2
5.849	-1.018	-1.82E-2	7.57E-4	-4.41E-6	-8.48E-7	-1.28E-8	-2.57E-09	-3.86E-11	2.86E-12	7.595	42.7
7.245	-2.919	0.00E+0	1.04E-3	-1.95E-5	-1.31E-7	2.96E-8	4.02E-09	2.03E-11	-9.90E-12	7.248	14.1
11.193	-1.496	0.00E+0	1.71E-3	-6.12E-6	-4.57E-7	-5.37E-9	-3.75E-09	-2.94E-11	-9.46E-12	8.459	37.2
6.612	0.323	0.00E+0	1.65E-4	-1.93E-5	-1.02E-6	4.64E-8	-7.20E-10	-6.69E-11	1.09E-11	6.556	9.1
-7.237	-1.927	2.73E-3	-1.70E-4	3.66E-5	-8.93E-7	-9.11E-8	1.10E-09	2.01E-10	-6.68E-12	-8.723	39.5
9.493	-0.016	0.00E+0	-2.23E-4	7.21E-6	1.90E-7	-3.92E-8	2.41E-09	1.10E-10	-2.38E-12	9.733	26.5
-11.290	2.852	-1.61E-2	3.14E-4	-2.88E-6	1.82E-7	-3.31E-8	-2.12E-9	-6.87E-11	1.74E-11	-8.353	4.4
8.549	1.427	-1.41E-3	-2.07E-4	-1.35E-5	1.73E-6	2.67E-8	3.05E-10	-1.50E-10	-5.55E-12	8.187	42.9

TABLE 4.7 Examples of the aspheric test surface prescriptions for which multiple aspheric coefficients and the conic constant were allowed to have non-zero values.

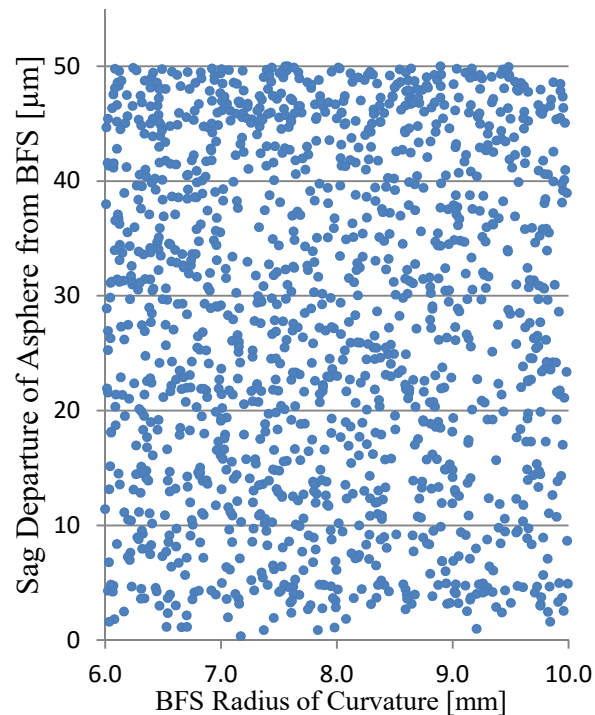


FIGURE 4.23 The distribution of the maximum aspheric departure for convex aspheric surfaces where multiple aspheric coefficients and the conic constant were allowed to vary versus the BFS radius of curvature.

A list of toroidal surfaces was also created where only the radii of curvature in the Y-Z and X-Z planes were varied. However, the Zemax BFSD operand does not work for non-rotationally symmetric surfaces. Therefore the curvature operand, CVVA, was used to calculate the average curvature of the surfaces. The Zemax model used two surfaces: a spherical surface and toroidal surface. The two radii of the toroidal surface were set to be variables, while the radius of the spherical surface was set to be the desired BFS radius of curvature of the toric surface. The departure between the two surfaces was found by calculating the sag of each surface at their edges along both the x and y axis, and subtracting. A macro was used in combination with the Zemax optimization algorithm to vary the two radii of curvature in order to generate toric surfaces with BFS radii of

curvature ranging from 6mm to 10mm and from -10mm to -6mm, in 0.25mm steps, with departures from the BFS ranging from -100 μ m to 100 μ m. Examples of the torodial surfaces are given in TABLE 4.8. A graph of the maximum departure of each convex surface versus the BFS radius of curvature is shown in FIGURE 4.24. A graph of the radii of curvature in the Y-Z plane versus the X-Z of the convex torodial surfaces are shown in FIGURE 4.25.

Radius in the Y-Z Plane [mm]	Radius in the X-Z Plane [mm]	Departure Y [μ m]	Departure X [μ m]	BFS Radius of Curvature [mm]
6.01	5.99	-4.96	5.00	6.00
7.22	6.98	-23.93	25.00	7.10
9.03	7.77	-79.14	95.00	8.40
10.66	9.34	-55.90	65.00	10.00
-6.50	-5.90	86.36	-100.00	-6.20
-7.13	-6.67	50.19	-55.00	-6.90
-7.79	-7.41	32.80	-35.00	-7.60
-10.36	-9.44	40.48	-45.00	-9.90

TABLE 4.8 Examples of the Torodial Test Surface Prescriptions

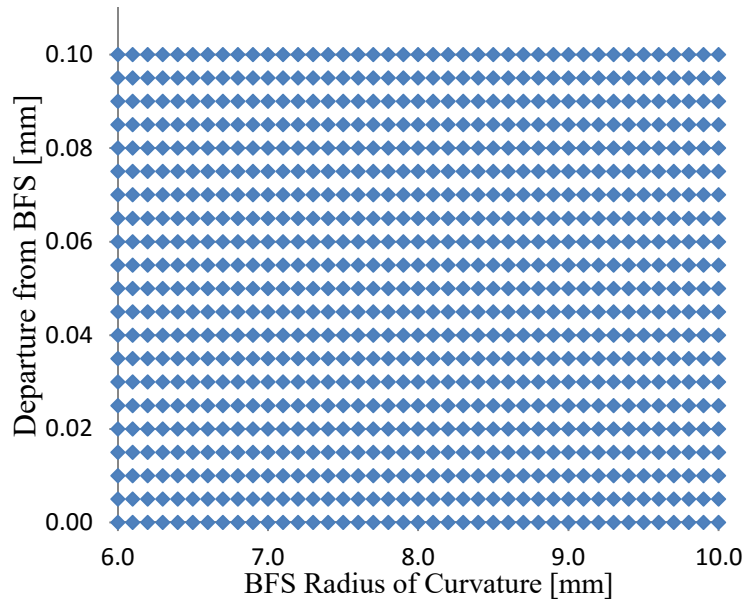


FIGURE 4.24 The maximum departure of each convex surface generated versus the BFS radius of curvature.

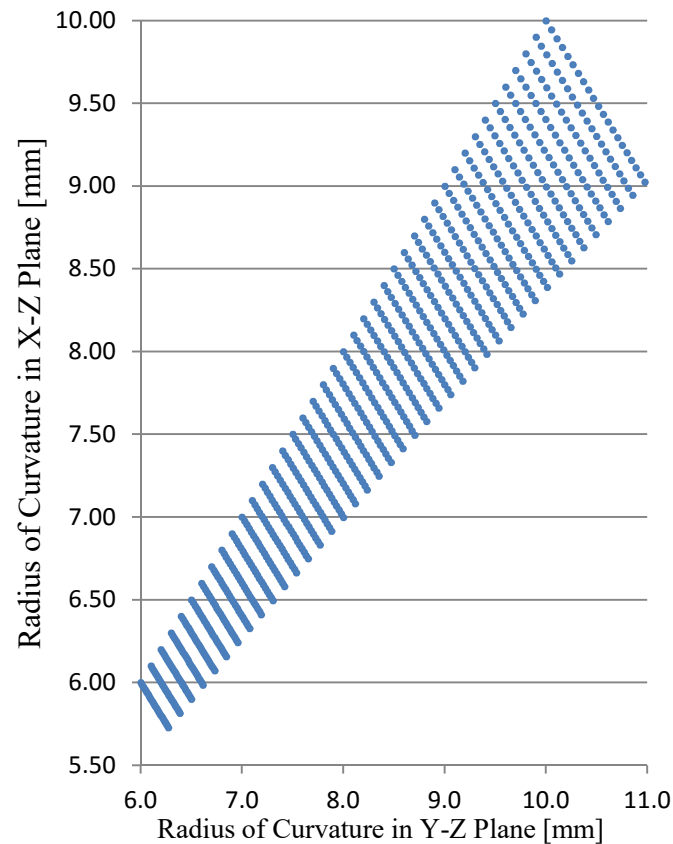


FIGURE 4.25 The radius of curvature in the X-Z plane versus the radius of curvature Y-Z for the convex torodial surfaces generated.

Next the aspheric surfaces were used to compare the performance of the three diverger lens designs. This was accomplished using Zemax models of each diverger lens in double pass. The test surface was placed at a distance from the diverger focus equal to the negative of the BFS radius of curvature. The test surface was set to be the stop of the system and the Zemax Aperture Type was set to float by the stop size. Ray aiming was turned on to ensure that the test entire surface remained illuminated. The distance between the diverger and the insert was set to be a variable and the distance from the light leaving the diverger to the last surface in the model was set using a pupil solve. This ensures that as the test part, or stop, is moved the last surface of the model will

remain at the exit pupil. The exit pupil of this simple system will become the intermediate pupil once the imaging lens is added to the model. The merit function operands used to compare the performance of the diverger lenses were UDO23, UDO43 and ZPL49 which were discussed in more detail in Chapter 3.3. Only two targets were used in the merit function. The wavefront slope at the intermediate pupil was targeted to zero, using UDO23, in order to minimize it. Additionally the UDO43 merit function operand was targeted to a value of over 0.1 in order to try to keep the intermediate pupil from drifting into a caustic region. The ZPL49 operand was placed in the merit function to keep track of the amount of pupil aberration at the intermediate pupil. The Zemax optimization procedure was then run to find the distance between the test part and the diverger which minimized the maximum wavefront slope. This process was repeated for all the rotationally symmetric test surfaces as well as the toroidal test surfaces. The percentage of surfaces that were testable was then calculated. In order to be testable the test wavefront at the intermediate pupil had to meet several criteria. First the wavefront slope needed to be less than 1150 waves/radius, which is the limit of the sparse array detector. Second the wavefront must not be confused, meaning it must not be in a caustic region as explained in Chapter 3.3.3. Third no portion of the wavefront should vignette. The percentage of the rotationally symmetric surfaces that were testable by each diverger is listed in TABLE 4.9. Additionally the average, maximum, and standard deviation of the WFS for all test surfaces are listed.

	Three Element Spherical Diverger	Single Element Aspheric Diverger	Two Element Aspheric Diverger
Percentage of Testable Surfaces	89.7%	79.2%	92.9%
Average MWFS [Waves/Radius]	559.2	733.3	528.4
Maximum MWFS [Waves/Radius]	2931.8	8109.5	2901.6
Standard Deviation MWFS [Waves/Radius]	412.1	683.9	357.1

TABLE 4.9 Results for the Rotationally Symmetric Test Surfaces

Additionally the pupil aberration generated by each aspheric surface, as calculated by ZPL49, was recorded. In order to make a fair comparison between the three divergers, only surfaces that were testable by all three divergers were used to calculate an average, maximum and standard deviation of the generated pupil aberration, as listed in TABLE 4.10.

	Three Element Spherical Diverger	Single Element Aspheric Diverger	Two Element Aspheric Diverger
Average	2.09%	3.36%	2.11%
Maximum	8.78%	17.59%	4.53%
Standard Deviation	1.13%	2.91%	0.80%

TABLE 4.10 Pupil Aberration of the Rotationally Symmetric Test Surfaces

The same comparisons were performed for the toroidal surfaces. In this case all three diverger lenses were able to test all of the generated torodial surfaces. The WFS performance of each diverger lens for the torodial surfaces is shown in TABLE 4.11. The corresponding pupil aberration is shown in TABLE 4.12.

	Three Element Spherical Diverger	Single Element Aspheric Diverger	Two Element Aspheric Diverger
Percentage of Testable Surfaces	100%	100%	100%
Average MWFS [Waves/Radius]	317.6	362.3	311.7
Maximum MWFS [Waves/Radius]	694.0	790.5	675.6
Standard Deviation MWFS [Waves/Radius]	193.6	224.0	188.6

TABLE 4.11 Results for the Toric Test Surfaces

	Three Element Spherical Diverger	Single Element Aspheric Diverger	Two Element Aspheric Diverger
Average	2.41%	3.63%	2.73%
Peak to Valley	4.13%	12.59%	4.56%
Standard Deviation	0.81%	3.67%	1.00%

TABLE 4.12 Pupil Aberration of the Toric Test Surfaces

From these tables it is clear that, for the surfaces tested, the three element spherical design and the two element aspheric design are superior then the single element aspheric design. The single element aspheric diverger was able to test over 10% fewer of the rotationally symmetric test surfaces, while the average MWFS and Maximum MWFS were significantly higher, TABLE 4.9. Additionally the pupil aberration generated by the single element diverger when testing both the rotationally symmetric aspheric surfaces and the toric surfaces was significantly higher than the other two divergers. The results for the toric surfaces do not show much difference between the performance of the two element aspheric diverger to the three element spherical diverger. The results for the number of testable toric surfaces and the associated MWFS statistics, TABLE 4.11 are very similar, as are the pupil aberration statistics for toric surfaces, TABLE 4.12. It is clear that the specifications given in TABLE 4.1 for the range of toric surfaces required

to be tested by the sub-Nyquist interferometer is within the measurement range of these divergers as all toric surfaces generated were testable. Finally the performance of the two element aspheric diverger is equal to or slightly better than the three element diverger when testing the rotationally symmetric test surfaces, TABLE 4.9 and TABLE 4.10.

4.6 Imaging Lens

As discussed in Chapter 3.2.4 the responsibility of the imaging lens in an interferometer is to make the test surfaces and detector conjugate planes. Since the imaging lens is the furthest optical element from the test part it is the element at which rays are most likely to vignette. In Chapter 4.6.1, the relationship between the focal length of the imaging lens and the diameter in order to avoid vignetting will be discussed. In Chapter 4.6.2 the effects of aberrations in the imaging lens will be discussed. Finally in Chapter 4.6.3, several different imaging lenses will be compared. Additionally it is important to remember that while many interferometers make use of ground glass or diffuse screens in order to convert the coherent imaging of the two wavefronts to incoherent imaging of the interferogram this has to be avoided due to the problems discovered by Palum and Greivenkamp (1990), and discussed in Chapter 2.4.2.

4.6.1 Paraxial Imaging Lens Design

Gaussian imaging equations were used to find the first order paraxial design of the imaging lens, in which the effects of induced mapping and phase errors are ignored. The

diameter of the imaging lens, as a function of its focal length, required to allow for the imaging of test wavefront slopes corresponding to the maximum fringe frequency resolvable by the sparse array sensor will be calculated. However, rather than considering the imaging of the test surfaces onto the detector, which would require including the diverger optics, the problem was simplified to consider only the imaging of the intermediate pupil onto the detector. As discussed in Chapter 3.1, in a conventional imaging system the location and size of the stop determines the angular distribution of rays from each point on the object that makes it through the imaging system onto the image plane. In an interferometer there is only one ray associated with each point on the test surface or in the test arm's intermediate pupil. However a ray bundle can be used to represent the angular range the test ray can have at a certain point in the intermediate pupil. The maximum fringe frequency supported by the detector can be used to define the allowed angular spread of the test rays about the reference ray. This is shown in FIGURE 4.26, where the interferometer's intermediate pupil serves as the object plane in the model and the detector is placed at the conjugate image plane. The definitions of the variables used in the Gaussian imaging equations for a paraxial thin lens are also shown in FIGURE 4.26, (Greivenkamp 2004)

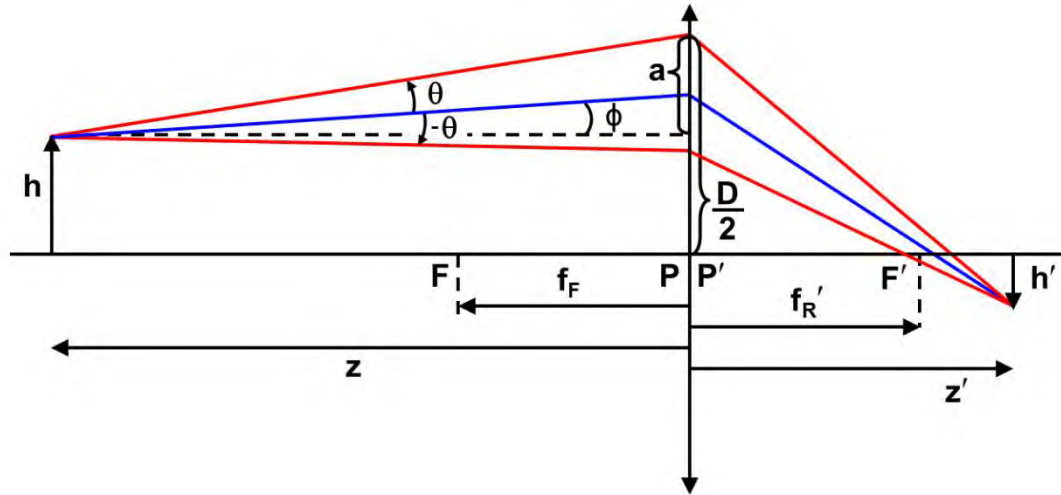


FIGURE 4.26 The paraxial imaging of intermediate pupil onto the detector, where the blue ray represents a generic reference ray and the red rays represent the possible angular spread of the test rays bound by the fringe frequency limits of the detector.

In the Gaussian image equations the distances, f_R' , h and z' are positive, and f_F , h' and z are negative. In this model of the interferometer imaging optics, the object and image heights, h and h' , are the semi-diameters of the intermediate pupil and detector, respectively. Since the object and image plane are in air the front and back focal lengths are equal, Equation 4.34.

$$f_E = -f_F = f_R' \quad 4.34$$

Additionally from the Gaussian imaging equations, the transverse magnification is defined as the ratio of the image height to the object height, Equation 4.35, and the object and image distances are related to the focal length of the lens and the transverse magnification by Equations 4.36 and 4.37. (Greivenkamp 2004)

$$m \equiv \frac{h'}{h} \quad 4.35$$

$$z = \left(\frac{1-m}{m} \right) f_E = - \left(1 - \frac{h}{h'} \right) f_E \quad 4.36$$

$$z' = (1-m) f_E \quad 4.37$$

The largest possible test wavefront footprint on the imaging lens would be generated by a ray leaving the edge of the intermediate pupil at a positive angle, θ , which corresponds to the maximum fringe frequency supported by the detector. This is shown in FIGURE 4.26 as the ray leaving the top of the object and angled away from the optical axis. The relationship between the fringe frequency and the angle of the two interfering rays was derived in Chapter 4.2.1 and given in Equation 4.19. It can be rewritten in terms of the test and reference ray angles of the imaging system, θ and ϕ with respect to the optical axis, Equation 4.38. In this equation ξ represents the fringe frequency in cycles/mm at the intermediate pupil.

$$\xi = \frac{(\sin \theta - \sin \phi)}{\lambda} \quad 4.38$$

As previously discussed using units of waves/radius for fringe frequency or wavefront slope is convenient since the fringe frequency at the intermediate pupil and the detector will not be the same in units of cycles/mm but will be the same in units of waves/radius. In order to scale the frequency or wavefront slope difference into units of waves/radius at a given position in space simply multiply the frequency in cycles/mm by the semi-diameter of the wavefront. Equation 4.39 gives the wavefront slope difference or fringe frequency, η , at the intermediate pupil and at the detector plane or exit pupil.

$$\eta = \xi_{IntPupil} h = \xi_{Det} h' \quad 4.39$$

The diameter of the imaging lens required to capture all test rays will depends on the maximum initial height of the ray, the angle at which it leaves the object and the distance to the lens, Equations 4.40-4.42.

$$\frac{D}{2} = h + a \quad 4.40$$

$$a = -z \tan(\theta + \phi) \quad 4.41$$

$$D = 2h - 2z \tan(\theta + \phi) \quad 4.42$$

In the case of the sub-Nyquist interferometer designed for this research a flat reference wavefront is used, thus, the reference angle, ϕ , is always zero. The relationship between the diameter and focal length of the imaging lens can be found by substituting Equation 4.36 into Equation 4.42.

$$D = 2h + 2 \left(1 - \frac{h}{h'} \right) f_E \tan(\theta) \quad 4.43$$

The angle of the test ray at the intermediate pupil which corresponds to the maximum fringe frequency supported by the detector, in waves/radius, can be calculated by solving Equations 4.38 and 4.39.

$$\theta = \arcsin \left(\frac{\eta \lambda}{h} \right) \quad 4.44$$

Thus the diameter of the lens required to completely avoid vignetting is given by Equation 4.45.

$$D = 2h + 2 \left(1 - \frac{h}{h'} \right) f_E \tan \left[\arcsin \left(\frac{\eta \lambda}{h} \right) \right] \quad 4.45$$

The maximum fringe frequency at the detector was found in Chapter 4.2.2 to be 300cycles/mm or 1150 waves/radius. The half width of the sub-Nyquist detector, h' , is 3.8325mm. The semi-diameter of the intermediate pupil is slightly more subjective since it depends on the aspheric surface being tested and the diverger lens used. However from the simulations performed in Chapter 4.5.6 the largest intermediate pupil semi-diameter encountered for the two element aspheric diverger lens was 21.8mm. Therefore for this analysis an intermediate pupil diameter of 44mm was used. TABLE 4.13 shows, for a range of focal lengths, the diameter and F/# of the lens required to prevent vignetting of any ray which corresponds to a wavefront slope less than or equal to the frequency limit of the detector. Additionally, the Gaussian object and image distances are shown along with the total length between the object and the image planes.

f_E [mm]	D [mm]	F/#	z [mm]	z' [mm]	Total Length [mm]
50	62.8	0.797	-337	59	396
100	81.5	1.227	-674	117	791
150	100.2	1.496	-1011	176	1187
200	119.0	1.681	-1348	235	1583
250	137.7	1.815	-1685	294	1979
300	156.5	1.917	-2022	352	2374
350	175.2	1.997	-2359	411	2770

TABLE 4.13 Paraxial imaging lens properties required to avoid vignetting and allow the imaging of fringes, up to the frequency limit of the detector, originating anywhere in the intermediate pupil.

It is important to note that the diameter of the lens, determined by Equation 4.45, for a given focal length, is not required to image every ray traveling at the angle corresponding to the wavefront slope limit of the detector. Rather, it is the largest possible diameter that is required so that all rays traveling at the wavefront slope limit can be imaged. Therefore it is worthwhile to examine the diameter which first allows a ray traveling at the slope limit to be imaged. This case would correspond to a ray leaving the edge of the intermediate pupil traveling at the slope limit towards the optical axis, as shown by the lower test ray in FIGURE 4.26. Then, assuming that at the intermediate pupil the slope of the rays continually decrease for rays closer to the optical axis, the diameter of the test wavefront at the lens would then be equal to Equation 4.46.

$$D = \left| 2h - 2 \left(1 - \frac{h}{h'} \right) f_E \tan \left[\arcsin \left(\frac{\eta \lambda}{h} \right) \right] \right| \quad 4.46$$

The absolute value is needed since it is possible that the test ray crosses the optical axis such that the ray height at the lens becomes negative. However, this is just the diameter of the test wavefront and the imaging lens would still have to pass the reference wavefront. Therefore, the diameter of the imaging lens must be at least, $2h$, the diameter of reference wavefront as shown in TABLE 4.14. If the diameter of the imaging lens, for a given focal length, is in between the values of TABLE 4.13 and TABLE 4.14 then some wavefronts with slopes corresponding to the detector limit will be able to be imaged and some will not.

f_E [mm]	D [mm]	F/#
50	44.0	1.136
100	44.0	2.273
150	44.0	3.409
200	44.0	4.545
250	49.7	5.026
300	68.5	4.381
350	87.2	3.774

TABLE 4.14 Paraxial imaging lens properties required to begin allowing for the imaging of fringes, corresponding to the frequency limit of the detector, in special cases.

The choice of the focal length of the imaging lens is based largely on the imaging distances. The upper limit on the total imaging length was determined by the space available on the optical table upon which the sub-Nyquist interferometer was built. In order to fit on the table the total length required for the imaging needed to be less than 1.75m. The addition of fold mirrors into the imaging arm of the interferometer was avoided since the consequences of the additional optical surfaces on the reverse ray tracing and reverse optimization procedures must be considered.

4.6.2 Imaging Lens Induced Errors

While the paraxial case serves as a good starting point for the lens design the aberrations of the imaging lens must be considered. As was the case with rest of the interferometer components there are two competing design strategies of the imaging lens. Either the number of elements can be reduced to keep imaging lens simple for the reverse optimization and ray tracing models, or a more complicated lens design can be used to

reduce the induced aberrations. The sub-Nyquist interferometer was designed with the emphasis placed on the former. However it is still beneficial to be able to calculate the range of induced mapping and phase errors generated by the imaging lens in the interferometer. Additionally, a future interferometer could be designed to make use of the latter approach of minimizing the induced errors.

Two different approaches were used to compare the induced errors introduced by potential imaging lenses. The first approach uses a model in which the rays are setup to match the range of possible test rays in the interferometer, similar to the paraxial design, discussed in Chapter 4.6.1. The induced errors are calculated for all the rays traced providing a fast way to compare the performance of multiple imaging lenses as well as a method of optimizing an imaging lens design to minimize induced errors. However this method does not provide much insight into the induced errors experienced by a given wavefront. This leads to the second approach in which the induced errors experienced by a range of potential wavefronts, or test surfaces, are calculated on a wavefront by wavefront basis using the programs discussed in Chapter 3.3. This approach allows the induced errors introduced into the test of a given test surface or wavefront to be predicted. Additionally, given a list of test surfaces, this approach allows for statistics on the induced errors and the percentage of the test surfaces that could be properly imaged by the proposed imaging lens to be calculated. A properly imaged test surface or wavefront is one that is imaged onto the detector without vignetting any rays,

where the fringe frequency of the interferogram is less than the frequency limit of the detector, and where the image plane is not located in a caustic region.

In the first approach, as in the paraxial case, the analysis will be simplified by only considering the imaging of the interferometer's intermediate pupil onto the detector. While the diverger will introduce its own errors into the imaging of the test part or aperture stop onto the intermediate pupil, these errors will already be present in the intermediate pupil regardless of the chosen imaging lens design. If the interferometer did not contain a diverger element then the following analysis would hold for the mapping of the aperture stop onto the detector. The induced errors of the imaging lens are calculated by modeling the pupil imaging optics of the interferometer as a conventional imaging system. The intermediate pupil of the interferometer now serves as the object plane of the model and the exit pupil of the interferometer is represented by the image plane of the model, FIGURE 4.27. The rays traced in the model are setup to match the range of test rays that could be generated in the non-null interferometer, as described in the paraxial case. The angular distribution on the rays leaving the object plane of the model can be set by changing the Zemax aperture type to "Object Cone Angle" and selecting the option for the model to have a "Telecentric Object Space". With the "Telecentric Object Space" option selected Zemax assumes the entrance pupil of the model is located at infinity regardless of the aperture stop location. The Zemax aperture value is then be set to equal the maximum expected test ray angle calculated from the frequency limit of the sparse array sensor, Equation 4.44. The size of the intermediate pupil of the interferometer is

then set by changing the Zemax field data mode to “Object Height”. The maximum object height is then set to the predicted maximum radius of the interferometer’s intermediate pupil. An example of this type of layout is shown in FIGURE 4.27. This method exploits the fact that in this non-null interferometer design all reference rays travel parallel to the optical axis in the intermediate pupil. In the model the chief ray for each object point also travels parallel to the optical axis. Therefore referencing the spot diagram and other Zemax calculations to the chief ray in the model is the same as comparing the test rays against the interferometer’s reference ray which originates from the same pupil location. If the reference wavefront was not a plane wavefront then a method of tilting the center ray of the cones to match the reference wavefront would have to be found.

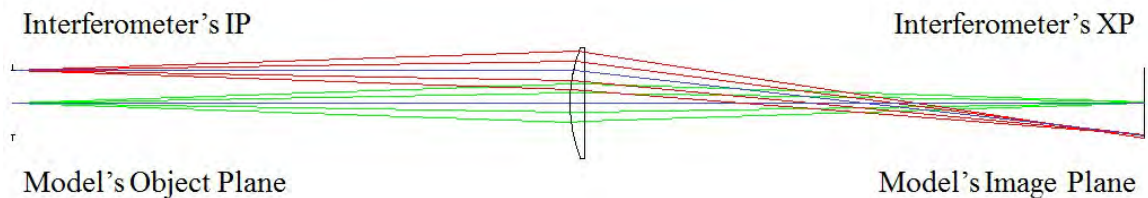


FIGURE 4.27 The intermediate pupil is imaged onto the detector by the interferometer’s imaging lens. In this example the magnification is -1 in order to make the rays visible. The reference rays are shown in blue.

One effect pupil aberration has on the interferometer imaging is that the magnification of the test wavefronts is dependent on the aberrations of the imaging lens and the slope of the wavefront at the edge of the aperture stop or the intermediate pupil. The relationship between the height of a test ray at the stop and its height at the exit pupil was derived by Murphy et al (2000a) and was discussed in Chapter 3.2.5 and given in Equations 3.4 and

3.5. However it can be seen graphically in FIGURE 4.28 which is an expanded view of the range of possible test rays at the edge of the detector plane in FIGURE 4.27.



FIGURE 4.28 The spread of the possible test rays at the edge of the exit pupil which all originated from the same point on the edge of the intermediate pupil. In a given interferogram there is only one test ray present for any point in the pupil. Therefore the size of the test wavefront at the exit pupil depends on the slope of the test ray in the intermediate pupil and the transverse ray error of the imaging lens.

This change in magnification means that the imaging distances may need to be adjusted based on the slope of the test wavefront at the edge of the intermediate pupil in order to make the image of the test part exactly fill the detector. This also makes it difficult to compare the performance of multiple imaging lenses to each other since the imaging distance shifts required to image all test rays with correct magnification will vary depending on the amount of aberration present. Therefore in the comparison of imaging lenses presented in Chapter 4.6.3, the imaging distances were set such that the reference ray from the edge of the intermediate pupil is imaged to the edge of the detector. Additionally the paraxial calculation of the required $F/\#$ of the imaging lens needed to avoid vignetting, Equation 4.45, is no longer accurate. It would have to be modified to incorporate the dependence of h' on h and the transverse ray aberration given in Equations 3.4.

An additional problem arises from this magnification change when testing non-rotationally symmetric wavefronts. In a non-rotationally symmetric wavefront, such as a torodial wavefront, the angle of the test rays with respect to the optical axis around the edge of the wavefront varies as a function of the polar angle. This means that in the presence of pupil aberration a circular stop or intermediate pupil may be mapped to a non-circular exit pupil. An example of this is shown in FIGURE 4.29 (Left) where a torodial test surface, with a circular aperture serving as the aperture stop of the interferometer, is distorted in the Zemax model by the pupil aberrations of both the diverger and the imaging lens to form an elongated exit pupil at the detector. FIGURE 4.29(Right) shows the same phenomenon in the interferometer where a torodial test surface is imaged utilizing a plano-convex imaging lens.

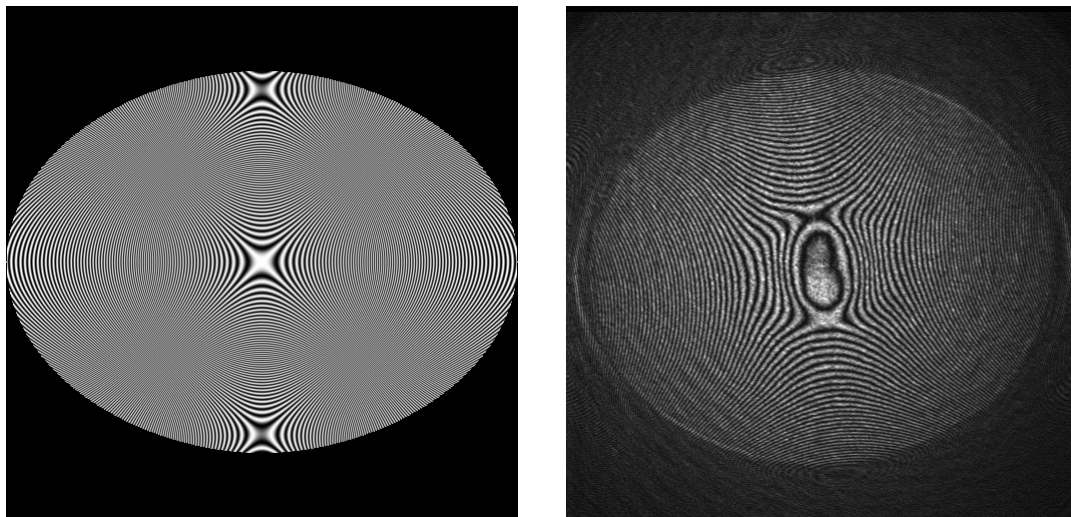


FIGURE 4.29 Interferograms, modeled (Left) and real (Right), where the induced mapping errors of the interferometer distort the circular stop into an elongated exit pupil when testing a torodial surface.

Now consider the phase errors generated in a non-null interferometer. The phase difference in an interferometer is the result of the difference in the optical path lengths of

the test and reference rays. Therefore the terms phase error and OPD error can be used interchangeably where the conversion between phase and OPD is given in Equation 1.5. A method of describing the phase error in a non-null interferometer through the use of aberration theory was demonstrated by Murphy et al (2000a, 2000b). In their derivation the phase error was first defined in terms of the OPL function Ψ , as a function of the test and reference rays pupil coordinates, ρ , and field coordinates, h , Equation 4.47.

$$\Delta\Psi = \Psi_{oi}(\rho_{\text{test}}, h) - \Psi_{oi}(\rho_{\text{ref}}, h) \quad 4.47$$

However as a result of mapping errors the test and reference rays which interfere in a non-null interferometer do not have to originate from the same the field point. Murphy et al (2000a, 2000b) show that the pupil coordinate of the test ray depends on the field coordinate of the test ray and that the pupil coordinate of the reference ray is always zero if the reference wavefront is free of aberrations, Equation 4.48.

$$\Delta\Psi(h_{\text{test}}) = \Psi_{oi}[\rho_{\text{test}}(h_{\text{test}}), h_{\text{test}}] - \Psi_{oi}(0, h_{\text{ref}}) \quad 4.48$$

In order to determine the field coordinate of the reference ray, h_{ref} , which interferes with the test ray, originating from h_{test} , the inverse of the mapping error function given in Equations 3.5 would have to be found, $h_{\text{ref}}(h'_{\text{ref}})$ as well as the conversion between h'_{test} and h'_{ref} . Additionally they discuss the difficulty in separating the phase errors generated by the mapping errors from the nominal phase errors. Finally Murphy et al derive the phase error in terms of the third-order wavefront coefficients and conclude with the following statement. “For lesser slope departures, third-order, aberration theory proves

extremely accurate. For larger departures, it is still a valuable evaluation tool, but real rays should be traced for high accuracy.” (Murphy et al, 2000a)

Therefore in selecting the imaging lens to be used with the non-null interferometer ray tracing was the desired method to calculate the phase errors. Additionally rather than calculating the total phase error of the interferometer a method of calculating the induced phase error of the imaging lens is desired. The induced phase error of the imaging lens can be defined as the difference between the relative phase of the test and reference wavefronts at the interferometer’s exit pupil and their relative phase at the interferometer’s intermediate pupil. The induced phase error can also be described as an induced OPD error, OPD_E , by Equation 4.49.

$$OPD_E = OPD_{XP} - OPD_{IP} \quad 4.49$$

Ideally a built-in feature would exist in Zemax that could be used to calculate the range of induced OPD errors in the same manner that the transverse ray aberration calculations were used to describe range of the induced mapping errors. Since the chief rays in the model of the imaging lens are also the reference rays of the interferometer it is tempting to use the Zemax OPD_Z plots to represent the range of induced OPD errors. In the absence of mapping errors this could be accomplished by setting the Zemax Reference $OPD_{[Z]}$ Mode to Exit Pupil, so that the OPL of the all the test rays originating from a given field point would be compared against the same chief ray which is also their corresponding reference ray. However, as discussed previously, in the presence of mapping errors test and reference rays which interfere at the exit pupil of the

interferometer do not necessarily originate from the same point in the interferometer's intermediate pupil. This means that all test rays, which originate at the same field point, may interfere with unique reference rays in the exit pupil and therefore a single chief ray does not exist which could be used in the OPD_Z calculation. Thus a different method of calculating OPD_E must be found. Since the normalized pupil and field coordinates for the test and reference wavefronts are not the same it is more straightforward to calculate the induced OPD error by using non-normalized Cartesian coordinates at the two conjugate planes, Equation 4.50. These coordinates are in units of length and correspond to the same physical location in both the test and reference arms of the interferometer. Additionally the test arm of the interferometer is used to determine the mapping of points in the intermediate pupil, represented by the coordinate (x, y) , to their corresponding points in the exit pupil, represented by the coordinate (x', y') .

$$OPD_E = [OPL_{Test}(x', y') - OPL_{Ref}(x', y')] - [OPL_{Test}(x, y) - OPL_{Ref}(x, y)] \quad 4.50$$

In order to calculate the induced OPD error a Zemax macro was written, OPDE.zpl. As stated in Equation 4.50 the OPL of each ray needs to be determined. In previous calculations the OPD_Z , which is stored by Zemax for all rays traced, was used as substitute for the OPL. However this only works because in those models the chief rays of both the test and reference arms were identical. In the model described above, and shown in FIGURE 4.27, all of the rays traveling parallel to the optical axis serve as chief rays for their respective field coordinate and therefore their OPD_Z equals zero. Thus in order to use the OPD_Z calculated by Zemax the OPL of the chief ray associated with each

ray traced must be taken into consideration. Therefore it is easier to simply calculate the OPL of each ray traced. The OPL of a ray traced by Zemax is only available if rays are traced one at a time and even then it is only available in the Zemax programming language, not the Zemax extensions. This leads to an increase in the time it takes to calculate a ray's OPL when compared to the time needed to calculate a ray's OPD_z . However in order to generate induced OPD error fans only a few hundred rays need to be traced and therefore the increased time associated with tracing them one at a time is negligible. The macro, OPDE.zpl, works by tracing tangential and sagittal ray fans, over the specified object cone angle, from a user specified point on the object surface of the model. These rays represent the range of possible test rays that could be present in the non-null interferometer's intermediate pupil. The macro keeps track of the real coordinates of the test rays and their OPL at both the intermediate pupil and the exit pupil. Next the OPL of the reference ray originating at the same point in the object plane is recorded. Since this non-null interferometer makes use of a plane reference wavefront at the intermediate pupil, this OPL is always zero. Then the ray aiming procedure, described in Chapter 3.3.1, is used to iteratively trace reference rays from different field coordinates in the model's object plane until the reference ray that intersects the test ray at the exit pupil is found. The OPL of this reference ray along with the previously recorded OPLs is used to calculate the induced OPD error by Equation 4.50.

The induced mapping errors cause one additional problem with implementing the OPD_E calculation in Zemax. Since the intermediate pupil of the test and reference arms are not necessarily the same size, the reference rays that interfere with the test rays at the edge of

the exit pupil might originate from a larger intermediate pupil than is defined in the model. Since Zemax will not trace rays with normalized field or pupil coordinates greater than one, the diameter of the object surface in the model is temporarily increased by the macro. Additionally the user is required to specify the coordinate at which the test rays should originate in real coordinates rather than normalized coordinates. Examples of the OPD_E fans for two lenses are shown in FIGURE 4.30 and FIGURE 4.31 along with the corresponding OPD_Z fans. The calculations were performed for tangential and sagittal rays originating from the center of the intermediate pupil and at a radial height of 18mm. In this example rays were traced over an object cone angle of $\pm 1^\circ$. The first lens, FIGURE 4.30, is a plano-convex lens and the second, FIGURE 4.31, is a well corrected multiple element lens.

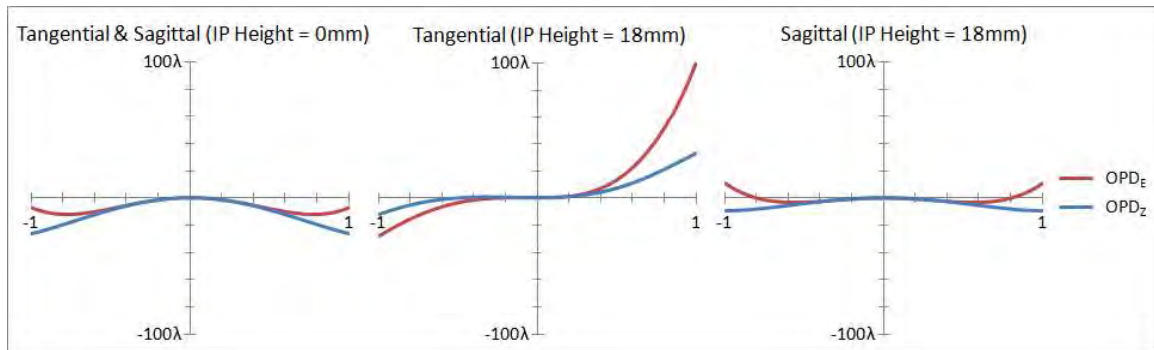


FIGURE 4.30 The induced OPD_E in the interferometers imaging optics, using a 200m plano-convex lens, from the center and the edge of an 18mm intermediate pupil (Red), along with the OPD_Z of the test arm (Blue).

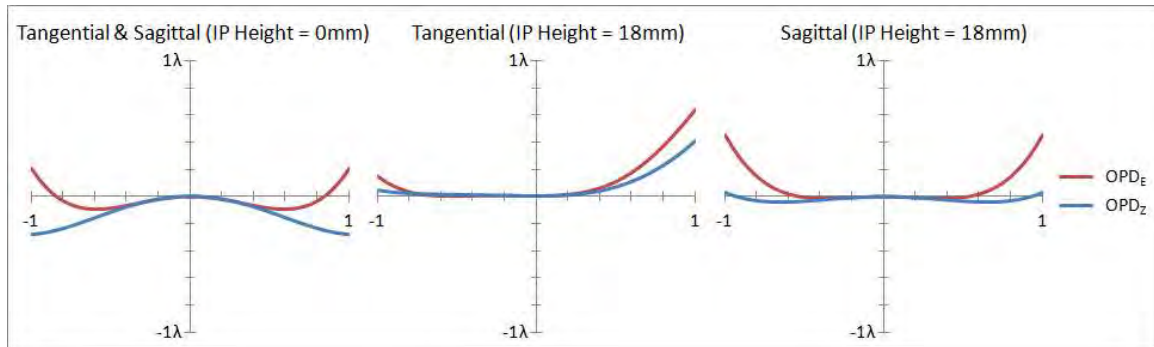


FIGURE 4.31 The induced OPD_E in the interferometers imaging optics, using a 200mm the three element lens, from the center and the edge of an 18mm intermediate pupil (Red), along with the OPD_Z of the test arm (Blue).

This technique allows induced errors to be calculated over the range of possible test rays. However when testing any given surface or wavefront only a small fraction of the rays modeled in these test will be present. The OPD_E fans shows a the range of induced OPD error experience by all the possible test rays from a given point in the intermediate pupil, but in the interferometric test of an aspheric surface only one test ray will exist from the same point in the intermediate pupil. Therefore it is also beneficial to look at the imaging lens performance when testing a range of aspheric wavefronts. This allows the induced aberrations generated in a single interferometric test setup to be calculated as they would when performing a test on a single aspheric surface. This was accomplished by evaluating the ability of several imaging lenses to properly image the rotationally symmetric and torodial surfaces generated in Chapter 4.5.6. The performance of the imaging lenses could be compared by calculating the percentage of test surfaces that each lens could image properly. Additionally the induced mapping and OPD errors generated by each imaging lens for each test surface can be calculated and compared. This was accomplished by adding the prescription of the imaging lens to the Zemax model of the

test surface and diverger lens which was described in Chapter 4.5.6. Then the distance between the intermediate pupil and the imaging lens was set to be the only variable in the model. The distance between the diverger and intermediate pupil as well as the distance between the imaging lens and the last surface of the model were set using the Zemax pupil solve. Since the test surface is the aperture stop in the model, the pupil solves ensure that the intermediate pupil is at the image of the test surface through the diverger and that the detector is located at the exit pupil. Additionally a second configuration, containing only a plane wave at the intermediate pupil and the imaging lens, was added to the model to represent the reference wavefront. The diameter of the test wavefront at the exit pupil was targeted to be the width of the detector using the Zemax merit function. Then the prescription of one of test surfaces was loaded into the model at its optimal distance from the diverger; which was found in the procedure described in Chapter 4.5.6. Then the Zemax optimization procedure was run to find the intermediate pupil to imaging lens spacing that produced the desired exit pupil diameter. Several of the macros discussed in Chapter 3.3 were then used to determine if the test surface had been imaged properly and to calculate the induced errors. First UDO23 was used to calculate the MWSD at the detector. In order for the detector to be able to resolve the interference fringes the MWSD at the detector needed to be less than 1150 waves/radius or 300 cycles/mm. The macro UDO23 was also used to calculate the change in the MWSD between the intermediate pupil and the exit pupil, Equation 4.51. This is simply a measure of the change in the maximum fringe frequency between the two conjugate pupils due the induced errors of the imaging lens.

$$\Delta MWSD = MWSD_{XP} - MWSD_{IP} \quad 4.51$$

Additionally UDO23 was also used to calculate the size of the test wavefront at the exit pupil in order to ensure that it was less than the width of the detector. The macro UDO43 was used to calculate if the detector plane was located in a caustic region or if any rays vignetted. If the test surface was imaged to the proper size, the fringe frequency was less than the limit of the detector, the exit pupil was not located in a caustic region and no rays vignette, then the test surface was considered to be testable by the combination of the diverger and imaging lens. The macro ZPL49 was then used to calculate the magnitude of the induced pupil aberrations, ε_{Mag} , experienced by the test wavefront as discussed in Chapter 3.3.2. Finally the induced OPD error, OPD_E , was calculated using a modified version of the macro OPDE.zpl discussed earlier in this chapter. Rather than tracing tangential and sagittal ray fans, the modified macro, ZPL55, traces a distribution of rays across the test wavefront. The rays are defined in the same method used by ZPL23, which was discussed in Chapter 3.3.1. The macro returns the minimum, maximum, peak to valley and average OPD_E experienced by the rays traced. The output of all of these macros was recorded to a file and the process was repeated for all the rotationally symmetric and torodial surfaces generated in Chapter 4.5.6. The results of this simulation will be discussed in the next section.

4.6.3 Comparing Imaging Lens Designs

A few different imaging lens options were considered for use in the non-null interferometer. The main goal was to find a simple commercially available lens which could be characterized for the reverse ray tracing procedure. However for comparison purposes more complicated lens designs were also analyzed, and will be discussed in this chapter.

Even though the imaging lens $F/\#$ requirement given by Equation 4.45, does not include the effects of induced errors it still serves as a good starting point for the basis of a lens design. The original design goal for the imaging lens was to find a plano-convex lens which satisfied the $F/\#$ requirement of Equation 4.45. That way any test surface which produced an intermediate pupil with a semi diameter less than 21.8mm and interference fringe frequencies less than the frequency limit of the detector could be tested. A singlet imaging lens would have twelve properties that need to be characterized for the reverse ray tracing procedure. These properties are the same as those discussed for the singlet diverger lens. However, in the case of a plano-conex lens, if the surface error of the plano surface is small enough that it can be ignored in the reverse optimization model, then the surface decenters will have no impact on the model and can also be ignored. This leads to the number of lens properties requiring characterization to be reduced to nine. Targeting an imaging lens diameter of approximately 75mm would be required to be less than 82.7mm from Equation 4.45. The closest commercially available lens that could be

found was a plano-convex lens, Newport® KPX223, which has a focal length of 100mm and a diameter of 76mm. The lens prescription is listed in TABLE 4.15.

Surf:Type		Radius	Thickness	Glass		Semi-Diameter	
1	Standard	51.68000	19.76300	BK7		38.10000	U
2	Standard	Infinity	0.00000			38.10000	U

TABLE 4.15 Plano-Convex Lens Prescription (F = 100mm)

However in trying to use this lens it quickly became apparent that the induced errors drastically limited the range of wavefronts or test surfaces that could be tested. When paired with the two element aspheric diverger only 29.2% of the rotationally symmetric aspheric test surfaces, generated in Chapter 4.5.6, could be properly imaged onto the detector.

In order to try to increase the number of surfaces that could be tested, without increasing the complexity of the lens, the focal length of the imaging lens was doubled. In the absence of induced errors the longer focal length would allow some rays to vignette at the imaging lens aperture thus decreasing the range of testable wavefronts. However if a large number of the wavefronts cannot be properly imaged due to the induced errors generated by the imaging lens; then decreasing the range of wavefronts that can be imaged without vignetting in order to improve the induced errors experienced by the remaining wavefronts may in fact lead to a net gain in the number of wavefronts that can be properly imaged. The prescription of this 200mm focal length lens, Newport® KPX223, is given in TABLE 4.16.

Surf:Type		Radius	Thickness	Glass	Semi-Diameter	
1	Standard	103.36000	10.27000	BK7	38.10000	U
2	Standard	Infinity	0.00000		38.10000	U

TABLE 4.16 Plano-Convex Lens Prescription (F = 200mm)

The percentage of the rotationally symmetric test surfaces which could be tested when combined with the two element aspheric diverger rose to 40.2%. The reduction in the induced errors of the longer focal length lens can also be seen in the spot diagrams and OPD_E fans of the two lenses when set up to model the range of expected test rays of the interferometer as discussed in Chapter 4.5.6. The rays traced through both models should be identical at the object planes of the models in order for a fair comparison of the induced errors generated by the two lenses. In the paraxial calculations the largest intermediate pupil semi-diameter generated by the random test surfaces, 21.8mm, was used to calculate the imaging lens F/#. However since neither of these lenses meet that requirement the range of test rays used for the comparison can be reduced. On average the semi-diameter of the intermediate pupil generated by the random surfaces is much smaller at only 14.9mm. Additionally, 85% of the test surfaces generated an intermediate pupil with a semi-diameter of 18mm or less. The maximum angle of the test rays, constrained by the 1150waves/radius frequency limit of the detector, for an intermediate pupil with an 18mm semi-diameter is 1.95°, from Equation 4.44. Therefore the optical models used to generate the spot diagrams and OPD_E fans presented in this chapter utilize an 18mm maximum object plane height and an object cone angle of $\pm 1.95^\circ$. The spot diagrams for the two plano-convex lenses show a clear reduction in the induced mapping error with the longer focal length lens, FIGURE 4.32 and FIGURE 4.33. The induced

OPD errors for the two lenses are shown to be comparable in FIGURE 4.34 and FIGURE 4.35. Rays from the center of the intermediate pupil experience less OPD_E from the 200mm focal length lens while rays near the edge of the pupil traveling at larger angles with respect to the optical axis experience less OPD_E with the 100mm lens. Additionally the 200mm lens is not able to image all of the specified test rays without vignetting. This shows up in FIGURE 4.35 as missing data for higher angle rays near the edge of the pupil.

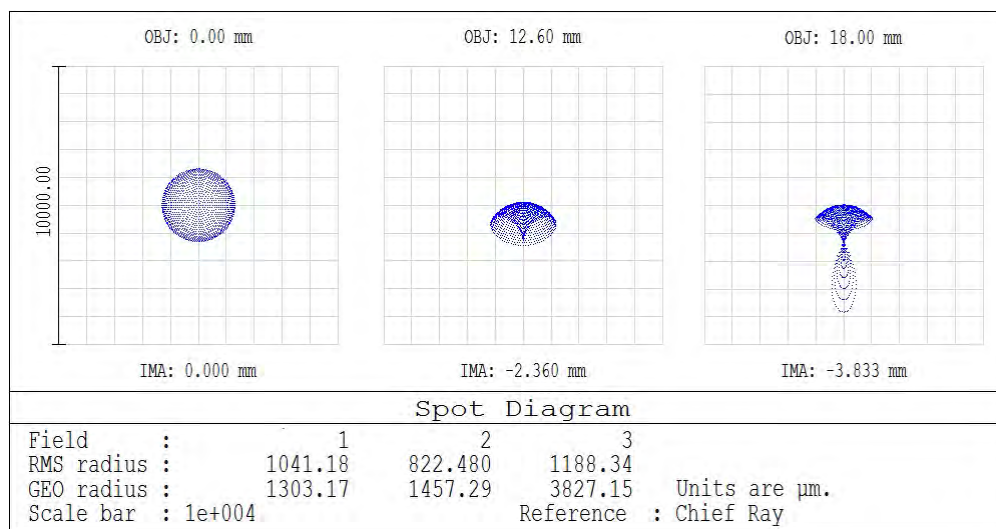
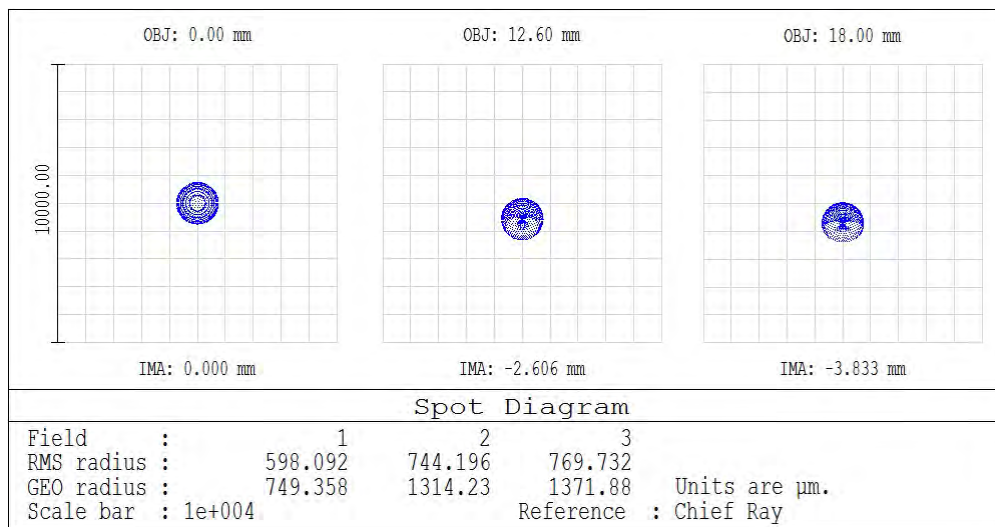
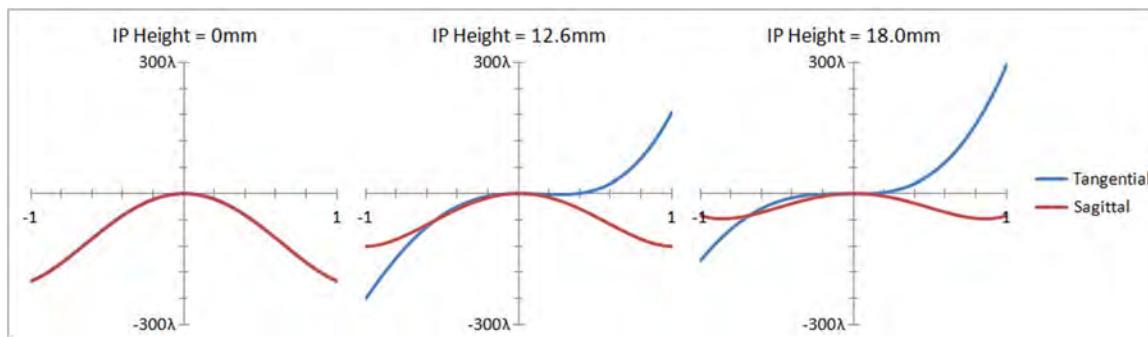
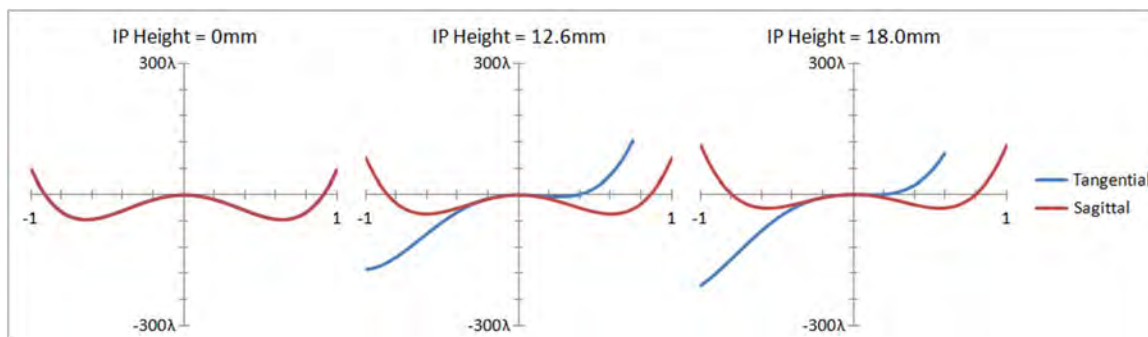


FIGURE 4.32 Induced Mapping Error: Plano Convex Lens ($F = 100\text{mm}$)

FIGURE 4.33 Induced Mapping Error: Plano Convex Lens ($F = 200\text{mm}$)FIGURE 4.34 Induced OPD Error: Plano Convex Lens ($F = 100\text{mm}$)FIGURE 4.35 Induced OPD Error: Plano Convex Lens ($F = 200\text{mm}$)

The percentage of the rotationally symmetric and toric test surfaces, generated in Chapter 4.5.6, which can be properly imaged by each lens, in combination with the two element

diverger, are given in TABLE 4.17. This data clearly shows the number of surfaces that can be properly imaged increases with the longer focal length lens. The peak to valley induced mapping, ε_{Mag} , and induced OPD error, OPD_E , calculated in the model of each test surface were recorded. The average and maximum peak to valley ε_{Mag} and OPD_E values, along with the average $\Delta MWSD$, generated by the test surfaces which could be properly imaged by each lens are given in TABLE 4.18 and TABLE 4.19.

Imaging Lens Type	f [mm]	D [mm]	F/#	Rotationally Symmetric Test Surfaces Imaged Properly	Toric Test Surfaces Imaged Properly
Plano-Convex	100	76.2	1.34	29.2%	44.7%
Plano-Convex	200	76.2	2.62	40.2%	66.7%

TABLE 4.17 The percentage of the rotationally symmetric and toric test surfaces that could be properly imaged with the plano-convex lenses tested

Imaging Lens Focal Length	Average P2V(ϵ_{Mag})	Maximum P2V(ϵ_{Mag})	Average P2V(OPD _E) [Waves]	Maximum P2V(OPD _E) [Waves]	Average Δ MWSD [Waves/Radius]
100mm	17.7%	77.8%	23.7	161.5	57.2
200mm	9.9%	41.2%	17.5	142.6	15.1

TABLE 4.18 Summary of the Induced Errors for Rotationally Symmetric Test Parts

Imaging Lens Focal Length	Average P2V(ϵ_{Mag})	Maximum P2V(ϵ_{Mag})	Average P2V(OPD _E) [Waves]	Maximum P2V(OPD _E) [Waves]	Average Δ MWSD [Waves/Radius]
100mm	19.1%	61.2%	12.2	134.0	31.9
200mm	13.0%	45.1%	16.5	104.2	29.7

TABLE 4.19 Summary of the Induced Errors for Toric Test Parts

From the data listed in TABLE 4.17 - TABLE 4.19 it is clear that the longer focal length lens reduced the induced errors and increased the number of test surfaces which could be properly imaged. Therefore the next logical step may have been to increase the focal length further. However, since the use of fold mirrors was to be avoided, the length of space available on the optical table to perform the imaging limited the focal length of the lens to 200mm. Another method of decreasing the induced errors of the imaging lens is to increase the number of elements. As was the case for the diverger lens, increasing the number of elements from one to two increases the number of lens properties to characterize in the reverse optimization model from twelve to twenty-five. Adding a third element increases the total to thirty-eight. In order to investigate the impact of further decreasing the induced errors three different lenses were investigated, all of which had focal lengths equal to 200mm and diameters of approximately 75mm. The first lens was a cemented achromatic doublet from Edmund Optics, Model# 45-417, the prescription of which is listed in TABLE 4.20 and the induced errors are shown in FIGURE 4.36 and

FIGURE 4.37. This lens was used as an example of a well corrected two element lens, in order to see the impact the reduced aberration would have on the number of test surfaces which could be properly imaged. This lens was not considered as a viable option for the non-null interferometer because, as Lowman (1995) pointed out, the buried surface cannot be characterized for the reverse ray trace and optimization models. The preferred two element solution is an air spaced doublet. However, a commercially available well corrected air spaced doublet with a comparable focal length and diameter could not be located. Therefore the second lens tested was an air spaced doublet constructed from two plano-convex lenses. The lenses used were a 500mm focal length lens from Edmund Optics, Model# 63-496, and a 300mm focal length lens from Newport Optics, Model# KPX232. Their combined prescription is given in TABLE 4.21 and the induced errors of the lens are shown in FIGURE 4.38 and FIGURE 4.39. The final lens tested was a custom designed three element air spaced lens. Due to budgetary and time constraints as well as the desire to keep down the number of elements in the interferometer this lens also was not an option for the final non-null interferometer. However it served as a good example of the imaging performance that could be achieved when the induced errors were pushed closer to zero. This lens was designed with a larger aperture so that the effects of using a well corrected lens with a $F/\#$ smaller than the requirement calculated from the paraxial model, $F/1.68$, could be investigated. In order to provide a fair performance comparison to the other lenses this lens was modeled twice. Once stopped down to $F/2.67$ to match the other lenses and once utilizing the full aperture with an $F/\#$ of $F/1.61$. The prescription of this lens is listed in TABLE 4.22.

The induced errors for the stopped down version of this lens are shown in FIGURE 4.40 and FIGURE 4.41. The induced errors calculated using the full aperture are listed in FIGURE 4.42 and FIGURE 4.43. The only difference between the two sets of plots is that the full aperture calculations include data for rays that vignette in the stopped down version.

Surf	Type	Radius	Thickness	Glass	Semi-Diameter	
1	Standard	118.81000	17.94000	BK7	37.50000	U
2	Standard	-96.37000	6.00000	SF5	37.50000	U
3	Standard	-288.97000	0.00000		37.50000	U

TABLE 4.20 Cemented Doublet Lens Prescription

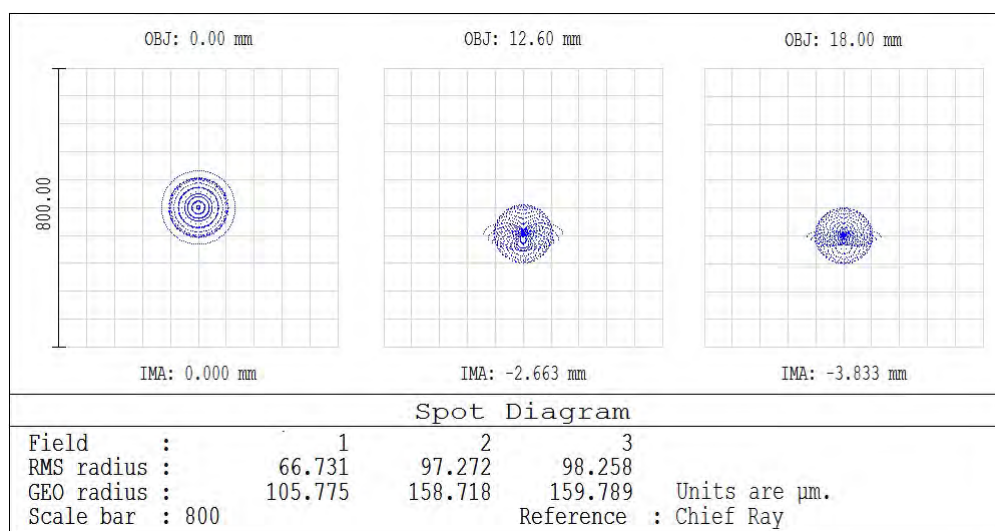


FIGURE 4.36 Induced Mapping Error: Cemented Doublet Lens

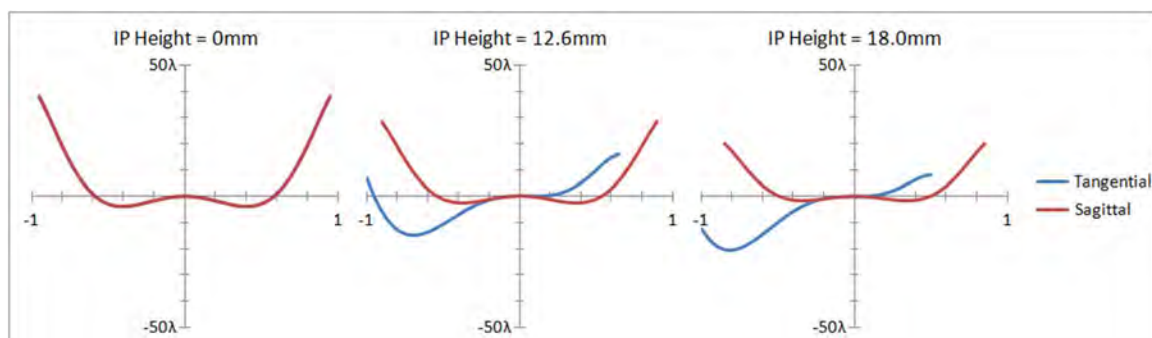


FIGURE 4.37 Induced OPD Error: Cemented Doublet Lens

Surf:Type		Radius	Thickness	Glass		Semi-Diameter	
1	Standard	258.40000	8.00000	N-BK7		37.50000	U
2	Standard	Infinity	47.83000			37.50000	U
3	Standard	155.04000	7.75400	BK7		38.10000	U
4	Standard	Infinity	0.00000			38.10000	U

TABLE 4.21 Air Spaced Doublet Lens Prescription

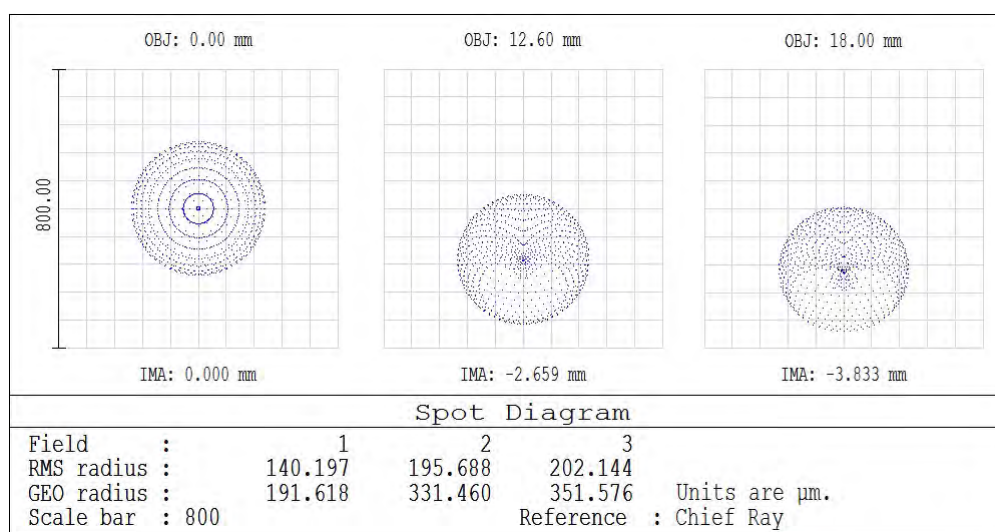


FIGURE 4.38 Induced Mapping Error: Air Spaced Doublet Lens

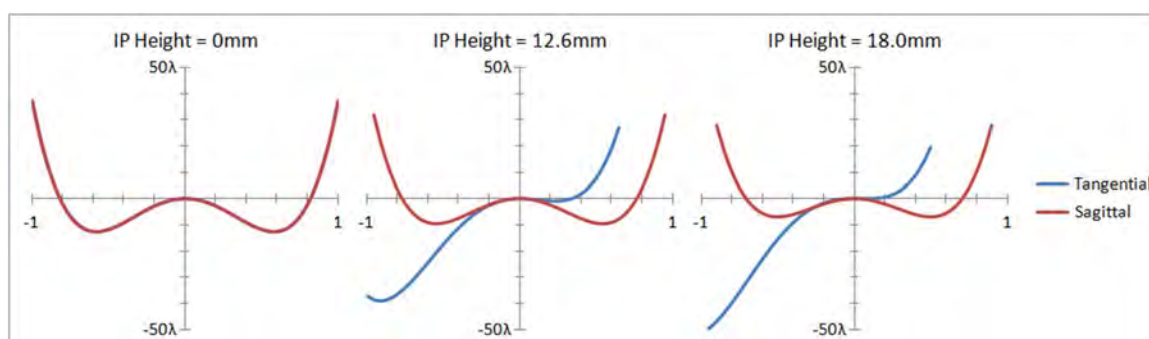


FIGURE 4.39 Induced OPD Error: Air Spaced Doublet Lens

Surf	Type	Radius	Thickness	Glass	Semi-Diameter	
1	Standard	357.73596	16.00000	N-SF11	62.00000	U
2	Standard	-438.48554	2.46000		62.00000	U
3	Standard	-283.39488	12.00000	N-SF11	62.00000	U
4	Standard	-578.83252	10.00000		62.00000	U
5	Standard	138.43876	17.00000	N-BK7	62.00000	U
6	Standard	407.40986	0.00000		62.00000	U

TABLE 4.22 Custom Three-Element Lens Prescription

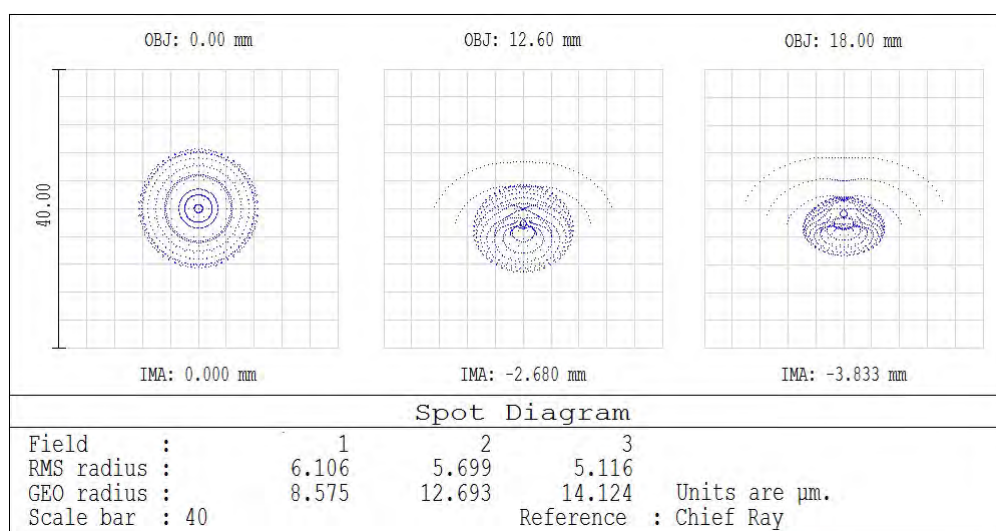


FIGURE 4.40 Induced Mapping Error: Custom Three-Element Lens (Small Aperture)

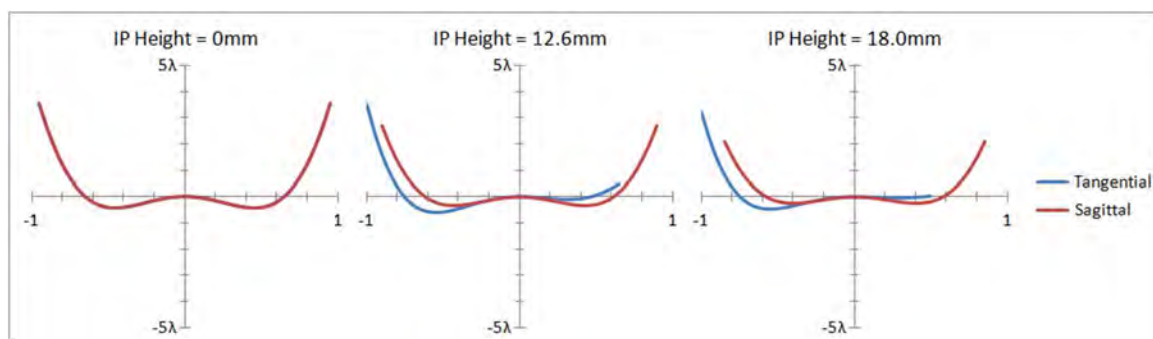


FIGURE 4.41 Induced OPD Error: Custom Three-Element Lens (Small Aperture)

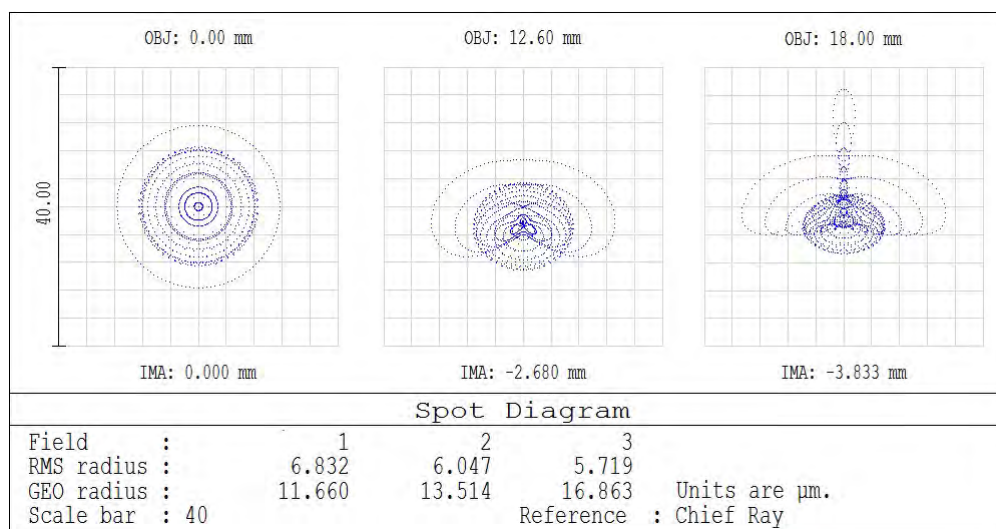


FIGURE 4.42 Induced Mapping Error: Custom Three-Element Lens (Full Aperture)

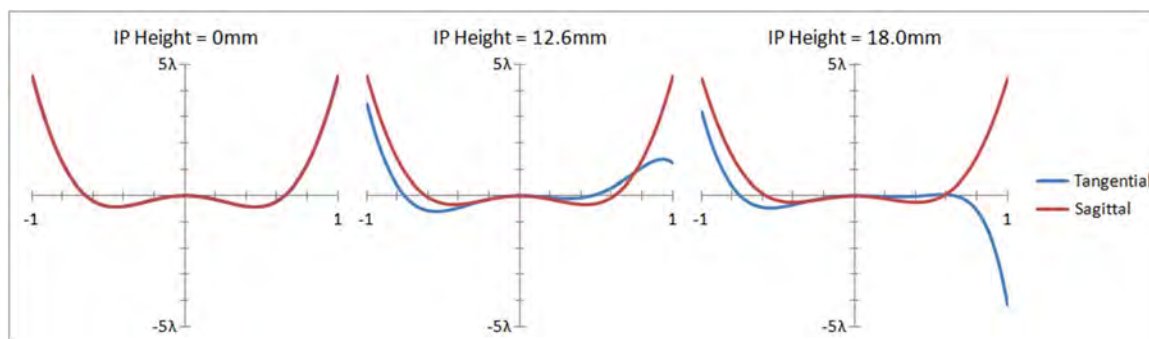


FIGURE 4.43 Induced OPD Error: Custom Three-Element Lens (Full Aperture)

It can be seen from the spot diagrams that the induced mapping error can be improved dramatically by the addition of extra elements to the imaging lens. The induced mapping errors of the air spaced doublet and cemented doublet are approximately one fourth and one eights respectively of the induced mapping errors of the 200mm focal length plano-convex lens. The mapping errors of the custom lens are two orders of magnitude less than those of the plano-convex lens. Additionally, the induced OPD errors show a similar pattern of improvement. The number of the rotationally symmetric and toric test

surfaces generated in Chapter 4.5.6 which could be properly imaged increased for these multiple element lenses, shown in TABLE 4.23. The cemented doublet and air spaced doublet saw an increase of approximately thirty and thirty-five percentage points over the comparable plano-convex lens. The fact that the performance of these two lenses was so similar was surprising since the cemented doublet showed less induced mapping and OPD error in its spot diagrams and OPD_E fans. However the air spaced doublet has a 47mm gap between the two lenses. When operating at a comparable magnification the first surface of the air spaced doublet is significantly closer to the intermediate pupil than the first surface of the cemented doublet. Therefore a ray, originating at or near the edge of the intermediate pupil, will be closer to the optical axis when it intersects the first surface of the air spaced doublet than it will be when it intersects the first surface of the cemented doublet. This leads to the improvement in the number of test surfaces which can be imaged properly despite the slightly higher induced aberrations. However the test surfaces which are imaged properly encounter less induced errors from the cemented doublet than they encounter from the air spaced doublet, TABLE 4.24 and TABLE 4.25. The custom triplet lens also showed significant improvement in the number of test surfaces that could be imaged properly over the plano-convex lens, TABLE 4.23. However, when operating at F/2.67, it only produced a six percentage point increase for rotationally symmetric test surfaces and five percentage point increase for the toric test surfaces over the performance of the doublet lenses. As expected the custom lens operating at F/1.61 was able to properly image the most surfaces, with a twelve percentage point increase over the stopped down version for the rotationally symmetric

test surfaces. It was also able to properly image all of the toric test surfaces. The induced aberrations experienced were also significantly reduced, TABLE 4.24 and TABLE 4.25. These tables make it appear that the induced aberrations generated by the large aperture version of the lens are slightly higher than the small aperture version. However this is simply due to the large aperture lens being capable of properly imaging test surfaces which generated higher wavefront slopes. These test surfaces could not be properly imaged by the F/2.67 version of the lens due to vignetting. The large wavefront slopes generated by these test surfaces and their corresponding large induced aberrations are included in the data for the large aperture lens but are not included in the data for the small aperture lens.

Imaging Lens Type	f [mm]	D [mm]	F/#	Rotationally Symmetric Test Surfaces Imaged Properly	Toric Test Surfaces Imaged Properly
Plano-Convex	200	76.2	2.62	40.2%	60.7%
Air Spaced Doublet	200	75	2.67	69.5%	94.8%
Cemented Doublet	200	75	2.67	69.0%	95.0%
Custom Triplet (Small Aperture)	200	75	2.67	75.6%	99.8%
Custom Triplet (Large Aperture)	200	124	1.61	87.6%	100%

TABLE 4.23 The percentage of the rotationally symmetric and toric test surfaces that could be properly imaged with the 200mm lenses tested.

Imaging Lens Type	Average P2V(ϵ_{Mag})	Maximum P2V(ϵ_{Mag})	Average P2V(OPD _E) [Waves]	Maximum P2V(OPD _E) [Waves]	Average Δ MWSD [Waves/Radius]
Plano-Convex	9.9%	41.2%	17.5	142.6	15.1
Air Spaced Doublet	5.2%	17.7%	12.9	87.2	9.0
Cemented Doublet	4.8%	17.1%	10.7	66.1	7.1
Custom Triplet (Small Aperture)	2.3%	6.5%	1.5	22.6	3.2
Custom Triplet (Large Aperture)	2.6%	16.2%	2.6	74.5	6.1

TABLE 4.24 Summary of the induced errors for the rotationally symmetric test parts

Imaging Lens Type	Average P2V(ϵ_{Mag})	Maximum P2V(ϵ_{Mag})	Average P2V(OPD _E) [Waves]	Maximum P2V(OPD _E) [Waves]	Average Δ MWSD [Waves/Radius]
Plano-Convex	13.0%	45.1%	16.5	104.2	29.7
Air Spaced Doublet	6.5%	17.7%	11.2	61.9	15.5
Cemented Doublet	6.0%	15.8%	8.8	47.5	12.9
Custom Triplet (Small Aperture)	2.9%	6.1%	0.95	10.2	2.9
Custom Triplet (Large Aperture)	2.9%	6.1%	0.95	10.2	2.9

TABLE 4.25 Summary of the induced errors for the toric test parts

The results presented in this chapter clearly call into question the idea that a simple imaging lens which is easy to characterize is the preferable solution for a non-null interferometer. Increasing the complexity of the imaging lens can both increase the dynamic range of the interferometer and decrease the induced errors. This reduction in the induced errors is even more apparent when only test surfaces which can be properly imaged by both the plano-convex lens and the custom triplet lens are compared, TABLE 4.26 and TABLE 4.27. An area of future research would be to determine the optimal

relationship between the complexities of the imaging lens designed, the induced errors and the effect on the reverse optimization and reverse ray tracing properties. This will largely depend on the tolerances to which the properties of a given lens can be measured and the net effect uncertainties in these properties have on the ability to calibrate the interferometer.

Imaging Lens Type	Average P2V(ε_{Mag})	Maximum P2V(ε_{Mag})	Average P2V(OPD _E) [Waves]	Maximum P2V(OPD _E) [Waves]	Average Δ MWSD [Waves/Radius]
Plano-Convex	9.9%	41.2%	17.5	142.6	15.1%
Custom Triplet (Small Aperture)	2.1%	4.4%	0.4	5.5	1.3%

TABLE 4.26 Summary of the induced errors for the rotationally symmetric test parts that can be imaged by both the plano-convex lens and the custom triplet

Imaging Lens Type	Average P2V(ε_{Mag})	Maximum P2V(ε_{Mag})	Average P2V(OPD _E) [Waves]	Maximum P2V(OPD _E) [Waves]	Average Δ MWSD [Waves/Radius]
Plano-Convex	13.0%	45.1%	16.5	104.2	29.7
Custom Triplet (Small Aperture)	2.4%	4.8%	0.23	1.6	0.9

TABLE 4.27 Summary of the induced errors for the toric test parts that can be imaged by both the plano-convex lens and the custom triplet

For the non-null interferometer designed for this research the air spaced doublet was chosen as the imaging lens. The improvements in the percentage of the test surfaces which could be imaged properly combined with the decrease in the induced mapping and OPD errors over those of the plano-convex singlet seemed to be worth the complications in characterizing the lens due to the addition of the second element. The characterization of this lens for the reverse ray tracing and reverse optimization model will be discussed in Chapter 5.4.3.

4.7 Beam Splitter

A large beam splitter was required in order to avoid clipping light in the interferometer due the previously discussed decision not to expand the beam in the test arm.

Additionally the interaction of the test beam, after reflecting off the test part, with the beam splitter should be minimized in order to decrease the induced aberrations of the aspheric test wavefront. Two common types of beam splitters used in interferometers are cube and plate. A cube beam splitter, FIGURE 4.44, consists of two right angle prisms cemented together along their hypotenuse with a partially reflective coated sandwiched in between them. The four external faces of the beam splitter are often anti-reflective coated. One benefit of a cube beam splitter is that light in both arms of the interferometer travel through glass equal to twice the width of the beam splitter. Therefore when used in a null interferometer the same OPL is introduced into each arm by the beam splitter. However, light propagating from the test arm into the imaging arm interacts with three surfaces of the beam splitter, two external faces and the interior surface, and travels through glass of thickness equal to the width of the beam splitter.

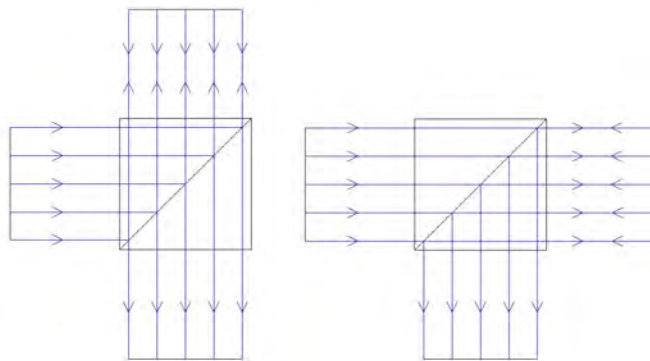


FIGURE 4.44 Twyman-Green Interferometer using a cube beam splitter.

A plate beam splitter consists of a single glass window with two parallel, or near parallel surfaces. Typically one surface has a partially reflective coating while the other has an anti-reflective coating. When a plate beam splitter is used in a Twyman-Green interferometer, light in one arm of the interferometer will make three passes through the beam splitter while the light in the other arm will only make one. Often the difference in OPL between the arms is balanced by placing a compensating glass plate of the same thickness and glass type as the beam splitter in the arm that contains only one pass through the beam splitter, FIGURE 4.45. This is especially important when a low coherence source is used in order to maintain the same OPL for all wavelengths and maximize the visibility of the fringes (Goodwin & Wyant 2006). However, when a long coherence light source, such as a laser, is used no compensating plate is needed to maintain high visibility fringes.

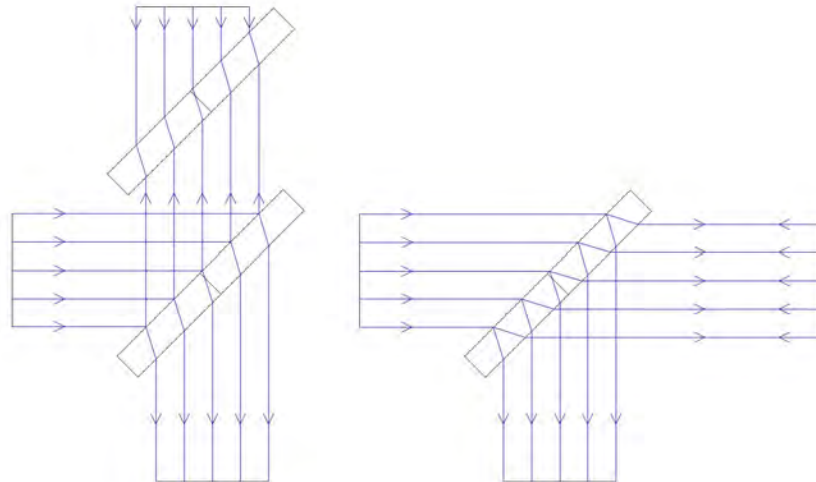


FIGURE 4.45 Twyman-Green Interferometer using a plate beam splitter and a compensating plate in order to balance the OPL of the two arms.

The interaction of the test wavefront with the beam splitter can be minimized by using the arm directly opposite the input beam as the test arm and by orienting the partially

reflective side of the beam splitter towards the test arm. Light returning to the beam splitter after reflecting off the test surface will only interact with the partially reflective surface on its way into the imaging arm. This is two less surface interactions and no propagation through glass for the reflected test wavefront, when compared to a cube beam splitter, making it the more desirable beam splitter type to use for a non-null interferometer. Therefore a plate beam splitter was used in the non-null interferometer with the anti-reflection (AR) coated side facing the reference arm. The AR coating was centered at 532nm and reduced the reflectance to 0.2%. The side facing the test arm had a 50% reflective 50% transmissive dielectric coating at 532nm applied. The beam splitter was made from BK7 with a center thickness of 12.7mm and a diameter of 101.6mm. The diameter was oversized in order to avoid being the limiting aperture in the test arm of the interferometer. In addition to the anti-reflective coating a 2° wedge angle between the surfaces was added in order to direct stray light from multiple reflections off the beam splitter surfaces out of the interferometer. The dihedral angle of the wedge was aligned to be parallel the optical table. Finally the angle at which the beam splitter was oriented relative to the input beam was designed to be 30° rather than the traditional 45°. This was done for three reasons, the first being that less surface area of the beam splitter is used with the smaller input angle. The second reason is that the optical mount used to hold the beam splitter would have blocked a portion of the 48mm reference beam at 45°, but at 30° the beam was unclipped. Lastly the length of the collimating optics, test arm optics and their supporting mounts and rails was longer than the 48" width of the optical table upon which the system was built. With a beam splitter input angle of 30° and the

imaging arm set to run down the length of the optical table the interferometer could fit on the table without the introduction of extra fold mirrors.

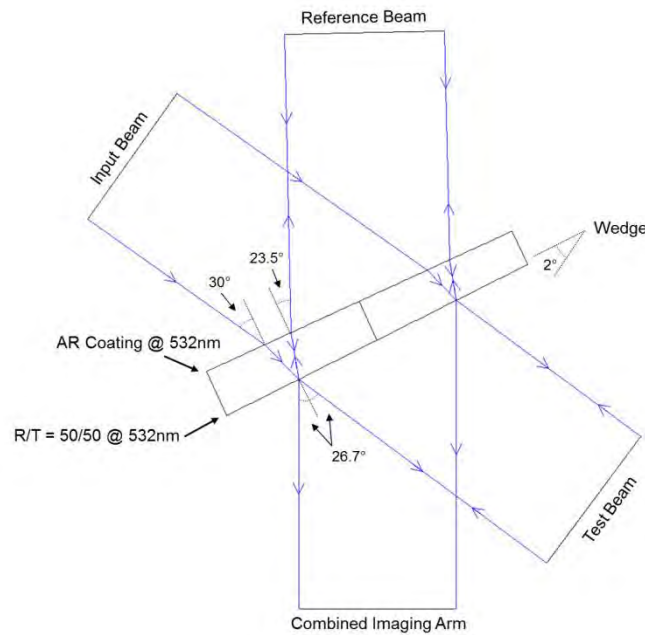


FIGURE 4.46 The beam splitter used in the sub-Nyquist Interferometer.

Malacara (Malacara 2007b) derives the surface quality tolerances for a beam splitter used in a null Twyman-Green interferometer with the reflecting side facing the input beam. It is shown that half of the errors a ray experiences on the anti-reflection coated side are common path to both test and reference rays. Also, imperfections on the reflecting surface add to the OPD at twice their size while the imperfections on the anti-reflection side add at $(n-1)$ their size. Therefore the reflecting face must be polished with approximately twice the interferometer accuracy while the AR side only needs to be polished to half the interferometer accuracy. However in the non-null interferometer there is no guarantee that a defect on either surface will be common path. Therefore, in order to be able to ignore the contribution of the surface errors in the final model of the

interferometer used for reverse optimization and reverse ray tracing both sides of the interferometer were specified to be twice the interferometer accuracy, $\lambda/20$, over 90% of the 101.6 mm diameter. A tighter, $\lambda/40$ tolerance was placed on the central 60% of the aperture. If the surfaces were truly flat then the other errors in the beam splitter, center thickness, decenters of the surfaces, the magnitude and alignment of the wedge angle and the index of refraction will have no impact on the final measurement as they will only add piston and tilt to the final measured OPD. One exception is the tilt added into the test wavefront incident on the diverger lens. Tilt in the wavefront at the diverger lens will change the wavefront produced by the diverger at the test part by introducing off axis aberrations such as coma. However in the optical model of the system used for reverse ray tracing this error can also be attributed to the misalignment of the diverger to the incoming wavefront.

Unfortunately, the manufacturer of the beam splitter did not meet the specified surface flatness. The manufacturer, Rocky Mountain Instrument Company (Lafayette, CO), claimed the surfaces were produced to $\lambda/10$ peak to valley error over 85% of the surface diameter. However measurements performed on a WYKO 6000 laser-based Fizeau interferometer (Wyko, Tucson, AZ) showed that the surfaces actually contained $\lambda/2$ peak to valley error over the entire surface, FIGURE 4.47 and FIGURE 4.48. Therefore the errors introduced by these surfaces have to be accounted for the optical model of the systems, which will be discussed in Chapter 5.4.2. The manufacturer measured the wedge angle with a dial indicator to be $2^\circ 1'$. However in order to verify the

manufacturer's number the wedge angle was measured on a prism table. The average of ten prism table measurements yielded a wedge angle of $2^{\circ} 0' 37''$ with a standard deviation of $2''$. The manufacture also provided melt data on the index of refraction at the C, d, F, g wavelengths, but not at 532nm. The Zemax glass fitting procedure and the melt data produced an index of refraction of 1.519683 at 532nm. The ten measurements on the prism table produced an index of refraction of 1.519697 with a standard deviation of 0.0002.

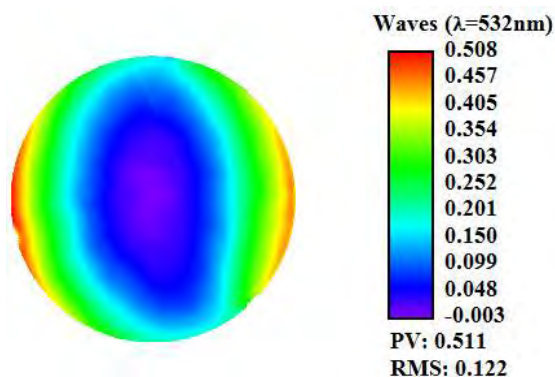


FIGURE 4.47 Partially reflective beam splitter surface measured on WYKO 6000 laser-based Fizeau interferometer.

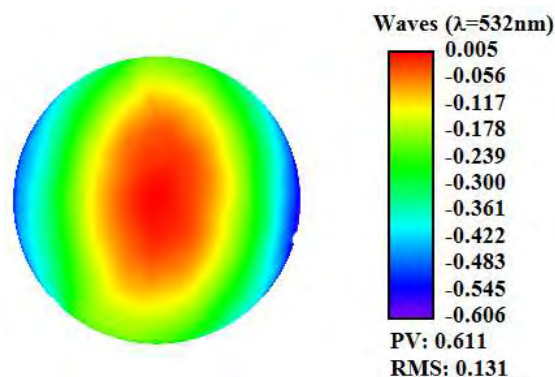


FIGURE 4.48 The AR coated beam splitter surface measured on WYKO 6000 laser-based Fizeau interferometer.

4.8 Reference Surface and Phase Shifter

The reference surface used in the interferometer was manufactured by Newport Corporation (Irvine, CA), model 20Z40AL.2. It was made of Zerodur glass coated with aluminum had a diameter of 50.8mm and a specified surface flatness of $\lambda/20$ at 632.8nm. It was measured on a WYKO 6000 laser-based Fizeau and had a peak to valley error of $0.028\mu\text{m}$ and an RMS error of $0.005\mu\text{m}$ over the full diameter, FIGURE 4.49. At 532nm the surface flatness is slightly less than twice the desired interferometer accuracy over the full diameter. In modeling the test surfaces generated in Chapter 4.5.6 with the two element diverger and the air spaced imaging lens the largest required reference wavefront diameter was 47.2mm. However 90% of the test surfaces required a reference wavefront diameter of 38.5mm or less, which corresponds to 76% of the reference surface diameter. Over the smaller diameter the surface flatness improves to approximately $\lambda/40$, or four times the target interferometer accuracy.

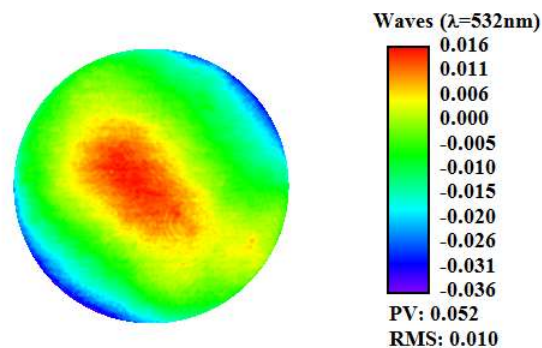


FIGURE 4.49 Reference Mirror Measured on WYKO 6000 laser-based Fizeau scaled to 532nm wavelength light.

The reference surface was phase shifted using a piezoelectric optical mount from EXFO Burleigh (Victor, NY), model PZ-91, FIGURE 4.50. At the maximum allowed voltage of 1000V the PZT mount was capable of shifting the mirror $2\mu\text{m}$, TABLE 4.28.



FIGURE 4.50 Piezoelectric Optical mount used to phase shift the reference mirror.

Specification	Value
Maximum Voltage	1000V
Translation at Max Voltage	2 μ m
Non-Linearity	< 1%
Hysteresis	< 1%
Frequency Response	5 KHz

TABLE 4.28 Specifications of the PZT used for phase shifting the reference mirror.

The voltage steps or ramp were produced using a high voltage digital to analog PCI card manufactured by Piezomechanik GMBH (Munich, Germany). This D/A card allowed the computer to directly output a 0V to +500V signal with 14bit resolution eliminating the need for an external high voltage amplifier. The 500V range of the card limited the phase shifter to 1 μ m or half of its full range of motion. However this is still approximately 4 times the minimum required travel range based on five phase steps generated by four $\lambda/8$ translations of the reference mirror.

4.9 Collimating Optics

The collimating optics consisted of a spatial filter and a doublet lens. The spatial filter consisted of a 60x microscope objective with a NA of 0.85, and a 5 μ m pinhole. The pinhole was created in a 25 μ m thick sample of 304 stainless steel. The back surface of the pinhole had a flat black absorbing coating applied to it in order to minimize stray reflections, the need of which will be discussed in Chapter 4.10. A lens with a 50mm clear aperture and 246.1mm focal length was used to collimate the light out of the spatial filter. The F/5 aplanatic air spaced doublet was manufactured by CVI Melles Griot (Rochester, NY), model LAP-250.0-50.0. The prescription of which is given in TABLE 4.29. All lens surfaces were coated with an anti-reflection coating to drop the reflectance at 532nm to under 0.25%. The theoretical transmitted wavefront from a perfect point source at 532nm is approximately $\lambda/50$ waves peak to valley, FIGURE 4.51. However the manufacturer only specifies the wavefront distortion of the lens to be less than $\lambda/4$ peak to valley. At the desired interferometer accuracy level a better lens would have been beneficial. Although since this was already on hand when the non-null interferometer was being designed, and a better off the shelf lens could not be located, it was hoped the wavefront distortion could be accounted for in the reverse ray tracing model. As was the case for the two element diverger and imaging lens twenty-five lens properties would have to be accounted for if this two element collimating lens was included in the reverse ray trace model. However, rather than including the physical description of the collimating lens in the reverse ray trace model, the transmitted

wavefront through the collimating lens was measured and included in the model. This measurement will be discussed in Chapter 5.4.1

Surf:	Type	Radius	Thickness	Glass	Semi-Diameter	Conic
1	Standard	Infinity	236.976674		0.000000	0.000000
2	Standard	1257.18000	7.000000	N-SF11	25.000000	U 0.000000
3	Standard	300.045000	0.346000		25.000000	U 0.000000
4	Standard	433.874000	9.000000	N-SF11	25.000000	U 0.000000
5	Standard	-187.05200	20.000000		25.000000	U 0.000000
6	Standard	Infinity	-		25.000099	0.000000

TABLE 4.29 Collimating Lens Prescription

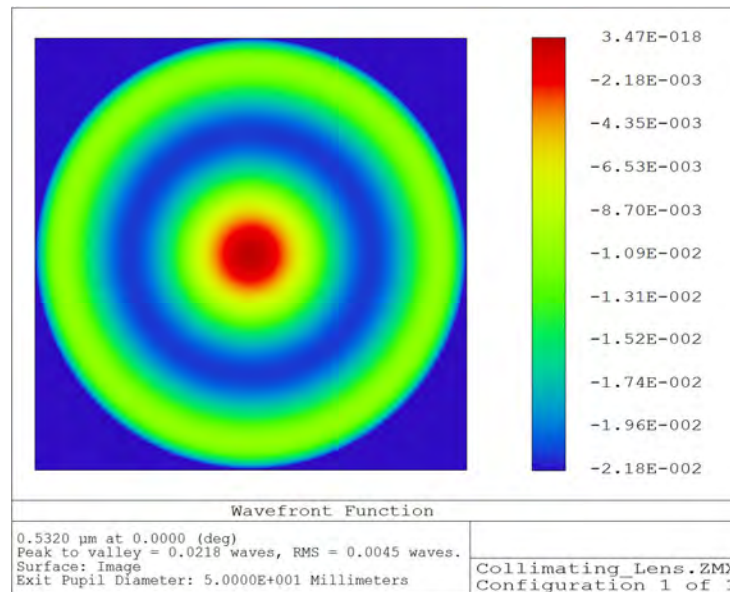


FIGURE 4.51 Theoretical Transmitted Wavefront from Collimating Lens.

4.10 Spurious Fringes

Stray reflections in an interferometer can lead to multiple beam interference and spurious interference fringes being produced on the detector. The effects of spurious fringes on phase shifting interferometry have been discussed by several sources. (Schwider et al, 1983) (Greivenkamp & Bruning, 1992) (Ai & Wyant, 1988). Spurious fringes introduce

phase errors into the measurement and can degrade the fringe modulation. The long coherence length of the laser allows high visibility spurious fringes to be produced even when the OPD between the stray light and main interferometer arms have become very long. Additionally, the sparse array sensor can resolve fringes produced by stray light incident at steep angles with respect to the main beam. There were two major sources of spurious fringes which were overlooked in the design of this system. The first was a stray reflection off the pinhole of the spatial filter. In a Twyman Green interferometer the beam splitter directs half the light into the test and reference arms. After the light reflects off the test surface or reference mirror, the beam splitter directs half each beam into the imaging arm and half back towards the collimation optics. The collimating lens will focus the reflected light onto the metal disc containing the pinhole in the systems spatial filter. Originally the spatial filter made use of a $5\mu\text{m}$ pinhole in a thin strip of molybdenum, which would create a highly specular reflection of this light. After reflecting off the metal surrounding the pinhole some of this light would be captured by the collimating lens and propagate back through the interferometer to the detector creating spurious fringes. The solution to this problem was to change the pinhole substrate from the reflective molybdenum to 304 stainless steel with in a black absorbing coating applied to the side which faces the collimating lens. The second major source of spurious fringes was the sparse array sensor. The aluminum layer in which the pinhole array is etched is highly reflective. Light incident on the detector would reflect off this aluminum layer and propagate backwards through the interferometer. However the raised electrodes on the sensor print through the aluminum mask and create a pair of

crossed reflective phase gratings. These electrodes can be seen in the SEM image of the sensor shown previously in FIGURE 4.3. The diffraction pattern created by the raised electrodes can easily be seen by illuminating the sensor with a coherent plane wave. To illustrate this, a flat mirror was inserted into the test arm of the interferometer and the imaging lens was removed from the system. The collimated light from both arms was directed onto the detector after passing through a hole in a piece of paper. The diffracted light was then visible on the back side of the paper and is shown by the diagram in FIGURE 4.52. The straight line tilt fringes present in the interference pattern at the detector can be seen in the diffraction pattern.

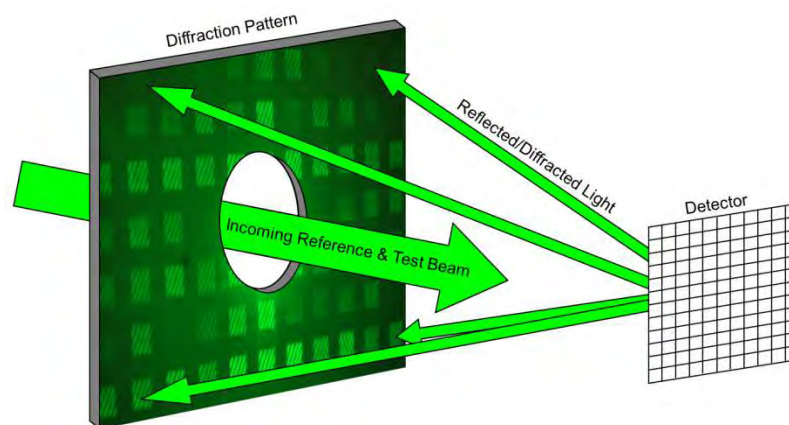


FIGURE 4.52 Diffraction of the test and reference wavefront off the sparse array sensor.

When the imaging lens is inserted into the system the diffraction pattern changes because the sensor is now illuminated with a converging or diverging wavefront. Additionally the imaging lens collects the light in a few of the diffraction orders and images them back into the interferometer. Often they can be seen by placing a screen in one of the interferometer arms, as shown in FIGURE 4.53. In this image light from the reference

arm is diffracted by the sensor and travels backwards through the imaging lens where it is brought to focus in the test arm of the interferometer.

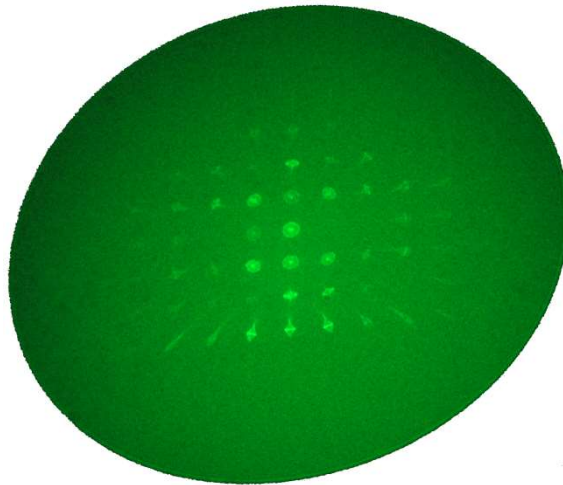


FIGURE 4.53 This is an example of a diffraction pattern present in the test arm of the interferometer which is generated by light from the reference arm diffracting off the sparse array sensor and brought to focus during its return trip through the interferometer by the imaging lens. The image appears skewed due to the angle at which the image was captured.

These diffraction patterns can be extremely disruptive if they are brought to focus at or near the sensor. This occurs when diffracted light is captured by the imaging lens, reflected off the reference mirror or the test part, and is then focused onto the sensor by a third pass through the imaging lens, as shown in FIGURE 4.54. There are four paths through the system which can generate a similar diffraction pattern on the sensor. The light can originate from either the reference or test arm and, after being diffracted by the sensor, can be re-imaged back onto the sensor after reflecting off either test or reference surface. Three of these paths make use of the reference surface so the light in them will phase shift as the reference surface is phase shifted.

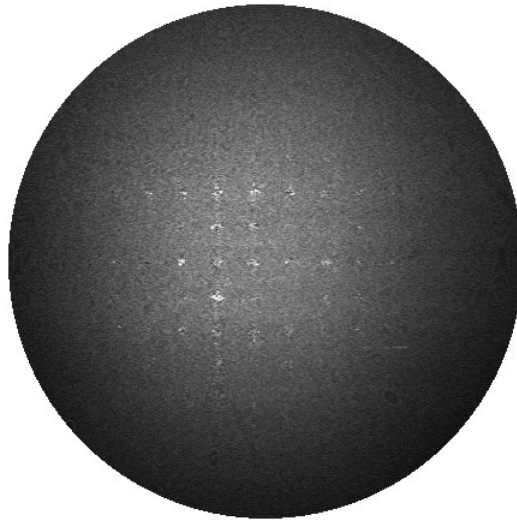


FIGURE 4.54 Diffraction pattern present on the sparse array sensor.

The ideal solution to this problem would be to design the camera with a pinhole mask that is non-reflective. However since this was not an option with the current interferometer another method of eliminating these patterns had to be found. An absorbing pellicle placed in the imaging arm of the interferometer was found to be a suitable solution. Light from both arms are attenuated equally by the pellicle, therefore the modulation of the desired fringes can be kept high by increasing the intensity of the laser. The light which diffracts off the sensor must make two additional passes through the pellicle before it returns to the detector. The pellicle used had a transmittance of approximately 20%. Interferograms captured without and with the pellicle in place are shown in FIGURE 4.55 and FIGURE 4.56.

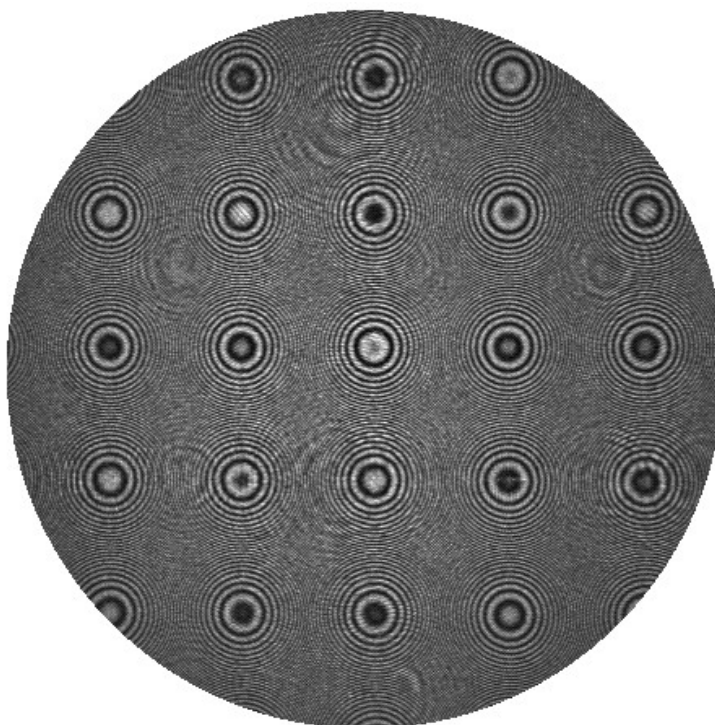


FIGURE 4.55 Spurious fringes are visible in this interferogram captured without the absorbing pellicle in the imaging arm of the sub-Nyquist interferometer.

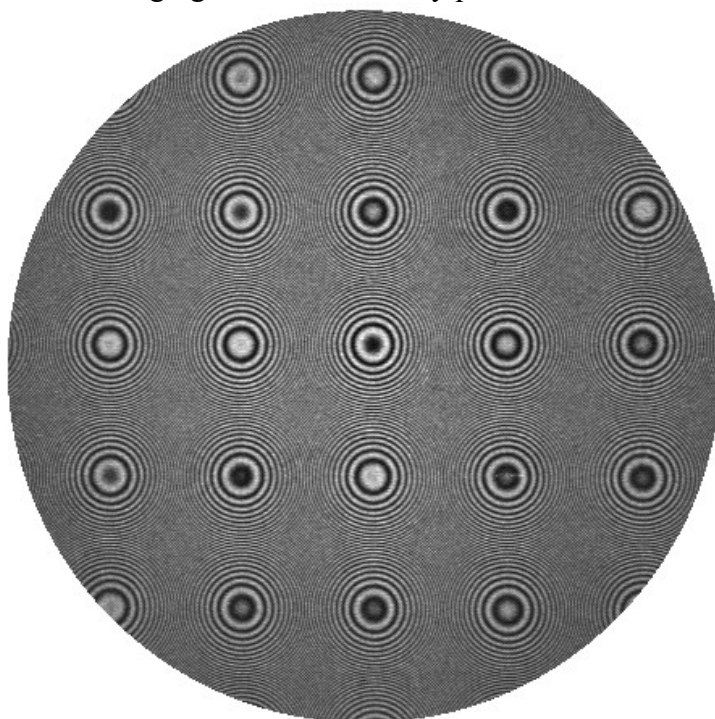


FIGURE 4.56 Spurious fringes are suppressed in this interferogram captured with the absorbing pellicle in the imaging arm of the sub-Nyquist interferometer.

The effect these spurious fringes have on the SNI unwrapping algorithm, which will be discussed in Chapter 5.2.3, can be seen in FIGURE 4.57. After the phase unwrapping algorithm is completed pixels that do not satisfy slope continuity requirement are dropped from the measurement and appear white. The unwrapped wavefront in FIGURE 4.57(Left) shows failures in the SNI algorithm due to spurious fringes present in the interferogram, FIGURE 4.55, which was recorded without the pellicle. These failures typically occur around odd multiples of the Nyquist frequency where the modulation of the recorded phase shifted interferograms is already low due to the MTF of the sparse array sensor. The spurious fringes push the modulation below the threshold needed for proper sub-Nyquist phase unwrapping. The unwrapped wavefront in FIGURE 4.57(Right) shows the improvement in the number of pixels that are properly unwrapped due to the reduction of spurious fringes in the interferogram, FIGURE 4.56, by the addition of the absorbing pellicle. Using an absorbing pellicle in the system provided an additional advantage when glass surfaces were tested. Then the pellicle could be moved from the imaging arm to the reference arm in order to balance the intensities of the test and reference wavefronts. The reduction in the diffracted light intensity is the same since the pellicle is still present between the reference surface and the sensor in the reference arm. In the test arm the glass test part will reduce the intensity of the diffracted light.

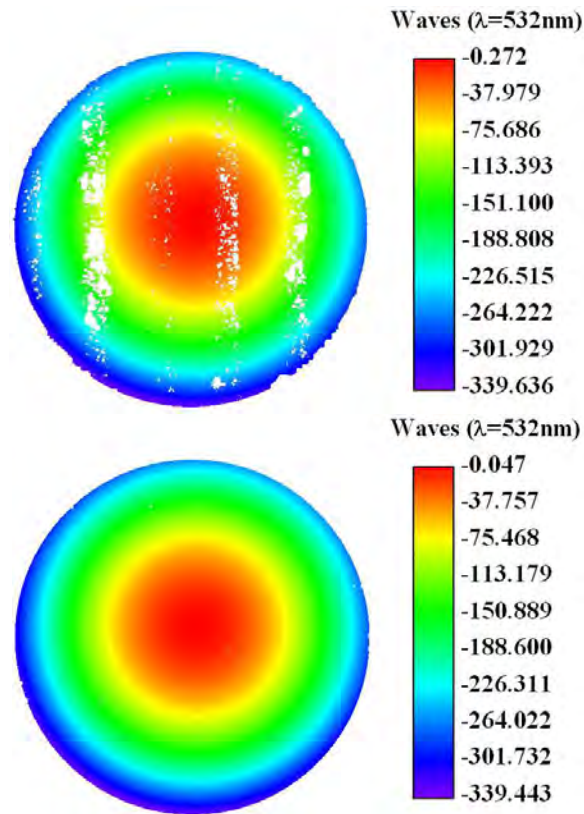


FIGURE 4.57 The unwrapped wavefront from the interferogram recorded without the pellicle shows missing data where the SNI unwrapping algorithm failed (Left). The addition of the pellicle clearly improves the result of the SNI phase unwrapping algorithm (Right).

However as previously stated the introduction of additional optics into the system, especially after the test surface, should be avoided to avoid increasing the complexity of the reverse optimization and ray tracing model. Therefore it had to be determined if the pellicle would introduce significant OPD error into the measurements. The pellicle used was roughly 150mm in diameter with a thickness of $2\mu\text{m}$ and a specified transmitted wavefront distortion of less than $\lambda/2$ over the full aperture. However, only a small fraction of the pellicle's full diameter was used in the sub-Nyquist interferometer measurements. The transmitted wavefront error of the pellicle was measured using PSI

by placing it in the test arm of the interferometer followed by a flat mirror.

Measurements were made using PSI with and without the pellicle in the interferometer.

The transmitted wavefront error of the pellicle was found by taking half of the difference between the two measurements as shown in FIGURE 4.58. Over the full diameter of the reference wavefront the error was on the order of $\lambda/18$, however over the smaller 38.5mm diameter the peak to valley error was $\lambda/36$.

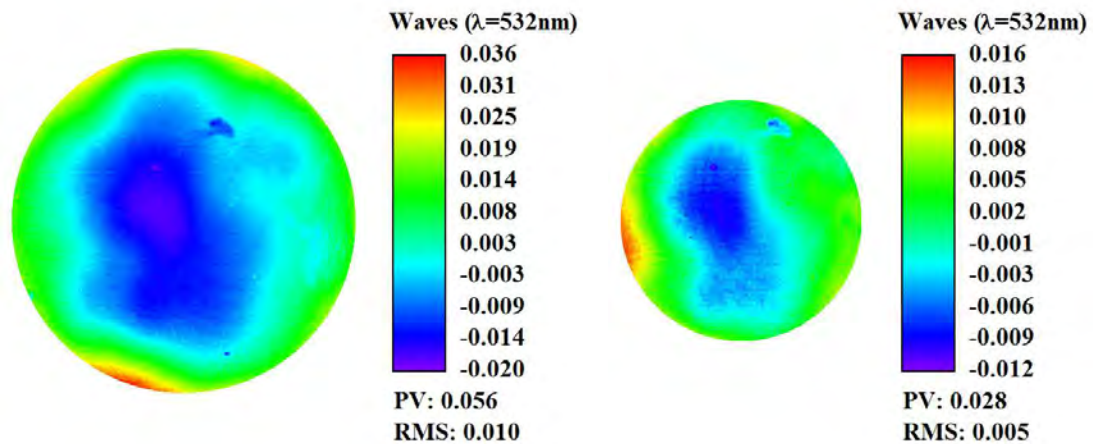


FIGURE 4.58 The transmitted wavefront error of the pellicle over a 50mm diameter (Left) and over a 38.5mm diameter (Right)

In a non-null measurement, in order for the pellicle to induce the maximum OPD error into a given point in the interferogram the test ray and reference ray would have to pass through the pellicle at regions corresponding to the smallest and largest transmitted wavefront error. While the transmitted wavefront error map, shown in FIGURE 4.58, does contain some high spatial frequencies, for the most part the change across the wavefront is gradual. Therefore if the test and reference rays strike the pellicle at approximately the same location the induced OPD error will be much smaller than measured peak to valley transmitted wavefront error. This means that the placement of

the pellicle can influence the OPD error that is introduced into the measurement.

Consider a converging test wavefront being measured against a flat reference wavefront,

FIGURE 4.59. In plane A the test and reference rays, which eventually interfere at the detector, are close to overlapping. The induced OPD error would depend on the variation in the pellicle over the small distance between the test and reference ray. In plane B, all the rays in the test wavefront pass would through the same point. However, the reference rays are spread out across the pellicle. In this case the OPD error across the test wavefront would take the shape of the negative of the transmitted wavefront error. In plane C it appears the test and reference rays are close to overlapping. However at this plane the test rays are flipped over the optical axis from the reference rays that they interfere with at the detector plane. In this case, the rotationally symmetric errors of the pellicle would not affect the induced OPD error but the non-rotationally symmetric transmitted errors would. Therefore in this example plane A would be the best location to insert the pellicle. In using the pellicle in the non-null interferometer care was taken to minimize the induced OPD error so that it could be excluded from the ray tracing models.

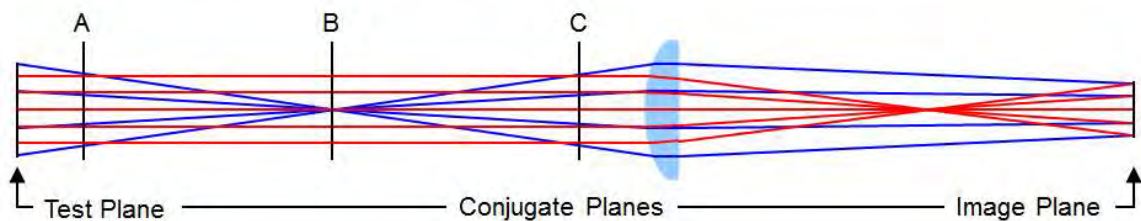


FIGURE 4.59 Possible locations for the pellicle in the interferometer imaging arm

5 SNI SOFTWARE, RAY TRACING MODELS & MEASUREMENT PROCEDURE

This chapter will outline the software, measurement procedures and ray tracing models used to make a non-null interferometric measurement of an aspheric surface using sub-Nyquist interferometry. The process makes use of three Zemax ray tracing models and software, written by the author, to collect and process data from both the physical interferometer as well as the ray tracing models. This Chapter will start with a high level overview of the measurement process, followed by a summary of the SNI control and analysis software. Then each process step will be discussed in more detail. The various Zemax models will be discussed in the order they are utilized in the measurement process. Additionally measurements of the interferometer and its components which were made to facilitate the reverse optimization procedure will be discussed.

5.1 Overview of the Measurement Process

The measurement process for making a measurement with the sub-Nyquist interferometer is outlined in flow chart shown in FIGURE 5.1. The process is separated into three categories; steps that use a model of the interferometer in Zemax, steps that use the physical interferometer and steps that use the data collection and analysis software. The process starts with the nominal prescription of the part to be tested being loaded into a simple model of the interferometer. The simple model is used to determine if the interferometer is capable of testing the surface. The simple model is also used to

determine the physical layout of the interferometer required to complete the test. The process of setting up the interferometer to match the simple model is started by analyzing magnification target. Phase shifted fringes are collected by the software of the magnification target. This data is fed into a model of the interferometer, which includes the magnification target, to determine the imaging lens to sensor separation. The physical interferometer alignment is adjusted until the measured magnification matches that specified by the simple model.

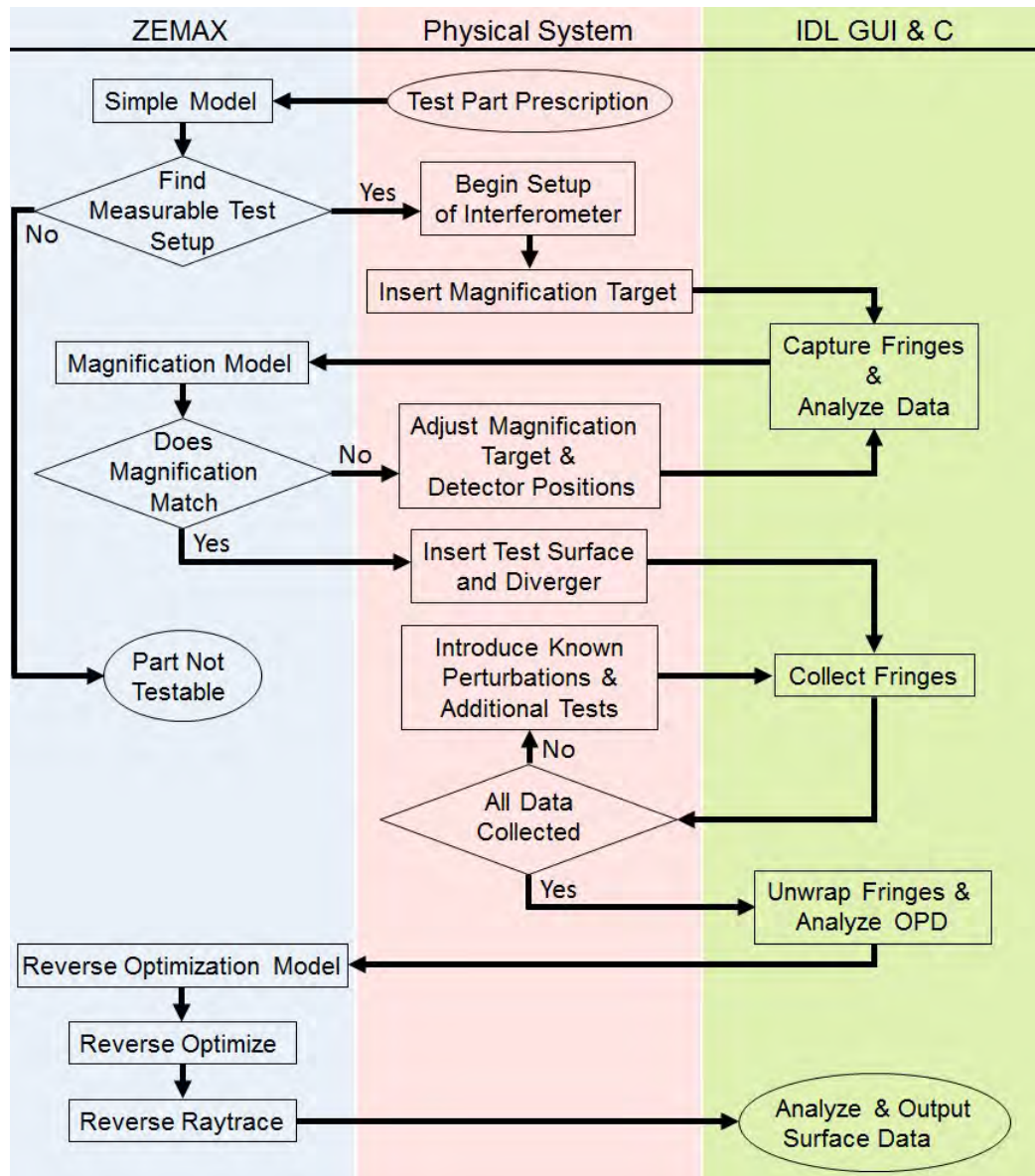


FIGURE 5.1 Flow chart of the SNI measurement process

Next the diverger and test part are inserted and into the interferometer and phase shifted interferograms of the test part are collected by the software. The known perturbations of the test part location are introduced in order to collect additional data for the reverse optimization procedure. Once all the measurements are completed the sub-Nyquist interferograms are unwrapped, and the OPD data is processed so it can be loaded in the

reverse optimization Zemax model. This model is used for both the reverse optimization and reverse ray tracing procedures. Finally the results of the reverse optimization and reverse ray tracing model are exported back to the software it can be analyzed and graphical representations of the surface can be generated.

5.2 SNI Software GUI

The acquisition, analysis, and management of data required for the function of the sub-Nyquist interferometer necessitated the writing of software. A Graphical User Interface (GUI) was written in IDL (Interactive Data Language) produced by Exelis Visual Information Solutions. (Boulder, CO) The GUI, FIGURE 5.2, allows the user to control the Sub-Nyquist Interferometer, analyze the recorded data, and pass data to and from the Zemax model. Many of the acquisition and data passing programs were written in C as executable and Dynamic-Link Libraries (DLLs), which are in turn called by the GUI. The analysis programs, written in IDL, allow the user to mask data, compare wavefronts, and perform mathematical manipulations; such as scaling or Zernike fitting. The following is a brief overview of the GUI options and programs separated into five categories; Menu Bar, Side Panel, Acquire Data Tab, Zernike Fitting Tab, and Math Tab. The vast majority of the software was written by the author; however, a few procedures were based on programs written by others or contain supporting functions written by others, which will be noted.

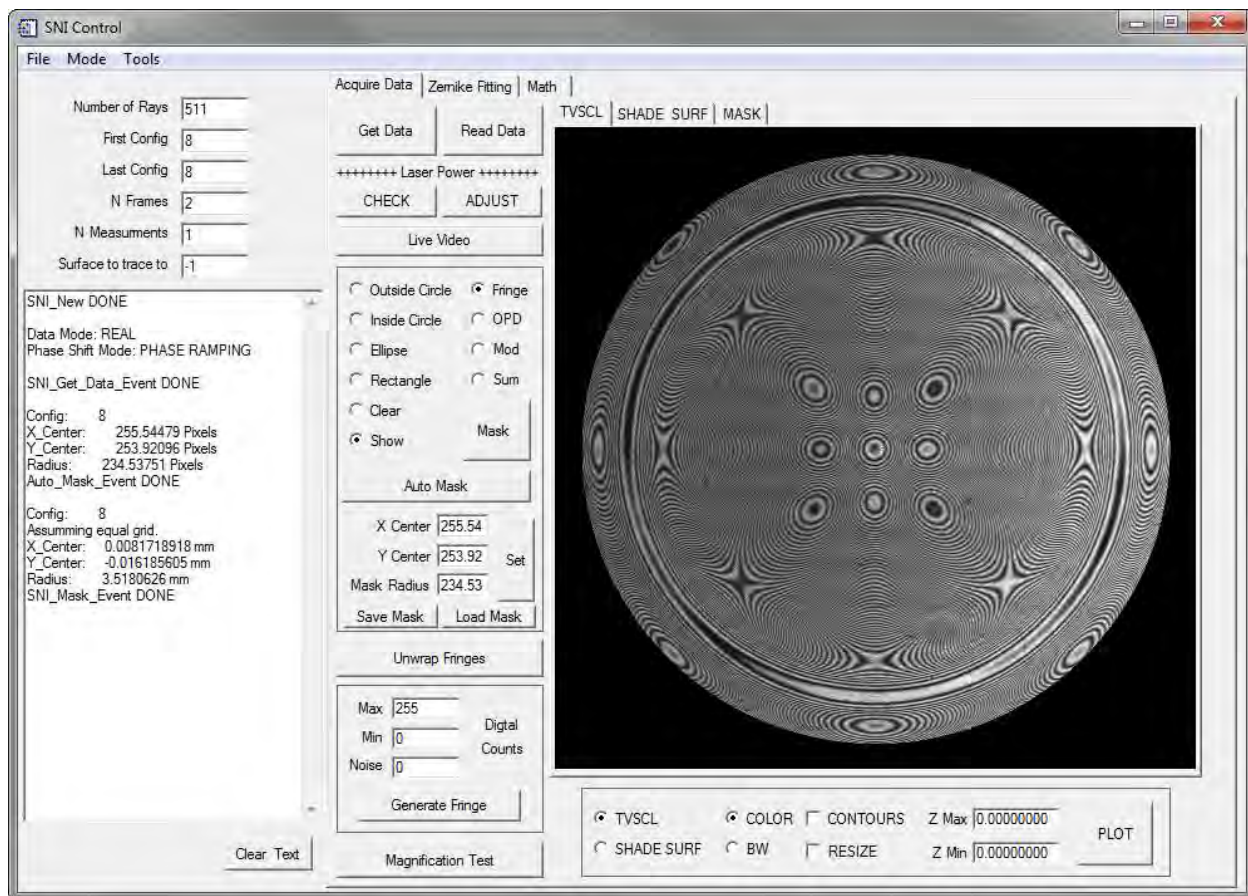


FIGURE 5.2 Image of the Graphical User Interface (GUI)

5.2.1 GUI Menu Bar

The first sets of options are located on the GUI menu bar, under the headings File, Mode and Tools, FIGURE 5.3.

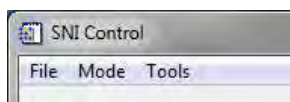


FIGURE 5.3 GUI Menu Bar

File: Under the file menu option there are procedures that allow for a new GUI session to be created as well as for saving the current session or loading a previously saved session. Creating, saving and loading sessions allows all of the data stored in the

computer's memory, which is utilized by the GUI, to be allocated, saved to a file, and subsequently loaded from a file.

Mode: Under the mode menu option there are procedures that allow the user to switch between different data acquisition modes. One option allows the user to select the source of the data; either **Real Data** mode, in which wavefront data is acquired by capturing interferograms from the interferometer, or **Simulated Data** mode, in which data is acquired by ray tracing the Zemax model. When the GUI is in Real mode, there are two options for the type of phase shifting to use in acquiring the interferograms; phase stepping or phase ramping, as discussed in Chapter 2.1.1.

Tools: There are three procedures under the Tools menu option, Test Phase Shift, Modulation Map and Send Data to Zemax.

Test Phase Shift: This procedure is used to check and calibrate the phase shift produced by the PZT. It calculates the phase shift at each pixel from a previously recorded series of phase shifted interferograms utilizing Equation 5.1 (Greivenkamp and Bruning 1992), which can be derived from Equations 2.12-2.16. The procedure displays a map of the calculated phase shift at each pixel, FIGURE 5.4, and calculates a histogram of the phase shift at each pixel, FIGURE 5.5. If the peak of the histogram drifts off 90° , the voltage per phase step or the slope of the voltage ramp signal generated by the high voltage DAC card in the computer should be adjusted.

$$\alpha(x, y) = \cos^{-1} \left[\frac{I_5(x, y) - I_1(x, y)}{2(I_4(x, y) - I_2(x, y))} \right] \quad 5.1$$

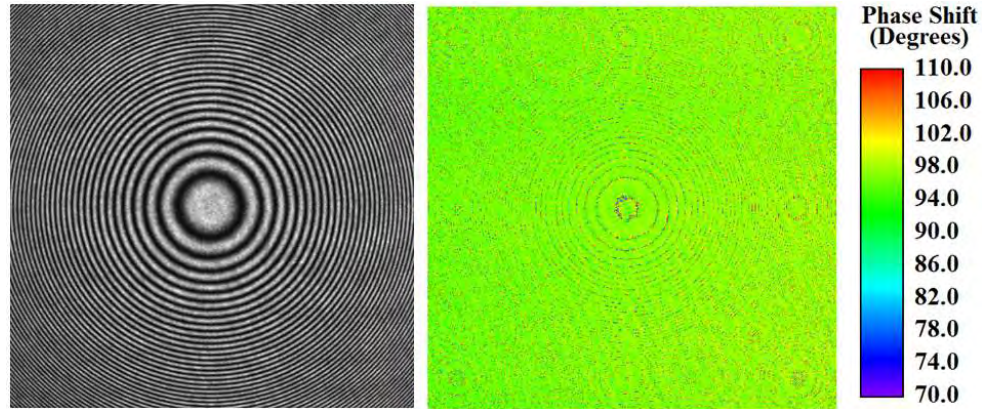


FIGURE 5.4 One phase shifted interferogram (Left) and the calculated phase shift at each pixel (Right)

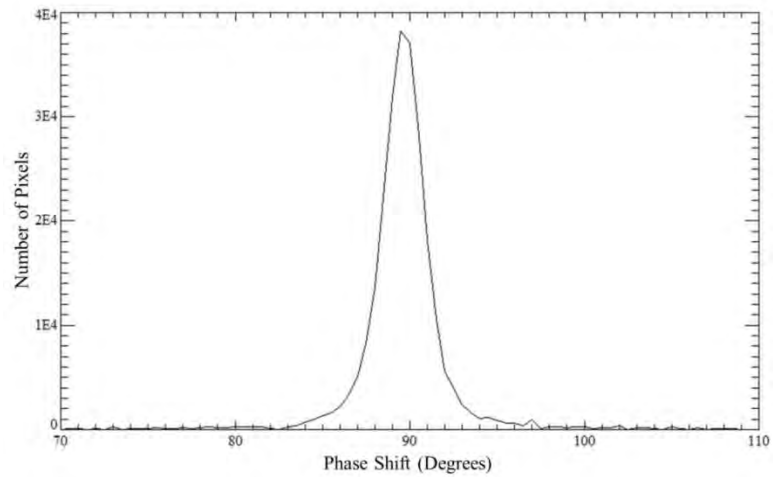


FIGURE 5.5 Histogram of number of pixels at each phase shift in degrees

Modulation Map: This procedure calculates the modulation at each pixel across the wavefront by utilizing Equation 2.20 as shown in FIGURE 5.6. This is useful for identifying the un-aliased fringes in order to determine the starting point for the phase unwrapping procedure. It also serves as a method of identifying areas of the

interferogram where the modulation may drop below the threshold needed for proper unwrapping, around 20%. The modulation can occasionally be improved by changing the laser intensity or by recalibrating the phase shifter.

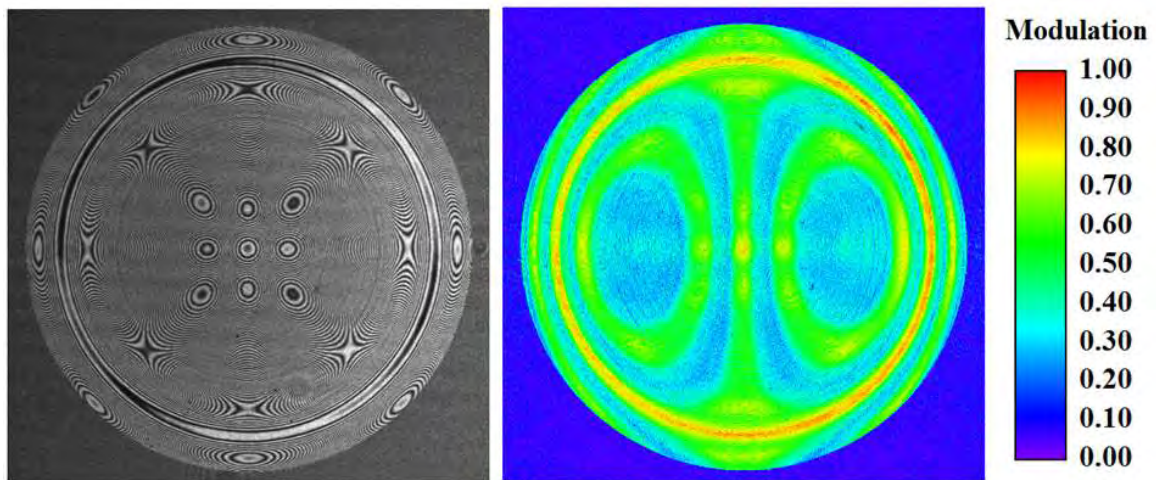


FIGURE 5.6 An example of a sub Nyquist Interferogram and the Modulation Map calculated from 5 phase shifted interferograms.

Send Data to Zemax: The last procedure under the Tools menu option is a second GUI, which controls the exporting of Zernike data to the Zemax model, FIGURE 5.7. The user is required to input some basic information about the surface and then the GUI exports the data into the Zemax model as either a Zernike Sag Surface or a Zernike Phase Surface. The program is capable of reading in and exporting data from the original SNI GUI, as well as measurements made using a WYKO laser-based Fizeau interferometer. The WYKO measurements are fit to Zernikes, using the Zernike fitting procedure which will be discussed in Chapter 5.2.4. This procedure was used to transfer surface measurements of the interferometer components, such as the imaging lens and beam splitter, into the Zemax models of the interferometer.

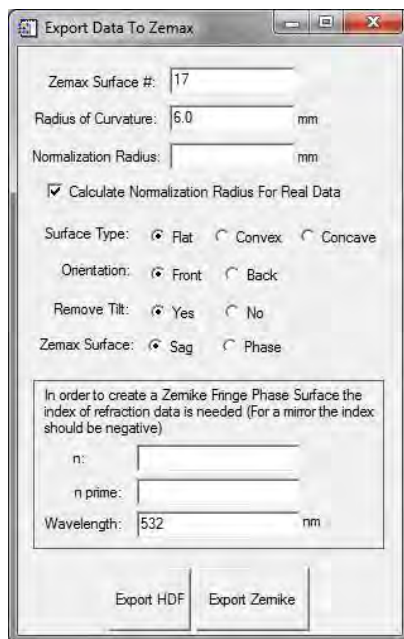


FIGURE 5.7 GUI used to export Zernike Phase or Sag data into Zemax.

5.2.2 GUI Side Panel

The GUI Side Panel, FIGURE 5.8, is the area on the left hand side of the GUI which is always visible. It contains several small text boxes for user input as well as a large text box used by the various procedures to output information back to the user. Below is a description of the purpose of each of the text boxes.

Number of Rays: In **Simulated Data** mode, it is the number of sampled points across the pupil over which the wavefront will be calculated. The number should be odd since Zemax uses an odd number of data points across the wavefront to ensure there is always a chief ray corresponding to the data point (0,0). In **Real Data** mode the value is initially set to 511, which is the number of pixels across one dimension of the recorded interferograms. The sparse array sensor used in the system consists of a 512x512 array of

pixels; however one row and column are dropped in order to maintain the odd number of elements across the array required by Zemax. The number may become smaller after the initial capturing and unwrapping of the interferograms by down sampling the resulting wavefront.

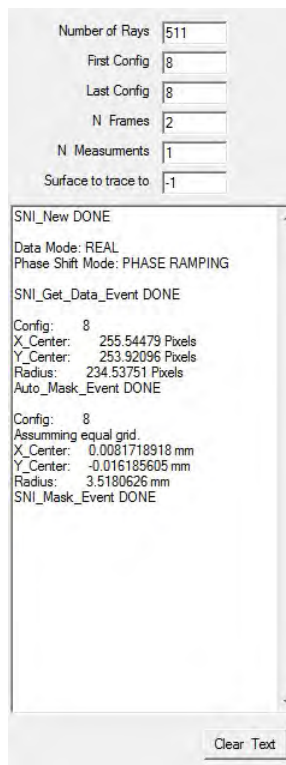


FIGURE 5.8 GUI Side Panel

First Config & Last Config: The data array for each wavefront is stored in IDL in a data structure delimited by a configuration number. While in **Simulated Data** mode the configuration number corresponds to the Zemax configuration from which ray data will be imported. When collecting **Real Data**, these values are simply an indexed label for each consecutive measurement. Multiple wavefront measurements, with different configuration values, can be stored in the program's memory, simultaneously allowing for comparisons and mathematical operations to be performed between wavefront data

arrays. Several of the GUI procedures can be run on multiple configurations specified by the **First Config** and **Last Config** text boxes.

N Frames: This value is only used when collecting **Real Data** in phase stepping mode. It is the number of camera frames to capture and average at each phase step.

N Measurements: This value is only used in **Real Data** mode. It represents the number of phase shifted measurements to be recorded and averaged for a given wavefront.

Surface to Trace to: This value is only used in **Simulated Data** mode. It represents the Zemax surface number at which the OPD_z should be calculated. It can be any positive integer that has a corresponding valid Zemax surface in the currently opened Zemax lens file. Additionally a value of -1 can be used to specify the last surface of the Zemax lens file.

Clear Text: The clear text button at the bottom of the side panel simply erases the information previously written to the output text box.

5.2.3 GUI Acquire Data Tab

The Acquire Data tab, shown in FIGURE 5.2, contains procedures for acquiring, masking and plotting data. It also contains the main display window of the GUI which is used to display interferograms, masks and OPD data back to the user.

Get Data: This button starts the process for acquiring data. When the GUI is set in **Simulated Data** mode this procedure gathers the user inputs from the GUI calls a Zemax Extension, SNI_Get_Opd_Zemax.exe, which commands Zemax to trace rays. Rays are traced by pupil coordinates therefore; the location of the stop and ray aiming setting in Zemax will alter the rays which are traced. The OPD_Z data is then saved by the executable to a text file to be read into the GUI. In **Real Data** mode, this button signals the interferometer to begin collecting data. The program commands the high voltage output card to begin ramping or stepping the PZT attached to the reference mirror. Simultaneously, it triggers the frame grabber to begin recording images from the sub-Nyquist camera. The images are saved to the computer's hard drive to be unwrapped.

Read Data: This Procedure reads text files containing OPD data into the computer's memory so that it is accessible to all the GUI procedures. The data can either be generated from the Zemax model or previously saved real OPD data from the interferometer. When OPD_Z is read in from Zemax the sign discrepancy mentioned in Chapter 3.2.1 is taken into consideration.

Laser Power: There are two buttons under the laser power label **Check** and **Adjust**. The **Check** procedure simply captures one image from the camera and displays a false color image to verify that the sensor is not saturating before recording a set of phase shifted interferograms. The **Adjust** procedure opens communications with the laser via a

terminal emulation program PROCOMM PLUS® by Symantec (Mountainview, CA).

The terminal interface allows the laser properties, such as output power or operating temperature to be viewed and adjusted. The terminal program runs in a DOS shell which supersedes all other operations by the computer and must be terminated before using other GUI functions.

Live Video: This procedure opens a window displaying live video from the sub-Nyquist camera via the frame grabber. However, only one program is allowed to communicate with the frame grabber at a time. The window must be closed before phase shifted images can be captured by the GUI. Usually viewing the live video on the computer monitor isn't necessary as it can always be seen on a monitor attached directly to the camera's analog output.

Mask: The software allows for several masking options in order to define the size of the test wavefront at the detector and to remove bad data points. Masks are stored in the data structure as a separate binary array that is used by subsequent programs to determine which pixels contain valid wavefront data. The function of the mask button depends on which of the nearby radio buttons are highlighted, FIGURE 5.9. There are two lists of options; one selects the type of data to use for the masking algorithm while the other selects the type of mask to generate. The types of data that can be used are Fringe, OPD, Modulation, or Sum, which is the simple addition of the 5 phase shifted images. The

types of masks that can be generated are **Outside Circle**, **Inside Circle**, **Ellipse**, **Rectangle**, **Clear** and **Show**.

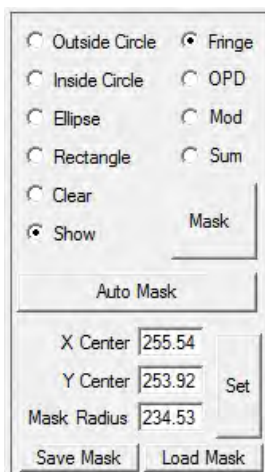


FIGURE 5.9 GUI Mask Panel

The **Outside Circle** and **Ellipse** options have the same basic operation. The user is shown an image corresponding to the data type selection, FIGURE 5.10 (Left). The user manually selects pixels around the edge of the wavefront, FIGURE 5.10 (Right).

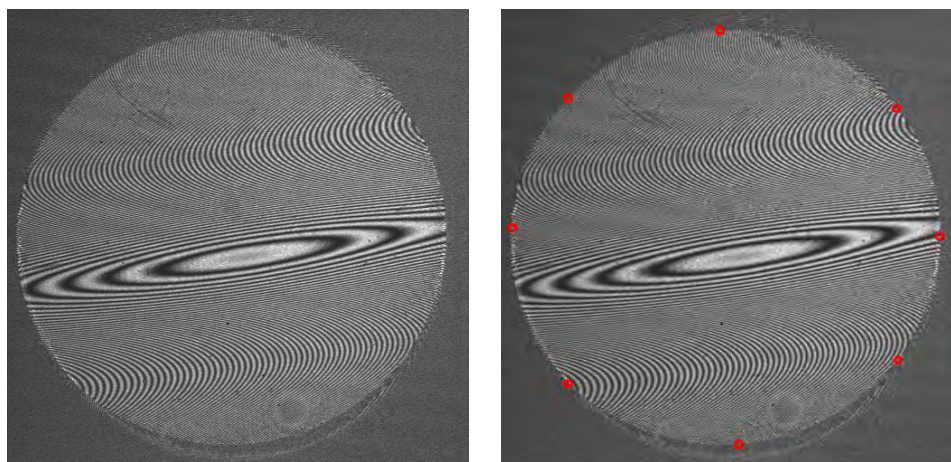


FIGURE 5.10 An interferogram generated by a cylindrical surface (Left) The user selected edge pixels shown in red (Right)

The (x, y) coordinates of these pixels are then fit to a circle or ellipse using least squares fitting, FIGURE 5.11 (Left), (Strebel et al, 1994). Finally, mask is applied to the data type selected, FIGURE 5.11 (Right). For a circle the (x,y) coordinate of the center of the mask as well as the mask radius in pixels and millimeters are printed out to the user and are displayed in their corresponding text boxes on the GUI. These values can then be modified by the user and updated using the “Set” button. These values are used by other procedures in the GUI such as the Zernike fitting procedure. In the case of an ellipse the coordinates of the center, the length of the major and minor semi-axes as well as the rotation of the major axis in radians is printed back to the user. The elliptical masking is useful when measuring toric parts, which either do not have a circular edge, or have a circular edge but due to aberration of the imaging lens map to an ellipse on the detector. The mask radius for elliptical masks is set to be the length of the major semi-axis. The program that performs the least squares fitting of the selected points to an ellipse was written by Craig B. Markwardt of NASA/GSFC. (Markwardt)

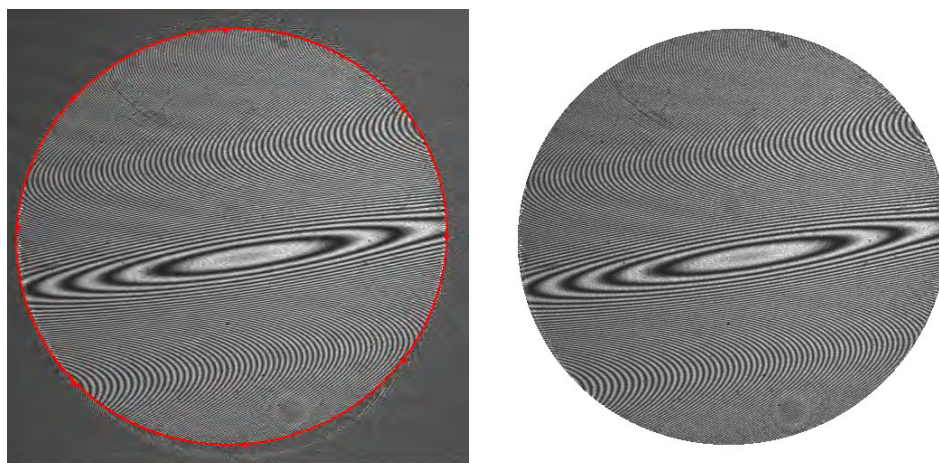


FIGURE 5.11 Ellipse calculated by least square fit of selected pixels (Left) Elliptical mask applied to interferogram (Right)

The **Inside Circle** calculation is the same as the Outside circle, but the mask center and radius properties are not updated. It is simply used to mask off a circular interior obstructions. For the **Rectangle** option two pixels corresponding to the diagonal corners of a rectangle are selected by the user and pixels on the interior are blocked. The **Clear** option simply resets the mask array so no data points are masked, while the **Show** option displays the current mask and displays center and radius information to the user. Additionally masks can be saved as bitmaps and text files and then loaded back into the GUI using the **Save Mask** and **Load Mask** buttons respectively.

Auto Mask: The auto mask program fits a circular mask to the edge of the test beam. It is used to reduce the time required and the variability associated with the user manually selecting the edge of the interferogram. The input to the auto mask procedure is an image of the test wavefront where the reference arm of the interferometer has been blocked.

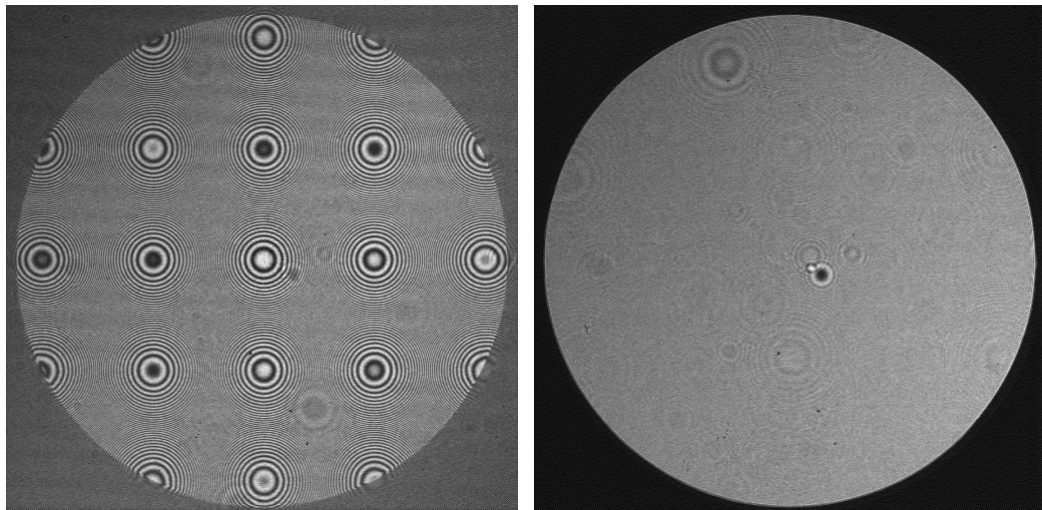


FIGURE 5.12 An example of a sub-Nyquist interferogram (Left), an image of test wavefront with the reference arm blocked (Right)

The program starts with an image of the test wavefront, FIGURE 5.12 (Right). A histogram of the intensity of the pixels in the image of the test wavefront is calculated, where intensity is measured in digital counts of the camera ranging from 0 to 255. This histogram is smoothed with a moving average of five digital counts as shown in FIGURE 5.13.

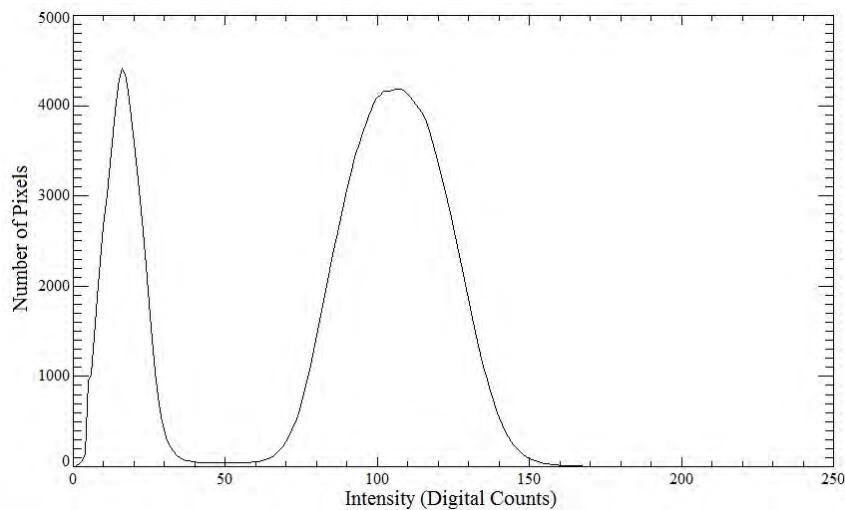


FIGURE 5.13 Histogram of the intensity of the test wavefront.

The peak on the left represents the dark pixels around the outside of the test wavefront, where the peak on the right represents the bright pixels from inside the wavefront. The histogram array is scanned from both directions to find the locations of the two peaks. The minimum value between these two peaks is then found, which in this example is at 50 digital counts. This value becomes the threshold value for the image of the test wavefront, shown in FIGURE 5.15 (Left), where the white regions are pixels with intensity less than the threshold and black regions have intensity greater than the

threshold. Next, a Sobel edge detection filter is applied to the image. The Sobel edge detection filter uses the convolution of two 3x3 kernels, FIGURE 5.14, with the input image in order approximate the gradient of the image in two orthogonal directions, Equation 5.2, 5.3. (Acharya & Ray, 2005)

-1	0	1
-2	0	2
-1	0	1

1	2	1
0	0	0
-1	-2	-1

FIGURE 5.14 Sobel Horizontal (Left) and Vertical (Right) Convolution Kernels

$$\nabla I_{x,y} = \begin{bmatrix} \frac{\delta I_{x,y}}{\delta x} \\ \frac{\delta I_{x,y}}{\delta y} \end{bmatrix} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} \quad 5.2$$

The convolution of the two kernels with the input image yields the two components of the gradient G_x for the horizontal and G_y for the vertical directions, Equation 5.3.

$$\begin{aligned} G_x &= I_{i-1,j-1} + 2I_{i-1,j} + I_{i-1,j+1} - [I_{i+1,j-1} + 2I_{i+1,j} + I_{i+1,j+1}] \\ G_y &= I_{i-1,j-1} + 2I_{i,j-1} + I_{i+1,j-1} - [I_{i-1,j+1} + 2I_{i,j+1} + I_{i+1,j+1}] \end{aligned} \quad 5.3$$

The magnitude of the gradient is then calculated using the approximation shown in Equation 5.4. The result of the Sobel edge detection is then scaled to be a binary image of the edges of the test wavefront as shown in FIGURE 5.15 (Right).

$$|\nabla I_{x,y}| = \sqrt{G_x^2 + G_y^2} \approx |G_x| + |G_y| \quad 5.4$$

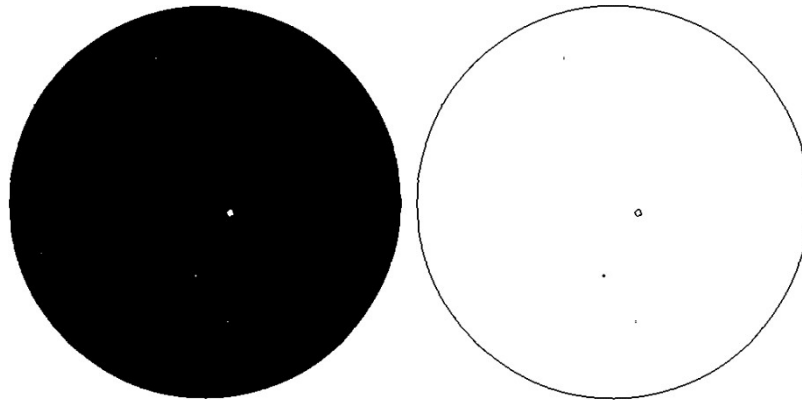


FIGURE 5.15 Image after the threshold (Left), Edges highlighted by the Sobel filter (Right)

Next the radius and center the test wavefront are calculated by least squares fitting the highlighted edge pixels from the Sobel filter to a circle, which is shown as the blue line in FIGURE 5.16 (Left). The poor fitting is due to detected edges inside the test wavefront being included in the least squares fit. These internal edge points shift the calculated center and radius of the circle. In order to solve this problem, two additional circles are calculated one with a twenty pixel larger radius and one with a twenty pixel smaller radius but the same center location; represented by the red and green circles respectively in FIGURE 5.16 (Left). Then all edge points inside the smaller green circle or outside the larger red circle are removed and the least square fitting is repeated. On the second iteration the radius of the smaller and larger circle are only nineteen pixels from the circle calculated by the least squares fit. The process of fitting and throwing away edge points is repeated until the radius of the larger and smaller circles are within one pixel of the least squares fit. The solution to the least squares fitting converges on the edge of the test wavefront as shown in FIGURE 5.16 (Right). Using a starting value of twenty pixels for the difference between the radii of the circles was found experimentally to be a practical

starting point. It is large enough that under normal circumstances no points on the actual edge are removed, but is also small enough that the entire process excessively time consuming. The final mask is then applied to the original image of the test wavefront and the captured interferograms, FIGURE 5.17.

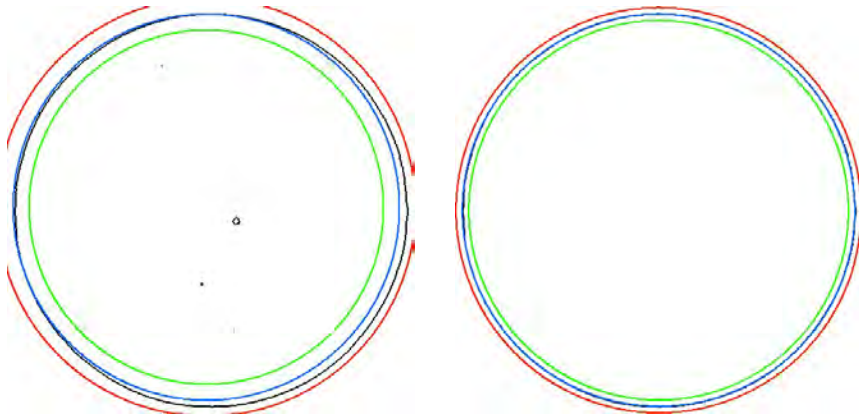


FIGURE 5.16 Initial least squares fit (Left), and after a few iterations (Right)

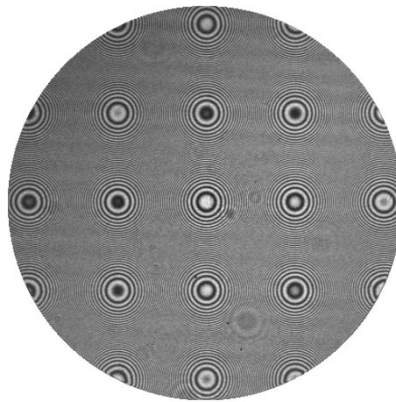


FIGURE 5.17 Final mask applied to the interferogram

To test the accuracy of the auto masking procedure simulated wavefronts were generated using the Zemax model. The margin of error for the auto mask procedure prediction of the mask radius and center location was less than $\pm 1\mu\text{m}$, compared to

around $\pm 5\mu\text{m}$ from the manual process. The manual process however depends heavily on the number of points selected by the user and how carefully those points are selected.

Unwrap Fringes: This button will start the sub-Nyquist unwrapping program. The basic process is as follows; first the wrapped phase is calculated from 5 phase shifted interferograms utilizing the Schwinder-Hariharan algorithm discussed in Chapter 2.1.2, Equation 2.18. Next, a simple path dependent PSI unwrapping procedure is applied to the wrapped phase. This ensures that the next step, a path dependent SNI unwrapping procedure, starts in a region free of discontinuities. Finally, a path independent SNI unwrapping procedure is run to clean up errors produced by the first routine. The two sub-Nyquist unwrapping procedures are used, because while prone to errors, the path dependent phase unwrapping routine is several orders of magnitude faster than the path independent routine. The directional PSI and SNI procedures were based on those written by Andrew Lowman and Rob Gappinger, (Gappinger 2002) (Lowman 1995).

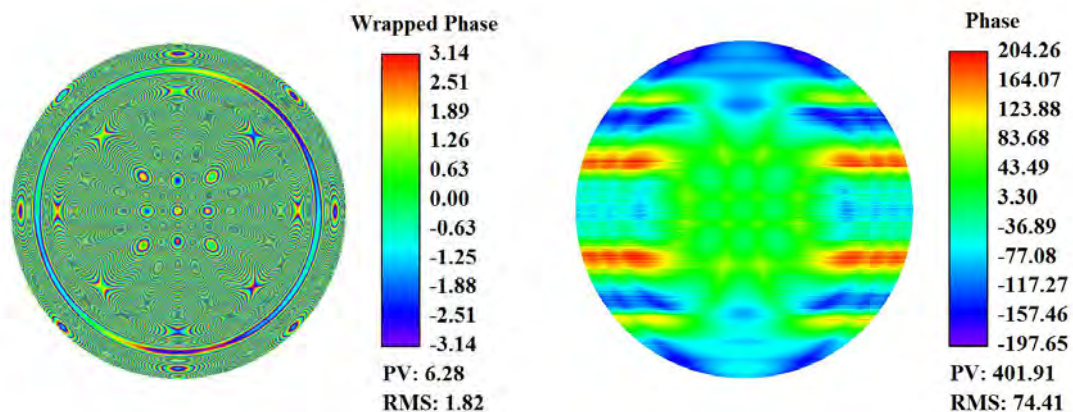


FIGURE 5.18 Wrapped phase produced from a sub-Nyquist sampled interferogram (Left) After PSI unwrapping process (Right)

At the start of the procedure the user is presented with an image of the wrapped phase calculated using the Schwinder-Hariharan algorithm, FIGURE 5.18 (Left). Where the unwrapped phase value, Φ_i at a given pixel is equal to the wrapped phase value, ϕ_i , plus an integer multiple, n_i , of 2π , Equation 5.5.

$$\Phi_i = \phi_i + 2n_i\pi \quad n_i = 0, \pm 1, \pm 2, \dots \quad 5.5$$

The user is prompted to identify a pixel in an unaliased section of the wavefront to serve as the starting point of all the unwrapping algorithms. In the pattern shown in FIGURE 5.18 (Left), the zero order fringes are located at the center of the interferogram as well as the continuous ring near its edge. Next, the path dependent PSI unwrapping algorithm is applied. It selects solutions for n_i , in Equation 5.5, such that the phase of the pixel being unwrapped, Φ_i , is within $\pm\pi$ of the phase of previously unwrapped pixel, Φ_{i-1} , Equation 5.6.

$$n_i = \text{Round} \left[\frac{\Phi_{i-1} - \phi_i}{2\pi} \right] \quad 5.6$$

This algorithm fails when the fringe frequency is greater than the Nyquist frequency, as shown in FIGURE 5.18 (Right). In the area around the starting pixel, where the fringe frequency is near zero, the PSI algorithm produces the correct phase value. The PSI algorithm only needs to provide the properly unwrapped phase over a small 3x3 pixel box in order to provide a starting location for the SNI unwrapping. The path followed by the PSI algorithm is the same as the SNI algorithm which will be discussed next.

As discussed in Chapter 2.4, the sub-Nyquist phase un-wrapping algorithm assumes the slope of the wavefront is continuous in order to unwrap phase changes of greater than π/pixel . The path dependent SNI unwrapping algorithm calculates the slope of the wavefront from the two previous unwrapped pixels. It then uses this slope to find the projected phase value, Φ_i^* , at the current pixel by assuming the phase at the three pixels are collinear, Equation 5.7 & 5.8, where Δx is the pixel spacing.

$$\frac{\Phi_i^* - \Phi_{i-1}}{\Delta x} = \frac{\Phi_{i-1} - \Phi_{i-2}}{\Delta x} \quad 5.7$$

$$\Phi_i^* = 2\Phi_{i-1} - \Phi_{i-2} \quad 5.8$$

The algorithm then selects the solution for n_i , in Equation 5.5, such that the phase of the pixel being unwrapped, Φ_i , is within $\pm\pi$ of the projected phase value, Equation 5.9.

$$n_i = \text{Round} \left[\frac{(2\Phi_{i-1} - \Phi_{i-2} - \phi_i)}{2\pi} \right] \quad 5.9$$

The SNI unwrapping will fail when the projected value and actual phase value are separated by more than π . This occurs when the slope changes by more than $\pi/\text{pixel/pixel}$. This algorithm uses the first derivative of the wavefront to calculate the projected phase value. The range could be extended by assuming that the second derivative is also continuous. Then the projected phase value could be calculated using the phase of the last three unwrapped pixels and a quadratic rather than linear fit (Greivenkamp 1987).

The SNI unwrapping algorithm first unwraps the phase vertically along 3 columns, as shown in FIGURE 5.19 (Left). Then the horizontal rows are unwrapped, first to the right and then left, outward from the unwrapped columns, as shown in FIGURE 5.19(Right)-FIGURE 5.20 (Right). The reason the path dependent algorithm is fast is because the slope continuity assumption is only applied along the path of the unwrapping algorithm. However if an error is made in phase unwrapping process it will propagate to the edge of the wavefront creating a streak, as shown in FIGURE 5.20 (Right). These errors generally occur at pixels with low modulation. This is also why the vertical direction is unwrapped first since the MTF of the SNI sensor is higher in the vertical direction. Additionally when deciding on an orientation for testing a non-symmetric interferograms the higher fringe frequencies should be aligned with the vertical sensor direction.

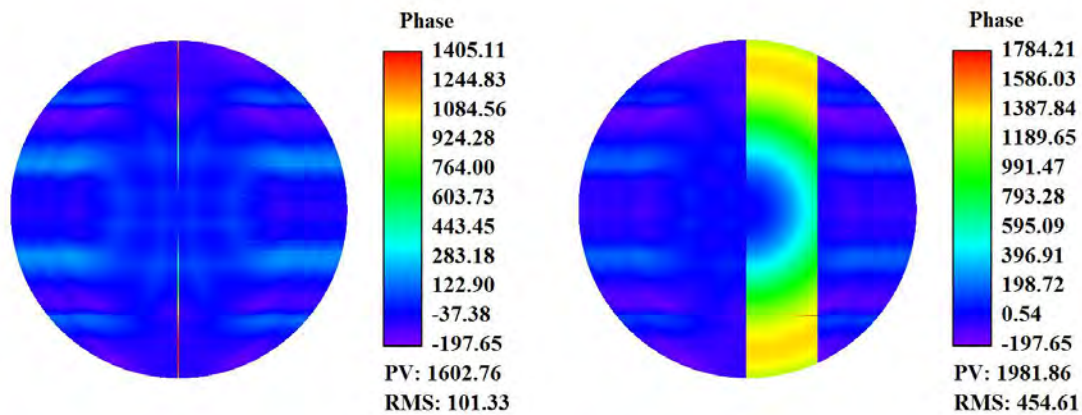


FIGURE 5.19 Unwrapped three central columns (Left), next unwrap all rows to the right (Right)

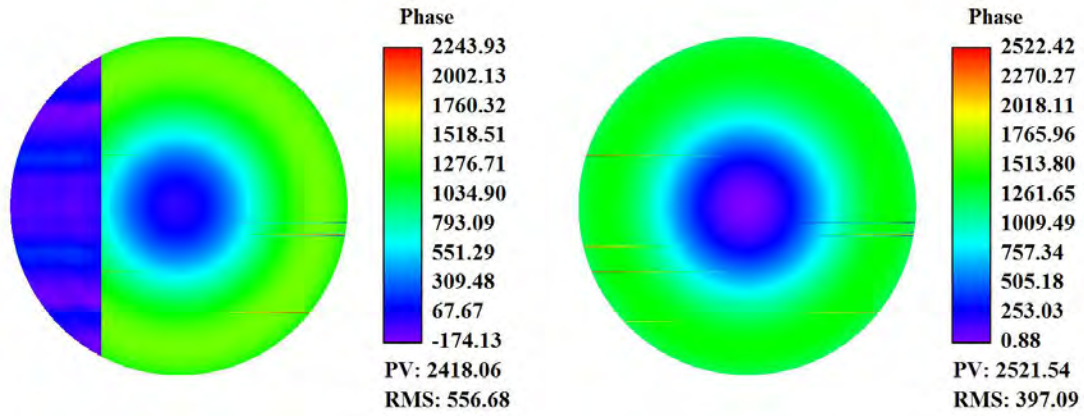


FIGURE 5.20 Unwrap all rows to the left (Left), Output of the path dependent SNI unwrapping procedure (Right)

Next, the path independent procedure is run. This procedure differs from the previous unwrapping procedure because the slope continuity assumption is applied in multiple directions simultaneously in order to unwrap around problem pixels. In this algorithm, a given pixel has eight possible directions from which it could be unwrapped; two horizontal, two vertical and four diagonal. The algorithm first separates the pixels into two groups; good pixels that have been properly unwrapped and bad pixels which have not. It does this by calculating the number of 2π 's needed to make the wavefront slope continuous at each pixel from all eight directions, Equations 5.10-5.17. Any pixel that has a non-zero $n_{i,j,k}$ is assumed to be improperly unwrapped and added to the bad pixel group.

$$n_{i,j,1} = \text{Round} \left[\frac{(2\Phi_{i-1,j-1} - \Phi_{i-2,j-2} - \phi_{i,j})}{2\pi} \right] \quad 5.10$$

$$n_{i,j,2} = \text{Round} \left[\frac{(2\Phi_{i-1,j} - \Phi_{i-2,j} - \phi_{i,j})}{2\pi} \right] \quad 5.11$$

$$n_{i,j,3} = \text{Round} \left[\frac{(2\Phi_{i-1,j+1} - \Phi_{i-2,j+2} - \phi_{i,j})}{2\pi} \right] \quad 5.12$$

$$n_{i,j,4} = \text{Round} \left[\frac{(2\Phi_{i,j-1} - \Phi_{i,j-2} - \phi_{i,j})}{2\pi} \right] \quad 5.13$$

$$n_{i,j,5} = \text{Round} \left[\frac{(2\Phi_{i,j+1} - \Phi_{i,j+2} - \phi_{i,j})}{2\pi} \right] \quad 5.14$$

$$n_{i,j,6} = \text{Round} \left[\frac{(2\Phi_{i+1,j-1} - \Phi_{i+2,j-2} - \phi_{i,j})}{2\pi} \right] \quad 5.15$$

$$n_{i,j,7} = \text{Round} \left[\frac{(2\Phi_{i+1,j} - \Phi_{i+2,j} - \phi_{i,j})}{2\pi} \right] \quad 5.16$$

$$n_{i,j,8} = \text{Round} \left[\frac{(2\Phi_{i+1,j+1} - \Phi_{i+2,j+2} - \phi_{i,j})}{2\pi} \right] \quad 5.17$$

Additionally, any island or group of pixels that isn't connected to the starting pixel of the directional unwrapping routine by properly unwrapped pixels in the horizontal or vertical directions is added to the bad pixel group. Finally, the bad pixel group is expanded to include all pixels within two pixels of a bad pixel. This is done in order to capture all of the pixels which were used in the unwrapping of a bad pixel. FIGURE 5.21 (Left) shows the binary array where properly unwrapped or good pixels have a value of one and are shown in black. While improperly unwrapped or bad pixels have a value of zero and are shown in white. This array can be applied to the previously unwrapped phase from FIGURE 5.20 (Right) to suppress the streaks as shown in FIGURE 5.21 (Right).

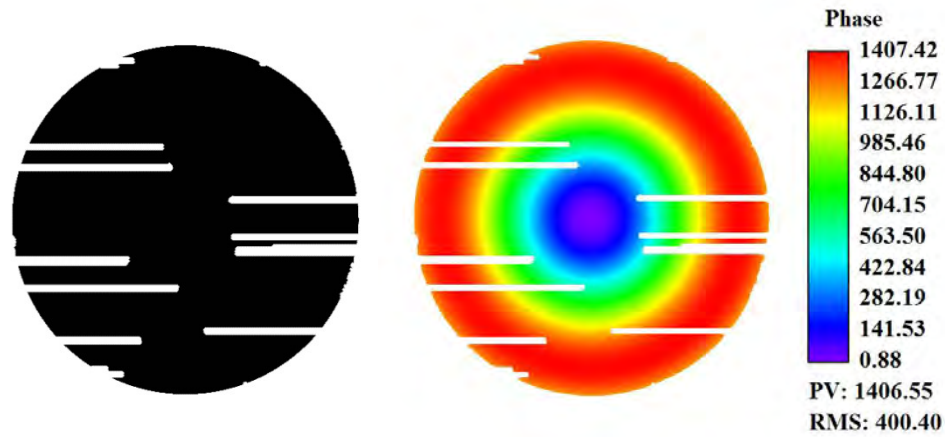


FIGURE 5.21 Bad pixels binary array (Left), The unwrapped phase with bad pixels removed (Right)

Now that the properly unwrapped pixels have been separated from the improperly unwrapped pixels, the algorithm begins the process of unwrapping the bad pixels. First the phase for all the bad pixels is set back to their wrapped phase value from the Schwinder-Hariharan algorithm. Next, all of the wrapped pixels which border good pixels in at least three directions are identified as shown in FIGURE 5.22 (Left). These pixels are unwrapped by calculating $n_{i,j,k}$ from all the neighboring directions which contain properly unwrapped phase values. If the calculated unwrapped phase values from all the available directions match then the phase value is accepted and the pixel is added into the good pixel group, expanding the area of the wavefront that has been properly unwrapped. The algorithm then identifies the wrapped pixels located on the new border and the process is repeated. With every iteration the unwrapped area of the wavefront grows, FIGURE 5.22 (Right), until all pixels have been unwrapped or until no new pixels are successfully unwrapped. The initial check is repeated and points that do not have a continuous slope in all available directions are masked producing the

$$I(x, y, \delta) = \frac{1}{2} + \frac{1}{2} \cos \left[\frac{2\pi}{\lambda} OPD(x, y) + \delta \right] \quad \delta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \quad 5.18$$

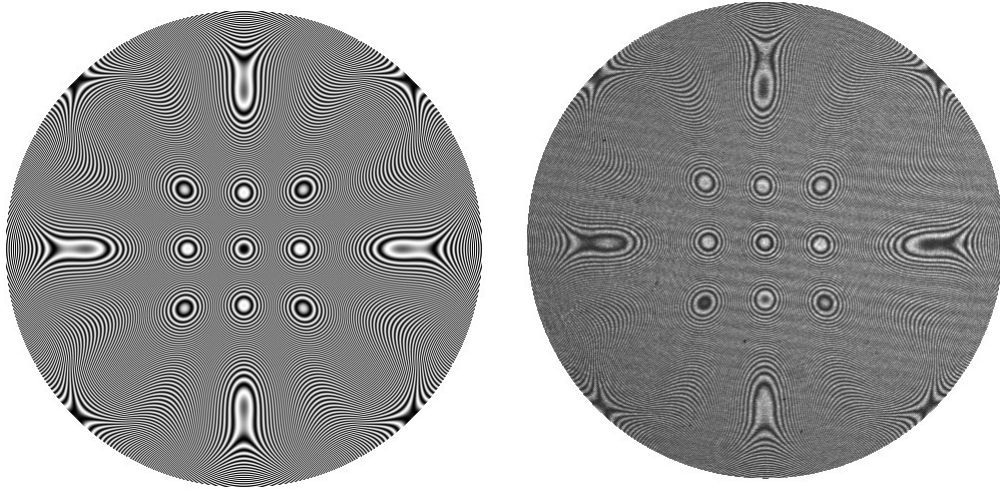


FIGURE 5.24 Calculated interferogram from the OPD generated by ray tracing the Zemax model (Left) and from the interferometer (Right).

Magnification Test: The magnification test program is an automated process for determining the distance from the imaging lens to the detector for use in the reverse optimization process. The magnification target is an aluminum diamond turned mirror. It consists of 20 alternating flat and convex rings 1.2mm wide, FIGURE 5.25.

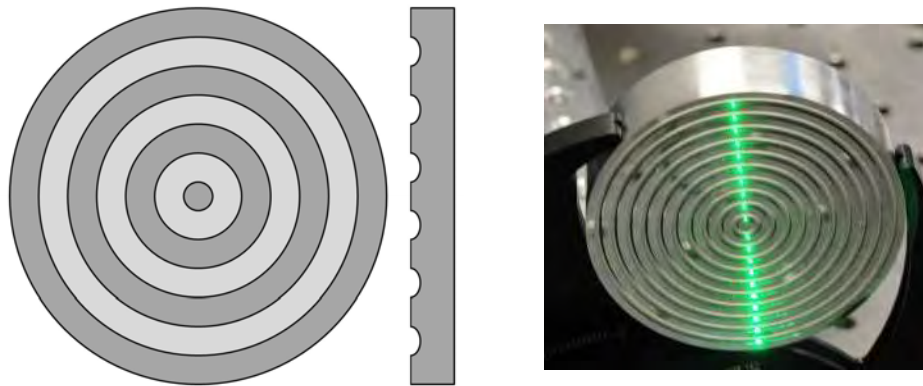


FIGURE 5.25 A cartoon of the flat magnification target as viewed from the front and in cross-section (Left), and an image of the actual magnification target (Right).

The procedure requires a set of phase shifted images, where the magnification target is placed in the test arm of the interferometer at a conjugate plane to the detector. When testing a part directly against the flat reference the target is placed at the test plane. When the diverger lens is used in the test arm, in order to collect light off the test surface, the magnification target must be placed at the plane conjugate to the test part through the diverger lens, which is the intermediate pupil location. Once the phase shifted interferograms have been collected the magnification target procedure detects the edges of the rings, calculates their size on the sensor, passes this data to Zemax, and runs the Zemax optimization procedure to calculate the distance from the imaging lens to the detector.

The process of detecting the edges of the rings is similar to the auto mask routine, however the modulation calculated from the phase shifted images, Equation 2.20, is used rather than the intensity. The modulation image is used because it contains high contrast between the concave rings and the flat rings. The high contrast is the result of the concave rings focusing and then sending the light out of the interferometer while the flat rings reflect the light back into the imaging arm of the interferometer, FIGURE 5.26. It is easy to determine when the magnification target is conjugate to the detector by observing when light from the center of the concave rings is visible in the image of the target on the detector. A Sobel edge enhancement filter is used to detect the edges of the rings in the modulation image. After the edge enhancement the image is scaled to produce a binary image FIGURE 5.27 (Left). The edges of the rings are clearly visible

but light from the bottom of the concave portions of the magnification target is also enhanced. Next the center of the target is found by first isolating the largest connected area in the binary array FIGURE 5.27 (Right). These pixels, which all lay on the edge of one of the rings, are then fit to a circle using least squares fitting algorithm. One of the outputs of this algorithm is the location of the center of the ring and thus the center of the magnification target.

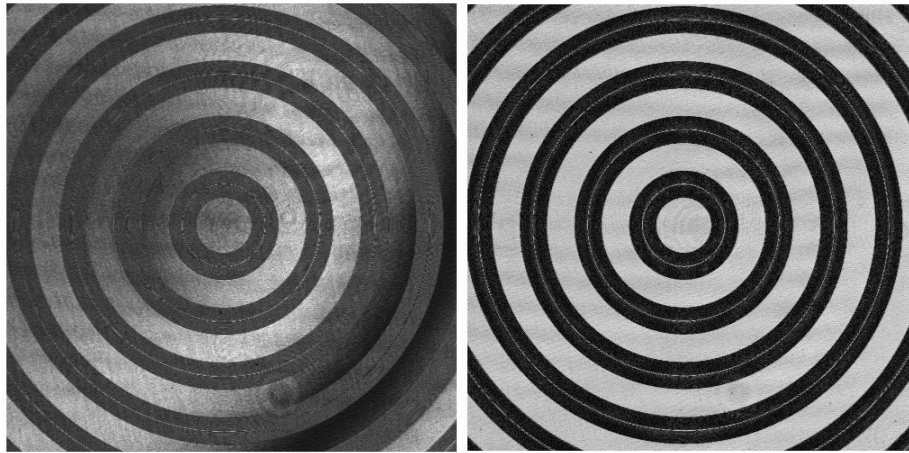


FIGURE 5.26 A single interferogram produced by the magnification target (Left) and the modulation image (Right)

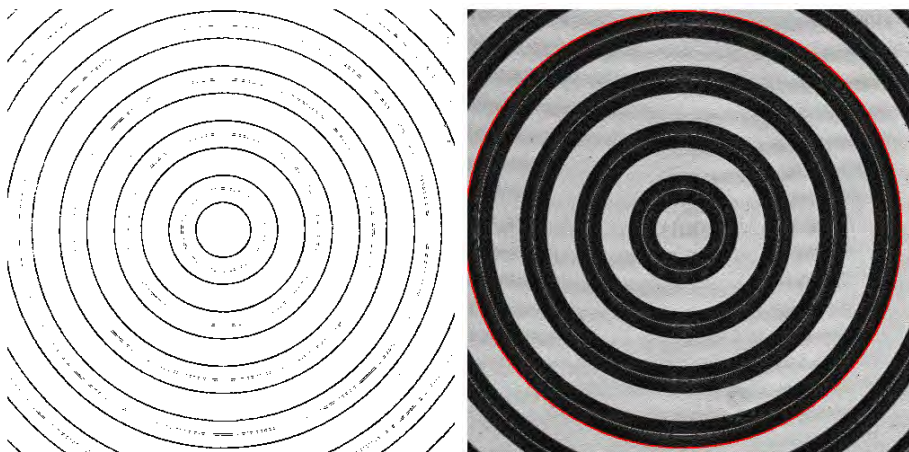


FIGURE 5.27 Binary array of the edges detected using the Sobel filter (Left) and the largest connected region of the binary array overlaid on the modulation map (Right).

Next a histogram of the distance of each edge pixel in the binary image from the center of the target is calculated, FIGURE 5.28. The histogram is then compared to a curve representing the threshold at each radial distance that must be met in order for a ring to be detected. This threshold will remove the false edge points created by the light that is captured from the bottom of the concave rings. Additionally it allows the edge pixels of each ring to be separated from one another based on their distance from the center of the target. The radius in millimeters of each ring at the sensor is then calculated by least squares fitting the pixels associated with each ring edge to a circle, FIGURE 5.29.

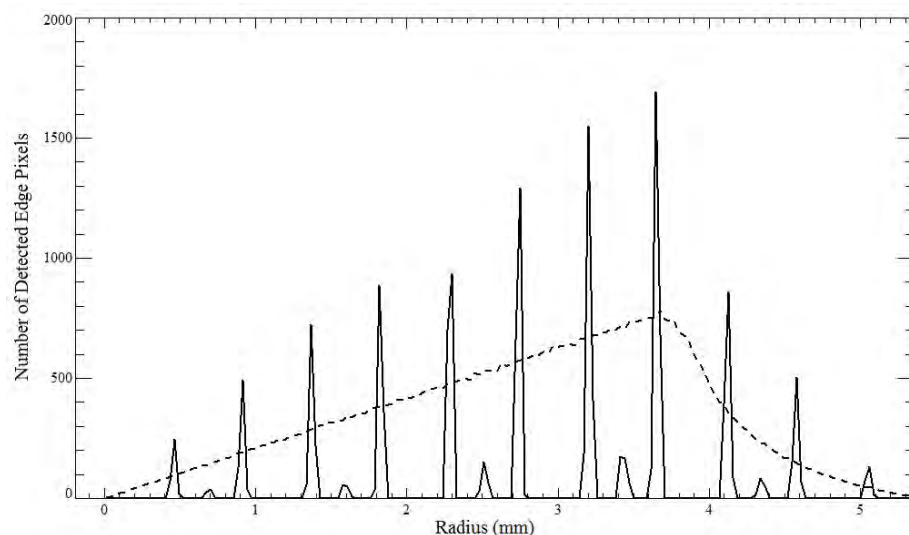


FIGURE 5.28 Histogram of the radius of the edge pixels. Only edge points which correspond to peaks above the dashed threshold curve are kept. The small peaks represent light reflecting off the center of the concave rings, which are ignored.



FIGURE 5.29 The final detected rings color coated in order of size and overlaid on top of the modulation data (Left), and an image generated by the Zemax model after optimization (Right).

The procedure then opens a Zemax model of the interferometer containing only the magnification target, the imaging lens and the detector. It exports the measured radius of each ring at the detector into the Zemax merit function. In the Zemax model, rays are traced parallel to the optical axis from the edges of the magnification target rings through the imaging lens and onto the detector. The magnification target is set to be the aperture stop of the system. The distance between the magnification target and the imaging lens is set to be a variable while the distance between the imaging lens and the detector is set to be a pupil solve, which ensures that the detector is conjugate to the magnification target. The merit function is setup to minimize the distance between the measured radius of each ring and the radius achieved through ray tracing. Then by running the Zemax optimization procedure the imaging distances which minimize the error between the measured and known magnification ring diameters is found. The resulting modeled image of the magnification target after optimization is shown in FIGURE 5.29 (Right).

Ideally, the spacing of a known imaging setup could be measured with this procedure in order to determine the accuracy of this technique. However this would require a second more accurate method of measuring the spacing. Since another method wasn't available three different experiments were conducted. The first experiment tested the procedure's ability to recover a known separation of the imaging lens and the detector generated using the Zemax model of the interferometer. A simple model of the interferometer was set up utilizing two configurations, one with the magnification target and one with a flat mirror representing the reference arm. Both configurations contained the imaging lens and the detector surface. The **Get OPD** procedure was then used to generate OPD_Z from both configurations at the detector plane. The OPD_Z data was then read into the GUI and the configurations were subtracted from each other to generate the OPD between the test and reference wavefronts. From this data, a set of interference fringes was generated using the **Generate Fringes** procedure. Then the **Magnification Test** procedure was run on these interferograms as if they had been captured with the interferometer. This process was repeated over the range of expected imaging lens to detector spacing of 210mm to 250mm. Over this range the procedure was able to recover the distance between the lens and the sensor to within $10\mu\text{m}$, FIGURE 5.30. The error was biased towards overestimating the distance with a mean of $+5\mu\text{m}$ and a standard deviation of $3\mu\text{m}$. There is also a trend towards increasing error as the distance between the imaging lens and detector is increased. This is probably due to the fact that fewer rings are available for the test procedure to analyze as the magnification is increased.

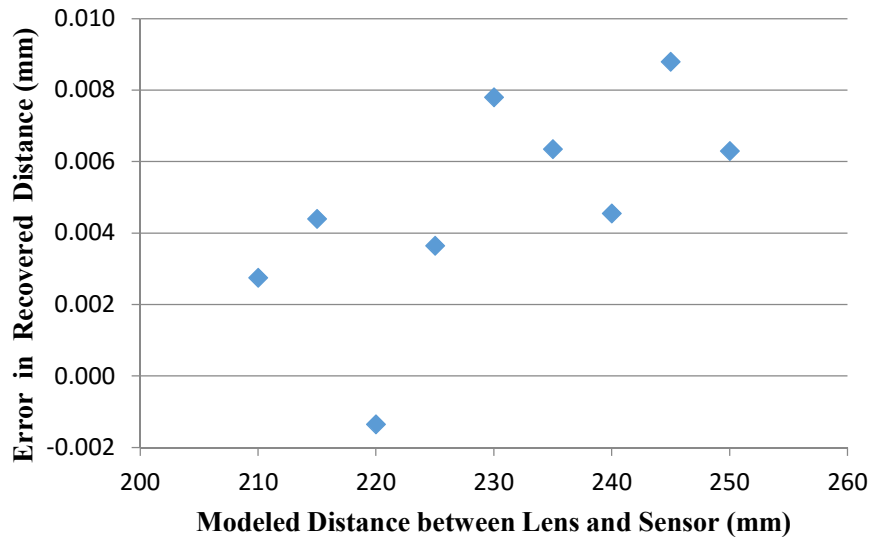


FIGURE 5.30 Error in the recovering the lens to detector distance from modeled data.

The second experiment, performed using data collected from the real interferometer, tested the procedure's sensitivity to error in the location of the magnification target. Ideally, the procedure's ability to measure the distance between the imaging lens and detector would not depend on the accuracy to which the magnification target is positioned relative the imaging lens. For this test, the lens and the detector were fixed in place while the magnification target was shifted over a $\pm 100\text{mm}$ range. It is fairly easy to align the magnification target to within a couple of millimeters of the conjugate plane of the sensor by looking for the presence of the bright light ring formed at the bottom of the concave rings in the interferograms. Therefore the range of magnification target position error tested in this procedure is significantly larger than would be experienced in a real measurement. The shifts introduced into the magnification target were only coarsely measured using the millimeter demarcations on the test arm optical rail. Over the 200mm range of motion the recovered imaging lens to detector spacing changed by less than

$\pm 10\mu\text{m}$ from the measured spacing with no shift of the magnification target, FIGURE 5.31. Additionally over a $\pm 30\text{mm}$ range of magnification target motion the measured spacing changed by less than $\pm 3\mu\text{m}$, FIGURE 5.31

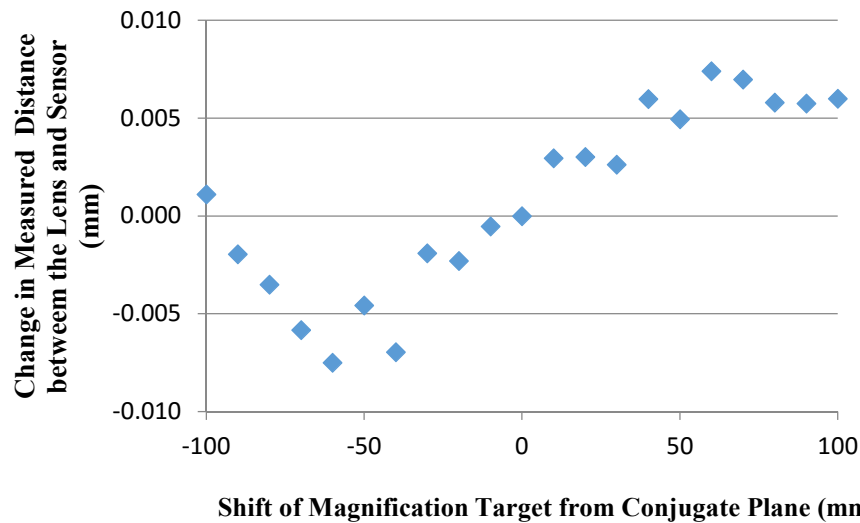


FIGURE 5.31 Sensitivity to error in placement of the magnification target.

The third test was performed to check the ability of the program to measure a known change in the lens to detector spacing. For this test, the sensor was shifted 9 times in 2.5mm increments from approximately 250mm to 227.5mm. The shifts were introduced and measured using the micrometer on the linear stage mounted to the camera, which had a $1\mu\text{m}$ Vernier scale. For each sensor position the magnification target was moved to new the conjugate plane. The mean error in the measured shift was $3\mu\text{m}$ with a standard deviation of $1\mu\text{m}$, FIGURE 5.32, which is on the order of the minimum incremental motion of the stage. Therefore the measured error in the introduced shifts could be the result of the inability to accurately position the stage.

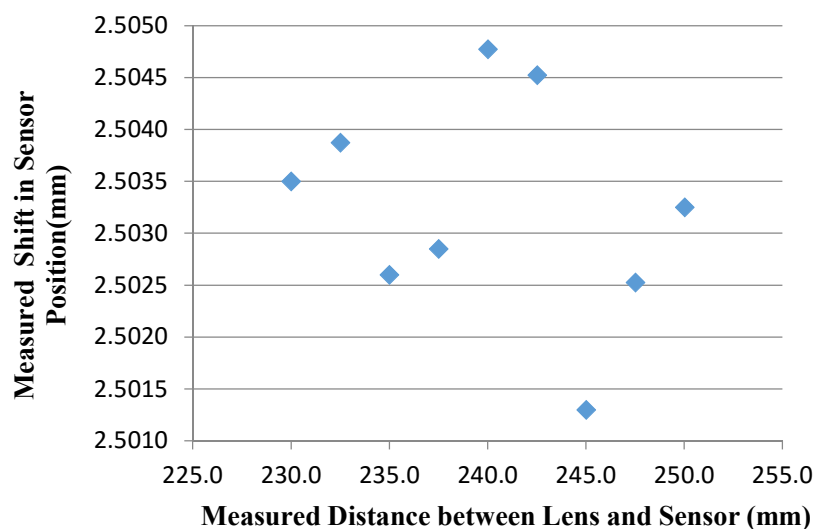


FIGURE 5.32 Measured shift introduced between the lens and the detector.

Plot: The plot button simply generates figures from the stored OPD data and the Zernike fit. The user has the option for the type of figure to generate, a scaled image or a shaded surface model, as well as the color and the plotting range, FIGURE 5.33. Some of the plotting procedures used were modified versions of programs written by Dan Smith and Greg Williby (Smith 2008) (Williby 2003). Examples of figures generated by the plotting procedures are shown in FIGURE 5.34.



FIGURE 5.33 Plotting Options Commands

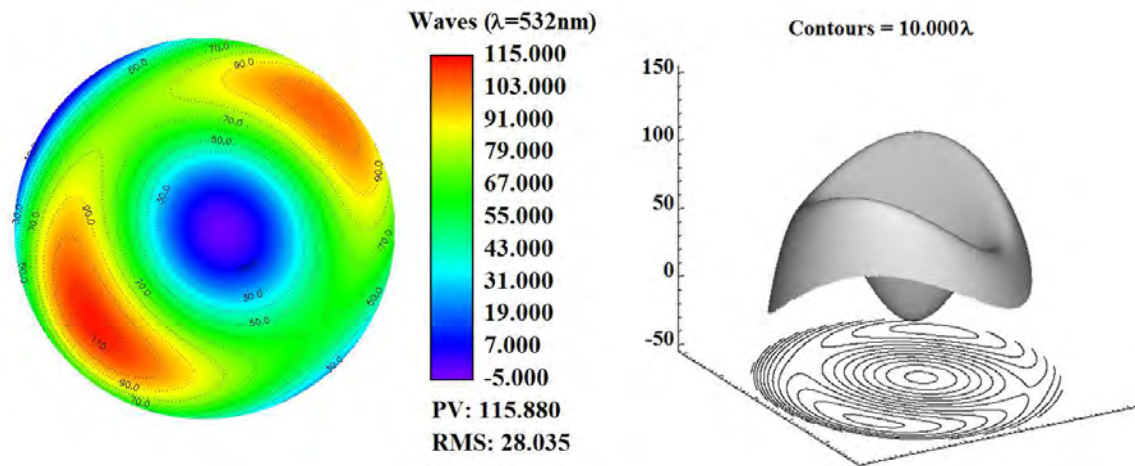


FIGURE 5.34 Examples of figures that can be generated using the plotting procedure

5.2.4 GUI Zernike Tab

The second tab of the GUI, FIGURE 5.35, contains all the procedures related to fitting the stored OPD data to Zernike polynomials. Zernike polynomials form a complete orthogonal basis over the interior of a unit circle and their use to represent wavefront data is well established (Born & Wolf 1999). Zernike polynomials are represented in polar coordinates as the product of a radial function and angular function, Equations 5.19 and 5.20.

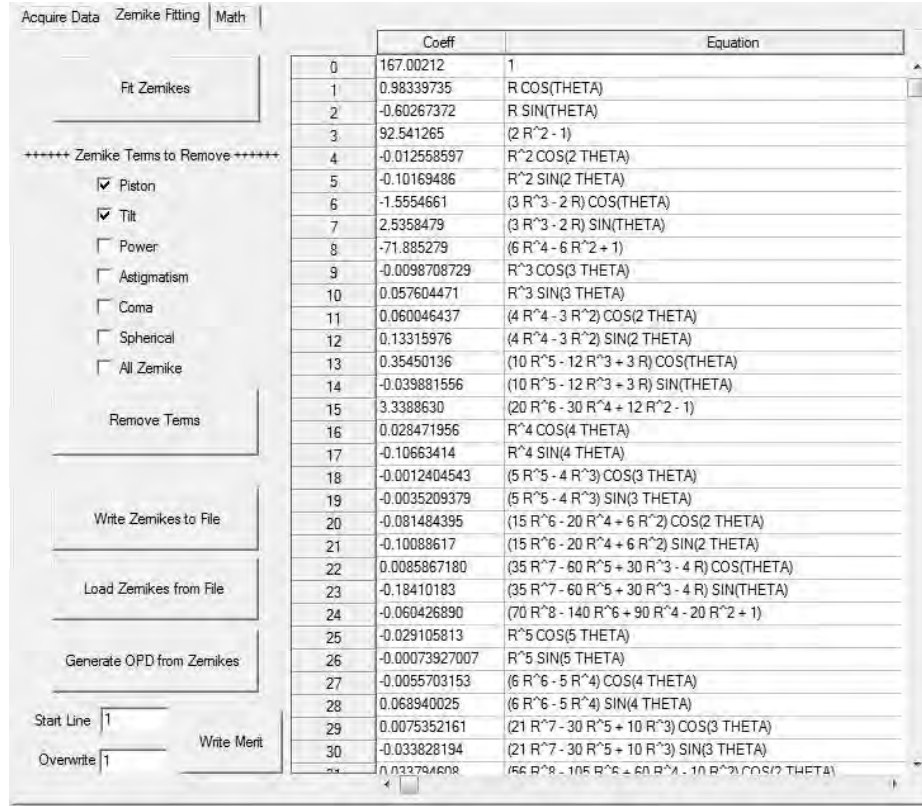


FIGURE 5.35 GUI Zernike Fitting Tab

$$\iint Z_n^l Z_{n'}^{l'*} \rho d\rho d\theta = \frac{\pi}{n+1} \delta_{ll'} \delta_{nn'} \quad 5.19$$

$$Z_n^l = R_n^l(\rho) e^{il\theta} \quad 5.20$$

There are several different conventions used for both ordering and normalizing Zernike polynomials. The Zernike polynomials used by the GUI were selected to match those used by Zemax. The Zemax manual refers to them as the University of Arizona or FRINGE Zernikes, after the software package in which they first appeared (Zemax LLC, 2011). They do not represent a complete set of polynomials through a specific order; rather they are made up of a low-order complete set with additional higher order radial polynomials to permit better fitting of errors commonly encountered during the

fabrication of large aspheric optical components (Shannon 1997). The thirty-seven Zemax Zernike Fringe polynomials differ slightly from those outlined by Shannon in that they are normalized to have unity magnitude at the edge of the pupil, TABLE 5.1. Additionally, θ is measured counter clockwise from the +xaxis, Equations 5.21-5.22,, and ρ is normalized to the edge of the pupil (Zemax LLC, 2011).

$$x = \rho \cos(\theta) \quad 5.21$$

$$y = \rho \sin(\theta) \quad 5.22$$

A Zernike polynomial fit of the OPD_Z is a convenient way for representing measured surface and wavefront data in the Zemax model for reverse optimization and ray tracing, utilizing the Zemax surface types Zernike Fringe Phase and Zernike Fringe Sag. These surface types allow for much faster ray tracing when compared to modeling data using the Zemax Grid Sag or Grid Phase surface types. This is because the Grid type surfaces require an N x N array of data to be stored in memory, from which the ray intercept and surface normal, needed for ray tracing, must be calculated by interpolating between points. Whereas a Zernike surface uses a relatively small number of coefficients, 37, to provide a closed form description of the surface. However, Zernikes do not do a very good job of representing high frequency errors such as fabrication errors associated with diamond turned optics (Wyant & Creath 1992). However, the primary interest of this work is surface form or shape. In spite of this, utilizing a Zernike polynomial representation of the measured wavefront is useful in the initial stages of reverse optimization process when the high frequency components are ignored. Additionally, a

Zernike surface can be combined with a grid surface in Zemax so that the bulk of the phase or sag is modeled by the Zernike surface and the unfit residual data is modeled with the grid surface.

#	$Z(\rho, \theta)$	#	$Z(\rho, \theta)$
0	1	19	$(5\rho^5 - 4\rho^3)\sin(3\theta)$
1	$\rho \cos(\theta)$	20	$(15\rho^6 - 20\rho^4 + 6\rho^2)\cos(2\theta)$
2	$\rho \sin(\theta)$	21	$(15\rho^6 - 20\rho^4 + 6\rho^2)\sin(2\theta)$
3	$2\rho^2 - 1$	22	$(35\rho^7 - 60\rho^5 + 30\rho^3)\cos(\theta)$
4	$\rho^2 \cos(2\theta)$	23	$(35\rho^7 - 60\rho^5 + 30\rho^3)\sin(\theta)$
5	$\rho^2 \sin(2\theta)$	24	$70\rho^8 - 140\rho^6 + 90\rho^4 - 20\rho^2 + 1$
6	$(3\rho^3 - 2\rho)\cos(3\theta)$	25	$\rho^5 \cos(5\theta)$
7	$(3\rho^3 - 2\rho)\sin(3\theta)$	26	$\rho^5 \sin(5\theta)$
8	$6\rho^4 - 6\rho^2 + 1$	27	$(6\rho^6 - 5\rho^4)\cos(4\theta)$
9	$\rho^3 \cos(3\theta)$	28	$(6\rho^6 - 5\rho^4)\sin(4\theta)$
10	$\rho^3 \sin(3\theta)$	29	$(21\rho^7 - 30\rho^5 + 10\rho^3)\cos(3\theta)$
11	$(4\rho^4 - 3\rho^2)\cos(2\theta)$	30	$(21\rho^7 - 30\rho^5 + 10\rho^3)\sin(3\theta)$
12	$(4\rho^4 - 3\rho^2)\sin(2\theta)$	31	$(56\rho^8 - 105\rho^6 + 60\rho^4 - 10\rho^2)\cos(2\theta)$
13	$(10\rho^5 - 12\rho^3 + 3\rho)\cos(\theta)$	32	$(56\rho^8 - 105\rho^6 + 60\rho^4 - 10\rho^2)\sin(2\theta)$
14	$(10\rho^5 - 12\rho^3 + 3\rho)\sin(\theta)$	33	$(126\rho^9 - 280\rho^7 + 210\rho^5 - 60\rho^3 + 5\rho)\cos(\theta)$
15	$20\rho^6 - 30\rho^4 + 12\rho^2 - 1$	34	$(126\rho^9 - 280\rho^7 + 210\rho^5 - 60\rho^3 + 5\rho)\sin(\theta)$
16	$\rho^4 \cos(4\theta)$	35	$252\rho^{10} - 630\rho^8 + 560\rho^6 - 210\rho^4 + 30\rho^2 - 1$
17	$\rho^4 \sin(4\theta)$	36	$924\rho^{12} - 2772\rho^{10} + 3150\rho^8 - 1680\rho^6 + 420\rho^4 + 42\rho^2 -$
18	$(5\rho^5 - 4\rho^3)\cos(3\theta)$		

TABLE 5.1 Zemax Zernike Fringe Polynomials

Fit Zernikes: This procedure fits Zernike polynomials to the OPD data, and was based on a program written by Dan Smith (Smith 2008). While Zernike polynomials are orthogonal over a unit circle of continuous data they are not orthogonal for discrete data. However, this problem can be overcome by oversampling the wavefront and utilizing a

least squares fit method (Wang & Silva 1980). The matrix method of least squares fitting minimizes the sum of the squares of the difference between the measured OPD and Zernike Polynomial fit. In which, the elements of the Vandermonde matrix, Z , represent each of the 37 fringe Zernikes evaluated at every data point location inside the unit circle. This matrix maps a vector of unknown Zernike coefficients, a , onto a vector containing the OPD value of every pixel inside the unit circle, Equations 5.23-5.24.

$$Za = OPD \quad 5.23$$

$$\begin{pmatrix} Z_0(\rho_0, \theta_0) & Z_1(\rho_0, \theta_0) & \cdots & Z_{36}(\rho_0, \theta_0) \\ Z_0(\rho_1, \theta_1) & Z_1(\rho_1, \theta_1) & \cdots & Z_{36}(\rho_1, \theta_1) \\ \vdots & \vdots & \ddots & \vdots \\ Z_0(\rho_N, \theta_N) & Z_1(\rho_N, \theta_N) & \cdots & Z_{36}(\rho_N, \theta_N) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{36} \end{pmatrix} = \begin{pmatrix} OPD(\rho_0, \theta_0) \\ OPD(\rho_1, \theta_1) \\ \vdots \\ OPD(\rho_N, \theta_N) \end{pmatrix} \quad 5.24$$

The unit circle is defined by the mask radius calculated using the previously discussed masking procedures. The assumption of oversampling the wavefront means that $N \gg 37$ so that the Zernike coefficients representing the least squares solution can be calculated by computing the pseudo inverse of Z , Equation 5.25. An example of the OPD from a measured wavefront, the Zernike fit and the difference between them showing the unfit data is shown in FIGURE 5.36.

$$a = (Z^T Z)^{-1} Z^T OPD \quad 5.25$$

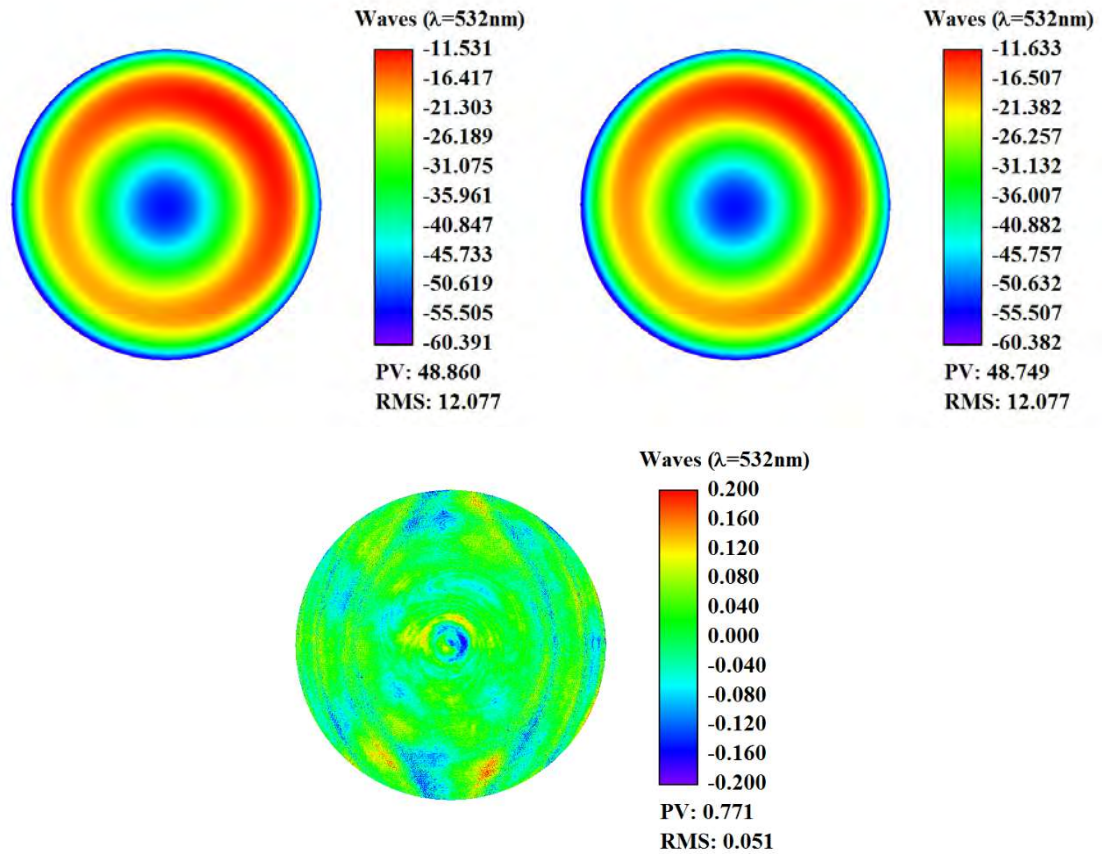


FIGURE 5.36 OPD (Top Left), Zernike Polynomial Fit (Top Right), Difference (Bottom)

One crucial aspect of this Zernike fitting procedure is that it must match the Zernike fitting performed by Zemax, so that when the coefficients are exported from the IDL GUI into Zemax they represent the same surface. To show this, a grid phase surface was used to generate several random wavefronts in Zemax. These wavefronts were then fit to Zernike polynomials in Zemax using the ZERN merit function operand. Additionally, the Get OPD procedure was used to import the OPD_Z data produced by ray tracing the grid phase surface into the GUI. This data was then fit to Zernike polynomials using the Fit Zernikes procedure. The individual coefficients generally matched to $1 \times 10^{-8} \lambda$ with the exception of the piston term. Since the chief ray always has an OPD_Z equal to zero

the piston term is lost when the ray data is exported to Zemax. The Zernike polynomials from both the Zemax fit and the IDL fit were then loaded into a new Zemax file as Zernike phase surfaces. One set of coefficients was then multiplied by negative one so that the phase of the second surface would cancel the phase of the first surface yielding the difference between the two fitting procedures. An example of this process is shown in FIGURE 5.37.

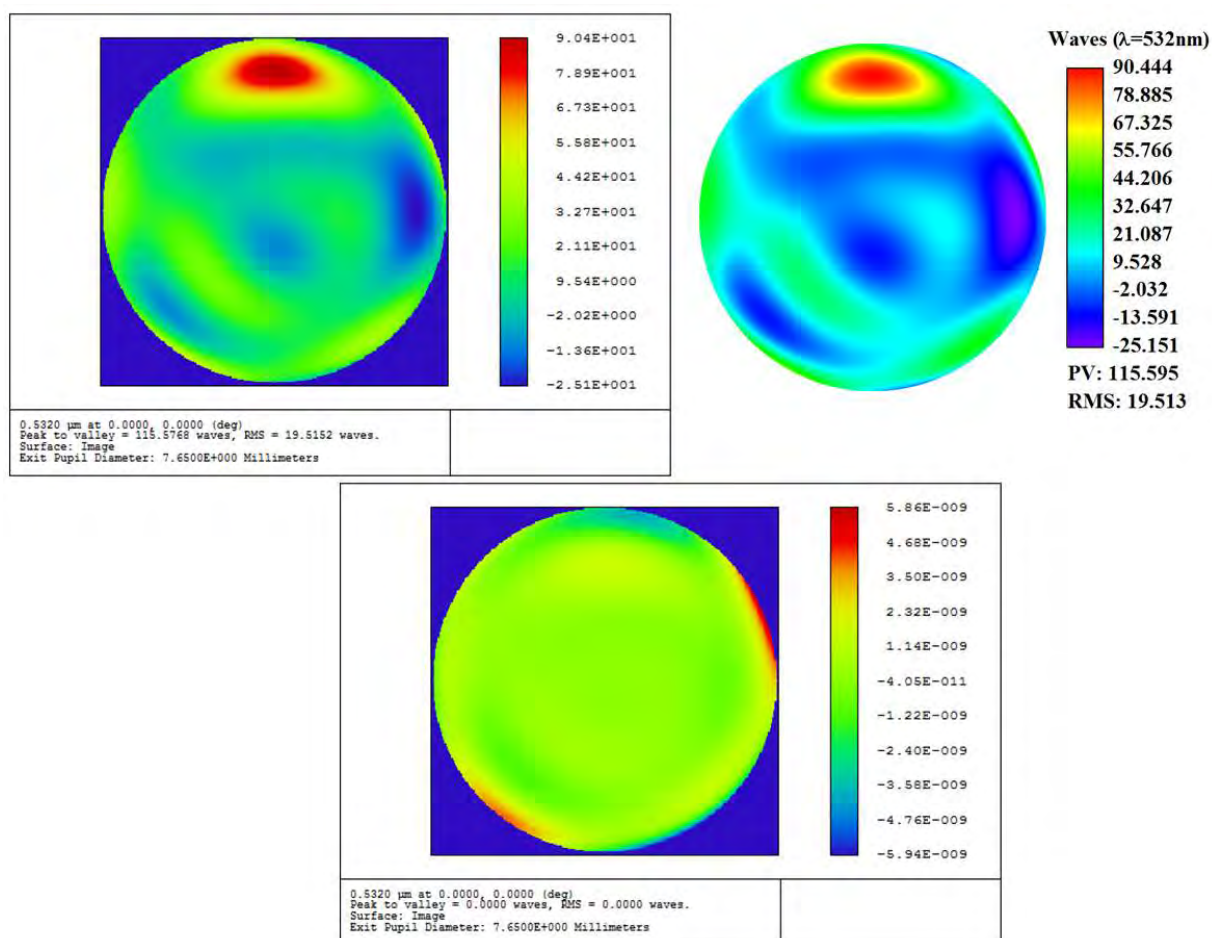


FIGURE 5.37 Zemax Zernike Fit (Top Left), IDL GUI Zernike Fit (Top Right), Difference (Bottom), all plots are in units of waves at 532nm.

Remove Terms: This procedure allows the terms selected on the left, such as piston or tilt to be removed from the OPD data. After the OPD data has been fit to Zernike polynomials a Zernike surface is constructed from the terms selected to be removed by Equation 5.24. This Zernike surface is then subtracted from the OPD. Additionally, there is an option to remove all Zernikes from the OPD data, which is useful to create a OPD array containing only the high frequency components of the wavefront.

Write Zernikes to File: Saves a text file containing the Zernike coefficients and the information on the mask used to set the nomination radius that the Zernikes were fit over.

Load Zernikes From Files: This procedure is used to load a previously saved set of Zernike coefficients, either from Zemax or from the GUI, into the system memory.

Generate OPD from Zernikes: This procedure overwrites the stored OPD data with the data that exactly matches the wavefront represented by the Zernike coefficients. It is used to be able to recreate the OPD data after loading a set of Zernike coefficients or to remove the high frequency components of the wavefront.

Write Merit: This procedure exports the Zernike coefficients to the Zemax merit function. Each coefficient is loaded into the merit function as the target value of a ZERN merit function operand, starting at the line number indicated by **Start Line**. See the Zemax manual for more information (Zemax LLC, 2011). If the **Overwrite** option is set

to one, the entire merit function will be overwritten, if it is set to zero the new lines will be appended onto the end of the merit function.

5.2.5 GUI Math Tab

The last tab on the GUI is the math tab, FIGURE 5.38. It contains procedures that allow for the manipulation of the stored OPD data and comparison of multiple OPD data sets. Most of the buttons on this tab are self-explanatory.

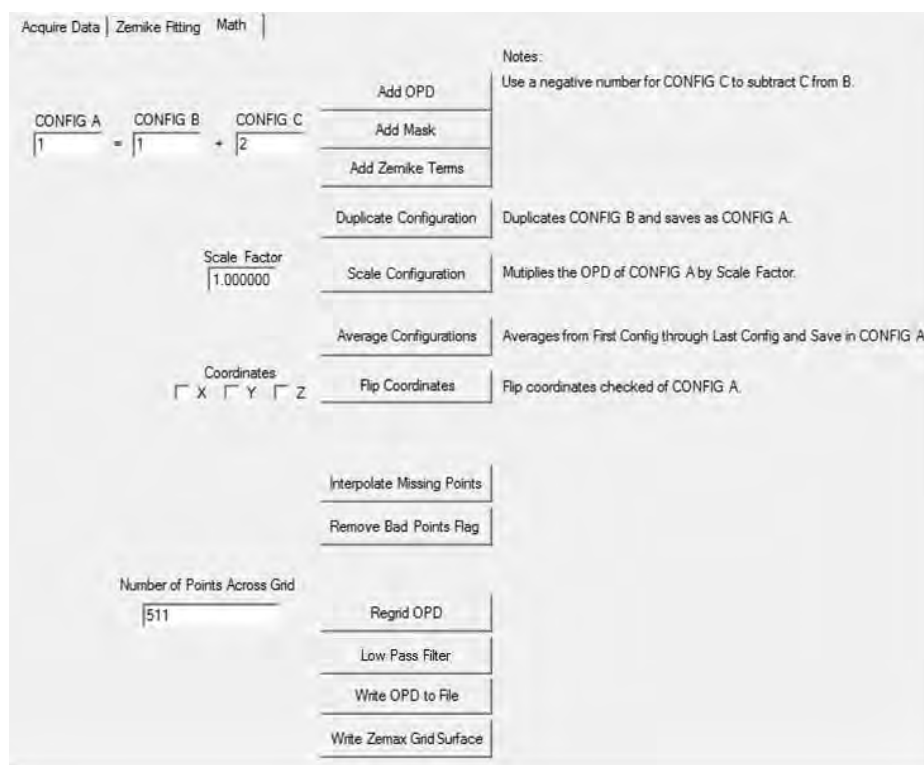


FIGURE 5.38 GUI Math Tab

Add OPD: The procedure is used to add or subtract the OPD data stored in two configurations. The configuration numbers are specified in the **CONFIG A**, **CONFIG B** and **CONFIG C** text boxes.

Add Mask: This procedure is used to combine the masks of two configurations.

Add Zernike Terms: This procedure adds or subtracts the Zernike coefficients of two configurations.

Duplicate Configuration: This button simply copies all the data stored in the structure for **CONFIG B** and saves to the structure for **CONFIG A**.

Scale Configuration: This button multiplies the OPD data stored in **CONFIG A** by the value written in the **Scale Factor** text box. By default, the OPD data is stored in units of waves at 532nm, this procedure is useful for scaling the wavefront data into other units, such as mm, before outputting the data to Zemax. It is also used to convert the OPD data to surface sag data.

Average Configurations: The procedure simply averages the OPD data from multiple configurations.

Flip Coordinates: This procedure is used to flip the OPD data about the x, y or z axis.

Interpolate Missing Points: Data points that were not successfully unwrapped are masked off using a separate bad pixel binary array than the normal mask array. This is

done so that the improperly unwrapped data points are not used for subsequent calculations such as Zernike fitting. When exporting data back to Zemax these points should be filled in. This procedure uses the surrounding OPD data and the Zernike fit to interpolate the missing OPD data for the bad pixels.

Remove Bad Points Flag: This procedure simply drops the bad pixel flag that is placed on non-properly unwrapped pixels. It is meant to be used after the pixels are replaced using the **Interpolate Missing Points** procedure.

Regrid OPD: This procedure is used to take the data array and scale it down so that Zemax can more easily use the data. For the reverse optimization process the OPD for several configurations is loaded into Zemax as either a Zernike phase surface or as a Grid Phase surface. Using the entire 511 pixel descriptions of the wavefront at the detector causes the optimization procedure to slow down and stop at local minima, especially during the initial stages of the reverse optimization. This procedure allows smaller array sizes to be generated by interpolating the measured OPD data.

Write OPD to File: This procedure simply saves the OPD data for the configurations specified by **First Config** to **Last Config** to a text file. This can be read back in at a later time with the **Read Data** button.

Write Zemax Grid Surface: This procedure writes out the OPD data into a file format that is compatible with the Zemax surface types Grid Sag and Grid Phase. The OPD data is stored in units of waves at 532nm. For a Grid Phase surface at 532nm the units will match those of Zemax, if a Grid Sag surface is required the OPD must first be scaled using the appropriate **Scale Factor**.

5.3 Simple Ray Trace Model

Before an aspheric surface can be tested in the non-null interferometer a test setup, consisting of the relative location of the interferometer elements, must be found. The requirements for a good non-null testing configuration are that the maximum fringe frequency present in the interferogram is within the measurable range of the detector as outlined in Chapter 3, the test part is imaged onto the detector, there is no vignetting of the rays and that the image plane is not in a caustic. In order to determine the spacing of the optical elements a simplified Zemax model of the interferometer is used consisting of the aspheric surface to be tested, the diverger lens, the reference mirror and the imaging lens. The optical elements used in the simple model, FIGURE 5.39, are put into Zemax in the following order.

- 1) Reference Surface
- 2) Diverger Lens – Light traveling toward the Test Part
- 3) Test Part
- 4) Diverger Lens – Light traveling away from the Test Part
- 5) Imaging Lens

Zemax has the ability to ignore surfaces which means they are not considered in the ray trace. Additionally, each configuration can specify different surfaces to ignore. Using these different features the simplified model was set up to consist of three configurations. The first configuration, FIGURE 5.39 (Top), consists of just the test part and the two passes through the diverger lens. It is used to find the distance between the focus of the diverger and the test surface which produces the minimum MWS at the intermediate pupil position. The second configuration, FIGURE 5.39 (Middle), represents the test arm of the interferometer and contains the test part, the diverger and the imaging lens. It is used to calculate the diverger to imaging lens and the imaging lens to sensor separations required to properly image the test surface onto the sparse array sensor. It also checks that no test rays are vignetted and that the image is not in a caustic. The third configuration, FIGURE 5.39 (Bottom), represents the reference arm of the interferometer and contains only the reference mirror and the imaging lens. The third and second configurations are used together to verify the maximum fringe frequency on the detector, and to produce an image of the interferogram that will be obtained when the physical system is set up properly. For all three configurations the starting wavefront is assumed to be a plane wave.

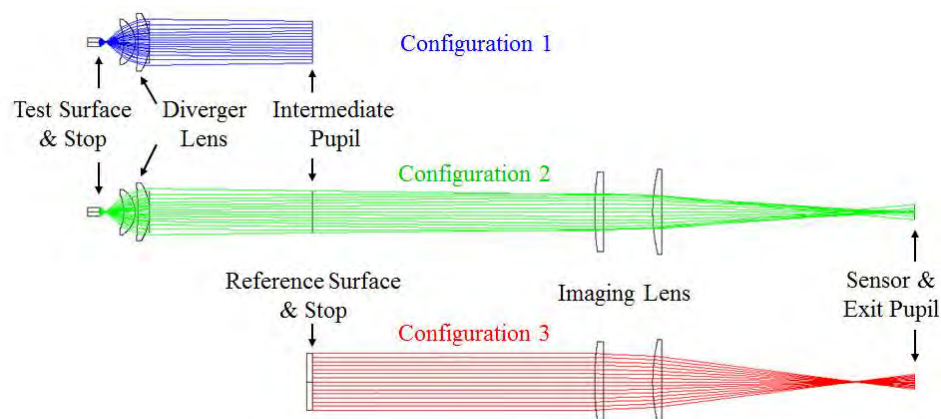


FIGURE 5.39 The three configurations that make up the simple Zemax model of the non-null interferometer. The image size and imaging distances in this figure are not to scale.

There is a multistep process to determine the interferometer setup for a given aspheric test surface, which makes use of several different built in and user defined merit functions. The merit function used for this process is shown in FIGURE 5.40.

Oper #	Type	Prog#	Data	Hx	Hy	Px	Py		Target	Weight	Value	% Contrib
1: CONF	CONF	1										
2: GOTO	GOTO	3										
3: BFSD	BFSD	9	0	0.0000	0.00				0.0000	0.000	-0.15337	0.000000
4: BFSD	BFSD	9	1	0.0000	0.00				0.0000	0.000	-6.52010	0.000000
5: UDOP	UDOP	23	0	111.00	0.00	0.00	0.00		0.0000	1.000	391.4844	100.0000
6: UDOP	UDOP	23	1						0.0000	0.000	0.540541	0.000000
7: UDOP	UDOP	23	2						0.0000	0.000	0.000000	0.000000
8: UDOP	UDOP	23	3						0.0000	0.000	18.54460	0.000000
9: UDOP	UDOP	23	4						0.0000	0.000	21.11042	0.000000
10: UDOP	UDOP	23	5						0.0000	0.000	0.643447	0.000000
11: ENDX	ENDX											
12: BLNK	BLNK											
13: CONF	CONF	2										
14: UDOP	UDOP	43	0	111.00	0.00	0.00	0.00		1.0000	0.000	0.000000	0.000000
15: UDOP	UDOP	43	1						3.8250	1.000	0.000000	0.000000
16: UDOP	UDOP	43	2						0.0000	0.000	0.000000	0.000000
17: BLNK	BLNK											
18: UDOP	UDOP	23	0	111.00	0.00	3.00	0.00		0.0000	0.000	0.000000	0.000000
19: UDOP	UDOP	23	1						0.0000	0.000	0.000000	0.000000
20: UDOP	UDOP	23	2						0.0000	0.000	0.000000	0.000000
21: UDOP	UDOP	23	3						0.0000	0.000	0.000000	0.000000
22: UDOP	UDOP	23	4						0.0000	0.000	0.000000	0.000000
23: UDOP	UDOP	23	5						0.0000	0.000	0.000000	0.000000
24: BLNK	BLNK											
25: TTHI	TTHI	21	22						0.0000	0.000	0.000000	0.000000
26: TTHI	TTHI	15	17						0.0000	0.000	0.000000	0.000000
27: TTHI	TTHI	8	8						0.0000	0.000	0.000000	0.000000

FIGURE 5.40 The merit function for the simple interferometer model

The process starts by using only the first configuration in which rays are traced through the diverger, to its focal point, and then onto the test surface. The distance between the focus of the diverger and the test surface is set to be a variable and the test surface is set as the aperture stop of the system. After the rays reflect off the test surface they are traced backwards through the diverger to the intermediate pupil location. The distance between the first surface of the diverger and the intermediate pupil location is found using pupil position solve. When a new part is to be tested its surface prescription is entered into the Zemax model at the test surface. The distance from the focus of the diverger to the test surface is then set to be equal to the negative of its base radius of

curvature. In the Zemax model a concave test surface will have a negative radius of curvature so it is placed a positive distance, or outside, the diverger focus. Whereas a convex test surface will have a positive radius of curvature so it must be placed a negative distance, or inside, the diverger focus. The base radius is only used as the starting location as it will ensure that the Zemax merit function can be calculated. An example is shown in FIGURE 5.41 (Top), for a concave conic aspheric surface with a -7mm radius of curvature and a conic constant of 0.7. It is initially placed 7mm behind the diverger focus. This location yields 316waves of departure from a plane wave at the intermediate pupil position and a maximum wavefront slope of 1428waves/radius. The next step is to manually move the aspheric surface to a location closer to the solution that minimizes the MWS at the intermediate pupil plane. This is done using a built-in Zemax merit function operand, BFSD, which calculates the radius of curvature of the best fit sphere, BFS, to the aspheric test surface.

“The BFS is determined by the radius of the sphere that minimizes the volume of material that would need to be removed from a spherical surface to yield the aspheric surface.” (Zemax LLC, 2011)

The distance between the test surface and the diverger focus is then changed to be the negative of the radius of the best fit sphere. This location minimizes the peak to valley wavefront departure from a plane wavefront at the intermediate pupil. For the example shown in FIGURE 5.41 (Middle), the test surface is moved to 6.520mm from the diverger focus producing a wavefront departure of 95 waves and a MWS of 768 waves/radius. The next few lines of the merit function contain the user defined operand

that calculates the maximum wavefront slope of the wavefront at the intermediate pupil location as discussed in Chapter 3.3.1. In this example UDO 23 is used since the test surface is rotationally symmetric. The calculation of the MWS at the intermediate pupil plane is the only line in the merit function with any weight. Running the Zemax optimization routine will change the distance between the diverger focus and the test part to minimize the MWS at the intermediate pupil. In the example this occurs when the surface is 6.277mm from the diverger focus creating a wavefront departure of 223 waves but only a MWS 391 waves/radius, FIGURE 5.41 (Bottom). Now that the location of the test surface has been found it is fixed in the Zemax model by removing the variable on the separation of the diverger focus and the test surface.

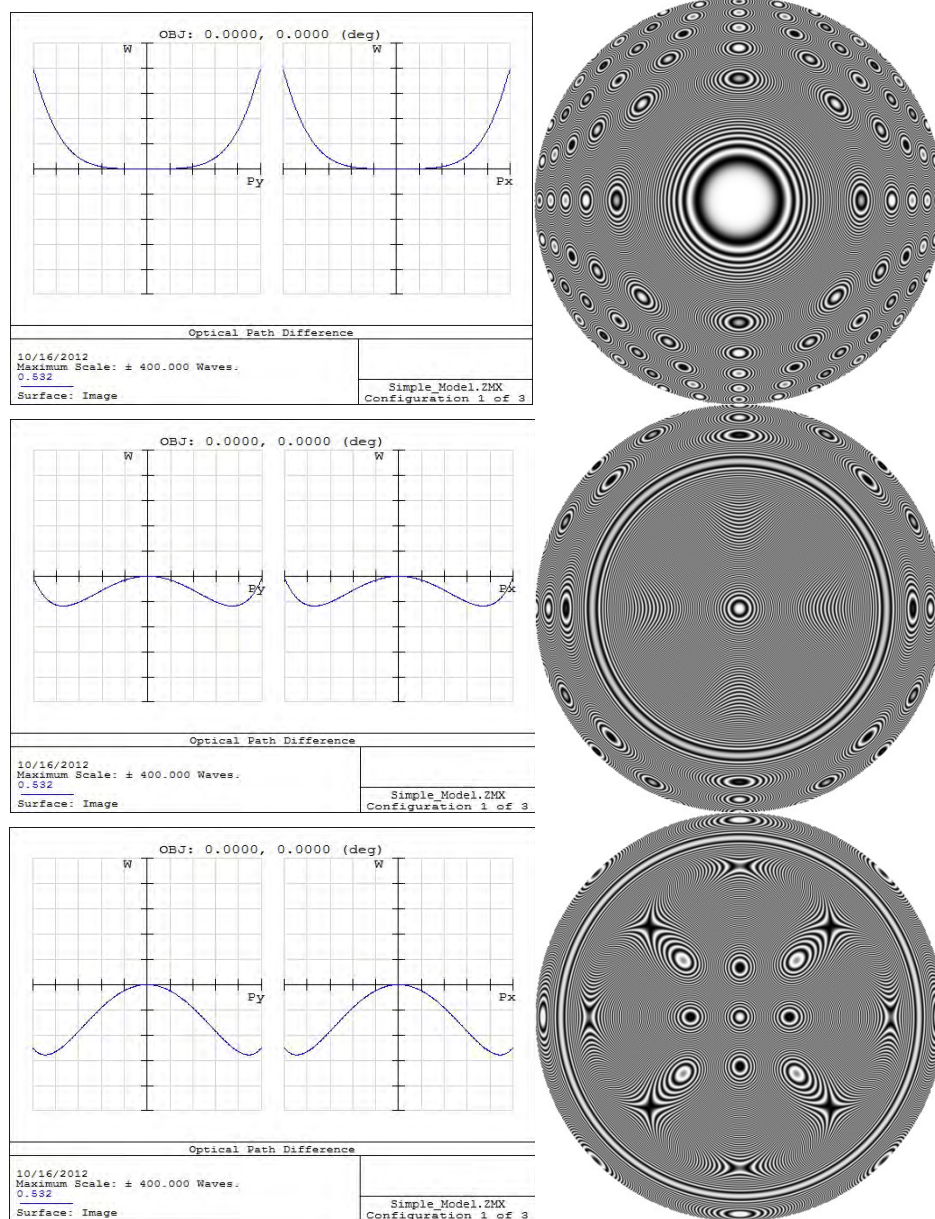


FIGURE 5.41 The OPD_z and interferogram of the wavefront at the intermediate pupil plane, when the distance between the diverger focus and test surface is equal to the negative of the base radius of curvature (Top), the negative of radius of curvature of the BFS (Middle) and distance that minimizes the maximum wavefront slope (Bottom).

At this point in the process it may be possible to decide if a given aspheric test surface will not be testable in the non-null interferometer. If the MWS is more that the limit of

the sub-Nyquist sensor of 1152 waves/radius, as discussed in Chapter 4.2.2, then surface will not be testable.

The next step is to determine the appropriate imaging distances in order to image the wavefront at the intermediate pupil plane, and thus the aspheric test surface, onto the detector. Additionally it must be checked that the image is not formed in a caustic, that no part of the wavefront vignettes, and that the fringe frequency at the detector is measureable. The imaging distances are found using the second configuration of the simplified interferometer model. The second configuration is identical to the first configuration except it also contains the imaging lens and detector. For this step the distance between the intermediate pupil and the imaging lens is set to be a variable and the distance between the imaging lens and the detector is calculated using a pupil position solve. As an initial guess, the distance between the intermediate pupil and the imaging lens is set to -400mm or twice the imaging lens focal length, in order to avoid the system from optimizing to the solution with a virtual object distance.

The third configuration does not contain the diverger or the test part, but instead contains the reference mirror, which is set to be the configurations aperture stop. The reference mirror is assumed to be at the same distance from the imaging lens as the intermediate pupil in the second configuration. While this is not necessarily true in the physical, since the reference surface and reference wavefront are both assumed to be perfectly flat in the simple model, their position along the optical axis doesn't affect the shape of the

wavefront at any subsequent plane. However by setting the reference surface to be the aperture stop of the third configuration, Zemax will force it to overlap with the intermediate pupil of the second configuration. Additionally the pupil solve placed between the imaging lens and the detector in both the second and third configurations will calculate the same distance which in turn ensures that the detectors are held at the same location in both configurations.

The GOTO merit function operand on the second line of the merit function is used to skip the merit function operands used for the first configuration. The second half of the merit function is now used which contains two user defined operands, UDOP 23 or 29 and UDOP 43 which will calculate the maximum wavefront slope at the detector, the size of the wavefront at the detector, whether or not any rays have vignetted and if the detector is in a caustic region, as discussed in Chapter 3.3.3. The only weighted part of the merit function is image size on the detector. The target for the radial size of the wavefront on the detector can be set by the user. Theoretically the largest the wavefront that can be inscribed inside the detector has a radius of 3.825mm. However, in order ease the alignment a smaller target generally around 3.6mm is used. Additionally, to aid the reverse optimization process, multiple measurements of the test part are made where a small axial shift of the test part is introduced between measurements. It is advantageous to initially under fill the detector as these shifts will change the size of the wavefront at the detector. The Zemax optimization is then run to find the imaging distances which produce the desired wavefront magnification at the detector. At this point the values

returned by the merit functions UDOP 23 or 29 and UDOP 43 can be checked to see if a successful test setup was found.

If a viable test setup was found a simulated interferogram can be calculated to act as a guide when the physical system is set up. In order for the calculated interferogram to match the interferogram recorded by the sparse array sensor the OPD must be calculated over a regular grid of points corresponding to the pixel separation of the sparse array sensor. A few different techniques can be used to obtain a regular grid of rays at the detector plane, including using a macro with a simple ray aiming algorithm as discussed in Chapter 3.3.1 for ZPL 23, using interpolation to convert from a uniform grid by pupil coordinates to a uniform grid by real coordinates, or by simply moving the aperture stop to the detector plane for both the second and third configurations of the model. Moving the aperture stops to the detector plane is the most straightforward method. This is done by first calculating the semi-diameter of the test wavefront at the detector, while the stop is still at the test part. This calculated semi-diameter should then be rounded down to a value corresponding to an integer number of detector pixels. This semi-diameter of the detector for both the test and reference arms should then be set to this value. Next the pupil solves need to be turned off to prevent the imaging distances from changing once the stop is moved. The detector surface can then be set as the aperture stop for both the test and reference configurations. At this point changes to the element separations in the model should not be made in order to avoid under filling the test part. The IDL program discussed in Chapter 5.2 can then be used to initiate the trace rays through the simple

model of the system and produce a simulated interferogram, as shown in FIGURE 5.24 and FIGURE 5.41. Since the reverse optimization procedure requires multiple measurements of the test part at shifted along the optical axis, these perturbations can be introduced into the model at this point in order to check the imaging at each test part location and to generate additional simulated interferograms. Additionally, the user defined macros ZPL49 and ZPL55 can be added to merit function in order to calculate the expected induced mapping and phase errors.

If this procedure fails to find a good imaging setup the optimization process can be repeated after making a slight modification to the merit function and the lens design. The process is started over at the point at which the MWS was minimized using the first configuration. However this time both the distance between the diverger and test part and the distance between the intermediate pupil and imaging lens are set to be variables, while the other distances are still found using pupil solves. This allows the optimization procedure to increase the MWS in order to try to find a setup in which the imaging of the test part onto the detector is free of vignetting. This process requires that the merit functions from both the first configuration, the system without the imaging lens, and the second configuration, the system with the imaging lens, be used simultaneously. This can be accomplished by removing the ENDX merit function on merit function line number 11. The relative weights on the image size and the MWS in the merit function can be adjusted in order to find an acceptable solution. However, if element separations

cannot be found which yield a good imaging setup while the MWS stays below the limit of the detector then the test part simply cannot be tested with this non-null interferometer.

Finally, if a surface is to be tested without the use of the diverger lens the same process can be used to determine the imaging distances. In such a case there is no need to locate the intermediate pupil as the test surface will be imaged directly onto the detector by the imaging lens. Therefore the first configuration reduces to just the test surface, which acts as the aperture stop, illuminated by the incoming plane wave. The MWS at the test part can be calculated using the merit function operands of the first configuration. However without the diverger optic there is no way to change or reduce the MWS since translating the test part along the optical axis inside the planar test wavefront will not affect the MWS. The process for finding the imaging distances are the same as the diverger case, except there is no need for the pupil solve before the imaging lens.

5.4 Reverse Optimization and Reverse Ray Tracing Model

The model of the interferometer that is used to convert measured wavefront data at the sensor plane to surface errors on the test part is the reverse optimization and reverse ray tracing model. It will be referred to simply as the reverse optimization model, or RO model, for brevity. The goal of the reverse optimization procedure is to alter the RO model until the OPD predicted by the model matches the OPD measured by the physical system. This is accomplished by utilizing the ray tracing programs optimization routine and a merit function to alter the prescription of the RO model until the difference

between the simulated test wavefront and reference wavefront is identical to, or within a small tolerance of, the measured OPD recorded with the physical system.

$$OPD_{Meas}(x, y) = OPL_{Test}(x, y) - OPL_{Ref}(x, y) \quad 5.26$$

$$OPD_{Meas}(px, py) = OPD_{Z\_Test}(px, py) - OPD_{Z\_Ref}(px, py) \quad 5.27$$

The assumption is then made that the model of the test arm accurately represents the physical test arm of the system, at which point reverse raytracing can be performed to determine the surface errors of the test part from the measured OPD at the detector.

Lowman (1995) demonstrated that because there is an infinite number of test and reference wavefronts that produce the same wavefront difference, multiple measurements are needed in which known perturbations are made to the system in order to accurately recover the test wavefront and test surface error. In this system these perturbations take the form of known shifts to the test part location.

There are a few different functions that the reverse optimization model should be able to perform. First, the RO model needs to be able to simulate the OPD present at the detector plane for a given aspheric test surface. This has already been demonstrated for the simple model in Chapter 5.3. Second, the RO model needs to provide a method of comparing this simulated OPD data to real measured OPD data and provide a mechanism for reducing the difference between these two data sets. Finally, it should have the ability to perform reverse ray traces, in which the test wavefront at the detector is separated from the measured OPD data and is then propagated backwards through to the test surface.

This section will give a general overview of the reverse optimization model and

measurements that were made for the characterization of individual components. The procedure for its use and a discussion of the measurements made using this model, as well as a discussion of some of the issues and possible improvements, will be discussed in Chapters 6 and 7.

In order to compare real data to simulated data in the RO model the measured OPD, the modeled test wavefront and the modeled reference wavefront must all be known at the same point on the detector surface. The measured OPD from the real interferometer is already sampled at discrete pixel locations on the detector plane. In order to calculate the simulated OPD at a given point on the detector plane the path lengths of the test and reference rays, which intersect each other at the desired point, must be found. The RO model therefore must contain accurate representation of both the test and reference arms of the interferometer. In the previous simple interferometer model discussed in Chapter 5.3 the two arms of the interferometer were modeled as distinct configurations in the ray tracing software. With the two arms of the interferometer separated in the model a method is needed to ensure that the two modeled detector planes are identical, such that the coordinates of a ray on the test arm detector correspond to the same point on the reference arm detector. Since rays are generally traced by pupil coordinates the solution to this problem is to force the pupil coordinates of the two configurations to correspond to the same real coordinates at the detector plane. While there are a few different ways this could be accomplished, one easy implementation in Zemax is to make the detector surface the aperture stop, for both the reference and test configurations, setting the

aperture mode to float by stop size, and to then turn on ray aiming. The diameter of the aperture stop can be set as the diameter of the measured wavefront at the detector rounded to the nearest pixel spacing. This method suppresses the effects of different pupil aberrations in the two arms of the interferometer by defining the pupil coordinates at the common detector plane. If properly set up the test rays, reference rays, and detector pixels can all be defined over the same uniform grid. This method was method used by Lowman (1995) and Gappinger (2002) for their reverse optimization models.

One problem with this strategy is that the detector should not be the aperture stop of the interferometer. Rather the test part should serve as the aperture stop. When Zemax is set up as described above, it will ensure that rays are traced over an equally spaced grid that completely fills the wavefront at the detector. However, these rays may under or over fill the test part. In reverse optimization procedures used by Gappinger a set of merit function operands were used to overcome this problem, by confining the ray bundle at the test part and the detector plane to match their measured values. In the Mach-Zender interferometer built by Gappinger, the test parts were measured in transmission and were located in a collimated beam. Changes made the model during the reverse optimization, such as the location of the test part, would have minimal impact on the diameter of the test wavefront at the test part. However, in the non-null interferometer designed for contact lens insert testing the test parts were located in either a converging or diverging wavefront. With the detector set as the aperture stop, changes made to the position of the test part during the reverse optimization procedure significantly impact the size of the ray

bundle at the test part. This results in the ray tracing software struggling to keep both the test part and the aperture stop fully illuminated as the test part position is shifted during the RO procedure. Additionally, placing the aperture near the end of the model and utilizing ray aiming can greatly slow down the ray tracing procedure, as discussed in Chapter 3.2.6.

With the test and reference arms separated between two configurations a method of subtracting the OPD_Z of the test arm from the OPD_Z of the reference arm and comparing the result to the measured OPD of the system must be found. The brute force approach would be to use two merit function operands to calculate the OPD_Z of the test and reference arms at every location of interest across the detector. A third operand could be used to subtract the reference OPD_Z from the test OPD_Z at each location. This difference merit function operand could then be targeted to match the measured OPD value at the corresponding pixel location. This means the measured OPD data needs to be loaded into the merit function by the pupil coordinates of the pixels.

However, this wasn't the approach taken by Lowman, or Gappinger. They each used a Zernike Phase Surface setup to match the negative of the reference wavefront, inserted into the test arm configuration just prior to the reference wavefront. This allows the OPD_Z of the test arm configuration to be targeted to match the measured OPD directly in the merit function, thereby reducing the required number of merit function operands. The advantage of this approach is that if multiple measurements are to be used, in which a

small perturbation has been made to the interferometer, then only a single reference configuration is needed. Additionally, the grid of traced reference rays and test rays no longer have to overlap at the detector. This is because the phase of the reference wavefront can be calculated at any test ray location from the calculated Zernike coefficients. The Zernike phase surface used to represent the reference wavefront at the detector simply needs to be defined over a diameter equal to or larger than that of the largest diameter test arm configuration. Additionally, this approach assumes that the reference wavefront can be adequately nulled by a set of Zernike polynomials.

$$OPD_{Z\_Ref}(px, py) - Z(px, py) = 0 \quad 5.28$$

$$OPD_{Z\_Test}(px, py) - Z(px, py) = OPD_{Meas}(px, py) \quad 5.29$$

For this technique to work a method of fitting the reference wavefront to a Zernike phase surface must be found. For this step both Lowman and Gappinger used the same approach. The Zernike phase surface was inserted into all configurations, including the reference configuration, and its Zernike coefficients were set up as variables. The OPD_Z of the reference configuration was targeted to zero in the merit function. This allows the Zemax optimization procedure to solve for the Zernike coefficients required to null the reference wavefront. However, the problem with this approach is that during the Zemax optimization procedure, data is not continuously updated between configurations, rather it cycles through configurations. During the reverse optimization procedure if a variable in the model is changed which impacts both the test and reference wavefronts, such as the spacing between the imaging lens and the detector, the reference phase surface must be updated. Additionally, the Zemax optimization algorithm could leave a non-zero

difference between the reference OPD_Z and the Zernike phase surface in order to improve the overall merit function. To avoid these issues, Lowman used a much larger weight, 100 times greater, on the reference merit function operands than the operands for the test arm or arms. Gappinger utilized an iterative approach in which the reference configuration and test configurations were not optimized simultaneously. As a side note, newer versions of Zemax can calculate the Zernike coefficients for a given wavefront directly via least squares fitting, without having to rely on the optimization algorithm. A user written macro could load the calculated coefficients into the reference phase surface for all the test configurations. This would resolve the dilemma of relative weighting between the test and reference configurations in the merit function, but not the issue of non-constant updates.

The reverse optimization model designed for this work uses a slightly different approach. While the ability to model the test and reference arms as separate configurations was maintained for the purpose of generating simulated measurement data or interferograms, this feature wasn't used for reverse optimization procedures. Rather during the reverse optimization procedure the test and reference arms are combined into a single configuration. In this RO model light is first traced forward through the test arm to the detector plane. At which point a phase surface representation of the measured wavefront difference is encountered, rather than phase representation of the reference wavefront. When the forward propagating test rays intersect the measured OPD phase surface their phase is converted to those of the reference rays, which can be seen by rewriting

Equation 5.27 as Equation 5.30. These reference rays are then traced backwards through the reference arm of the interferometer to the plane representing the collimated input wavefront. The modeled wavefront at the input to the reference arm of the interferometer will be nulled when the RO model matches the physical system.

$$OPD_{Z\_Test}(px, py) - OPD_{Meas}(px, py) = OPD_{Z\_Ref}(px, py) \quad 5.30$$

The negative of the measured OPD data can be loaded into Zemax as a Zernike phase surface or as a grid phase surface. The Zernike fit is useful because it smooths out the measured data and creates a closed-form solution to the phase surface so that the required phase at any point in the surface can easily be calculated. The grid phase type is useful since it allows higher frequency content to be incorporated into the model. However, the ray tracing algorithm needs to interpolate between grid points in order to find the phase for a given ray, which substantially slows down the ray tracing especially for dense grids. Additionally, noise and missing data in the measured OPD can be problematic. Finally, in this model multiple configurations can be used to represent different measurements of the same part, with known part shifts introduced, or even of different parts. Each configuration simply needs its own unique phase surface to represent the measured OPD for the particular measurement it represents. These phase surfaces can all be co-located with the detector surface in the model and are simply setup to be ignored by all the configurations, except for the one to which they pertain.

This approach has several advantages. First the reference and tests arms of the interferometer are optimized together. Any change made to the model of the system that

would affect both arms is immediately taken into consideration by the RO procedure without having to wait for the next cycle of the optimization routine. Therefore, there is no need for an iterative optimization approach, or a merit function in which heavier weights are placed on the reference arm, in order to ensure the RO procedure doesn't leave an error in the model of the reference arm in order to compensate for a discrepancy in the model of the test arm. A second advantage is that since the final surface of the model, during the RO procedure, is the collimated input wavefront to the reference arm of the interferometer the default Zemax merit function operands can be used in which all rays are targeted to have zero OPD_z . Additionally, this technique solves the problems that arise from needing to force rays, traced by pupil coordinates, between the two interferometer arms, which have different pupil aberrations, to overlap with each other and the measured data. This is because rays are converted from test to reference ray at the point at which they intersect the measured phase surface based on their physical coordinates, not by normalized pupil coordinates. Since ray tracing program can also interpolate between points on the measured phase surface there is no need for the test, reference and measured data points to all fall on the same grid. Additionally since, all rays are targeted to end up with an OPD_z equal to zero in the RO procedure, there is no need to tie a specific measured OPD value to a specific pupil coordinate in the RO merit function. Ultimately this means the RO model doesn't need to have the aperture stop placed on the detector with ray aiming turned on. Instead the aperture stop can remain at the test part, leading to faster ray tracing. While ray aiming is not required with this RO procedure, it can be used to ensure that the test part is uniformly sampled in the model.

This prevents the RO model from using an uneven distribution of rays which could lead to an unequal weighting on different regions of the test surface during the RO procedure.

The approach used to implement this RO model was to separate the interferometer into groups of optical elements. With the exception of the diverger lens element, each group of elements was inserted in the sequential ray tracing program twice, once for forward propagating rays, those traveling from the collimated input wavefront to the detector, and once for backwards propagating rays, those traveling from the detector to the input wavefront. The diverger lens elements need to be inserted into the RO model, four times because both forward and backwards propagating test rays would pass through the diverger lens twice. If a fully physical representation of the interferometer was used, in which each surface of the interferometer has a corresponding sag surface in the model, then the beam splitter surfaces would also need to be inserted into the model more than twice. However, many of the beam splitter interactions were reduced to phase surfaces in order to simplify the model. The beam splitter modeling will be discussed in more detail in Chapter 5.4.2. Additionally, while the design prescription of the air spaced doublet used for the collimating lens was known, the manufacturing errors, such as form errors for the internal lens surfaces, the surface misalignments, and the index of refraction of the glasses were not known. Therefore, the collimated input wavefront into the interferometer was measured and incorporated into the model as a phase surface, which will be discussed in Chapter 5.4.1. The imaging lens and diverger optics will be

discussed in Chapters 5.4.3 and 5.4.4. The groups of the reverse optimization model are given in TABLE 5.2.

#	Direction	Group
1	Forward	Reference input wavefront, Reference Surface, and Beam Splitter.
2	Forward	Test input wavefront (After first pass through the beam splitter)
3	Forward	Diverger Lens – Light traveling toward the Test Part
4	Forward	Test Part
5	Forward	Diverger Lens – Light traveling away from the Test Part
6	Forward	Beam Splitter Reflective Surface
7	Forward	Imaging Lens
8		Detector plane & Phase surfaces for loading measured data.
9	Backward	Imaging Lens – Backwards Light Propagation
10	Backward	Beam Splitter Reflective Surface
11	Backward	Diverger Lens – Light traveling toward the Test Part
12	Backward	Test Part
13	Backward	Diverger Lens – Light traveling away from the Test Part
14	Backward	Test input wavefront
15	Backward	Reference input wavefront, Reference Surface, and Beam Splitter.

TABLE 5.2 Reverse optimization and reverse raytracing model organized components into groups that can be turned on and off to enable forward and backward raytracing.

Each of these groups contains the surfaces and the coordinate break surfaces required to tilt and decenter the various surfaces. The individual surfaces can then be turned on and off using the IGNORE surface operand in the Zemax Multi-Configuration Editor. When a surface, or group of surfaces, is turned off they are completely ignored by the ray traced algorithm. Turning on and off the various groups allows the model to be set up in a variety of different ways in order to accomplish the different objectives. For instance, turning on groups 2-8 traces rays forward through the test arm to the detector, while using groups 1, 7 and 8 traces rays forward through the reference arm to the detector. These combinations are useful for generating simulated interferograms and wavefront data. In

order to set up the system for reverse optimization groups 2-9 and 15 are used to trace rays forward through the test arm and then backwards through the reference arm. In the reverse raytracing process the goal is to determine the OPD introduced by the unknown test surface. This can be accomplished by comparing the wavefront before and immediately after the test part. For the moment, assume that the model of the interferometer perfectly matches the physical interferometer. Then the test wavefront, after the test part, can be found by tracing reference rays forward through the system to the detector and then backwards through the test arm to the test part, using groups 1, 7 and 8-12. This ray trace picks up the errors, created by the test surface, but recorded at the detector, and propagates them back to the test part. In order for reference rays to be converted into test rays the sign of the measured OPD phase surface must be reversed from what is typically used to convert test rays to reference rays, so that the measured OPD data is added to the reference wavefront. In Zemax this is as simple as changing the scaling factor on the phase surface from negative one to one.

The test wavefront immediately before the test surface can be calculated by using groups 2, 3 and 4. Now if the full OPD introduced by the test surface is the desired outcome of the reverse ray trace, then the wavefront immediately after the test surface is compared rays traced right up to, but not reflecting off of, the test surface. However, often the deviation of the test part from the design is the desired outcome. In this case, the forward propagating rays are allowed to reflect off the test part. At this point the forward propagating wavefront and backwards propagating wavefront are co-located at the same

point, just after light reflects off the test part. The forward propagating wavefront does not contain the surface errors, while the backwards propagating wavefront does, comparing the two wavefronts yields the OPD introduced by the test part surface errors. Since this RO model uses multiple copies of the same surfaces it can be difficult to set up the model such that the each instance of a repeated surface is collocated at the same plane with the same orientation. In general, pickups are used to ensure subsequent copies of a surface stay in alignment with the first as it is altered in the reverse optimization process. These pickups are used on the majority of the lens properties, such as, radius of curvature, thickness, glass type, Zernike coefficients, etc. But ensuring that the copies overlap the original surface can become especially onerous to manage once all of the coordinate breaks required to tilt and decenter the various surfaces are added to the model. Fortunately, there is one additional advantage to setting up the RO model in the method described. There is a built in mechanism to check if the forward and backward propagating portions of the model are set up the same way. Rays can be traced forward through either arm of the interferometer to the detector plane and then backward through the same arm of the interferometer to the input surface. The groups 2-14 can be used for the test arm and 1, 7-9, and 15 for the reference arm. In this situation, no phase surface is used at the detector plane. If the model has been setup properly, any change made to the forward propagating components in the model will be mimicked by the backward propagating components. As such, every ray will then follow the same path forward and backwards through the system resulting in zero OPD_Z. A non-zero OPD_Z is an indication that the model is not properly set up.

5.4.1 Characterization and Modeling of the Collimated Input Beam

The collimating optics consisted of a commercially available lens and spatial filter. The design prescription of the collimating lens was known, as described in Chapter 4.9, but the manufacturing errors were unknown. Rather than disassembling the lens in order to measure its physical properties for inclusion in the reverse optimization model, measurements of the transmitted collimated wavefront were made. Depending on the magnitude of the measured error it could either be included in the reverse optimization model as a phase surface or ignored. The collimating optics were originally aligned with the aid of qualitative measurements made using a shear plate collimation tester produced by Melles Griot. (Murty 1964).

One method of measuring the aberration in the collimated input beam is the use of a Shack-Hartmann wavefront sensor (SHWS). A SHWS consists of a pixelated detector placed at the rear focus of a positive lenslet array and is an extension of the Hartmann screen test. (Ghozeil 1992). The basic principle behind a SHWS is that the slope of the wavefront at each lenslet position can be calculated from the position of the focal spot on the detector. For a plane wavefront incident normal to the lenslet array each spot will be located directly behind its corresponding lenslet. When an aberrated wavefront is incident on the lenslet array the positions of the spots shift on the detector plane by distances of δx and δy , FIGURE 5.42. Generally, it is assumed that the shift of each spot is proportional to the average wavefront slope across the corresponding lenslet and that

the spots are diffraction limited. (Smith 2008) The average wavefront slope, or the average phase gradient, across each lenslet can then be calculated from the spot shifts and the focal length of the lens, using Equation 5.31. (Smith 2008)

$$\langle \nabla \Phi \rangle = \frac{2\pi}{\lambda} \frac{\hat{x}\delta x + \hat{y}\delta y + \hat{z}f}{\sqrt{\delta x^2 + \delta y^2 + f^2}} \quad 5.31$$

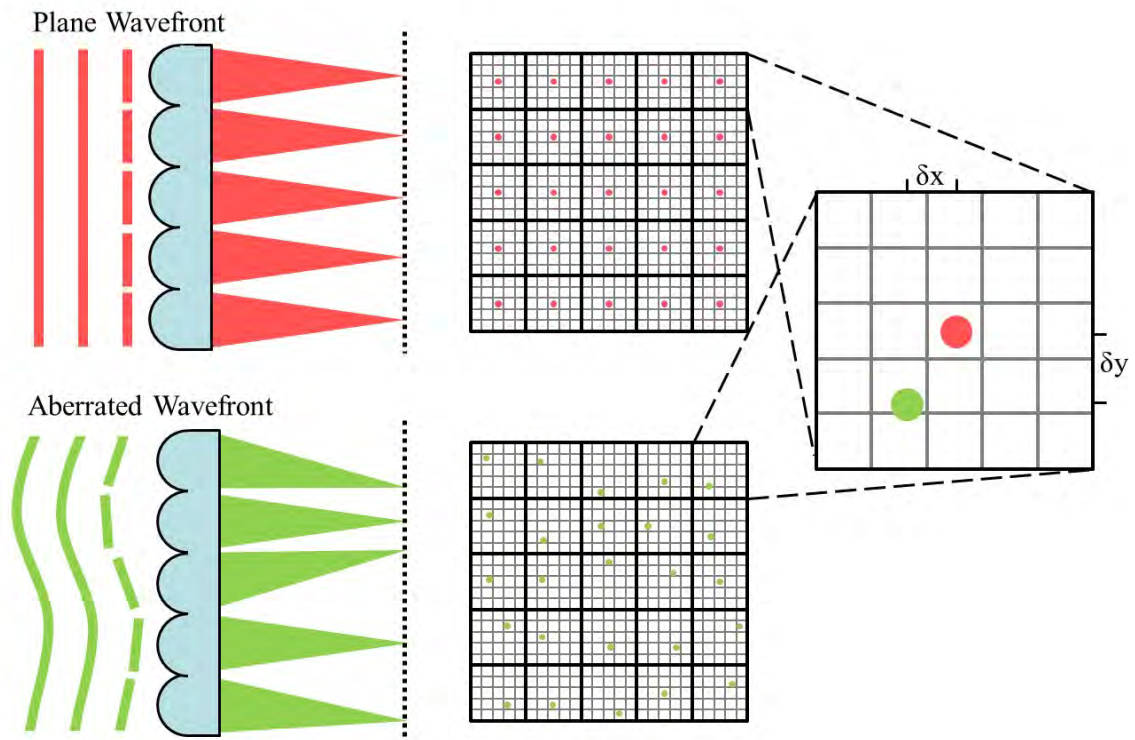


FIGURE 5.42 A Shack-Hartmann wavefront sensor measuring a plane-wave incident normal to the sensor top and an aberrated wavefront bottom. The side view of the SHWFS is shown on the left. Center view is looking along the optical axis, the outline of the lenslets are represented by the dark lines, the gray lines illustrate the individual pixels of the detector, and the dots represent the focal spots. A blown-up view of the focal spots produced by a single lenslet, for both a plane wavefront and an aberrated wavefront, is shown on the right.

Ideally, a SHWS could be placed in the interferometer just after the collimating lens in order to measure the aberration present in the collimated wavefront. This test would require that the width of the lenslet array in the SHWS be larger than the diameter of the

collimated input beam. Alternatively, a telescope can be used to reduce the diameter of the collimated wavefront so that a SHWS with a smaller lenslet array can be used. In this case the measurement of the wavefront aberrations introduced by the telescope will also be included in the measured wavefront. In attempting to measure the input wavefront in the non-null interferometer neither a large SHWS nor an aberration free telescope were available. However, since the wavefront to be measured was part of an interferometer the aberrations introduced by the telescope can be measured by the interferometer and subtracted from the SH wavefront measurement. The process for measuring the collimated wavefront involved three measurements, two null interferometer measurements and one SHWS measurement, as shown in FIGURE 5.43.

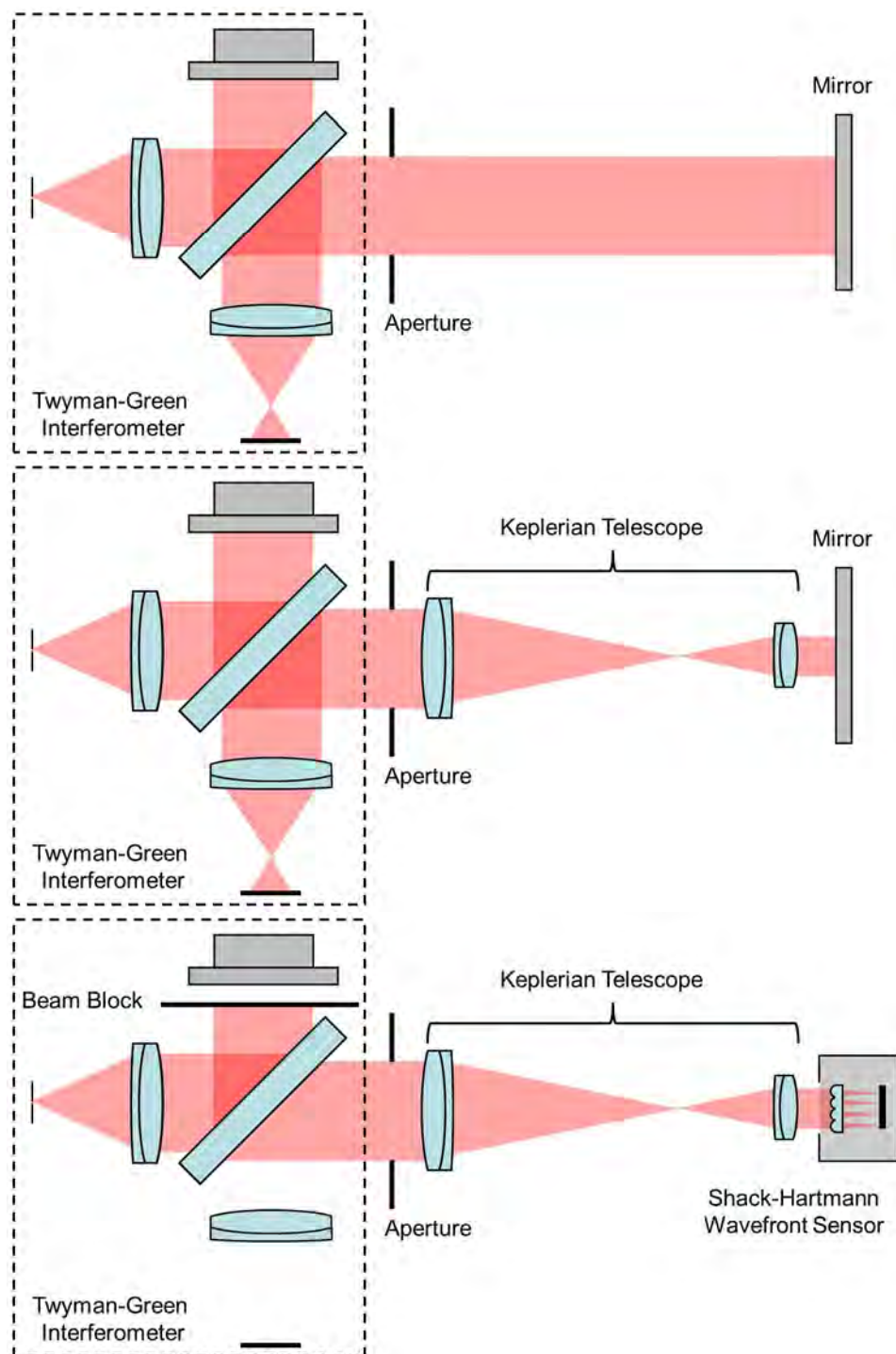


FIGURE 5.43 Procedure for measuring collimated wavefront with a Shack-Hartmann Wavefront Sensor and Keplerian telescope.

First, an aperture which is slightly smaller than the incoming beam is placed in the test arm just after the beam splitter, FIGURE 5.43 (Top). This aperture is imaged on the detector and is used to scale the size of the measured wavefront to real units. Additionally, a flat mirror is placed in the test arm of the interferometer and is aligned to produce a null interferogram. At this point the optical path difference, OPD_1 , is recorded using a PSI measurement. The optical path length contributed by the mirror is separated from the optical path length of the rest of the test arm in Equation 5.32.

$$OPD_1 = (OPL_{Test} + OPL_{Mirror}) - OPL_{Ref} \quad 5.32$$

Next, a telescope is inserted into the test arm in-between the aperture and the flat mirror, FIGURE 5.43 (Middle). The telescope lenses are then aligned to the interferometer so that a null interferogram is again produced at the detector. This is done without making any changes to the rest of the interferometer optics. A second PSI measurement is made to get the optical path difference, OPD_2 , which includes the OPL introduced by the double pass through the telescope as well as the OPL introduced by the smaller beam footprint on the flat mirror.

$$OPD_2 = (OPL_{Test} + 2OPL_{Telescope} + OPL_{Mirror\_Small}) - OPL_{Ref} \quad 5.33$$

The OPD introduced by the telescope can now be calculated, Equations 5.32 - 5.33. The OPD introduced by the test and reference arm cancel as they are the same for both measurements, however the OPL introduced by the mirror do not, Equation 5.34. Therefore a high quality mirror should be used so that its OPL contributions can be assumed to be negligible.

$$OPL_{Telescope} = \frac{1}{2} \left(OPD_2 - OPD_1 + OPL_{Mirror} - OPL_{Mirror\_Small} \right) \quad 5.34$$

Finally, the mirror is removed from the test arm and replaced with the Shack Hartman wavefront sensor, FIGURE 5.43 (Bottom), and the wavefront shape is recorded. The wavefront error introduced by the telescope is subtracted from the wavefront recorded by the SHWS. Since the wavefront is measured after the first pass through the beam splitter it is a combination of the collimated wavefront produced by the collimating lens and the error introduced by a single pass through the beam splitter. The error introduced by the beam splitter will be discussed in Chapter 5.4.2.

The mirror used for this test on the non-null interferometer was measured using a WYKO 6000 phase shifting interferometer. The surface error was less than $\lambda/10$ waves peak to valley and $\lambda/60$ waves rms over the full 94mm aperture. However, across the central 50mm of the mirror the surface error was measured to be less than $\lambda/38$ peak to valley and $\lambda/100$ rms. The aperture used to restrict the test beam, for the OPD_1 measurement, had a diameter of 47.1mm. Over the approximately 4.1mm beam diameter after the telescope in the OPD_2 measurement the surface error was even smaller. Therefore, the effects of the mirror were not taken into consideration for the wavefront measurement. The Keplerian telescope was constructed using two achromatic doublets. The exact prescriptions of the lenses were not known. However, the objective lens had an approximate focal length of 250mm and a diameter of 75mm while the second lens had a focal length of 21.5mm and diameter of 12mm. One advantage to the Keplerian design is

that the inversion of the wavefront in the test arm results in the orientation of the wavefront at the SHWS matching the orientation of the wavefront at the detector. If a Galilean telescope was used, the data from the interferometric measurements would have to be inverted before it could be compared to the SHWS data. The SHWS used for the test was manufactured by Thorlabs,(Newton, NJ) TABLE 5.3.

Thorlabs Shack-Hartmann Wavefront Sensor	
Aperture Size	5.95 mm X 4.76 mm
Number of Lenslets	39 X 31
Lenslet Pitch	150 μm
Effective Focal Length	3.7 mm
Camera Resolution	1280 x 1024 Pixels
Wavefront Accuracy	$\lambda/15$ rms
Wavefront Sensitivity	$\lambda/50$ rms
Wavefront Dynamic Range	$>100\lambda$

TABLE 5.3 Thorlabs Shack-Hartmann Wavefront Sensor Specifications (Thorlabs)

The entire measurement procedure was repeated ten times. After each measurement, the telescope was removed and the procedure was started over from the beginning.

Additionally, each OPD measurement was the average of ten separate PSI measurements.

In the software provided by Thorlabs for the SHWS the reported wavefront would occasionally change from consecutive measurements by as much as quarter wave from frame to frame. In order to stabilize the measured wavefront the software allowed for one-hundred consecutive frames to be averaged for each Shack-Hartmann measurements.

The peak to valley and rms error of each of the ten measurements is shown in TABLE 5.4 along with the peak to valley and rms wavefront error of the average of the ten measurements. Additionally TABLE 5.4 lists the peak to valley and rms wavefront error for each measurement minus the average wavefront. The average peak to valley

measured wavefront error over the entire 47.1mm was 0.72 waves, 0.13waves rms, and is shown in FIGURE 5.44 (Left). As discussed in Chapter 4.6.3 the average wavefront diameter needed to test random aspheric insert designs is only 29.8mm. Over this diameter the peak to valley error is reduced to 0.26λ , 0.06λ rms, and is shown in FIGURE 5.44 (Right).

One issue with this measurement process is that the Shack-Hartman wavefront sensor used only measures thirty-one points across the wavefront and thus only the low spatial frequency content of the wavefront is recorded. Additionally, in order to compare the interferometric data to the Shack-Hartman data both data sets were fit using the fringe Zernike polynomials. The normalizing radius used for both measurements was assumed to be equal and half the 47.1mm limiting aperture that was placed in front of the telescope.

Measurement	Measured Wavefront		Measured Wavefront Minus Average Wavefront	
	PV	RMS	PV	RMS
1	0.582	0.108	0.176	0.031
2	0.648	0.114	0.197	0.036
3	0.640	0.119	0.156	0.030
4	0.734	0.133	0.102	0.019
5	0.912	0.164	0.219	0.041
6	0.814	0.156	0.172	0.034
7	0.679	0.124	0.217	0.040
8	0.657	0.122	0.137	0.026
9	0.823	0.150	0.177	0.033
10	0.756	0.141	0.088	0.017
Average WF	0.716	0.130		

TABLE 5.4 The peak to valley of the measured wavefront for each of the ten measurements as well as the peak to valley and the rms of each wavefront minus the average wavefront.

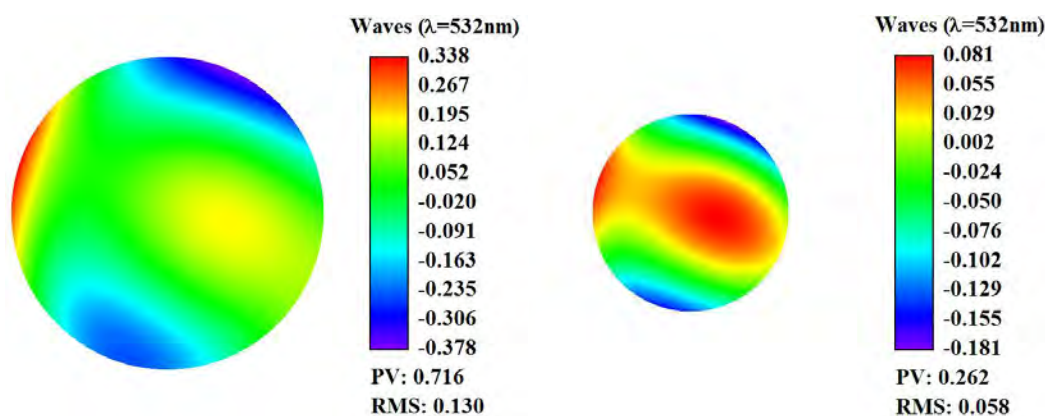


FIGURE 5.44 The average of ten measurements of the error in the collimated wavefront over the full 47.1mm diameter beam (Left) and over the smaller 28.9mm beam needed to test the average aspheric insert as discussed in Chapter 4.6.3 (Right)

The data in TABLE 5.4 illustrates that the precision of a single measurement using this technique is only on the order of a quarter wave peak to valley. The measurement with the largest deviation from the average, number five, yielded 0.22 waves peak to valley and 0.04 waves RMS. Ideally this technique could have been performed on a known wavefront in order to test the accuracy of this method. However, a known wavefront or another more accurate method of measuring the wavefront error was not available. Thus an attempt to approximate accuracy of this measurement technique was performed by testing its ability to measure a small perturbation of the wavefront's shape. This was accomplished by repeating the measurement for several shifted positions of the collimating lens and then comparing the change in the measured wavefront to the expected change in the wavefront generated from the Zemax model. First, the collimating lens was aligned to produce the best collimation by visual observation of the shear plate. The collimating lens was then to be shifted towards the pinhole by 125μm in

order to introduce one wave of defocus into the wavefront. The lens was then to be shifted four times in $62.5\mu\text{m}$ steps away from the pinhole. The actual shifts of the collimating lens were measured using a Heidenhain gauge and input into the Zemax model. The wavefront measured at the zero shift position was then subtracted from the other measured wavefronts. Finally, the modeled wavefront at each lens position was subtracted from the measured wavefronts. An example of one of these measurements is shown in FIGURE 5.45. This process was repeated five times and the average peak to valley error between the measured and predicted wavefront at any given step was found to be 0.136λ with an average rms of 0.031λ .

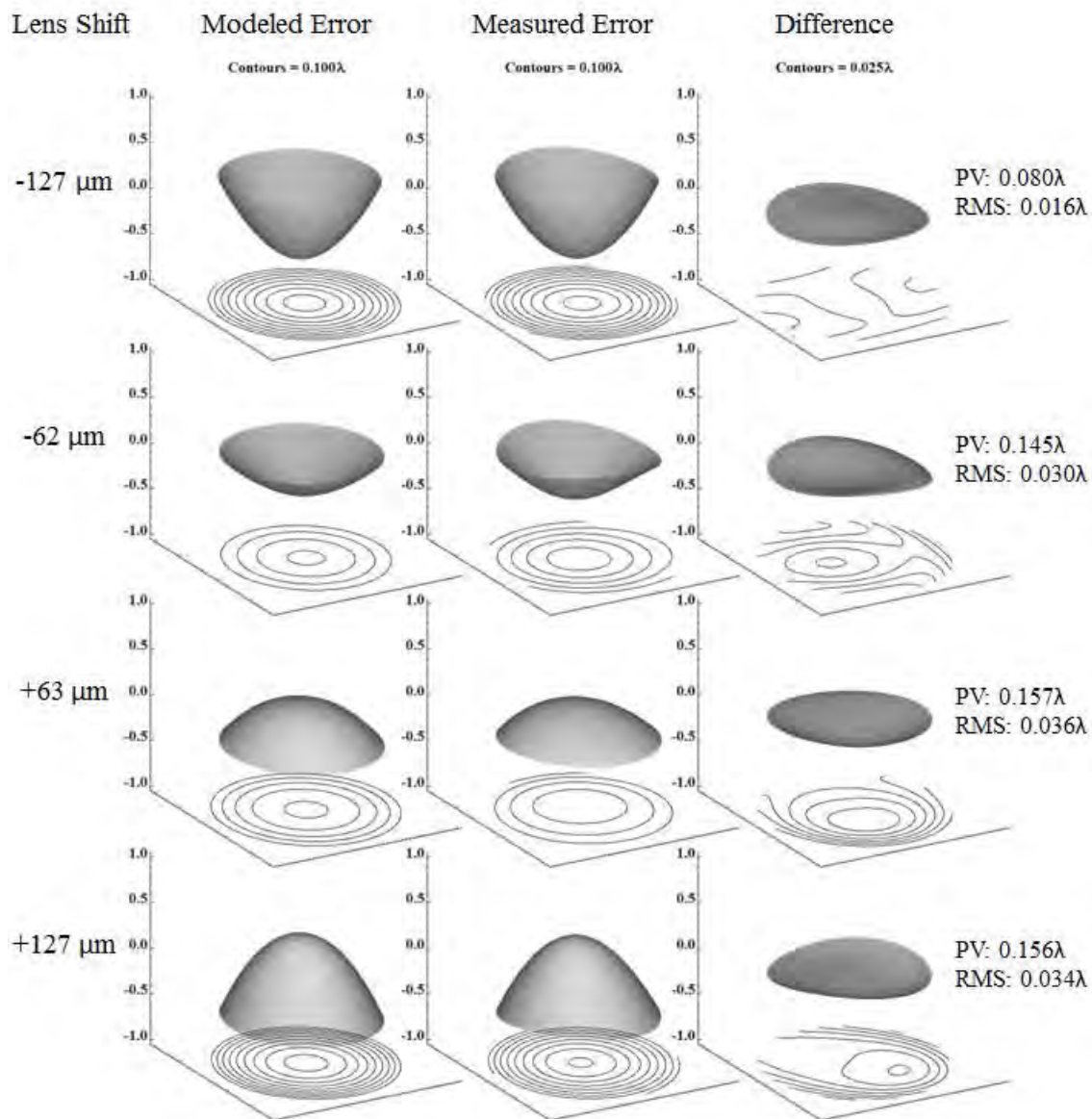


FIGURE 5.45 Difference between modeled wavefront error and measured wavefront error introduced by shifting the position of the collimating lens.

Next, the impact this level of wavefront error will have on the non-null interferometer measurements must be considered. In order to determine if the error in the collimated wavefront will have a significant impact on the RO procedure, the modeled aspheric inserts discussed in Chapter 4.5.6 were used in conjunction with the RO model of the interferometer. Initially, the model contained a perfectly flat input wavefront. Rays

were traced through the test arm, consisting of the beam splitter, diverger lens, aspheric insert and imaging lens to the detector. The aspheric insert was used as the stop of the test arm to ensure that the aspheric test part was completely illuminated. In this model the surfaces and alignment errors of the individual interferometer components were ignored, so that only the effect of the errors in the collimated wavefront would be present. A second configuration was used to model the reference arm which consisted of the beam splitter, reference surface and the imaging lens. The reference mirror was used as the stop of the reference arm. A macro was then written to load each of the aspheric test surfaces into the model one at a time, along with the element separations required for imaging the test part onto the detector, as previously calculated and discussed in Chapter 4.6.3. The macro then used a simple ray aiming procedure, similar to the one discussed in Chapter 3.3.1 for ZPL29, to produce a uniform square grid rays at the detector. The width of the grid was 511 rays on each side and they were spaced $15\mu\text{m}$ apart to match the layout of the sparse array sensor. Next, this process was repeated for the reference arm in order to produce a set of test and reference rays that intersect over a uniform grid of points at the detector. The OPD between the test and reference arms was calculated for each point on the grid and output to a text file as a Zemax Grid Phase surface. This grid phase surface represents a simulated measurement of the wavefront difference. At this point the RO model of the interferometer was changed to use only one configuration in which light was first traced forward through the test arm to the detector plane where the previously recorded Zemax Grid Phase surface was inserted. This phase surface converts the forward propagating test wavefront into the backward propagating reference

wavefront. The rays were then traced backwards through the reference arm to the collimated input wavefront, at which point a null wavefront should be obtained. Finally, the measured collimated wavefront error was inserted into the model, as a phase surface, at both the beginning of the test arm and the end of the reference arm. If the collimated wavefront error was a common path error the phase added on to the rays in the test arm would be canceled as the rays are traced through the reference arm leading to no change in the null wavefront. However, if the collimated wavefront error is not common path then there will be some error in the null wavefront.

In order to illustrate this process the following figures were generated for an aspheric surface which had a radius of 6.3583mm and a 4th order aspheric coefficient equal to - 1.016E-3. FIGURE 5.46, shows the calculated Zemax grid phase surface used for the simulated measurement. FIGURE 5.47, shows the resulting wavefront obtained after tracing rays forward through the test arm and backwards through the reference arm without the measured grid phase surface at the detector. The null wavefront that is obtained once the grid phase surface is inserted at the detector is shown in FIGURE 5.48. One problem with this approach is that there is often ringing at the edge of the wavefront, leading to a large residual peak to valley error in the null wavefront, FIGURE 5.48 (Left). This is the result of a step change in the phase surface at the edge of the exit pupil. To overcome this problem the phase surface would either have to be calculated for rays outside the exit pupil of the interferometer or the data can simply be stopped down to trim off the outside edge. Reducing the aperture stop at the test part by 1%, from 8mm to

7.92mm, the null wavefront shown in FIGURE 5.48 (Right) was obtained. Finally the error introduced by the errors in the collimated wavefront for this particular aspheric surface are shown in FIGURE 5.49.

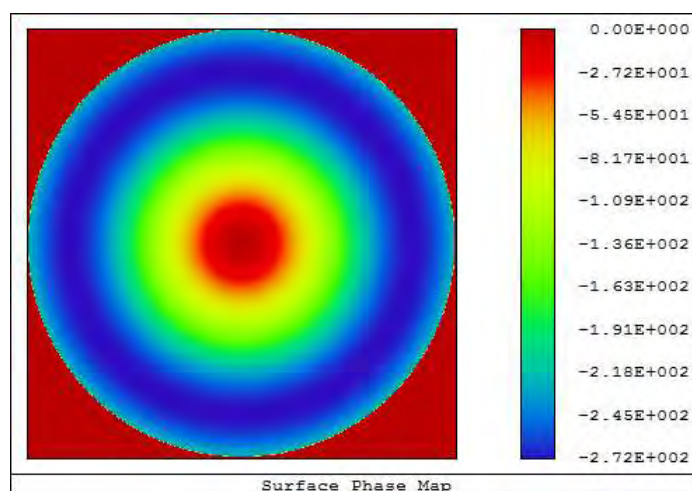


FIGURE 5.46 Zemax grid phase surface representing a simulated measurement

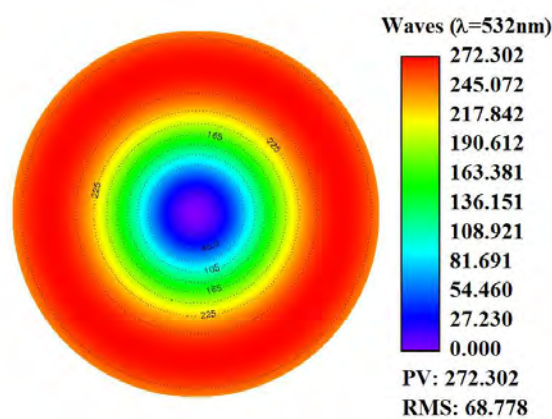


FIGURE 5.47 Wavefront obtained by tracing forward through the test arm and backwards through the reference arm, without the Zemax grid phase surface inserted at the detector.

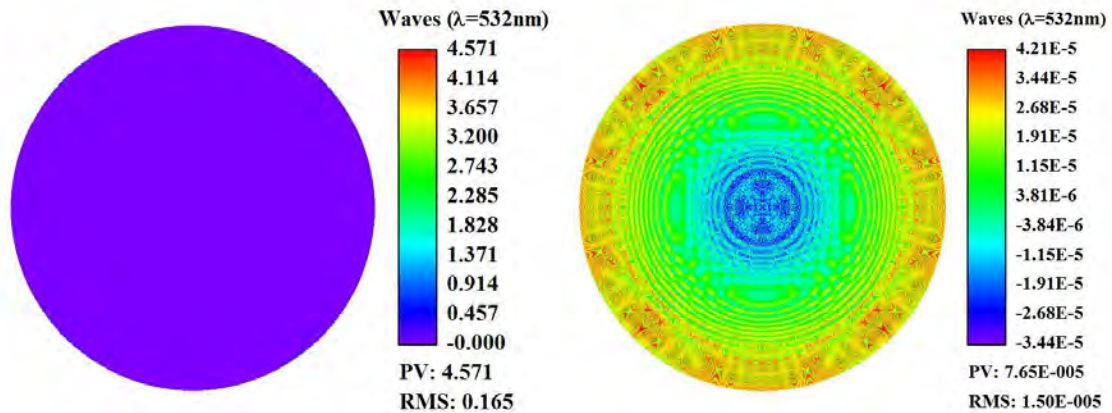


FIGURE 5.48 Wavefront obtained by tracing forward through the test arm and backwards through the reference arm, with the Zemax grid phase surface inserted at the detector. A large peak to valley error is encountered at the edge of the pupil (Left), which removed by stopping the aperture down by 1% (Right).

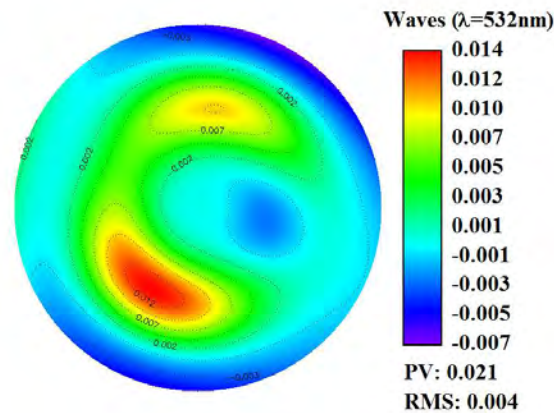


FIGURE 5.49 Error introduced into the simulated measurement by the presence of the errors in the collimated input wavefront.

In this example, the error in the collimated wavefront was on the order of $\lambda/50$ peak to valley, FIGURE 5.49. The error shown here is not the exact error that would show up in a measurement of this part, as the RO procedure would try to minimize this wavefront error by perturbing the RO model. However if the RO model doesn't have a method of directly manipulating the shape of the collimated input wavefront then it would attempt

to minimize this error by making other changes to the system such as shifting the location of the test part.

Additionally, this is only the result for one specific aspheric surface. This process was repeated for all of the aspheric surfaces that were previously deemed as discussed in Chapter 4.5 and 4.6. After the measured grid phase surface was generated for each of the aspheric surfaces, but before the error in the collimated wavefront was introduced, the residual peak to valley and rms wavefront error at the last surface of the RO model was calculated as shown in FIGURE 5.48 (Right). If the residual wavefront error was less than $\lambda/1000$ peak to valley, than the combination of the RO model, the aspheric test surface and calculated phase surface was considered adequate to produce a null wavefront. Of the original 4169 aspheric surfaces that were found to be testable with the two element diverger lens and the air spaced doublet imaging lens, only 3912 or 93.8% could be successfully nulled to less than $\lambda/1000$ peak to valley over 99% of the aperture stop utilizing a grid phase surface with 511 x 511 pixels. If the grid phase surface didn't adequately null the RO model without the collimation errors present, it would be difficult to separate the effects of the collimation error from the errors introduced by the non-nulled model once the collimation errors were introduced. Therefore, the remaining aspheric surfaces were dropped from the rest of this analysis.

Next the error in the collimated wavefront was introduced into the model for the remaining 3912 aspheric surfaces. The induced peak to valley and rms wavefront error

by the error in the collimated wavefront was recorded for each test surface, as shown in FIGURE 5.49. FIGURE 5.50, shows the cumulative percentage of the aspheric surfaces for which the induced wavefront error was less than the magnitude displayed on the x axis. This graph shows that for 42% of the aspheric surfaces tested the peak to valley wavefront error induced by the measured collimated wavefront error was less than $\lambda/100$. Likewise for 97% of the aspheric surfaces tested the rms wavefront error induced by the measured collimated wavefront error was less $\lambda/100$. Which means for many aspheric surfaces the measured wavefront error, shown in FIGURE 5.44, will have negligible impact on the final measurement. However, FIGURE 5.50 also shows that for some aspheric surfaces the peak to valley error induced by the error in the collimated wavefront would be greater than $\lambda/10$. Therefore, the choice to include or ignore the measured error in the collimated wavefront has to be made based on the aspheric part being tested.

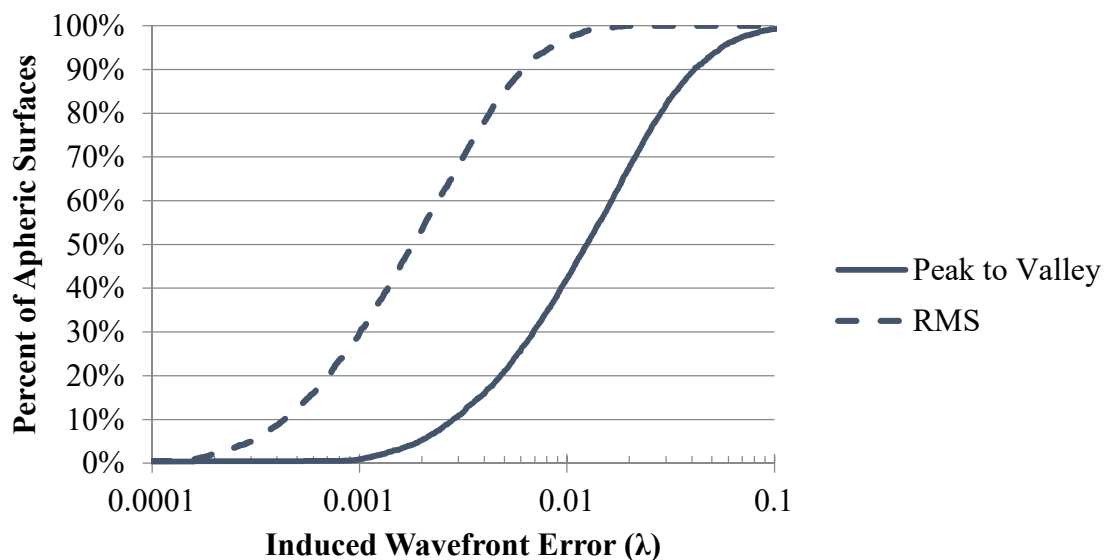


FIGURE 5.50 The cumulative percentage of the aspheric surfaces for which the wavefront error, induced by the error in the collimated wavefront, is less than the magnitude displayed on the x axis.

In the absence of mapping errors test and reference rays that interfere at the detector would originate from the same point in the collimated input wavefront of the interferometer. Therefore, errors in the input wavefront would be common to both arms of the interferometer and would have no impact on the measurement. However, as already discussed, if pupil aberration is present in the non-null interferometer than test rays and reference rays that interfere at the detector do not necessarily originate from the same point in the input wavefront. Therefore, the two rays do not necessarily encounter the same wavefront error since the wavefront error introduced into a given ray depends on its location in the input wavefront. This suggests that the impact the error present in the input wavefront will have on the final non-null measurement will increase as the difference between the wavefront error encountered by the test and reference rays increases. For each aspheric surface modeled the maximum separation of rays test and reference rays in the collimated input wavefront that would eventually interfere at the detector was calculated along with the absolute value of the OPD between these two rays. The Pearson product-moment correlation coefficient, which is a measurement of the linear correlation between two variables, was then calculated to see if either of these properties are correlated to the induced peak to valley or rms wavefront error. TABLE 5.5 lists the coefficients for these properties as well as for the sag departure of the asphere from the best fit sphere, the maximum wavefront slope difference and the peak to valley OPD.

Property	Correlation Coefficients	
	Induced PV Wavefront Error	Induced RMS Wavefront Error
Maximum separation, in the input beam, of interfering test and reference rays	0.894	0.915
OPD of the interfering rays with the largest separation in the input beam	0.854	0.728
Sag departure of the asphere from BFS	0.618	0.628
Maximum wavefront slope difference	0.573	0.638
Peak to valley OPD	0.550	0.674

TABLE 5.5 Pearson Product-Moment Correlation Coefficients

5.4.2 Characterization and Modeling of the Beam Splitter and Reference Surface

One element that proved difficult to set up in the RO model as a series of only sag surfaces was the beam splitter. It was difficult to keep the multiple instances of the beam splitter surfaces located at the same points in space due to the fact that the beam splitter interacts with both arms of the interferometer on multiple occasions. This is especially true once the interferometer is set up to allow for both forward and backwards ray tracing through the system. Therefore, the beam splitter was set up in the RO model as a combination of both phase and sag surfaces. This allowed for the beam splitter's contribution to each arm of the interferometer to be separated from one another. The layout of the beam splitter in the interferometer was discussed in Chapter 4.7. The original specifications of the beam splitter along with the measurements of the manufacturing errors were discussed in Chapter 4.8. A model of the beam splitter was constructed in Zemax, which incorporated the data from these measurements. This model was used to estimate the wavefront error that the beam splitter will introduce into each arm of the interferometer, to determine if uncertainties in the beam splitters

properties or alignment errors need to be included as variables in the RO model, and to calculate phase surface representations of the beam splitter's interactions with light in each arm of the interferometer.

In the test arm, light interacts with the beam splitter on two occasions. First, the collimated input beam of the interferometer transmits through both surfaces on its path towards the test surface. After the rays reflect off the test surface, they encounter the beam splitter the second time and are directed into the imaging arm of the interferometer by reflecting off the beam splitter's 50% reflective surface. These two interactions will be considered independently. The wavefront error introduced into the test arm after transmitting through the beam splitter, over a 48mm diameter, is shown in FIGURE 5.51. The beam splitter introduced $\lambda/40$ peak to valley and $\lambda/166$ rms wavefront error into the collimated wavefront, over the full 48mm aperture. However, 90% of the aspheric surfaces generated in Chapter 4.5.6, can be measured with a wavefront diameter of only 38.5mm. Over this smaller aperture the wavefront error is $\lambda/58$ peak to valley and $\lambda/250$ rms.

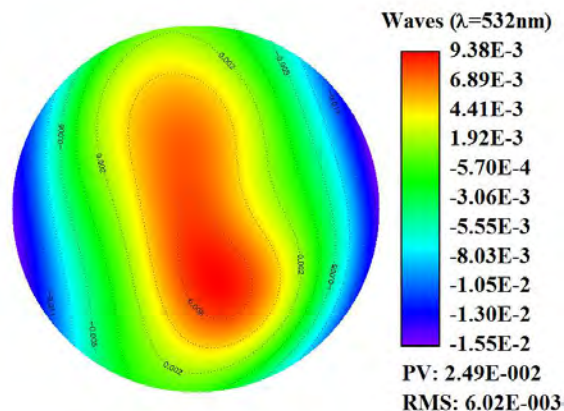


FIGURE 5.51 The wavefront error introduced by the test beam's initial transmission through the beam splitter.

The wavefront error map shown in FIGURE 5.51 would be generated if the model of the beam splitter exactly matched the position, orientation and properties of the actual beam splitter. In order to determine if errors in the alignment or the measured properties of the beam splitter would have to be included as variables in the reverse optimization procedure, perturbations were made to the modeled beam splitter and the resulting change in the wavefront error map were calculated. The properties that were changed in the model, included decenter of the beam splitter along both the x and y axes, the tilt about the x and y axes, the rotation about the z axis, the wedge angle between the two surfaces, the index of refraction of the glass and the center thickness. These properties were changed one at a time to see the impact each property would have on a given light interaction with the beam splitter. At the end of this chapter the properties were all allowed to change simultaneously in a Monte Carlo experiment in order to estimate the net effect of the uncertainty in each measurement would have on the final measurement. The magnitude of the perturbations for each property were determined from the

uncertainty in the measurement of each property made prior to, or during, the construction of the interferometer. The wedge angle of the beam splitter was measured using a prism spectrometer. The peak to valley range over ten measurements was 7.4 arc seconds or 0.002° . In order to show the minimal impact the wedge angle has on these measurements it was allowed to vary by ten times this range at $\pm 0.01^\circ$ for these simulations. The index of refraction was also measured using the prism spectrometer and over the ten measurements a peak to valley range of 0.00058 was observed. Again a much larger range was selected as the index of the beam splitter in the simulations was allowed to vary by ± 0.005 . The center thickness of the beam splitter was determined using a dial indicator and was reported by the manufacturer to the nearest .001 inches or 0.025mm, a range of ± 0.25 mm was used for the simulations. The decenter of the beam on the beam splitter had to be measured by visual inspection with a ruler. As such it could only be known to the nearest half millimeter. In the simulations, the decenter, in both the x and y axes, were allowed to vary by ± 2 mm. The tilt about the x axis of the beam splitter was determined by how well the beam could be aligned to be parallel to the optics table, before and after the insertion of the beam splitter. Stopping down the input beam and aligning the height of the beam to change by less than a millimeter over a meter should yield a beam splitter angle within 0.06° of perpendicular to the optics table. For the simulations, the tilt about x was allowed to change by $\pm 0.5^\circ$. Looking at the reflections off of both surfaces allowed for the wedge of the beam splitter, and the rotation about the z axis, to also be aligned to the optics table. However, the rotation about the z axis also includes the uncertainty in the rotation of surface measurements

made with the WYKO 6000 interferometer, based on the rotation of its camera, to the camera in the non-null interferometer. To estimate and minimize this error a 2" flat mirror was placed in the test arm of both interferometers along with a machinist square which blocked half the beam. The rotation angle of the camera was estimated by counting the change in the number of horizontal pixels masked by the square across the aperture. The sub-Nyquist camera was then shimmed in-order to match rotation of the WYKO interferometer camera. At best with this technique the cameras could only be aligned to a single pixel over approximately 500 pixels or 0.11° . However, it's unlikely that the agreement was achieved to this accuracy, therefore an angle of $\pm 1^\circ$ was used for the following simulations. Additionally, if there is an error in the rotation of the beam splitter surface maps it would be approximately the same, and in the same direction, for both surface measurements. This is because each surface was measured facing the Fizeau interferometer by rotating the beam splitter in its final mount 180° . However once the surface data is loaded into the model, the errors will be in opposite directions. Therefore for the following simulations the rotational error was always assumed to be in equal and opposite directions for the two beam splitter surfaces. The tilt about the y axis was determined by measuring the angle between the stopped down beams in each arm of the interferometer with a protractor. This measurement could be made to about 0.25° , but a range of $\pm 2^\circ$ was used for the simulations. TABLE 5.6 shows the maximum change in the peak to valley and rms OPD error, after subtracting tilt, for each perturbation in the beam splitter properties over the ranges indicated.

		ΔOPD_Z (Diameter = 48mm)		ΔOPD_Z (Diameter = 38.5mm)	
Property	Range	PV	RMS	PV	RMS
Test Arm (First Pass)	-	2.49E-2	6.02E-3	1.73E-2	4.16E-3
Wedge	$\pm 0.01^\circ$	2.69E-5	6.61E-6	1.81E-5	4.25E-6
Index of Refraction	± 0.005	2.13E-4	4.97E-5	1.47E-4	3.49E-5
Center Thickness	$\pm 0.25\text{mm}$	1.16E-4	2.35E-5	1.07E-4	2.18E-5
Decenter in X	$\pm 2\text{mm}$	1.59E-3	2.43E-4	1.10E-3	1.97E-4
Decenter in Y	$\pm 2\text{mm}$	1.45E-3	2.67E-4	1.35E-3	2.11E-4
Tilt about X	$\pm 0.5^\circ$	2.42E-4	4.06E-5	1.37E-4	2.03E-5
Tilt about Y	$\pm 2.0^\circ$	9.63E-4	2.52E-4	7.37E-4	1.70E-4
Tilt about Z	$\pm 1.0^\circ$	4.28E-3	8.06E-4	2.67E-3	5.14E-4

TABLE 5.6 The OPD_Z error introduced by the first pass of the test arm through the beam splitter, and the change in the OPD_Z error for perturbations of the various beam splitter properties.

The first pass through the beam splitter by the test arm introduces a small error, four times smaller than the desired interferometer accuracy, into the collimated wavefront, over the full 48mm aperture. This error is reduced when the input beam diameter required to test a given aspheric surface is smaller than the full 48mm. The changes in the wavefront error introduced based on the model of the test arm not matching the physical system, inside the predicted error in the measurement of each property, are significantly smaller than desired interferometer accuracy. Additionally, it is important to remember that this is wavefront error introduced into the test wavefront only. While the reference wavefront will have several more interactions with the beam splitter, which will be discussed later in this chapter, for now only consider its first pass through the AR coated side of the beam splitter and its final pass through the 50% reflective side of the beam splitter. If the pupil aberrations in the test and reference arms were equal then the test and reference rays, which interfere at the detector would pass through the same

points on each of the beam splitter surfaces. In this case, the same error would be introduced into both arms and there would be no contribution to the final measured OPD. Since non-null testing introduces different pupil aberrations into each interferometer arm, the errors will not be the same. However, in Chapter 5.4.1, the maximum separation of test and reference rays in the collimated input wavefront, that eventually interfere at the detector, was calculated for each of the 3912 modeled aspheric surfaces. Over all of these test surfaces the average maximum separation of test and reference rays in the input wavefront was only 0.48mm, while the largest separation was found to be 2.67mm. Looking back at FIGURE 5.51 the change in the wavefront error over any 2.7mm window was calculated to be less than $\lambda/150$. Therefore the error introduced into the final measurement for a single pass through both beam splitter surfaces is insignificant provided it is properly accounted for in the RO model for both interferometer arms. Finally, the measurement of the collimated wavefront, discussed in Chapter 5.4.1, was made after the beam transmitted through the beam splitter. Therefore, the wavefront error introduced into the test arm is already included in the phase surface representation of the collimated wavefront error. Rather than trying to separate these two error sources they are simply left as a single phase surface in the RO model.

The second interaction the test beam has with the beam splitter occurs when the aspheric test wavefront is directed into the imaging arm. The beam splitter is set up so that this is an external reflection, as such the errors in the index of refraction, wedge and center thickness do not impact the wavefront error. The error introduced into a collimated

wavefront over a 48mm diameter beam is shown in FIGURE 5.52, $\lambda/3$ peak to valley and $\lambda/14$ rms. This error is significantly larger than the error encountered on the first pass since it is generated by a reflection off the surface. This is the only interaction the beam splitter has in the interferometer in which the shape of the incident beam will change depending on the part being tested. Therefore the exact contribution into the test arm will depend on the diameter of the test beam and the distribution of the test rays as they encounter the surface. For instance, if the wavefront returning from the test surface is converging, the diameter of the beam at the beam splitter can be significantly smaller, reducing the induced error.

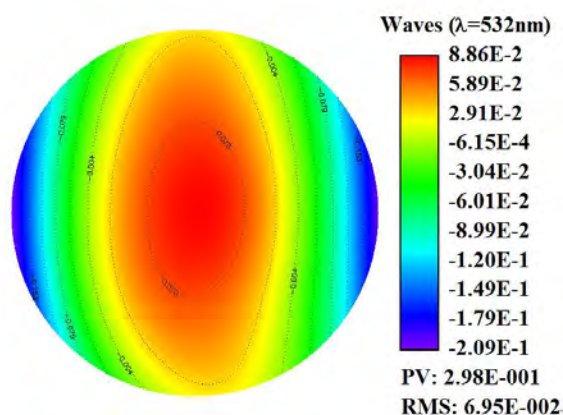


FIGURE 5.52 The wavefront error introduced into the test arm over a 48mm diameter collimated wavefront.

Property	Range	ΔOPD_Z (Diameter = 48mm)		ΔOPD_Z (Diameter = 38.5mm)	
		PV	RMS	PV	RMS
Test Arm (Second Pass)	-	2.98E-1	6.95E-2	1.90E-1	4.37E-2
Decenter in X	$\pm 2\text{mm}$	3.78E-3	7.66E-4	3.54E-3	6.91E-4
Decenter in Y	$\pm 2\text{mm}$	5.05E-3	7.97E-4	3.69E-3	5.39E-4
Tilt about X	$\pm 0.5^\circ$	9.48E-4	1.69E-4	6.59E-4	1.14E-4
Tilt about Y	$\pm 2.0^\circ$	1.09E-2	2.54E-3	7.65E-3	1.72E-3
Tilt about Z	$\pm 1.0^\circ$	5.73E-3	1.03E-3	3.40E-3	6.25E-4

TABLE 5.7 The OPD_Z error introduced by the second interaction of the arm with the beam splitter, and the change in the OPD_Z error for perturbations of the various beam splitter properties.

The same perturbations used for the first pass through the beam splitter were applied to the model for this second interaction. From TABLE 5.7, it is clear that the nominal OPD_Z error, must be taken into account in the RO model. This second interaction of the test beam with the 50% reflective surface of the beam splitter was included in the RO model as a Zernike Sag surface since the distribution of rays and the shape of the aspheric wavefront incident on this beam splitter will be different for every surface tested. The perturbations to the surface result in a small changes to the induced OPD_Z , all are less than $\lambda/130$ peak to valley over the 38.5mm diameter. The decision to include these properties as variables in RO model can be made on case by case basis, depending on the aspheric surface under test, the diameter of the test beam incident on the beam splitter, and the magnitude of the disagreement between the RO model and physical system. While these properties are not needed at the beginning of the RO process they could possibly be brought in at the end in order to fine tune the result.

Next consider the interaction of the reference arm with the beam splitter and the reference surface. The reference beam transmits thorough the AR coated surface of the beam splitter 3 times, both reflects off and transmits through the 50% reflective surface of the beam splitter, and reflects off the reference mirror. The path of the reference beam is shown below in FIGURE 5.53. This portion of the interferometer is not changed after the initial setup. The collimated input beam always follows the same path through the beam splitter and off the reference surface regardless of the aspheric surface tested. The only thing that changes between measurements is the portion of the reference beam that is used to produce the interferogram at the detector. The diameter of the reference beam that is required to fill the test beam at the detector will depend on the aspheric surface under test and the magnification at which the test surface is imaged. Additionally, the center of the test beam at the detector might be shifted from the center of the reference beam.

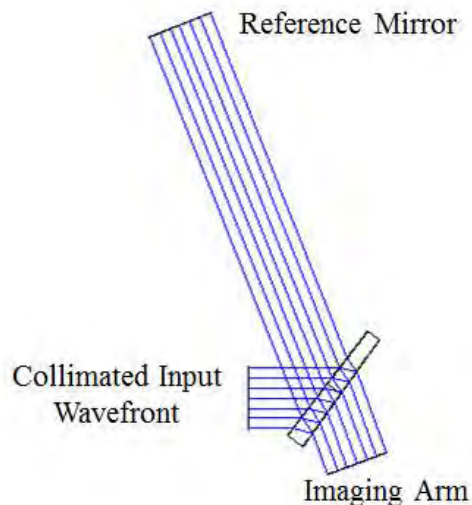


FIGURE 5.53 The interaction of the reference beam with the beam splitter and reference mirror.

First consider the contributions of the reference mirror. The measurement of the reference mirror surface error was discussed in Chapter 4.8 and shown in, FIGURE 5.53. This measured data was fit to Zernike fringe polynomials and added into the model of the beam splitter as a Zernike sag surface. Since the reference mirror is just a single surface the only properties that may need to be included are the various surface decenters and tilts. The reference mirror was set up to be perpendicular to the incoming light by placing a 1mm aperture in the beam after the collimating lens and adjusting the tip and tilt of the reference mirror so that the light was reflected back through the aperture, at a distance of slightly over 350mm. The tilt of the reference mirror, about the x and y axes, was assumed to be within $\pm 0.125^\circ$ of perpendicular to the incoming light. This angle corresponds to the range required to shift the reflected light from one side of the aperture to the other. The $\pm 2\text{mm}$ tolerance range on the surface decenters as well as the $\pm 1.0^\circ$ range on the rotation about the z axes used for the beam splitter were also applied to the reference mirror. The nominal error introduced by the reference surface is shown in FIGURE 5.54, and listed in TABLE 5.8 along with the change in the OPD_z introduced by perturbing the reference surface. The data in the table shows that while the nominal contribution to the OPD_z of reference wavefront by the surface errors of the reference mirror should be taken into account in the RO model, the change in the OPD_z due to misalignment of the surface can be ignored as they are all less than $\lambda/200$ peak to valley over the 38.5mm diameter.

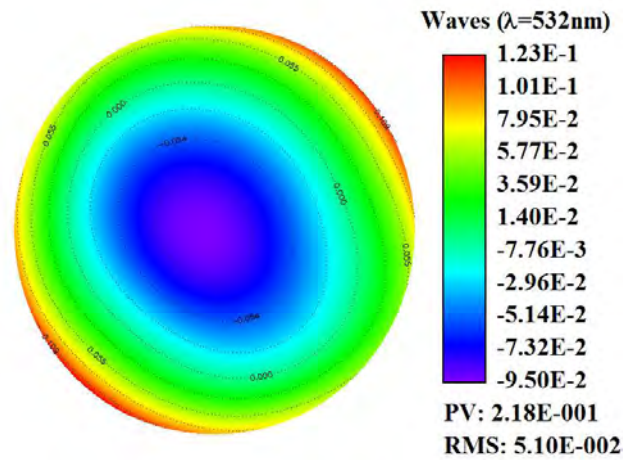


FIGURE 5.54 The wavefront error introduced into the reference arm by the reference surface over the full 48mm diameter aperture.

Property	Range	ΔOPD_Z (Diameter = 48mm)		ΔOPD_Z (Diameter = 38.5mm)	
		PV	RMS	PV	RMS
Reference Surface	-	2.18E-1	5.10E-2	1.40E-1	3.46E-2
Decenter in X	$\pm 2\text{mm}$	1.10E-2	1.14E-3	4.67E-3	8.87E-4
Decenter in Y	$\pm 2\text{mm}$	1.10E-2	1.15E-3	3.20E-3	8.22E-4
Tilt about X	$\pm 0.5^\circ$	2.44E-5	3.07E-6	8.89E-6	1.67E-6
Tilt about Y	$\pm 2.0^\circ$	3.18E-4	4.53E-5	1.30E-4	2.61E-5
Tilt about Z	$\pm 1.0^\circ$	2.37E-03	4.02E-4	1.47E-3	2.86E-4

TABLE 5.8 The OPD_Z error introduced by the reference surface, and the change in the OPD_Z error for perturbations of the reference surface orientation.

Next, consider the beam splitter's contribution to the wavefront error of the reference beam. The nominal error introduced into the reference arm by all the interaction of the reference beam with the beam splitter is shown in FIGURE 5.55. The nominal contribution, over the full 48mm diameter, is approximately $\lambda/4$ peak to valley and $\lambda/19$ rms. In addition to simulating the previously discussed perturbations for the beam splitter properties, the tilt of the reference mirror must also be considered. This is because a non-zero tilt in the reference mirror would cause the reference beam to follow a different path

back to the beam splitter and intersect a different portion of the beam splitter surface.

TABLE 5.7, shows the change in the OPD_z introduced into the reference beam by perturbing the beam splitter. The most significant contribution was due to the uncertainty of the rotation of the beam splitter about the z axis.

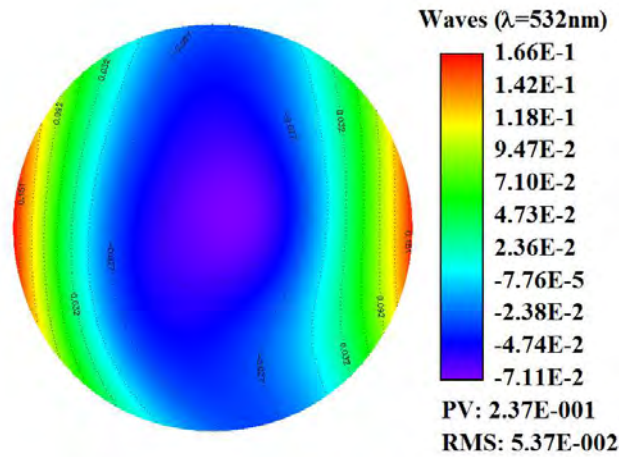


FIGURE 5.55 The wavefront error introduced into the reference arm by all of its interactions with the beam splitter over the full 48mm diameter aperture.

Property	Range	ΔOPD_z (Diameter = 48mm)		ΔOPD_z (Diameter = 38.5mm)	
		PV	RMS	PV	RMS
Reference Arm	-	2.37E-1	5.37E-2	1.68E-1	3.65E-2
Wedge	$\pm 0.01^\circ$	4.06E-5	8.66E-6	2.50E-5	5.48E-6
Index of Refraction	± 0.005	3.94E-4	8.51E-5	2.95E-4	7.28E-5
Center Thickness	$\pm 0.25\text{mm}$	7.36E-4	1.52E-4	7.18E-4	1.47E-4
Decenter in X	$\pm 2\text{mm}$	7.13E-3	1.50E-3	7.10E-3	1.48E-3
Decenter in Y	$\pm 2\text{mm}$	6.99E-3	1.49E-3	7.09E-3	1.49E-3
Tilt about X	$\pm 0.5^\circ$	8.36E-4	1.46E-4	6.23E-4	1.09E-4
Tilt about Y	$\pm 2.0^\circ$	4.69E-3	1.02E-3	3.34E-3	6.95E-4
Tilt about Z	$\pm 1.0^\circ$	1.84E-2	3.43E-3	1.12E-2	2.15E-3
Tilt of Reference about X	$\pm 0.125^\circ$	1.92E-3	3.41E-4	1.67E-3	2.89E-4
Tilt of Reference about Y	$\pm 0.125^\circ$	1.94E-3	4.12E-4	1.99E-3	4.14E-4

TABLE 5.9 The OPD_z error introduced by the beam splitter into the reference arm and the change in the OPD_z error for perturbations of the various beam splitter properties.

The wavefront error introduced into the reference beam by the combination of the reference surface and beam splitter is shown in FIGURE 5.56 (Left). As discussed earlier in this section the measurement of the collimated wavefront made in the test arm includes the error contributed by the first pass through the beam splitter. Rather than trying to determine the shape of the collimated light prior to entering the beam splitter, the measurement was simply used as the input into the test arm. If the same measurement is to be used as the input into the reference arm then two things must be determined. First it must be verified that the beam splitter would introduce the same error into the reference arm as is introduced into the test arm during a portion of their interaction. Secondly the source of these errors would have to be removed from the model of the reference arm so that their contribution is not doubled in the RO model. This was accomplished by taking the calculated wavefront error resulting from the test arm transmitting through the beam splitter, shown in FIGURE 5.51, and turning it into a Zernike phase surface. This phase surface was then placed in the model of the reference arm to simulate the error present in the collimate wavefront measurement. Next the sag errors responsible for this wavefront error needed to be removed from the model. This was done by removing the measured sag error on the first instance of the beam splitters AR surface and the last instance of the 50% reflective surfaces flat planes. It is important to note that the AR surface is encountered three times by the reference beam and only for the first of these encounters was the sag error removed. Likewise the sag error was only removed from the 50% reflecting surface when the reference beam transmits through it, the sag error is left in place for the instance when the reference beam reflects off this surface. The resulting

wavefront error map is shown in FIGURE 5.56 (Right) while the difference between the original model and the partial phase model is shown in FIGURE 5.57. Since the difference between the two models is negligible the contributions of the first and last interaction of the beam splitter with the test arm can be removed from the model and assumed to be taken into account by the phase surface representing the collimated wavefront measurement.

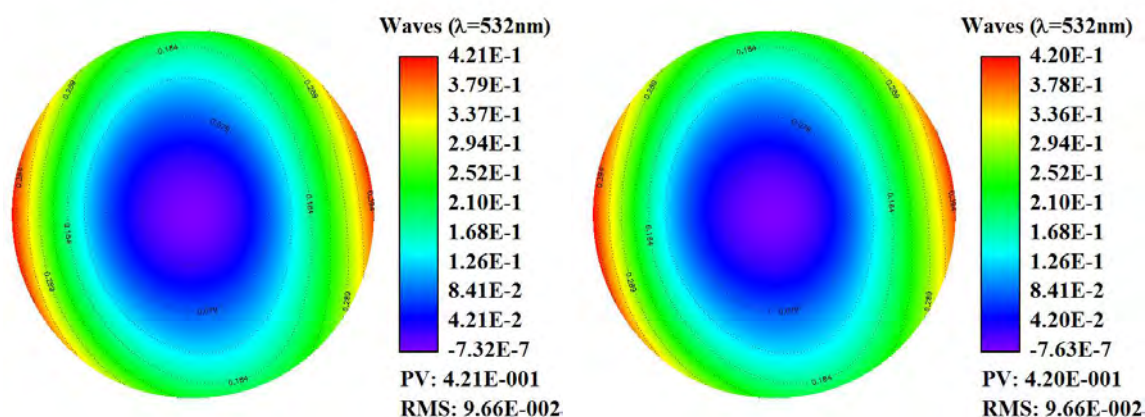


FIGURE 5.56 The wavefront error introduced into the reference arm by the combination of the beam splitter and reference surface (Left). The same error calculated after replacing two of the sag surface interactions with a phase surface representing the measured error in the collimated wavefront. (Right)

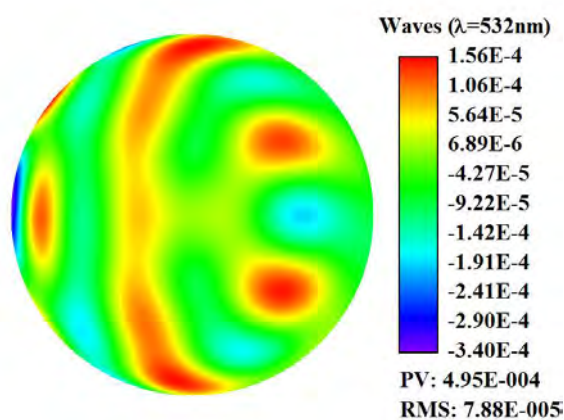


FIGURE 5.57 The difference between the original model and partial phase model, shown in shown in FIGURE 5.56.

The option remains to condense the entire reference beam, from the collimating lens to the last interaction with the beam splitter, into either a single phase surface that will produce the same wavefront in the imaging arm as the previously discussed models. This would significantly reduce the number of surfaces in the model, especially since all the coordinate break surfaces required to orient the beam splitter to the reference arm could be eliminated. However, this is only possible if there is no need for the RO procedure to make changes to the reference arm, by altering the position or alignment of the beam splitter and reference surface or if a method of altering the phase surface that mimics changes to the sag model of the beam splitter can be found. Changes to the diameter of the reference beam required to fill the test beam at the detector can still be made provided the reference phase surface is defined over a large enough diameter.

Additionally offsets in the center of the reference beam compared to the center of the test beam can still be modeled by shifting the phase surface. However this also assumes that the diameter of ray bundle incident on the reference phase surface plus the required shift is within the diameter over which the phase surface was defined. This phase surface could either be modeled as a Zernike phase surface or as a grid phase surface. The difference between the wavefront error introduced into the reference arm calculated from the sag representation of the beam splitter, shown in FIGURE 5.56 (Left), and the single phase surface representation are shown in FIGURE 5.58.

There is a very small amount of error, less than $\lambda/500$ peak to valley, from the Zernike fitting, utilizing 37 fringe terms, not being able to exactly match the transmitted wavefront, shown in FIGURE 5.58 (Left). While this is small enough to be ignored,

most of this error is on the edge of the wavefront and reducing the diameter by 2% yields peak to valley error of $\lambda/1000$. Likewise the FIGURE 5.58 (Right), shows the difference between the sag representation and a grid phase representation of the wavefront error introduced into the reference arm. In this case, the reference wavefront diameter needed to be reduced by 2% in order to avoid the ringing that occurs at the edges of the grid phase surface. However the residual wavefront difference over the remaining aperture is virtually non-existent. The grid phase surface clearly shows a better agreement with the original model. When tracing rays forward through the test arm and backwards through the reference arm this surface could be located at the final plane in the model. Since the model would not have to aim rays through this surface it probably wouldn't slow down the raytracing significantly. However, when the system is reversed for reverse ray tracing this surface would become the first surface in the model and it would impact the raytracing speed. Therefore, the Zernike representation was used for the RO model, even though it shows more error.

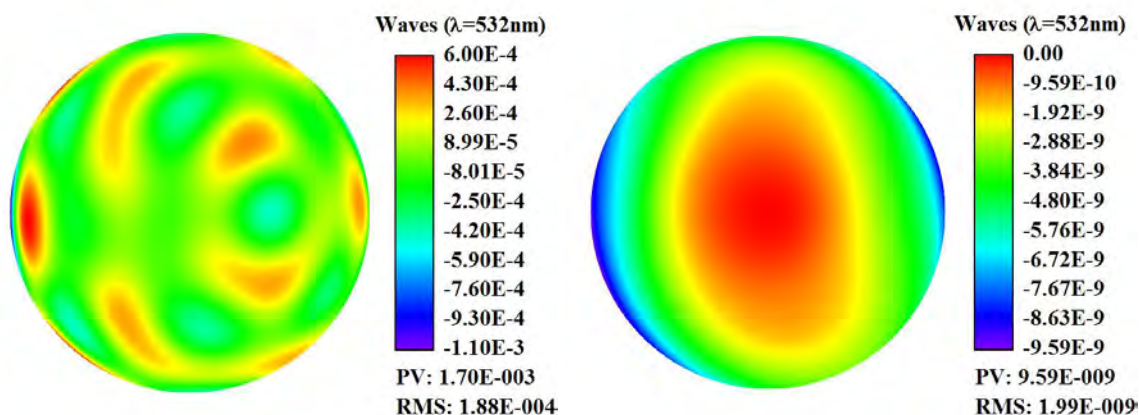


FIGURE 5.58 The difference between the original model and phase only model of the reference arm using a Zernike phase surface (Left) and a grid phase surface (Right).

Up until this point the wavefront error introduced into the individual test and reference wavefront have been considered independently of each other. However, it is a change in the OPD that will ultimately affect the measurement. In order to check if the phase surface model is comparable to the sag surface model in the presence of misalignments a Monte Carlo simulation was set up in which the sag model properties were assigned to a random number within their previously discussed ranges. The maximum change in the OPD between the sag model and the phase model was then calculated. This was repeated for 20,000 simulations and the percentage of the simulations in which the change was less than $\lambda/50$, $\lambda/100$ and $\lambda/200$ was tabulated, and shown in TABLE 5.10. However, the phase surface could also be allowed to shift, rotate and tilt in order to better match the sag model. While shifting and tilting the phase surfaces doesn't introduce the same error in the phase model as is introduced by perturbing the sag model, it does improve the agreement between them. Therefore in the RO model the beam splitter and reference surface were modeled as phase surfaces, except for the reflection in the test arm which was kept as a sag surface. The decenters and tilts can be allowed to vary to account for misalignments in the system, although since the change introduced by these misalignments is small, this is not typically done.

Δ OPD between Sag and Phase Models	$<\lambda/50$	$<\lambda/100$	$<\lambda/200$
Stationary phase surface	100%	78.7%	11.8%
Variable phase surface orientation	100%	99.8%	74.3%

TABLE 5.10 The percentage out of 20,000 simulations in which the change in the OPD between the sag and phase models is less than the indicated value when the beam splitter and reference mirror properties are perturbed to simulate misalignments.

5.4.3 Characterization and Modeling of the Imaging Lens

As discussed in Chapter 4.6.3, the imaging lens used was an air spaced doublet made up of two off the shelf plano-convex lenses, the design prescription was listed in TABLE 4.21. The imaging lens is a common element to both the test and reference arms of the interferometer. However, the test and reference rays will not travel the exact same path through it in a non-null test. Discrepancies between the physical imaging lens and the molded imaging lens will introduce errors into both arms of the interferometer.

Ultimately it is the difference between the errors introduced into each arm that will impact the final measurement. The position and orientation of the imaging lens will be set as variables in the RO process. However, it was uncertain if every property of the lens needs to be included as variables. These properties include the radii of curvature of the lenses, the form error of the surfaces, the index of refraction of the glass, the center thicknesses, and the various tilts and decenters of the individual surfaces.

Before making this determination, the expected uncertainty in each of the imaging lens properties must be estimated, either by direct measurements or by evaluation of the manufactures' tolerances. First, because these were off the shelf lenses the manufacture's tolerance had to be used to estimate the uncertainty in the index of refraction. Both lens manufacturers specified the same glass with the same ± 0.0005 tolerance on the index of refraction n_d , the Helium d-line 587.5618 nm, and a $\pm 0.8\%$ tolerance on the abbe number. Converting to 532nm light, this yields a tolerance of ± 0.00052 on the index of refraction. The center thicknesses of the lenses were specified to within $\pm 0.1\text{mm}$ of the designed

value by the manufacturer, which was used as the tolerance range. However, the center thickness of each plano-convex lens was also measured with a digital height gauge. The gauge had a specified resolution of 0.01mm and an accuracy of 0.04mm. The separation of the two lenses was determined by measuring the distance between the two mounting surfaces against which the planar surfaces rest. This was done with a dial indicator mounted to the previously discussed height gauge. The measurement was performed at three points around the circumference of the mount, and the average value was used for the model. The accuracy of this measurement should be similar to the glass thickness measurement, but because it involved a combination of devices the measurement range used for the simulations was doubled to $\pm 0.20\text{mm}$. The lens from Newport Optics had a radii of curvature tolerance of $\pm 0.3\%$ or approximately 0.05mm, while the planar surface had a tolerance of 1.5λ surface power. The lens for Edmund Optics had a tolerance of $\pm 0.1\text{mm}$ on the radius of curvature of the spherical surface and no specification of the power of the planar surface. Instead of relying on these tolerances the radii of curvature of the spherical lenses were characterized using the WYKO 6000 Fizeau interferometer and its digital slide. The uncertainty in the measurement of the surface radii is the result of Abbé errors introduced by the misalignment between the slide and the optical axis of the interferometer. This error on the radius of curvature measurement was estimated to be less than or equal to 0.01%. (Selberg 1992) A range of $\pm 50\mu\text{m}$ was used for both spherical surfaces, which corresponds to an uncertainty of approximately 0.02% for the first lens and 0.003% for the second. A range of 3λ was used for the power on both of the flat surfaces. The actual surfaces figure error of each surface measured with the

WYKO 6000 will be discussed later in this section. Finally, the tilts and decenters of the lens surfaces were measured with a Point Source Microscope, PSM, and centering station from Optical Perspectives Group. (Parks & Kuhn, 2005) The procedure, outlined by Parks (2007) (2012), involves using the PSM to observe the motion of the center of curvature of the each lens surface as the lens is rotated on a rotary air bearing table. In this case the lens was mounted with the spherical surfaces oriented down and viewed from the top down. FIGURE 5.59, drawn horizontally, shows the apparent center of curvatures formed by the two spherical surfaces and the center planar surface. The PSM, which is not shown in the figure, is mounted on a vertical slide to allow it to translate between various lens centers. When properly aligned, the light emitted from the PSM will follow the same path to and from the lens center of curvature. However, when a surface is tilted or decentered from the optical axis the returned focal spot will be displaced on its return to the PSM, FIGURE 5.60. The light focused by the lens will trace out a circular path when the rotatory air bearing table is turned when the center of curvature is not aligned to the center of the rotary air bearing table. Additionally, the orientations of previous surfaces will also affect the position of the returned spot, so only when all the surfaces are properly aligned to both the PSM and the center of rotation of the rotary air bearing table will all the returned focal spots be stationary as the air bearing table is turned.

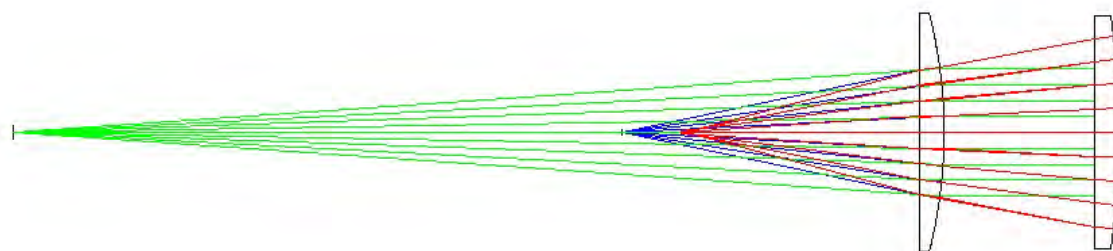


FIGURE 5.59 The locations at which light from the PSM is focused back on itself from the first three lens surfaces.

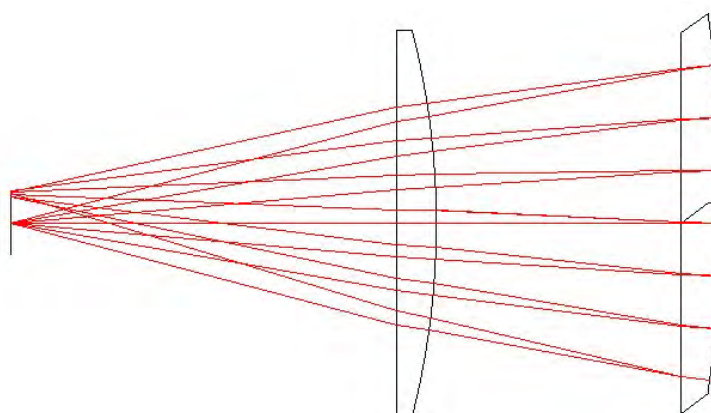


FIGURE 5.60 Example of a greatly exaggerated surface decenter in the lens causing a lateral shift in the location returned focal spot.

The mounting hardware used for the imaging lens contained holes along the outside edge which allowed for the lenses to be centered relative each other by tapping them into position. Unfortunately, the lenses could only be aligned such that the maximum displacement of the focal spots recorded by the PSM was just under $100\mu\text{m}$. Using the model shown in, FIGURE 5.60, the range over which each surface could decenter and tilt that would keep all the spots within $\pm 100\mu\text{m}$, and $\pm 200\mu\text{m}$, of each other was calculated, TABLE 5.11. The exception being that the centerations of the flat surfaces were not considered since a translation of the plane surface along the plane introduces no error.

This procedure could, and possibly should have been revisited after the surface measurements of the planar surfaces had been made.

Property	Maximum Spot Shift	
	<100 μ m	<200 μ m
Surface 1 Decenter	6.941E-2 mm	1.388E-1 mm
Surface 1 Tilt	1.538E-2°	3.077E-2°
Surface 2 Tilt	9.576E-3°	1.915E-2°
Surface 3 Decenter	5.003E-2 mm	1.001E-1 mm
Surface 3 Tilt	1.848E-2°	3.697E-2°
Surface 4 Tilt	1.883E-2°	3.766E-2°

TABLE 5.11 The maximum tilt and decenter of each surface that could produce the measured $\pm 100\mu$ m shift in the spots, as well as the required

In order to investigate the impact each property has on the RO model, each property was individually perturbed in the RO model by the maximum of its previously described range. The change in the OPD at the detector was then calculated using the same procedure discussed for the beam splitter simulations, Chapter 5.4.2. Then the model was optimized, where only the imaging distances, the position of the detector and the orientation of the entire imaging lens were allowed to vary. This was done to determine if an unaccounted for error in one lens property would be accounted for by a change in another property of the RO model such that the net effect on the OPD is negligible. As an example if the radius of curvature of one of the lenses in the model is incorrect then the RO process might adjust the imaging distance in order to match the magnification of the physical imaging lens. This process was repeated for all the 3912 aspheric test surfaces previously discussed. The maximum change, and the average change observed in the OPD after reverse optimization for each property over all of the aspheric surfaces is shown in TABLE 5.12. However having an error in only one property is unlikely so a

second test was performed in which each property was set to random number within its range, and the reverse optimization process was repeated. The test conducted for three groups of properties. In one group only the first nine properties listed in TABLE 5.12 were allowed to vary, those that affect the power of the imaging lens. In the second only the tilts and decenters of the lens surfaces were allowed to vary and in the final group all properties were allowed to vary. These simulations were each run for all the aspheric test surfaces, over three different perturbations of the imaging lens properties for each group. The results are shown in TABLE 5.13. While the average peak to valley and rms change to the OPD is small, some of the maximum observed errors are approaching the desired accuracy of the interferometer. One note on this is that because the tilts and decenters listed in TABLE 5.11 were determined from the magnitude that each individual property would have to change to see the changes observed in the PSM, allowing them all to simultaneously vary over these ranges represents a larger departure from the imaging lens than was measured.

Property	Range	Max. PV	Max. rms	Avg. PV	Avg. rms
RoC (Surface 1)	$\pm 50\mu\text{m}$	$\lambda/261$	$\lambda/1614$	$\lambda/4418$	$\lambda/21478$
Power (Surface 2)	$\pm 3\lambda$	$\lambda/171$	$\lambda/938$	$\lambda/1839$	$\lambda/8369$
RoC (Surface 3)	$\pm 50\mu\text{m}$	$\lambda/185$	$\lambda/845$	$\lambda/2033$	$\lambda/8309$
Power (Surface 4)	$\pm 3\lambda$	$\lambda/148$	$\lambda/555$	$\lambda/1146$	$\lambda/4344$
Index of Refraction (Lens 1)	± 0.0005	$\lambda/126$	$\lambda/462$	$\lambda/877$	$\lambda/3781$
Index of Refraction (Lens 2)	± 0.0005	$\lambda/114$	$\lambda/405$	$\lambda/691$	$\lambda/2857$
CT (Lens 1)	$\pm 0.1\text{mm}$	$\lambda/258$	$\lambda/1415$	$\lambda/2633$	$\lambda/10709$
CT (Air Gap)	$\pm 0.2\text{mm}$	$\lambda/148$	$\lambda/470$	$\lambda/1032$	$\lambda/3754$
CT (Lens 2)	$\pm 0.1\text{mm}$	$\lambda/229$	$\lambda/1309$	$\lambda/2234$	$\lambda/9630$
Decenter (Surface 1)	$\pm 0.14\text{ mm}$	$\lambda/225$	$\lambda/1799$	$\lambda/2912$	$\lambda/19855$
Tilt (Surface 1)	$\pm 0.03^\circ$	$\lambda/224$	$\lambda/1797$	$\lambda/2855$	$\lambda/19475$
Tilt (Surface 2)	$\pm 0.020^\circ$	$\lambda/242$	$\lambda/1833$	$\lambda/3852$	$\lambda/28274$
Decenter (Surface 3)	$\pm 0.1\text{ mm}$	$\lambda/237$	$\lambda/1826$	$\lambda/3529$	$\lambda/25448$
Tilt (Surface 3)	$\pm 0.037^\circ$	$\lambda/235$	$\lambda/1821$	$\lambda/3386$	$\lambda/24491$
Tilt (Surface 4)	$\pm 0.038^\circ$	$\lambda/213$	$\lambda/1341$	$\lambda/1893$	$\lambda/13628$

TABLE 5.12 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization for each imaging lens property varying over its listed range after testing the 3912 aspheric test surfaces previously described.

Property	Max. PV	Max. rms	Avg. PV	Avg. rms
Radii of curvature, Indices of refraction & Separations	$\lambda/59$	$\lambda/196$	$\lambda/815$	$\lambda/3213$
Tilts & Decenters	$\lambda/284$	$\lambda/1513$	$\lambda/3798$	$\lambda/23358$
All properties	$\lambda/45$	$\lambda/270$	$\lambda/594$	$\lambda/2630$

TABLE 5.13 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization when imaging lens properties are allowed to vary simultaneously.

The surface figure error of each lens surface was measured using the WYKO 6000 Fizeau interferometer. These measurements are shown in FIGURE 5.61 - FIGURE 5.63. These measurements were fit to Zernike Fringe polynomials and included in the RO model as Zernike sag surfaces. The average of 10 center thickness measurement of was 8.0772

mm for the first lens and 7.7514 mm for the second lens. The separation between the lenses was measured at 48.3814mm, again being the average of ten measurements.

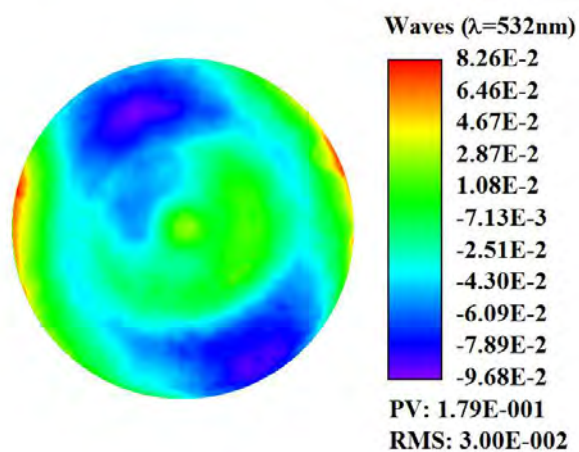


FIGURE 5.61 Measured surface error the spherical surface of Edmund Optics plano-convex lens 63-496.

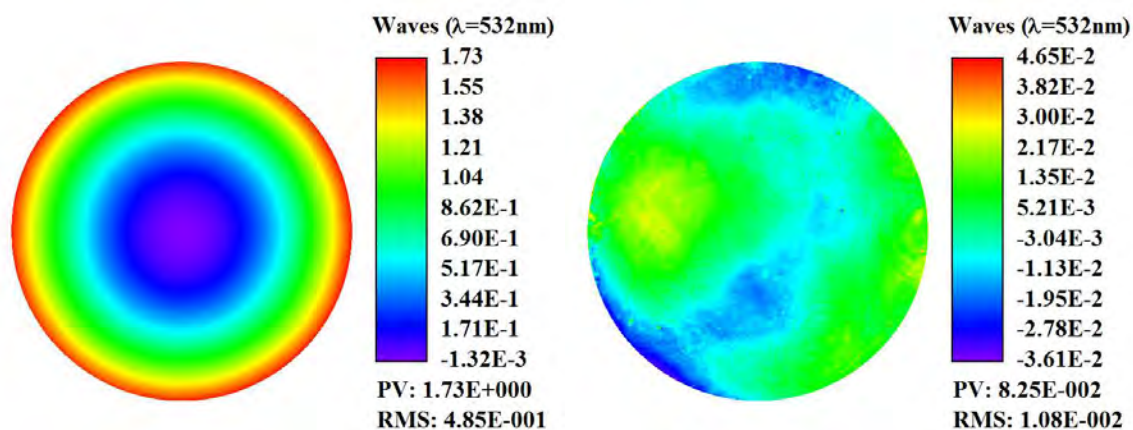


FIGURE 5.62 Measured surface error the planar surface of Edmund Optics plano-convex lens 63-49 (Left) and with 1.7λ of power removed (Right).

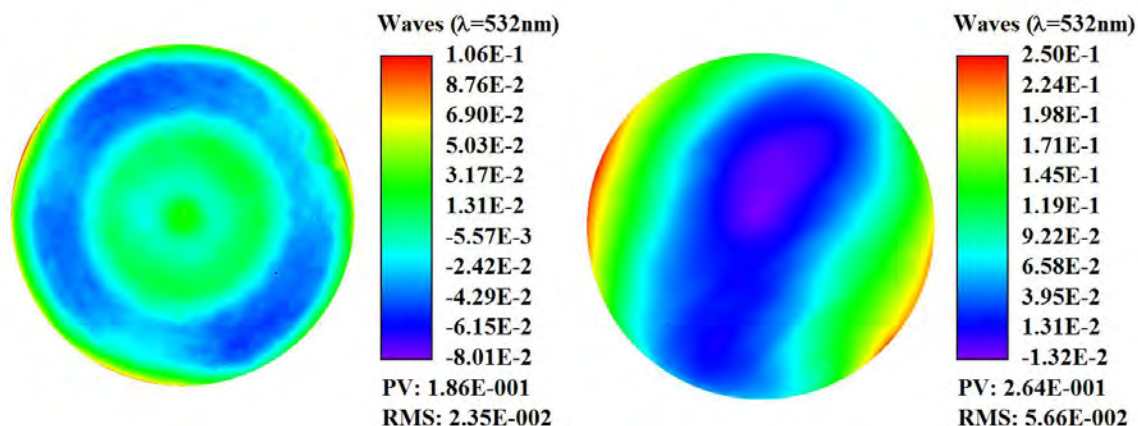


FIGURE 5.63 Measured surface error the spherical surface of Newport Optics plano-convex lens KPX232 (Left) and of the planar surface (Right).

5.4.4 Characterization and Modeling of the Diverger Lenses

Over the course of this research, attempts to use two different diverger lenses were made. One was an aspheric singlet, discussed in Chapter 4.5.4 with the prescription given in TABLE 4.4. The other lens was an aspheric doublet, discussed in Chapter 4.5.5 with the prescription given in TABLE 4.5. Both lenses were manufactured by Optimax Systems, Inc. (Ontario, NY). The advantages and disadvantages of each lens were discussed in Chapter 4.5.6. In this section, the measurements made to characterize the diverger lenses for the RO model will be discussed, starting with a brief description of the problem which precluded the use of the single element diverger. Then, the measurements of the aspheric doublet diverger lens properties will be presented, along with a basic analysis of each property's impact on the RO process, similar to the analysis performed on the beam splitter and the imaging lens.

The doublet diverger lens clearly outperformed the singlet in many key areas such as being able to test a higher percentage of the generated aspheric surfaces, generating lower average WFS, and inducing less pupil aberration. However, the singlet had one key advantage, fewer lens properties to characterize for the RO model. The properties of the singlet diverger that would need to be characterized or included as variables in the RO process include one index of refraction, one center thickness, two surface figures, and the position and orientation of those surfaces relative to one another. However, the position and orientation of the two surfaces are fixed relative to each other, so these values would not change after the lens has been measured. The lens properties could all be measured using similar techniques as discussed for the imaging lens. The center thickness of the lens was measured by the manufacturer and reported to the nearest micron, with a stated uncertainty of $\pm 2\mu\text{m}$. The glass index was determined by measuring a prism cut from the same blank the lens was manufactured from and measured using a prism spectrometer at 532nm. The manufacturer specified the edge thickness difference as a measure of the decenter of the two surfaces, but this also could have been measured using the PSM and alignment station. The concave spherical surface and radius of curvature was measured using the WYKO 6000 Fizeau interferometer and the digital slide. However, the one remaining property, the aspheric surface error, is the property that ultimately leads to this lens not being viable for use with the non-null interferometer. The aspheric lens surface was manufactured with a $1\mu\text{m}$ divot in the central 10mm of the surface. The surface error was measured using a contact profiler, shown in microns in FIGURE 5.64 (Left). The

surface was also measured using a Zygo Verifire Asphere as shown in FIGURE 5.64 (Right). In this plot the height data has been converted to waves at 532nm.

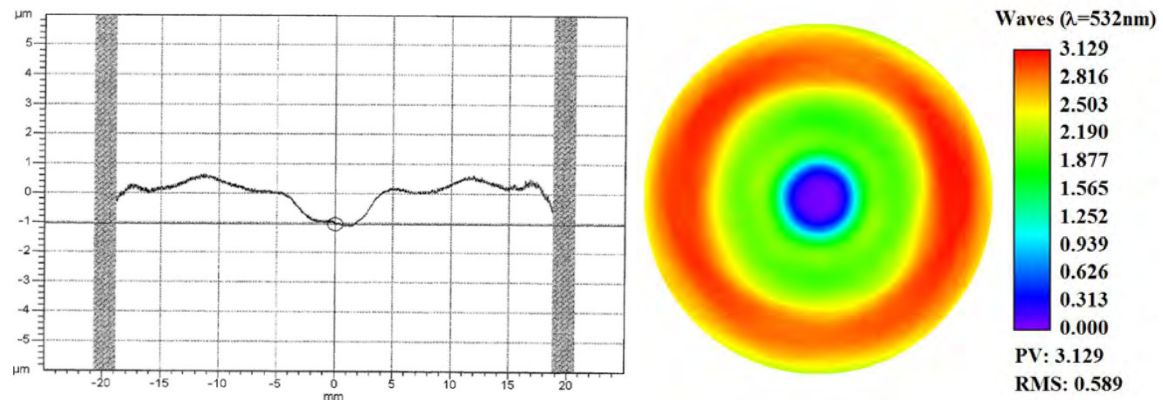


FIGURE 5.64 Measurements of the aspheric surface of the singlet diverger lens made by a stylus profiler (Left) and the Zygo Verifire Asphere (Right)

At 3.1λ peak to valley, the form error on this surface is over thirty times larger than the desired accuracy of the interferometer. Additionally, the lens was made out of the high index glass S-NPH2, that has an index of refraction of approximately 1.93 at 532nm, which means 0.93 times the surface error will be introduced into the test wavefront. Finally, the lens is used in double pass which doubles the surfaces contribution to the overall OPD error. A “null” fringe pattern generated by testing a spherical surface with the singlet diverger is shown in FIGURE 5.65 (Left), along with the measured OPD (Right). The reason the OPD is more than twice the surface error shown in FIGURE 5.64 (Right), is because the measurement of the spherical surface used a larger area of the singlet diverger’s aspheric surface than could be measured with the Zygo Verifire Asphere.

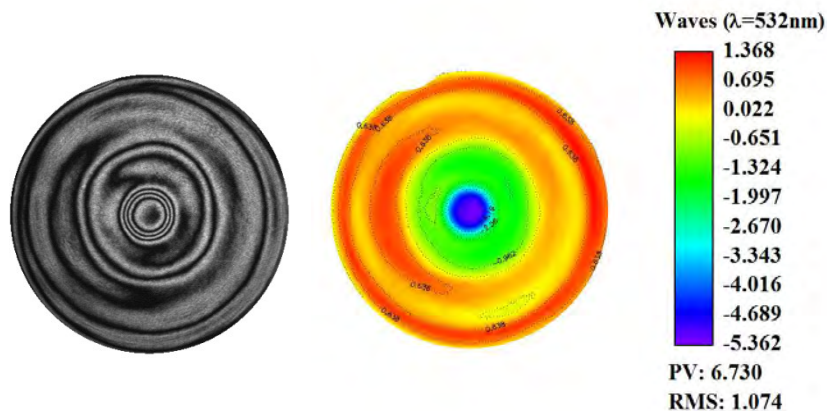


FIGURE 5.65 The “null” fringe pattern generated by testing a spherical surface (Left) and the resulting OPD produced by the aspheric surface in double pass (Right)

For the rest of the surfaces in the RO model the surface errors were modeled as Zernike fringe sag surfaces using 37 terms. However, with this surface because of the steep slopes near the central divot the difference between the measured surface data and the Zernike fit is large at 0.75λ waves peak to valley, FIGURE 5.66(Left). The difference is still over a half wave peak to valley if the measured surface data is fit using all 231 of the Zernike standard polynomials available for use in Zemax, FIGURE 5.66(Right). The impact that the defect on the surface of the diverger lens has on its performance in a non-null measurement will be discussed in Chapter 7.

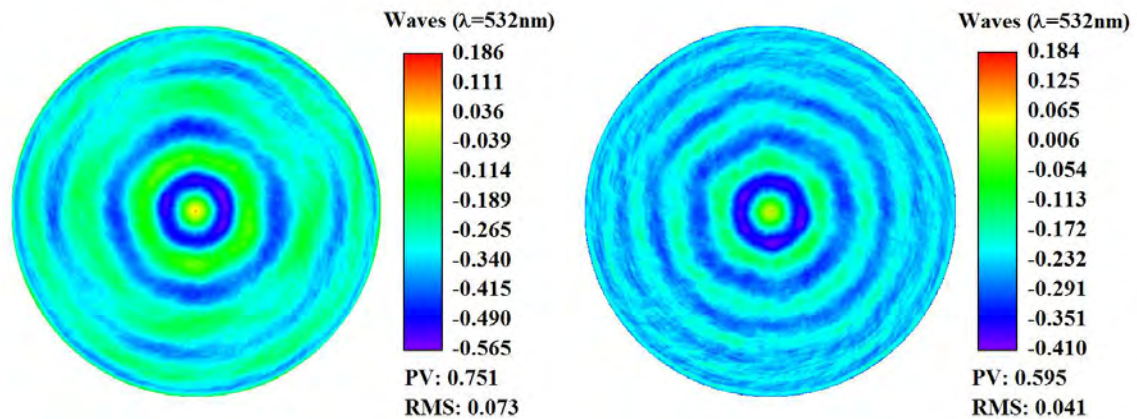


FIGURE 5.66 The difference between the aspheric surface of the singlet diverger lens measurement and the Zernike fit using Fringe terms (Left) and Standard terms (Right)

The doublet diverger lens was used for all the measurements that will be presented in Chapter 6. One issue with the doublet diverger lens was that it was not obtained until the very end of this research. As such many of the measurements used to characterize its properties were performed by either the manufacturer Optimax Systems, Inc or the original requester Optics 1, Inc. A similar analysis that was performed on the imaging lens to understand the impact of uncertainty in each property was performed on the diverger lens. The first property to characterize was the index of refraction of the glass. Both elements which make up the doublet diverger lens use the same high index glass, S-NPH2 from OHARA Corporation. (Branchburg, NJ) The index of refraction of the glass used to make each lens element was determined from melt data provided by the manufacturer. The melt data listed the index of refraction of each glass at n_C , n_d , n_F and n_g , out to the fifth decimal point. This data was then loaded into the Zemax glass catalog and fit to Sellmeier 1 formula. For more information on the process Zemax uses to fit melt data of glasses see the Zemax manual (Zemax LLC, 2011). The calculated index of

refraction of the first element was 1.93937 at 532nm while the index of the second element was 1.93874 at 532nm. The range of glass indices that was used in the tolerance analysis was $\pm 2 \times 10^{-5}$. The center thickness of each lens was measured by the manufacturer. The first lens had a center thickness of 10.528mm while the second had a center thickness of 10.047. A range of $\pm 2 \mu\text{m}$ was used in the tolerance analysis for these properties. The air gap spacing between the lenses as mounted was provided as 2.041mm, however a clear explanation of how this value was measured was never provided. Therefore, in the tolerance simulation the air gap separation of the lenses was allowed to vary by $\pm 20 \mu\text{m}$. The aspheric surface of the diverger lens was measured using a Zygo Verifire Asphere interferometer, FIGURE 5.67. The surface error was reported as the difference from the nominal surface prescription, which is a radius of curvature of 43.770 and a conic constant of -0.8929. The radius of curvature and the form error of each spherical surface were measured with a Zygo Fizeau interferometer, FIGURE 5.68 - FIGURE 5.70. The measured radii of curvature, starting with the second surface of the lens, were 234.495mm, 23.472mm, and 36.618mm. In the tolerance simulation the radius of curvature of each surface was allowed to vary by approximately $\pm 0.01\%$.

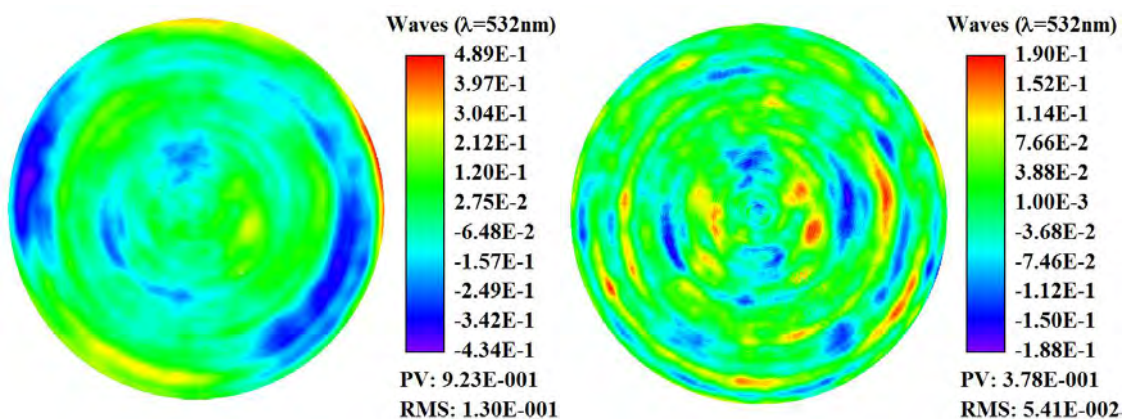


FIGURE 5.67 Error in the first surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right)

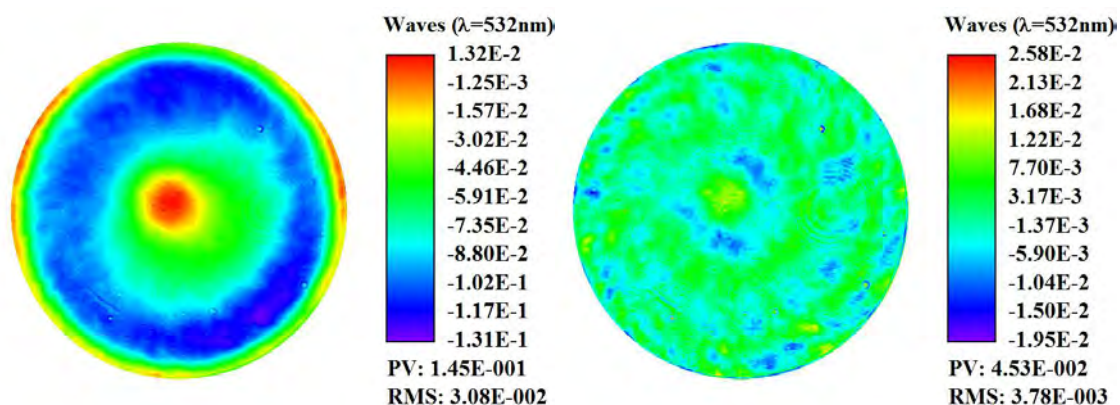


FIGURE 5.68 Error in the second surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right)

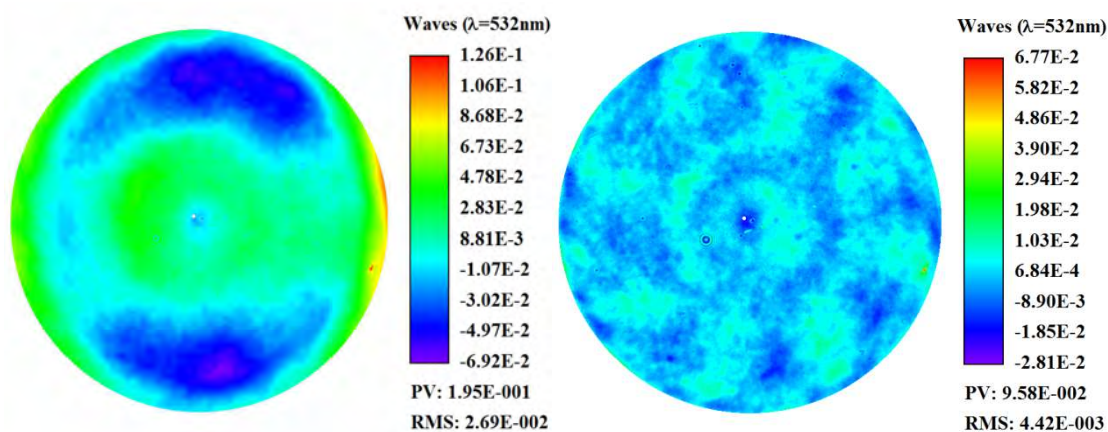


FIGURE 5.69 Error in the third surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right)

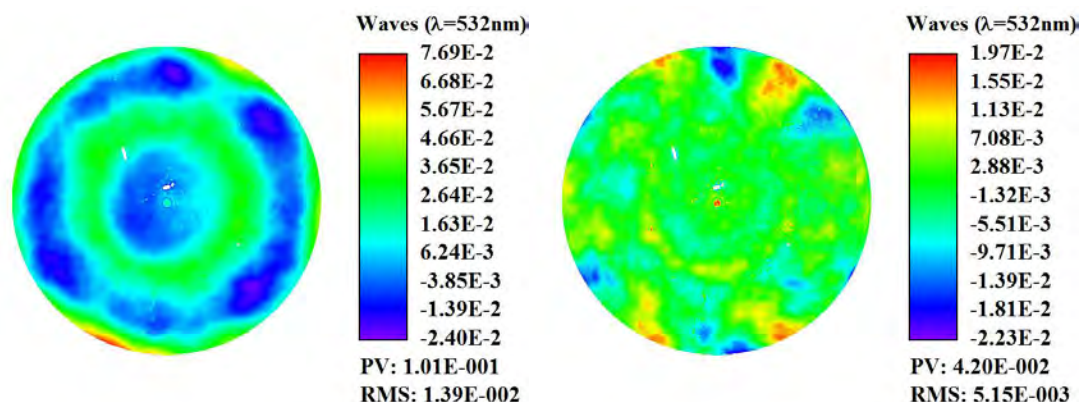


FIGURE 5.70 Error in the fourth surface of the two element diverger lens (Left) and the difference between the surface error and the Zernike fit (Right)

Finally, the alignment of the two elements in the diverger lens was done using a combination of interferometric measurements and the use of the PSM and alignment station. Unlike the imaging lens the diverger lens mount allowed for the two elements to be decentered relative to each other. However, it did not allow for the two elements to be tilted relative to each other. As such, the center of curvature of three of the four surfaces could be brought into tight alignment. In measuring the alignment with the PSM, the lens is illuminated from the back side, as shown in FIGURE 5.71. After the lens alignment was completed the reflected spot from both of the second element surfaces and the aspheric surface of the first element, were all aligned to within $1\mu\text{m}$ of each other. Since there was no way to tilt the lens with respect to each other the center of curvature of the spherical surface of the first element could only be brought to within $15.5\mu\text{m}$ of the other surfaces. The measured displacement of the center of curvature (CoC) for each lens surface is given in TABLE 5.14. Additionally, the positions of the centers of curvature were loaded into the merit function of the diverger lens model shown in FIGURE 5.71.

The optimization routine was then used to find the range of surface decenters and tilts that would match the measured locations of the center of curvatures, TABLE 5.14.

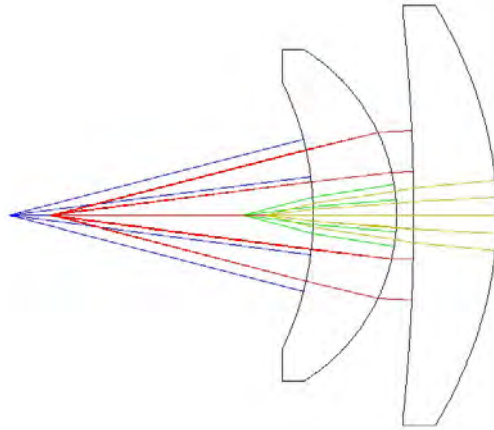


FIGURE 5.71 The locations at which light from the PSM is focused back on itself from the four surfaces of the aspheric doublet diverger lens.

Surface	Decenter of CoC	Surface Decenter	Surface Tilt
4	0.38 μm	$\pm 0.19\mu\text{m}$	$\pm 2.94\text{E-}4^\circ$
3	0.98 μm	$\pm 0.40\mu\text{m}$	$\pm 9.94\text{E-}4^\circ$
2	15.5 μm	$\pm 39.0\mu\text{m}$	$\pm 9.75\text{E-}3^\circ$
1	0.92 μm	$\pm 2.50\mu\text{m}$	$\pm 3.23\text{E-}3^\circ$

TABLE 5.14 The decenter of the center of curvature (CoC) of each surface as measured with the PSM. The decenter and tilt of the surfaces that could cause the measured shifts.

After all the diverger lens properties were measured the simulations previously discussed for the beam splitter and imaging lens were performed, Chapters 5.4.2 and 5.4. First the individual lens properties were varied over the maximum of their specified range. The change in the OPD at the detector from the nominal alignment is calculated. Then the model is optimized to reduce the change in the OPD, in which only the decenter and tilt of the entire diverger lens as well as the position and orientation of the test part are allowed to vary. This is then repeated for all of the 3912 aspheric test surfaces. The maximum and average changes to the peak to valley and rms OPD after the reverse

optimization for each property are shown in TABLE 5.12. As before, having an error in only one lens property is unlikely. Therefore a second test was performed in which each property of the lens was set to a random number within its range, and the reverse optimization process was repeated. The test was conducted for three groups of properties. In one group only the properties that affect the power of the diverger lens were allowed to vary. These are the first nine properties listed in TABLE 5.12. In the second only the tilts and decenters of the lens surfaces were allowed to vary and in the final group all properties were allowed to vary. These simulations were each run for all the aspheric test surfaces, over three different perturbations of the imaging lens properties for each group. The results are shown in TABLE 5.13.

Property	Range	Max. PV	Max. rms	Avg. PV	Avg. rms
RoC (Surface 1)	$\pm 5\mu\text{m}$	$\lambda/14$	$\lambda/51$	$\lambda/40$	$\lambda/163$
RoC (Surface 2)	$\pm 24\mu\text{m}$	$\lambda/37$	$\lambda/171$	$\lambda/159$	$\lambda/614$
RoC (Surface 3)	$\pm 3\mu\text{m}$	$\lambda/4$	$\lambda/19$	$\lambda/16$	$\lambda/64$
RoC (Surface 4)	$\pm 4\mu\text{m}$	$\lambda/8$	$\lambda/39$	$\lambda/36$	$\lambda/146$
Index of Refraction (Lens 1)	$\pm 2.0\text{E-}5$	$\lambda/62$	$\lambda/274$	$\lambda/216$	$\lambda/864$
Index of Refraction (Lens 2)	$\pm 2.0\text{E-}5$	$\lambda/45$	$\lambda/208$	$\lambda/175$	$\lambda/708$
CT (Lens 1)	$\pm 2\mu\text{m}$	$\lambda/92$	$\lambda/345$	$\lambda/274$	$\lambda/1130$
CT (Air Gap)	$\pm 20\mu\text{m}$	$\lambda/8$	$\lambda/32$	$\lambda/26$	$\lambda/106$
CT (Lens 2)	$\pm 2\mu\text{m}$	$\lambda/12$	$\lambda/56$	$\lambda/50$	$\lambda/203$
Decenter/Tilt (Surface 1)	TABLE 5.14	$\lambda/297$	$\lambda/1937$	$\lambda/3046$	$\lambda/16296$
Decenter/Tilt (Surface 2)	TABLE 5.14	$\lambda/27$	$\lambda/175$	$\lambda/281$	$\lambda/1499$
Decenter/Tilt (Surface 3)	TABLE 5.14	$\lambda/346$	$\lambda/2278$	$\lambda/4186$	$\lambda/22330$
Decenter/Tilt (Surface 4)	TABLE 5.14	$\lambda/706$	$\lambda/4631$	$\lambda/8396$	$\lambda/44896$

TABLE 5.15 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization for each property of the diverger lens varying over its listed range after testing the 3912 aspheric test surfaces previously described.

Property	Max. PV	Max. rms	Avg. PV	Avg. rms
Radii of curvature, Indices of refraction & Separations	$\lambda/4$	$\lambda/24$	$\lambda/14$	$\lambda/102$
Tilts & Decenters	$\lambda/53$	$\lambda/609$	$\lambda/242$	$\lambda/3221$
All properties	$\lambda/3$	$\lambda/24$	$\lambda/13$	$\lambda/102$

TABLE 5.16 The maximum and average, peak to valley and rms, change to the OPD after reverse optimization when imaging lens properties are allowed to vary simultaneously.

From the data displayed in TABLE 5.12 and TABLE 5.13 it is clear that uncertainties in several of the diverger lens properties may have a significant impact on the RO procedure. The largest contributors are the radius of curvature of the lenses, and the center thickness of the air gap and second surface. Therefore these properties were allowed to vary near the end of the RO process. The decenter and tilts of the surfaces had a much smaller impact than the properties that affect the power of the lens. The surface property that was not investigated was the rotation of the lens surfaces about the optical axis. The nominal shape of the lens surfaces are not affected by this property as they are all rotationally symmetric. However the surface errors shown in FIGURE 5.67 - FIGURE 5.70 will cause the OPD to change as the surfaces are rotated about the optical axis. The measurements provided are supposed to show the orientation of the lens surface as mounted. However since this was not verifiable the rotation of the lens surfaces were also included in the RO model as variables.

5.5 Data Collection Process

A brief overview of the data collection process will be given here, while a more detailed description will be provided with the measurements in Chapter 6. As discussed in Chapter 5.3, the simple model is used to determine the interferometer set up required to test a given aspheric surface. Now the interferometer must be set up to match, or at least approximate, the layout provided by the simple model. This process assumes that the basic alignment of the interferometer such as the, collimating optics, beam splitter and reference surface has already been completed. Therefore setting up the interferometer to test a given aspheric surface basically entails getting the other interferometer components, such as the diverger, test part, imaging lens and detector, separated by the proper distances along the optical axis. Optical rails, which were pre-aligned to the test and imaging arms of the interferometer, were used in both the test and imaging arm to aid in the coarse positioning of the optical components, FIGURE 5.72. Additionally, all components were placed on xyz translation stages utilizing micrometers for fine positioning, as well as tip/tilt stages for adjusting the orientation of the components. The original test part holder, shown on the right hand side of FIGURE 5.72, used a set of stacked goniometer stages in order to place the point of rotation close to the vertex of the test part. Additionally a rotation stage to allow the test part to be rotated about the optical axis. Unfortunately, the torque produced by cantilevering the mass of all these stages off xyz translation stage resulted in angular error motion being introduced as the test part was translated along the optical axis. In order to resolve this the goniometers and rotation stage were removed and replaced with a simple tip/tilt lens mount. However this resulted

in the loss of the ability to make fine rotational movements of the test parts about the optical axis. Rotating the test part in the new mount required unbolting the part from the mount which would change the xyz position of the part as well as its angular orientation relative to the interferometer. An iris placed after the collimating lens was used to stop down the input beam so that the back reflections off the various optics can be used to adjust the horizontal and vertical locations of the optical components.

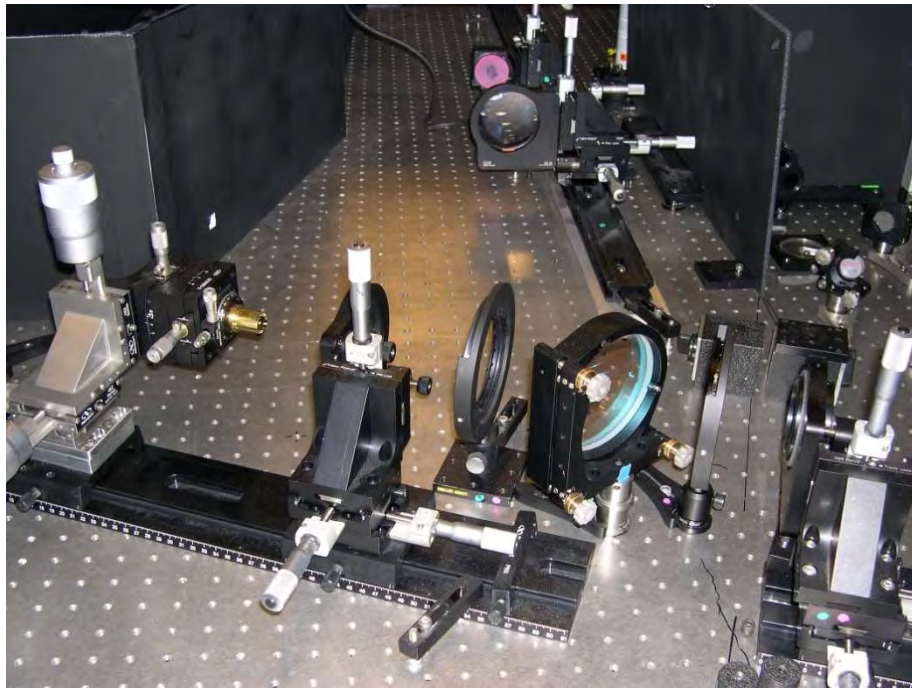


FIGURE 5.72 Image of the sub-Nyquist interferometer taken from above the reference surface (not pictured). The collimated beam comes in from the right, the test rail containing the diverger and test part is shown in the left foreground, and the imaging rail containing the imaging lens and detector is shown in the background.

The first step in setting up the interferometer is to place the magnification target into the test arm of the interferometer. The distance the magnification target is placed from the beam splitter, along the test arm rail, depends on the type and prescription of the surface

to be tested. If a surface is to be tested without utilizing the diverger lens the plane at which the magnification target is located would ideally become the test plane. If a surface is to be tested with the diverger lens, the optimal plane at which the magnification target is placed would eventually become the intermediate pupil, or the image plane of the test part through the diverger. The exact separation the magnification target and the beam splitter is somewhat arbitrary since the distance between the beam splitter and imaging lens can be adjusted to compensate. However, the future location of the diverger lens and test part must be accounted for when placing the magnification target. If a concave test part is to be tested with the diverger lens the intermediate pupil will be real. Which means the diverger and test part will need to be placed behind the magnification target. Therefore, the magnification target should be placed close to the beam splitter. However, if a convex test part is to be tested with the diverger lens the intermediate pupil will be imaginary. Thus the diverger lens will need to be placed in front of the magnifications targets location. Therefore, the magnification target should be placed further away from the beam splitter. The distance between the beam splitter and the magnification target and the distance between the beam splitter and the imaging lens are coarsely adjusted, on the order of 10mm, using the distance demarcations on the test arm and imaging arm optical rails such that the total distance between them is approximately equal to the specified distance from the simple model. The distance from the imaging lens to the detector is then adjusted to bring the magnification target into focus, and the tilt of the magnification target is adjusted to null the fringe pattern at the detector. Phase shifted interferograms are then collected and the IDL software and magnification model

are used to determine the separation of the imaging lens and the detector, as discussed in Chapter 5.2.3. The difference between the measured separation and the separation provided by the simple model is calculated. The detector location is then adjusted to reduce this difference. The separation is retested and this process is repeated until the measured separation agrees with the simple model to within a ± 10 microns. During this process the magnification target is repositioned so that its image remains in focus at the detector.

Unfortunately, the program used to recover the distance between the imaging lens and the detector does not accurately recover the distance between the magnification target and the imaging lens. This was discussed in Chapter 5.2.3 and highlighted in FIGURE 5.31, where the magnification target could be displaced by ± 30 mm and the spacing between the imaging lens and the detector changed by less than $\pm 3\mu\text{m}$. Therefore, once the imaging lens to detector separation is set another approach has to be used to find the plane that is conjugate to the detector. The reason locating this plane is important is that it is used as a reference for setting up the diverger lens and eventually the test part so that their locations match the simple model and ensure that the test part is conjugate to the detector. The approach used to find this plane was to use a test part with a known radius of curvature and the reverse optimization process. The known test part used for these measurements was a spherical mirror with a designed radius of curvature of 1000mm. During the system setup a simplified reverse optimization process is used to estimate the position, during the measurement data collected from this mirror can be used with the full

reverse optimization process. This mirror, and its use in the full RO process, will be discussed in more detail in Chapter 6.1. For the simplified process the mirror is placed in the test arm of the interferometer in place of the magnification target. Phase shifted interferograms are recorded and the unwrapped OPD is fit to Zernike polynomials and the tilt terms are removed. This measured OPD is placed into the RO model at the detector plane. In the model, the distance between the imaging lens and the detector is set to the distance determined from the magnification target test. The distance from the mirror to the imaging lens is set as the only variable in the system. Rays are traced forward through the test arm and backwards through the reference arm using the process described in Chapter 5.4. The merit function is set to reduce the rms wavefront error on the final surface of the model, which is the reference arm input wavefront. The optimization procedure is used to find the separation of the mirror and the imaging lens that minimizes the wavefront error at this plane. The separation returned by this simplified RO process is compared to the ideal separation calculated by the simple model. If they don't match the position of the mirror is altered and the process is repeated until the two distances agree to within $100\mu\text{m}$. At this point, multiple measurements of the spherical mirror can also be recorded at shifted axial positions, to aid the full reverse optimization process.

Next, if measuring a test surface without the use of the diverger lens the next step is to replace the spherical mirror with the test piece. While the diverger lens is not used for the final test it can be used to aid in the alignment process. The diverger lens is placed in

the test arm before the spherical mirror so that the spherical mirror is at its cat's eye position. This yields a null fringe on the detector. The spherical mirror is then removed and the test part is put in its place and aligned to be at the divergers cat's eye position. Then the diverger is removed and phase shifted interferograms can be recorded for the test part. Finally, multiple measurements of the test part can be made at different axial shifted positions. These part shifts are measured using a Heidenhain (Heidenhain Corporation Schaumburg, IL) length gauge.

If the diverger lens is going to be used to make a measurement of an aspheric test part it is inserted into the system so that the spherical mirror is at its cat's eye position. The diverger should now be setup so that its focus is at, or at least near, the final intermediate pupil location. The simple model provides the optimal distance between the focus of the diverger lens and the intermediate pupil location. The diverger is then shifted along the optical rail by the distance predicted by the simple model. The Heidenhain gauge is used to measure this shift, which for most of the parts tested was on the order of 20mm. Once at its final location the tilt and decenter of the diverger can be aligned using the stopped down input beam and the back reflections off the various diverger lens surfaces. Finally, the test part is added to the system. The simple model outputs the distance between the focus of the diverger lens and the location of the test part. So the test part is inserted behind the diverger lens at the cat's eye position and then shifted to its final testing location. The Heidenhain gauge is used to measure the distance it is displaced from the cat's eye position. Additionally, the predicted interferograms created by the simple model

can be used to aid in the alignment as shown in FIGURE 5.24. The phase shifted interferograms are then recorded from the test part at its nominal as well as at axial shifted locations. The size of these shifts depends on the test part, but is usually around 0.1 to 0.2mm.

6 MEASUREMENTS

This chapter contains the description and results of the measurements made with the non-null interferometer. As an initial test of the system and the reverse optimization and reverse raytracing process, a couple of cylindrical surfaces were tested directly against the flat reference mirror. The idea being that this would be an easier test to accomplish since the diverger lens is not used which simplifies both the interferometer and the model. Next, tests were performed on aspheric inserts. Initially, the interferometer and RO process were unable to produce repeatable measurements of the aspheric contact lens tooling inserts, especially with the original single element diverger. This was probably due to the OPD contributions of surface errors being smaller than the disagreement between the interferometer and the model. In order to see if a large defect could be measured two inserts were manufactured with an intentional surface error of approximately 2.5waves. These measurements will be discussed second, as they can be used to show the process as it was intended to function, even if the resolution is worse than anticipated. Finally, two measurements will be shown for aspheric inserts without the introduced error. After these measurements were taken, the RO model and process were improved to the point where repeatable results could be achieved. Discussion on these measurements, as well as other techniques that were investigated to try to improve the system performance will be covered in Chapter 7.

6.1 Measurement of Cylindrical Surfaces

Measurements were taken of the cylindrical surfaces of two ophthalmic lenses. These lenses were placed in the test beam directly without the use of the diverger lens. The lenses were approximately 18.6mm in diameter and had listed powers of -0.75 and +0.75 diopters. These surfaces are nominal cylinder in shape; however they do have a small amount of power along the crossed axis. The goal of these measurements was to determine if the lens shape, namely the radii of curvatures and the surface errors, could be measured without the use of any additional optics to account for the difference in power along the two axes of the cylindrical lens surface. The challenge with these measurements is how to determine the location of the test part relative to the interferometer without prior knowledge of the surface shape, since there are an infinite number of surfaces that could produce the same OPD at the detector. The method that was arrived at used a combination of axial part shifts and a known optical surface as a calibration artifact, in this case a concave spherical mirror. For these measurements, both the cylindrical test part and the spherical mirror were shifted to three to five part locations separated by 5mm, the data from each location is modeled as a separate configuration in the RO mode. A Heidenhain length gauge was used to measure the part displacements. The concave spherical mirror had a designed radius of curvature of 1000mm and which was measured at 1000.821mm using the WYKO 6000 interferometer and the digital slide. The semi-diameter of the mirror was measured at 12.134mm using an optical microscope with a translation stage outfitted with feedback from a linear encoder. The spherical mirror was measured at the same time as the cylindrical surface. The RO procedure was

then performed on both surfaces more or less simultaneously. Since the radius of curvature of the spherical mirror was known, it is fixed during the RO procedure, so that only its position is a variable. The RO procedure was used to determine the imaging distance that will best null the OPD from the spherical mirror, and in turn these imaging distances are used for the configurations containing the cylindrical surface. In the configurations containing the cylindrical surfaces the surface shape is allowed to vary in order to null the OPD.

A general outline of the procedure used will be given using the data from the spherical mirror and the -0.75 diopter cylindrical lens as an example. The first step in the process is to use the simple model to determine the system layout, as described in Chapter 5.3. Then the magnification target is measured, as discussed in Chapter 5.2. If the measured separation between the imaging lens and the detector does not match the separation called for by the simple model, the detector position is adjusted and the magnification test is repeated. Once the proper separation of the imaging lens and detector is found, within $\pm 10\mu\text{m}$, the magnification can be removed from the system and replaced with the test part. However, in order to ensure that the test part is inserted in the same plane as the magnification target, a lens is placed in front of the magnification target and aligned so that the magnification target is at the cat's eye position. The cylindrical lens is then installed and aligned to be at the lense's cat's eye position before the lens is then removed from the system. The test surface is then measured at three to five different axial positions. At each test positon, ten sets of phase shifted fringe images were recorded,

FIGURE 6.1. After the data is recorded from the test part, the spherical mirror is placed in the interferometer using the same procedure.

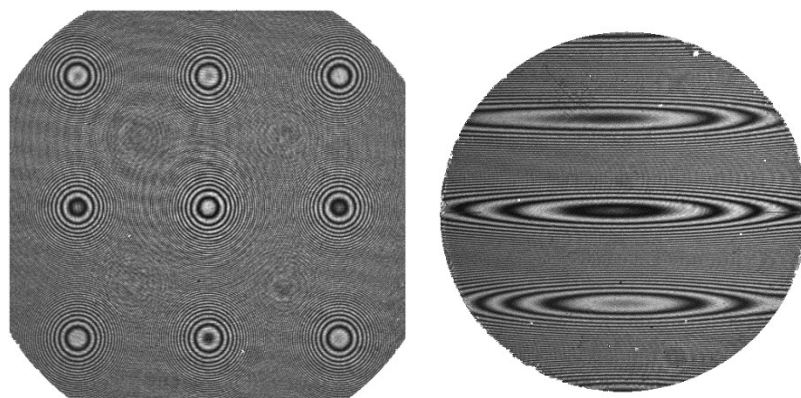


FIGURE 6.1 The interferograms produced by the spherical mirror (Left) and the -0.75diopter cylindrical lens surface (Right). The wavefront diameter produced by the spherical mirror is larger than the width of the detector and it is therefore cropped by the detector.

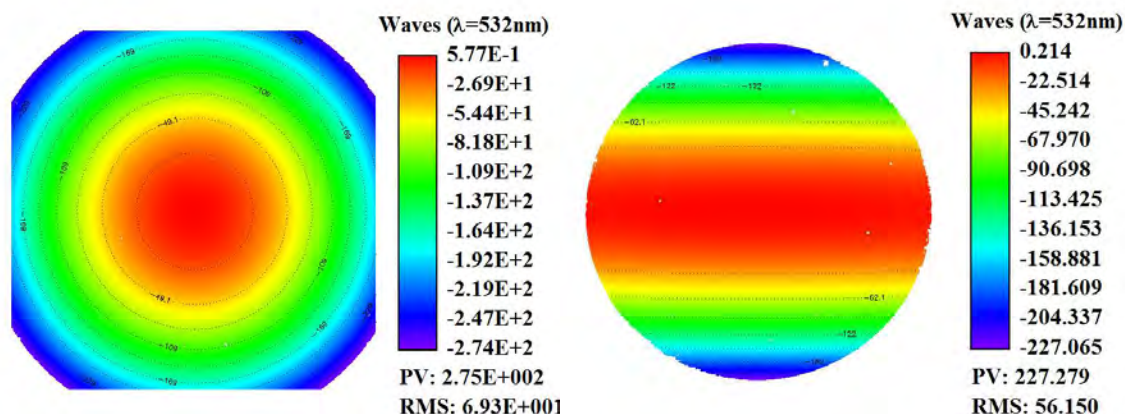


FIGURE 6.2 The unwrapped OPD recorded at the detector plane produced by the spherical mirror (Left) and the -0.75 diopter cylindrical lens surface (Right).

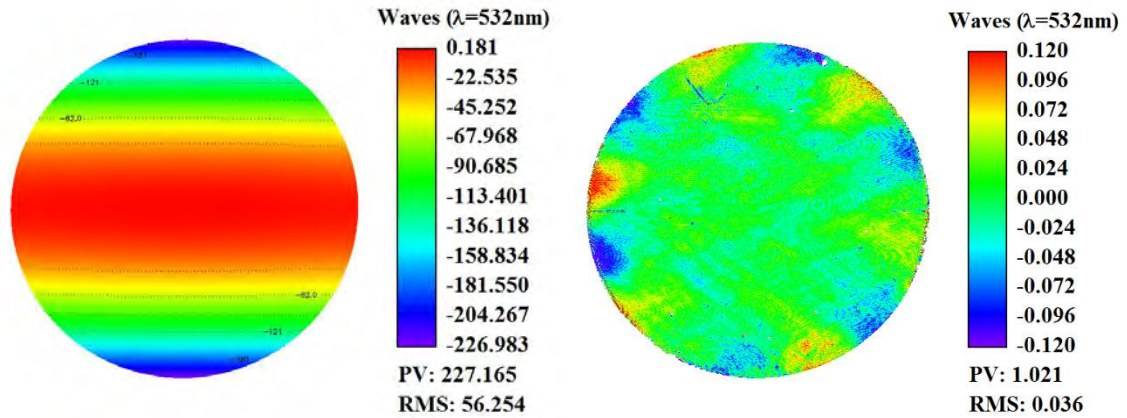


FIGURE 6.3 The Zernike polynomial fit to the OPD recorded from the cylindrical mirror (Left) and the difference between the OPD and the Zernike polynomial fit (Right).

The SNI control software is then used to calculate and unwrap the phase data, FIGURE 6.2. These OPD data sets are then averaged and fit to Zernike Fringe polynomials. The Zernike polynomial fit is shown in FIGURE 6.3, along with the difference between the measured OPD and the fit. While there are some spikes in the OPD data that drive up the overall peak to valley of the difference to just over one wave, the majority of the difference is within 0.24λ peak to valley and the rms of the difference is relatively small at 0.036λ . In the beginning of the reverse optimization process the Zernike fit is used as an analog for the measured OPD data and the Zernike coefficients are loaded into the RO model as a Zernike fringe phase surface located at the detector plane. The Zernike polynomial fit offers a closed-form solution, which the raytracing software can use to easily calculate the phase at any given point on the surface. This allows the measured OPD data to be incorporated into the model and raytracing procedure, rather seamlessly. If the difference between the measured data is small, or if the high frequency content that is not encoded in the Zernike fit is not of interest, then the RO process may also be finalized using the Zernike fit. However, if there is a significant difference between the

measured OPD and Zernike fit then the measured OPD data may need to be incorporated into the model near the end of the RO process as a grid phase surface. This is a judgment call that must be made by the user based on the quality of the fit and the frequency content of interest. There are challenges with incorporating the OPD data into the model as a grid phase surface, which will be discussed later in this chapter. First, the description of the process will be given here in its entirety, assuming that only the Zernike fit of the measured OPD will be used. Then the additional steps needed to utilize the OPD data as a grid phase surface will be discussed.

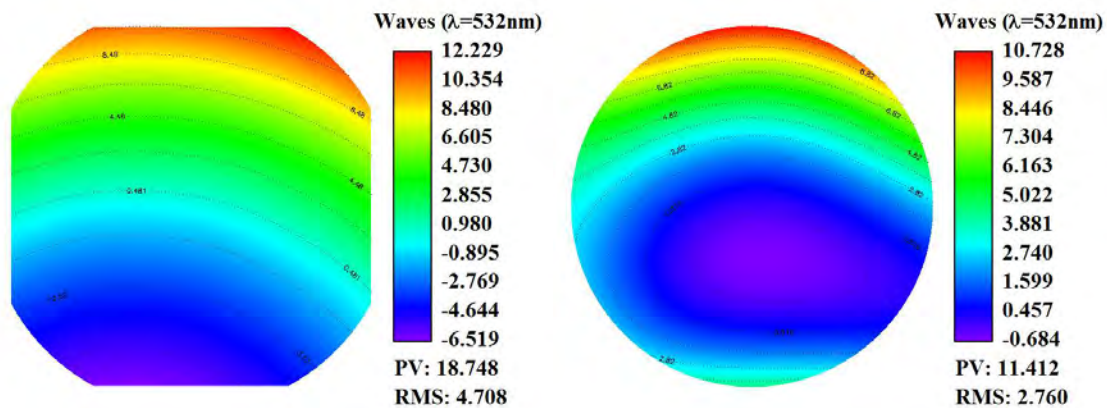


FIGURE 6.4 The wavefronts at the last surface of the RO model for two of the configurations just after the measured OPD data and surface properties are loaded. The wavefront for the spherical mirror is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.

The RO model is split into multiple configurations where each configuration represents a different shifted position of the test part, or the spherical mirror. For each configuration all of the phase surfaces are setup to be ignored except the one that contains the OPD data corresponding to the correct test surface and shifted position. Additionally, circular or elliptical apertures are set on the measured OPD data in order to block rays that land

outside the measured wavefront. Then rays are traced forward through the test arm to the detector and then backwards through the reference arm to the input plane, which produces the wavefronts shown in FIGURE 6.4. The last surface of the RO model represents the collimated wavefront at the start of the reference arm. When the difference between the test and reference wavefronts matches the measured OPD, the wavefront at the final surface of the model will be nulled. The default merit function targets minimizing the unreferenced RMS wavefront error over a square grid of rays serves as the base of the reverse optimization merit function. The option to delete the vignetted rays is used in order to remove rays from the merit function that land outside the measured OPD at the detector. This is important because rays which land outside the aperture over which the OPD data is defined can encounter very large OPD_Z values. This is the result of the phase values not being defined outside this aperture, as is the case with a grid phase surface, or Zemax trying to extrapolate phase values past the nominalization radius of the Zernike phase surface. These large phase values outside the measurement aperture can then prevent the optimization process from minimizing the OPD_Z for rays inside the measurement aperture. Along with the default merit function operands a few constraints are used on lens properties, like thicknesses and decenters, which will be discussed at the appropriate point in this description. However, in general constraining individual lens properties was avoided, because when the ray tracing software pushes a variable well beyond what could reasonably be expected from the physical system, it often offers insight into either problems with the model or an incompatibility of optimizing two variables simultaneously.

The first step of the RO process is to remove the tilt in the final wavefront by allowing the test surfaces and reference wavefront to tilt. The reference wavefront tilt is represented in the model by a Zernike phase surface in which only the two tilt terms are allowed to vary. This surface is common for all configurations, since it shouldn't change between measurements. The two test surfaces are allowed to vary independently since there is no guarantee that the surfaces were perfectly aligned. Additionally, even though the introduced shifts are intended to be purely an axial displacement, pitch and yaw motion of the stage has been observed. Next the rotation about the Z axis is allowed to vary for the cylindrical parts in order to align the cylinder axis of the part in the model to that of the interferometer. Then the decenter in x and y of the detector is allowed to vary for each configuration. The reason for this is to align the Zernike phase surfaces to the incoming wavefronts. While the detector does not move between measurements the center of each test wavefront does not necessarily overlap on the detector. Additionally, the masking procedure is used to define the center of the measured wavefront for the Zernike fitting procedure. The error in the calculated center point of the measured wavefront will depend on the user's ability to select points around the edge of the wavefront if the manual process is used and noise in the image if the automated process is used. Thus the origin of each Zernike phase surface does not correspond to the same point between configurations. However, this motion should be small so the decenters are constrained in the merit function to be less than 2 pixels, or 0.03mm. Generally, this was not observed to be an issue as the decenters were on the order of a single pixel. The results of these steps are shown in FIGURE 6.5. It should be noted that without the

additional constraint, on occasion Zemax would push these up to very large numbers in order to shift the measured wavefront data completely out of the incoming beam. This results in all rays being blocked and thus the merit function returns a small value.

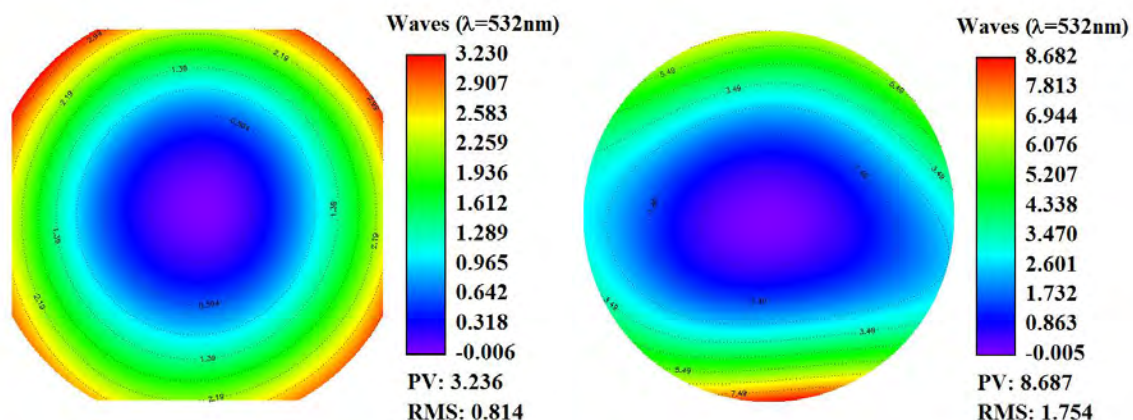


FIGURE 6.5 The wavefronts at the last surface of the RO model for two of the configurations after adjusting the tilt of the test parts. The wavefront for the spherical mirror is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.

At this point all of the previously established variables are turned off and the merit function is changed to only consider the wavefront produced by the spherical mirror. Then only the distances between the test part and the beam splitter and the beam splitter and the imaging lens are allowed to vary to remove the residual power from the spherical lens. After this optimization cycle is completed the previous mentioned tilts and decenters are again allowed to vary, along with the tilts and decenters of the imaging lens surfaces. The result of these steps on the wavefronts from both surface types is shown in FIGURE 6.6. The shifts to the phase surfaces representing the collimated wavefront and reference arm as well as the orientation of the beam splitter were found to have no significant impact on the wavefront for these test parts, so they were not altered.

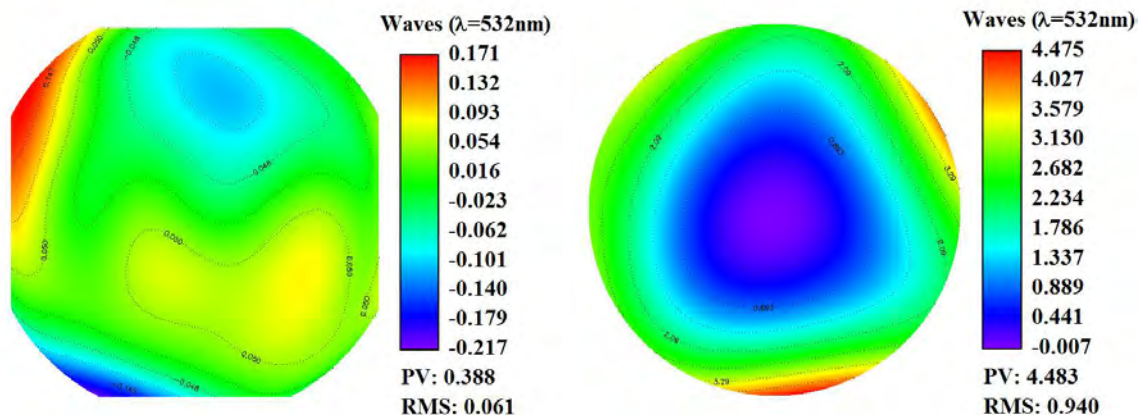


FIGURE 6.6 The wavefronts at the last surface of the RO model for two of the configurations after adjusting the distance between the test part and the imaging lens along with orientation of the imaging lens surfaces. The wavefront for the spherical mirror is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.

Next all the variables were again removed from the system, and the merit function is set up to only work on the cylindrical lens surfaces. The radii of curvature in both the x and y axis of the cylinder are allowed to vary along with the rotation about the z axis. This produces the wavefront at the last surface from the cylindrical surface configuration shown in FIGURE 6.7. Next the Zernike Standard Sag terms that are part of the torodial surface definition in Zemax are allowed to vary, starting with the 7th through the 37th. The lower terms, power and astigmatism are left at zero so that these surface shapes are accounted for by the two radii of curvature. At this point, the number of Zernike Standard terms can be increased, if the fit is insufficient. Everything up this point is simply to get the system close to a solution. The last step to complete this stage of the RO process involves turning all the previously discussed variables back on and reestablishing the merit function to take all configurations into account and running the optimization routine. At the completion of the RO procedure there is some amount of

residual error left over in the wavefront at the last surface of the model. The residual error is the error which the RO procedure could not assign to either the interferometer or the test part. Basically, the RO procedure cannot perturb the model of the interferometer, or alter the Zernike terms representing the test surface in order to compensate for this error. The resulting residual error for a configuration containing the spherical surface and the cylindrical surface are shown in FIGURE 6.8.

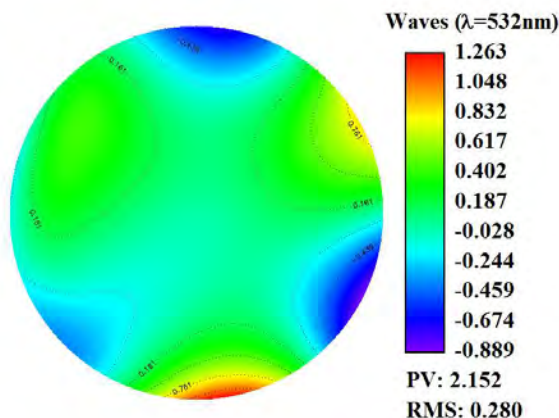


FIGURE 6.7 The wavefront at the last surface of the RO model for the -0.75 diopter cylindrical lens after allowing the radii of curvature to vary.

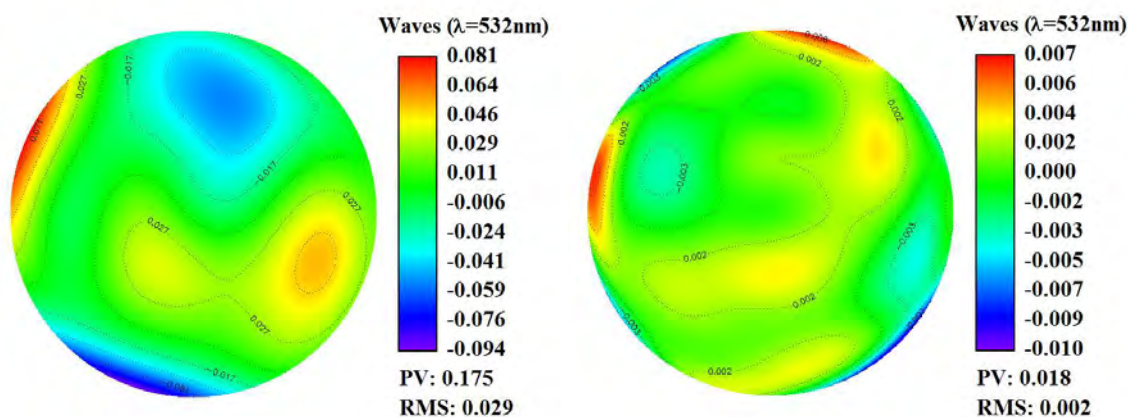


FIGURE 6.8 The wavefronts at the last surface of the RO model for two of the configurations after completing the RO procedure. The wavefront for the spherical mirror, stopped down to the same diameter as the cylinder lens, is shown on the left and the wavefront corresponding to the -0.75 diopter cylindrical lens is shown on the right.

In this case good agreement exists between the measured OPD data and the model of the interferometer as the peak to valley error was 0.018λ , FIGURE 6.8 (Right). However, the spherical mirror configurations still show a 0.388λ peak to valley residual error over the entire detector. Over the same aperture as the cylindrical lens this is reduced to 0.175λ , FIGURE 6.8 (Left), which is ten times the residual error of the cylindrical surface. The source of this error could be from error in the spherical mirror as this was not included in the model. However, the surface of the spherical mirror was measured with the WYKO 6000 Fizeau interferometer and was found to have less than 0.06λ of surface error peak to valley over the full 12.134mm aperture. Therefore unless the mirror was significantly distorted by the mount used to hold it in the non-null interferometer, error of the mirror could only be contributing approximately one third of the residual error. More likely is that some of the residual error in the cylindrical surface configurations is being absorbed into the surface error by the Zernike Sag surface terms. This would mean that the uncertainty of this measurement is at least as large as 0.175λ . At this point, the calculated Zernike surface could be taken as an estimate of surface measurement without bothering to do the reverse raytracing. This could be done using the sag surface plotting capabilities of Zemax, or by looking at the OPD introduced by only the surface by trimming the model down to only the test surface and looking at the wavefront. Removing the radii of curvature from the surface before calculating the OPD will remove the OPD introduced by the part prescription, as shown in FIGURE 6.9.

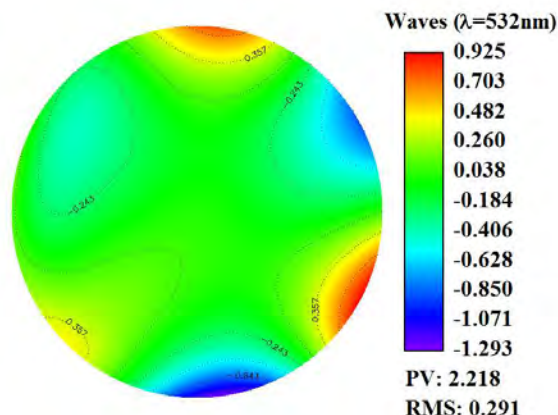


FIGURE 6.9 The OPD introduced by the lens surface minus the OPD introduced by the cylindrical part shape, or twice the surface error. This is the Zernike fit of the OPD introduced by the surface calculated by the RO procedure.

At this point the reverse ray tracing procedure, discussed in Chapter 5.4, can be performed. The reverse ray tracing procedure will assign the residual error of the RO model to the test surface. First the forward propagating wavefront at the test surface should be recorded. This can either be recorded immediately before the test surface, or immediately after the test surface, depending on if the full OPD introduced by the test surface is the desired outcome or if just the departure from the nominal part prescription is the desired outcome. For the latter procedure, the Zernike terms on the test surface need to be removed from the surface prescription before the reverse ray trace. The OPD immediately after the test surface for the forward propagating model is shown in FIGURE 6.11 (Left). Next, the system is setup to trace rays backwards through the system. This can be done by either modifying the existing configuration or by copying the existing configuration into a new configuration. The advantage of the later is that both the forward and backwards ray traces will be available at the same time, however it comes at the expense of an additional configuration for each backwards ray trace desired,

which can significantly slow down the model. Completely reversing the layout, so that rays are traced forward through the reference arm and then backwards through the test arm to the start of the interferometer, should produce the negative of the residual wavefront error present in the forward propagating model. The sign reversal is produced because the light is propagating in the opposite direction. This can be seen by adding the residual error shown in FIGURE 6.8 (Right) to FIGURE 6.10 (Left) in order to produce FIGURE 6.10 (Right).

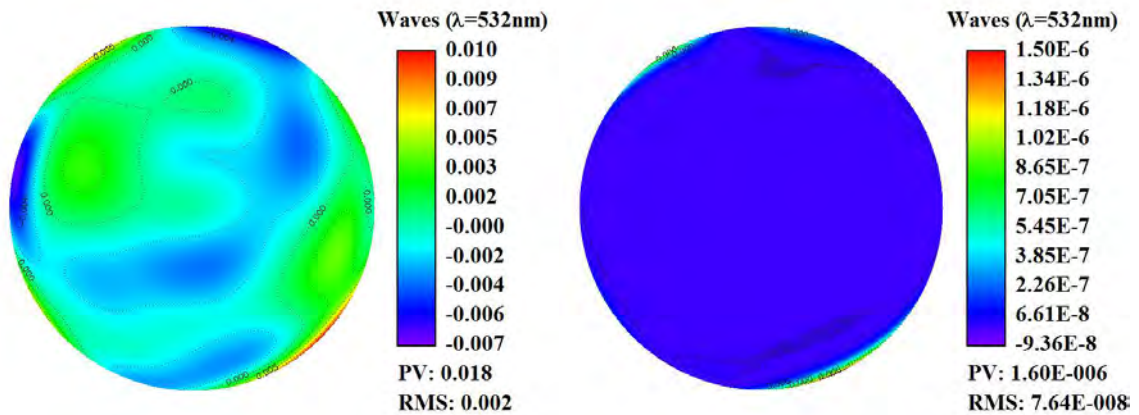


FIGURE 6.10 The residual error present in backwards ray trace through the model (Left), and the sum of the residual error from the forward and backwards ray trace (Right).

Now, the reverse ray trace model can be adjusted so that the last surface the model is the surface just after the cylindrical surface. The rays should be traced onto the surface of the test part. This is accomplished by setting the last surface of the model to match the nominal part prescription. In the backwards ray trace this is the point just prior to the test surface being encountered, while in the physical system and the forward ray trace this is the point immediately after the light reflects off the surface. The OPD at this plane is shown in FIGURE 6.11 (Right). The difference between the two plots shown in FIGURE

6.11, is the OPD introduced by the form error on the test surface, FIGURE 6.12. This is twice the surface error. As expected it is almost identical to the Zernike fit of the surface, shown in FIGURE 6.9.

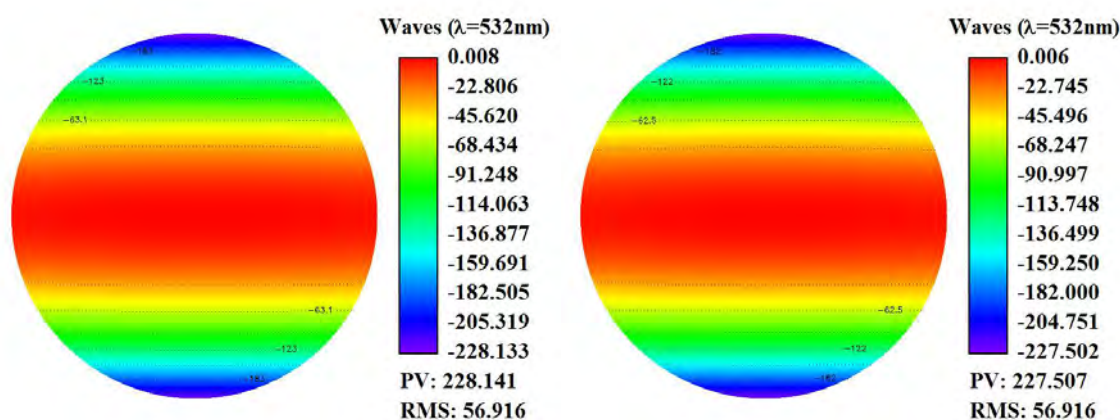


FIGURE 6.11 The OPD just after reflecting off the test surface for the forward propagating model (Left) and the backward propagating model (Right).

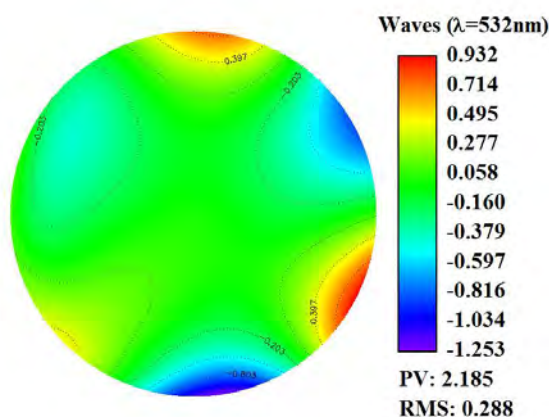


FIGURE 6.12 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure and the Zernike representation of the measured OPD data. This is twice the surface error.

A radius of curvature of -716.44mm, was recovered by the RO process for the short axis of the concave cylindrical, while the long axis radius was measured at $-5.48\text{E}+4\text{mm}$.

This measurement was then repeated 5 times each time the test part was removed from the system and the process was started over. Including a measurement done with the test

part rotated 90°, FIGURE 6.13. The short axis radii of curvature recovered using the RO process from these tests ranged the -718.87mm to -715.69mm, with an average of -716.83mm, which corresponds to a change in wavefront of just under 1λ across all the measurements. This highlights the RO process's inability to completely separate the position of the test part from the shape of the test part. After these measurements were completed the short axis of the cylinder was measured at -726.31mm with a stylus profiler. A ten millimeter change in the radius of curvature from -716mm to -726mm over the 18.6mm aperture would correspond to a 2.8λ change in the wavefront. Again this calls into question the system's ability to separate the position of the test part from the shape of the test part. However, if the cylinder is subtracted from each measurement then the change in the wavefront error of each test is much smaller, on the order of 0.15λ , FIGURE 6.14.

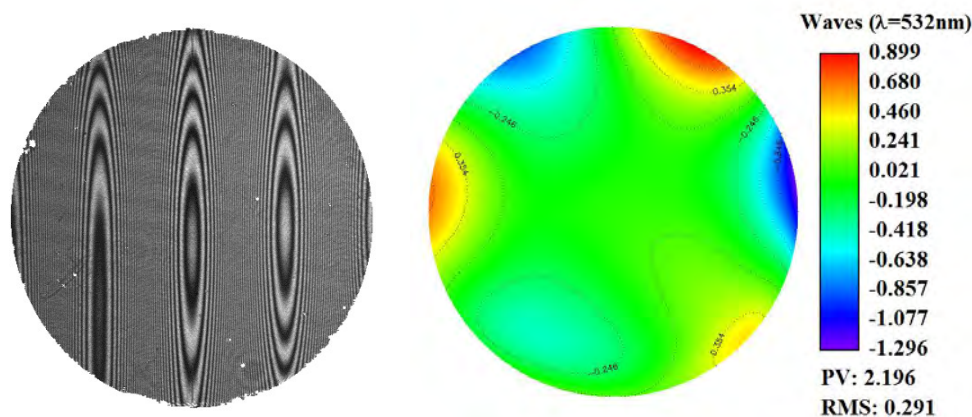


FIGURE 6.13 The fringes from a repeated measurement after rotating the test part 90° counter-clockwise (Left) and the recovered OPD introduced by the lens surface minus the cylindrical radii (Right). The measured OPD data is twice the surface error and has been rotated 180° to take into account inversion about the x and y axis introduced by the imaging lens.

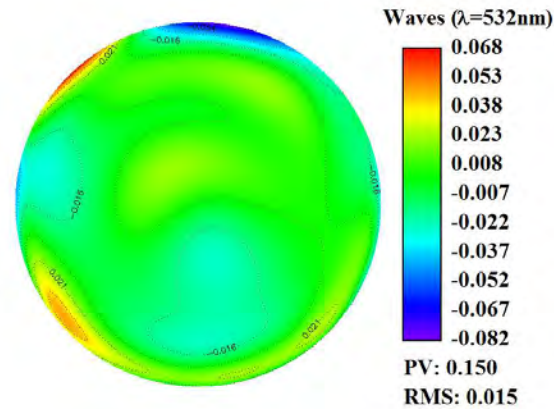


FIGURE 6.14 The difference between the recovered wavefront error shown in FIGURE 6.12 and FIGURE 6.13, after aligning the axes of the cylinders.

These data sets do not contain the high frequency information that was removed by the Zernike fit performed before the OPD data was loaded into the ray tracing model. The Zernike phase surface at the detector is replaced by a grid phase surface in order to add this content to the model. This grid phase surface could contain the raw measured OPD. However, there is typically missing data in the raw OPD data, caused by either bad pixels in the sensor or failure of the sub-Nyquist phase unwrapping procedure due to low modulation. The SNI software can fill in these missing points using the Zernike fit or by interpolation from the surrounding points. Additionally since the model will run slower as the number of grid points is increased the software can down select data from the 511x511 recorded by the sensor to a more manageable grid size, typically 127x127 pixels were used as a starting point. Finally the SNI software can apply a low pass filter to the data to smooth out some of the pixel to pixel noise that is present in the data. The data should be loaded into the model while it is still setup in the reverse optimization mode. The residual wavefront error in the model, immediately after loading the data is shown in FIGURE 6.15. On the left is the data over the full grid phase surface. This shows the

issues that appear at the edge of the grid phase surface, mainly large phase values being introduced on the edge of the wavefront, and even outside the pupil. On the right an aperture has been placed on the surface at 95% of the full diameter to block out these problem areas.

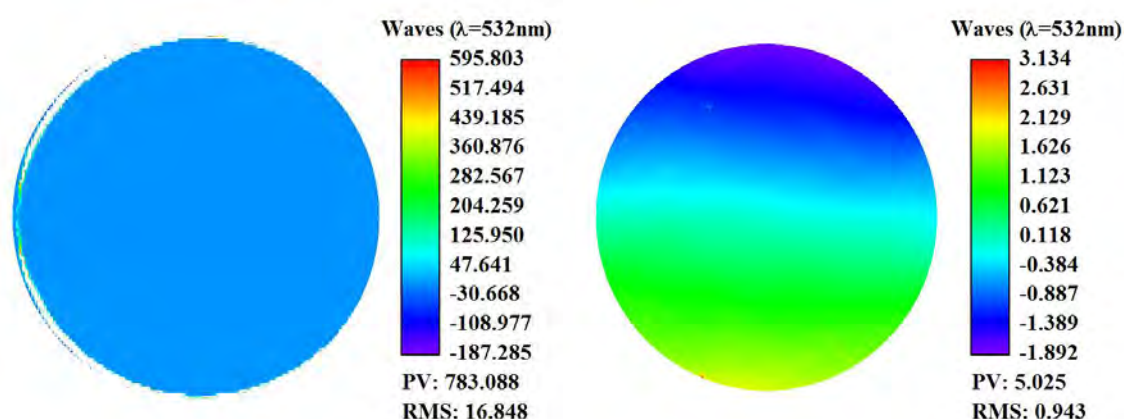


FIGURE 6.15 The residual wavefront error present in the model immediately after loading the grid phase measured OPD data (Left), and the same data set with an aperture placed to remove the outside 5% of the wavefront error (Right).

The wavefront error present in FIGURE 6.15 (Right) is caused by the grid phase data not being properly registered to the model. The detector surface had previously been allowed to shift in order to properly align the center the Zernike representation of the measured OPD data to the wavefronts in the model. The data can be re-centered by turning off all the variables of the RO model except the decenters on the detector surface. Then the down sampled grid phase data can be replaced with the data utilizing the full sensor resolution. The results of these steps are shown in FIGURE 6.16.

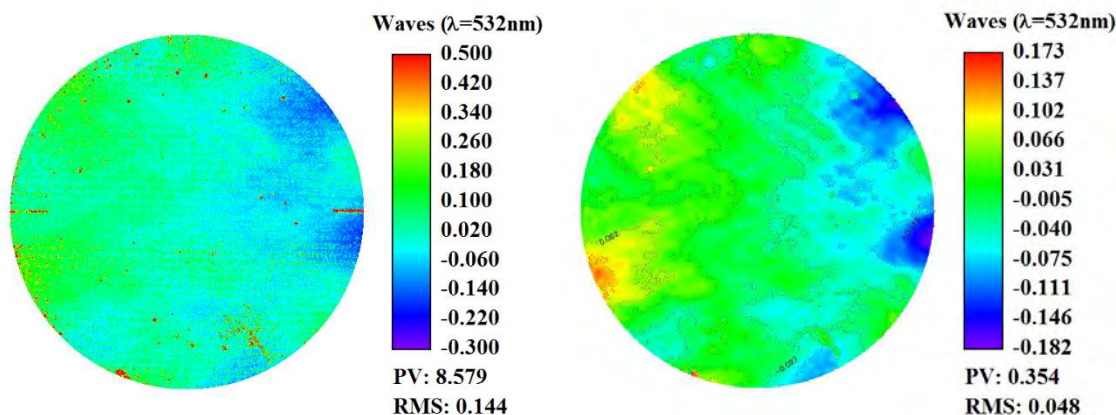


FIGURE 6.16 The residual wavefront error present in the model after aligning the grid phase data to the model, utilizing the raw OPD data (Left) and data that has been processed through a low pass filter (Right).

At this point another round of reverse optimization could be run. However, this can be extremely slow, and will often stall if the full resolution data is used or if the data contains too much noise or missing data. Finally, the model can be set up for reverse raytracing using the same process previously discussed. The results are shown in FIGURE 6.17 (Left), which basically shows the same result as utilizing the Zernike fit of the measured OPD data. Removing the Zernike fit, utilizing the 37 fringe terms, of this OPD surface yields FIGURE 6.17 (Right). The 180° rotation of this data set compared to that of FIGURE 6.3 (Left) is caused by the inversion of the test surface as it is being imaged onto the detector. Additionally the high peak to valley error in both images is the result of noise in the measured OPD data and the use of the grid phase surface in the Zemax model.

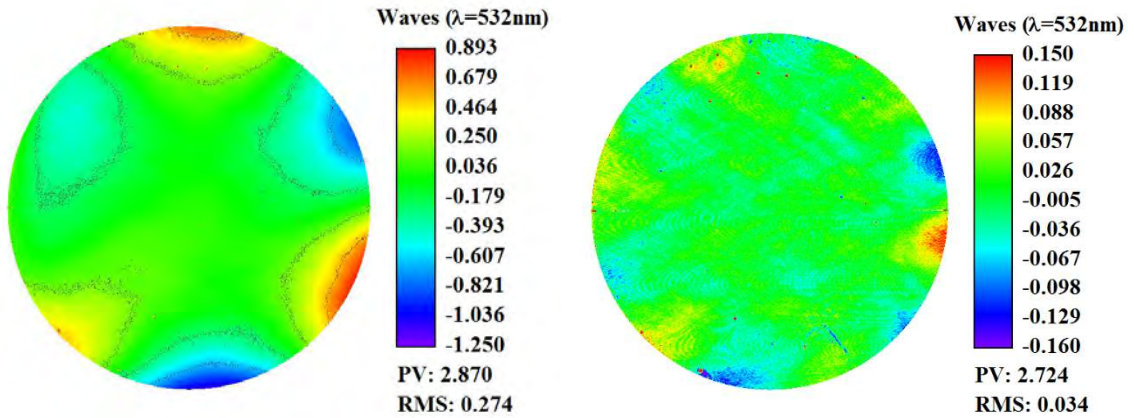


FIGURE 6.17 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure and the full resolution OPD data from the sub-Nyquist sensor (Left). This is twice the surface error. The difference between this data and the Zernike fit of this data (Right).

This process was then repeated for the convex surface of the +0.75 diopter lens, FIGURE 6.18. In this case the reflected wavefront off the cylindrical surface is diverging, and thus a larger portion of the interferometers surfaces are used, compared to the -0.75 diopter lens. The residual wavefront error present in the RO model for the spherical lens in this case was on the same order as the previous test at 0.37λ across the entire detector and 0.18λ over the aperture of the cylindrical lens. However, the residual error present in the RO model of the cylindrical surface was much greater than the previous case at 0.162λ peak to valley, FIGURE 6.19. This is ten times greater than the residual error present in the -0.75 diopter cylindrical lens. Five measurements made of the surface produced an average short axis radius of curvature of 709.05mm, with a range of 705.85mm and 712.67mm. This corresponds to a range of 2.1λ in the OPD introduced by the surface recovered by the reverse optimization and raytracing process. Like the negative diopter lens the measured radius of curvature was shorter than the value measured using a profiler, which was 718.28mm. A second measurement of the recovered form error along

with the difference between the first and second measurement is shown in FIGURE 6.20 (Left). This measurement was made with the surface rotated approximately 90° from the first. FIGURE 6.20 (Right), shows the comparison between the first and second measurement, after the second data set was rotated and the aperture was reduced by 2% in order to remove spikes at the edge of the measurement. This graph shows the difference between the two data sets is greater than a half a wave peak to valley.

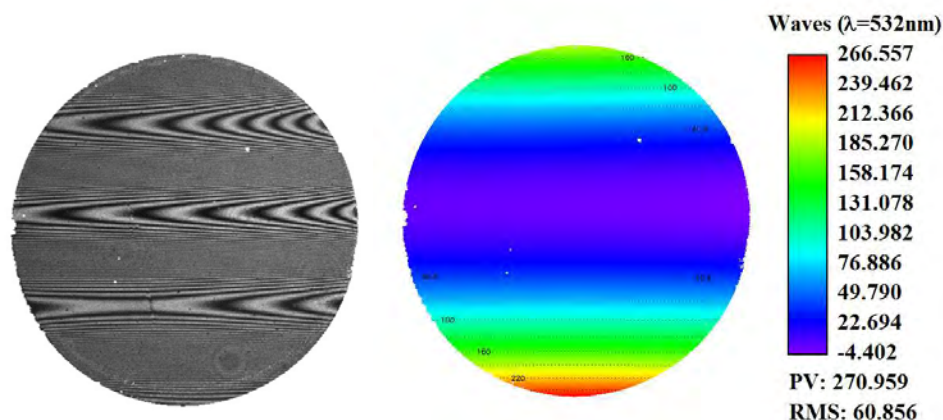


FIGURE 6.18 The interferogram (Left) and the unwrapped OPD (Right) for the cylindrical surface of the +0.75 diopter lens.

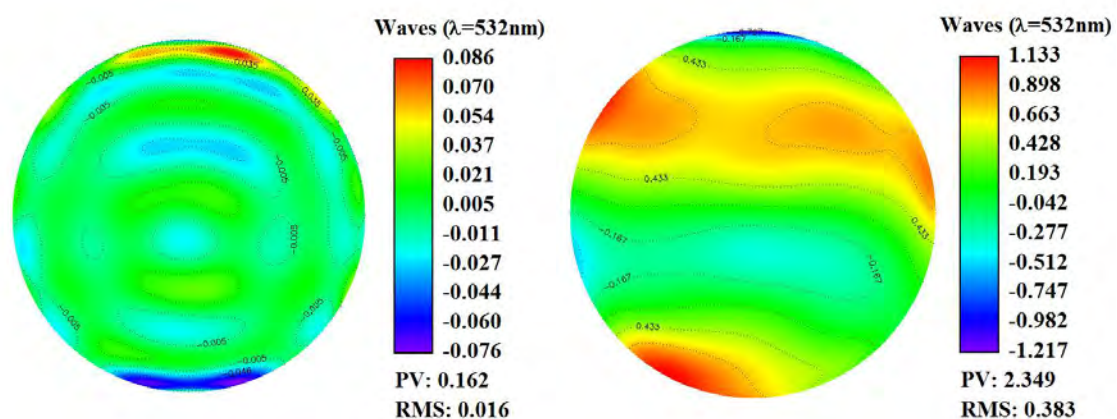


FIGURE 6.19 The residual wavefront error at the last surface of the RO model for the cylindrical surface of the diopter +0.75 lens (Left). The OPD error introduced by the lens surface recovered by the RO process minus the cylindrical radii of curvature (Right). This is twice the surface error.

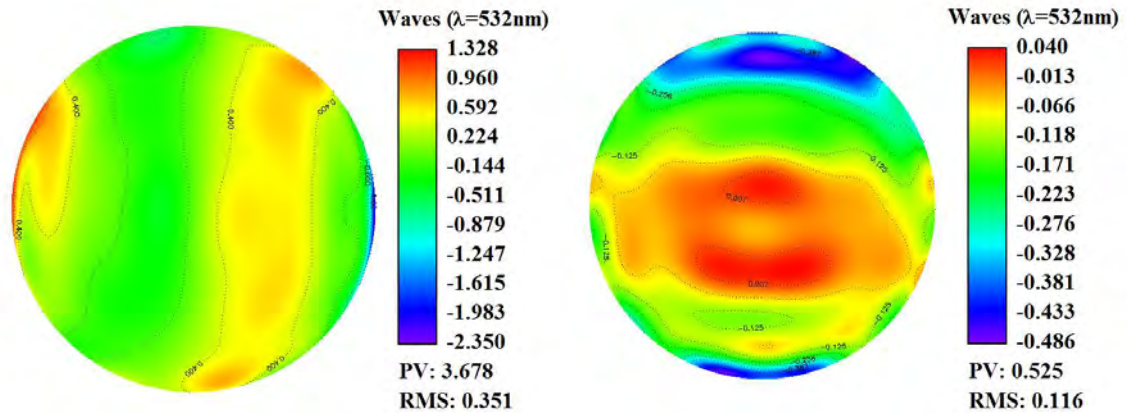


FIGURE 6.20 A second measurement of the OPD introduced by the cylindrical surface made at 90° with respect to the first measurement (Left). This is twice the surface error. The difference between this measurement and the previous measurement, after accounting for the rotation of the second measurement (Right).

6.2 Measurement of a Conic Aspheric Surface Containing a Designed Defect

Two inserts were fabricated with the same off-axis “defect”, which was added to the design file of the test part prior to fabrication. This defect was designed to be easily detectable with the non-null interferometer and RO process. An off-axis bump was used so that the part could be rotated and retested to see if error detected by the RO process would rotate with the part. The designed surface error added to the nominal test part shape is shown in FIGURE 6.21. The defect should introduce about 5λ of OPD into the measurement of the test parts. No prior knowledge of the shape or size of this defect was used during the RO process.

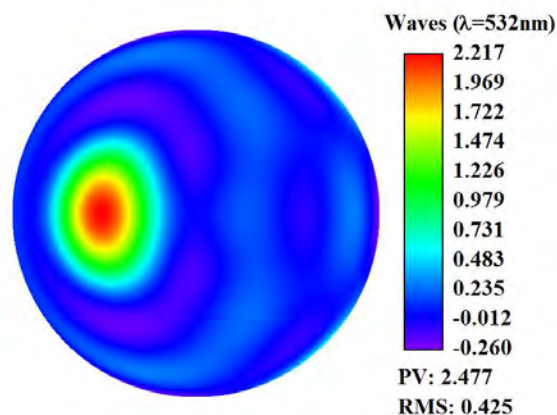


FIGURE 6.21 The designed surface error that was added to the two test parts.

A similar reverse optimization procedure as was described for the cylindrical surfaces was used on these test surfaces. However, data from measurements of a separate part were not used in this RO procedure, the reasoning behind this change will be discussed in Chapter 7. This process relied on multiple measurements of the shifted test part. In testing parts with the diverger lens, the test parts are located in a converging or diverging beam thus the change in the measured OPD can be hundreds of waves for a part shift of less than 1mm. Additionally, two assumptions were used at the beginning of the RO procedure. The first, which seems counterintuitive, was that the test part is equal to its design prescription. Basically this assumption simply means that the RO model was started with the design prescription for the part to be tested in the model, and that for the initial steps of the RO process it was not allowed to change. For the parts with the designed defect, this only includes the nominal prescription of the part and does not encompass any information about the shape of the defect. The nominal prescription of the part is only the base radius of curvature and the conic constant for the rotationally symmetric part and the two crossed radii of curvature for the toric part. The goal of the

RO procedure is to locate the defect. Without some prior knowledge of the test part shape, it would be difficult to find an initial starting point for the RO model. This assumption was lifted at the end of the procedure by allowing the Zernike terms placed on the test surface to distort its shape. The surface prescription could also be allowed to vary at the end of the RO process, but this was not implemented. The second assumption was that the measured shifts introduced into the part location were known perfectly. In practice with the use of the Heidenhain length gauge the displacement between part locations is only known on the order of $\pm 2\mu\text{m}$. Therefore, the size of the steps was chosen to maximize the difference in the OPD between positions, but still produce interferograms that could be both recorded with the sparse array sensor and unwrapped by the SNI software. Near the end of the RO process, this assumption is also relaxed and the position of the part for each measurement is allowed to vary, within the measurement uncertainty.

The first test part was a convex aspheric surface with a 9mm radius of curvature and a conic constant equal to 0.8, making it the surface of an oblate ellipsoid. The part was designed to have a diameter of 8mm with a sharp cut off at the edge. This was done so that the automated masking routine could be used, and so that there would be no ambiguity about the region of the surface over which the measurement was to be made. However, the manufacturer made the parts with a 10mm diameter. A 1mm ring outside the 4mm semi-diameter was created as a transition zone during the cutting process. This made it difficult to determine the edge of the aperture to be tested, however a slight line

or irregularity can be seen in the interferograms at intersection of the center region and the outside ring. The manual masking routine was used with this line to define the diameter of the test zone. After the part was loaded into the interferometer at the cat's eye position of the diverger it was shifted towards the diverger by 8.405 mm into its nominal testing position, as predicted by the simple model. The part was then shifted towards the diverger by an additional 0.5mm and then away from the diverger in 0.25mm steps in order to capture phase shifted interferograms at 5 different positions of the test part, shown in FIGURE 6.22. The phase data from each location is calculated, unwrapped, converted to OPD and fit to Zernike polynomials, before being loaded in Zemax for five separate configurations. The raw measured OPD for the first, third and fifth set of fringes are shown in FIGURE 6.23. Additionally, the measured test part shifts and the imaging distances calculated using the magnification target and the 1000mm radius of curvature mirror are loaded into the Zemax model.

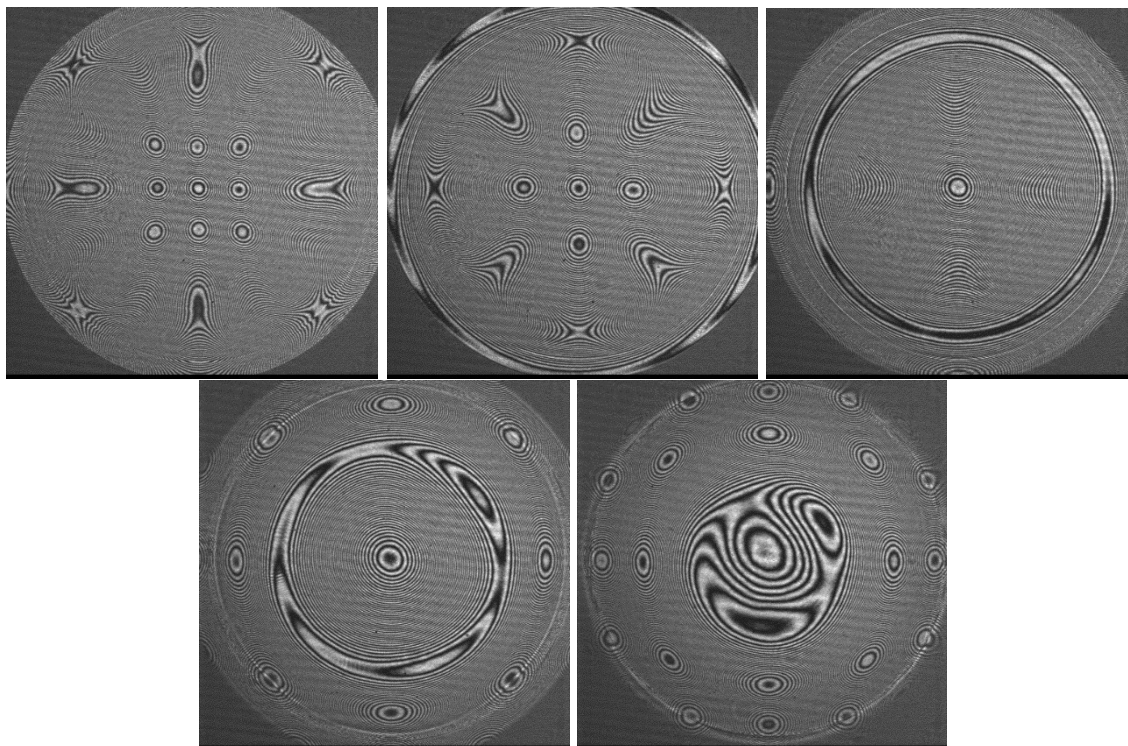


FIGURE 6.22 Interferograms recorded for the convex conic aspheric test surface recorded in 0.25mm steps progressing away from the diverger from the top left to bottom right.

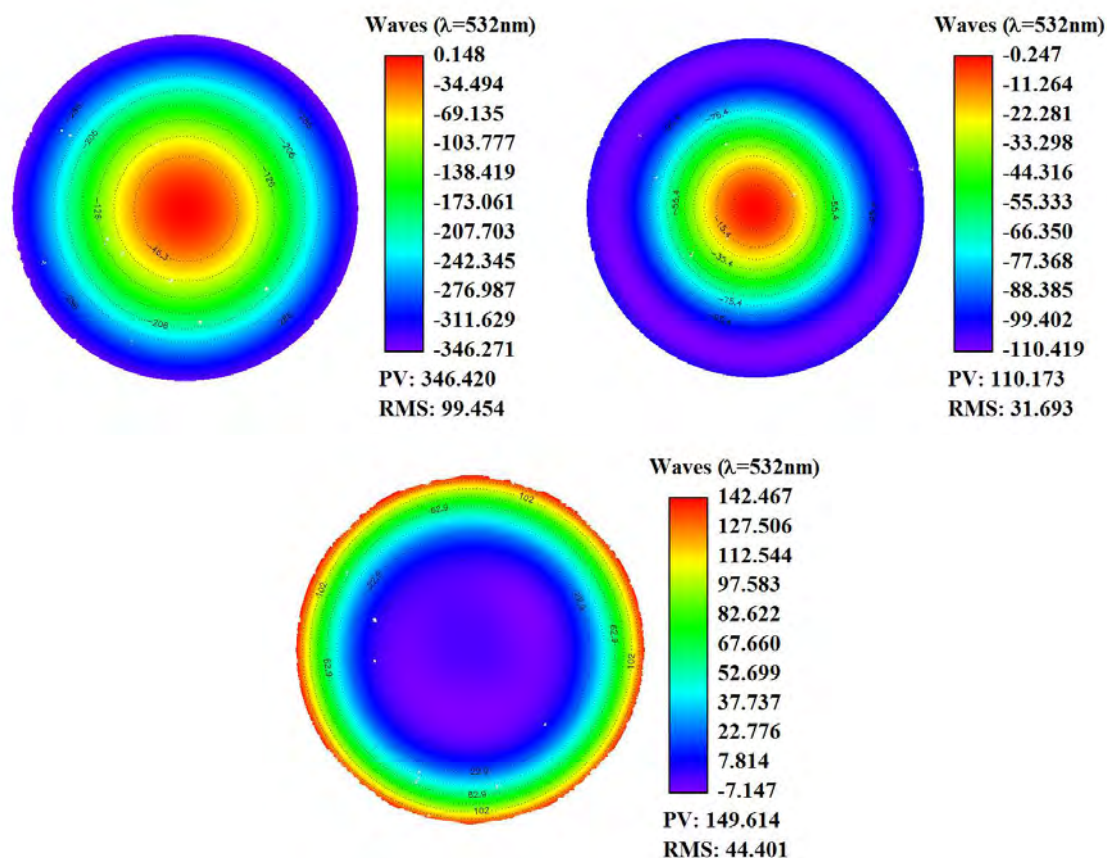


FIGURE 6.23 Unwrapped OPD for the first (Top Left), third (Top Right), and fifth (Bottom) interferograms shown in FIGURE 6.22.

The initial residual wavefront error for the center test part location obtained after loading the data into Zemax is shown in FIGURE 6.24 (Left). There is a significant amount of residual power present which is primarily due to the test part not being located exactly at the nominal position as well as disagreement in the imaging distances between the model and the interferometer. The same basic RO process as was previously described is then started. However, when testing parts with the diverger, only the configuration representing the test part at its nominal position, in this case the third measurement, uses the entire 8mm aperture of the test part. A merit function operand is used to target the

diameter of the wavefront at the detector in the model to be within two pixels of the measured diameter of the wavefront at the detector using the masking routine. For all the other configurations an aperture is placed on the test part that trims rays from the outside 2% to 5% of the surface. This means the data at the detector for these configurations is under filled. This is done for a couple of reasons. First, only the test part located in the non-shifted location is setup to be conjugate to the detector, while the other positions are slightly out of focus. This makes determining the edge of the test beam on the detector and relating it to a fixed diameter on the test part more difficult. Additionally, while not as significant in this case since the test optic actually extends past the test zone, diffraction can lead the measured OPD for the out of focus test part to curl up, or down at the edge of the pupil, which will not be predicted by the raytracing software. Finally, this allows the RO procedure to spend more effort finding the surface shape and location that matches the measured OPD without running into the previously discussed problems that can occur when rays are traced outside the defined aperture of the measured data.

The first step in the RO process is to remove the tilt by allowing the reference arm tilt, and the test part to decenter and tilt. This is followed by the centering of the data on the detector and then optimizing the distance between the test part and diverger to remove the majority of the residual power. Next, the separations of the diverger, beam splitter, imaging lens and finally the detector are allowed to vary one after another. At this point the shape of the designed defect should have emerged in the residual wavefront error, FIGURE 6.24 (Right). The Zernike standard surface terms on the test surface are

allowed to vary to begin to replicate the form error of the test part. After this the diverger lens properties, the orientation of the imaging lens, and the measured separations between the part shifts can be allowed to vary. The part shift locations should move by less than $\pm 2\mu\text{m}$ in order to account for error in the measured part locations. Finally, the position of the phase surfaces representing the collimated wavefront and the interaction of the reference arm with the beam splitter could be allowed to vary, along with the sag surface representing the interaction of the test arm with beam splitter. However, the residual wavefront error is typically much larger than the change that can be introduced by these surfaces. The final residual wavefront error produced after a long optimization run with all of these variables turned on is shown in FIGURE 6.25 (Left). The system is then setup for reverse raytracing as previously described. Tracing all the way back through the system produces the same residual error, only with the opposite sign, FIGURE 6.25 (Right). The Zernike phase representation of the measured OPD data is then replaced by the grid phase representation. Here the full sensor resolution of 511x511 pixels is used after the missing data points have been filled in using interpolation and the data has been run through a low pass filter. As with the cylinder lenses, after inserting the grid phase surface into the model the optimization procedure is run in which only the lateral position of the detector is allowed to vary in order to center the imported data to the model. The resulting residual wavefront error can be seen for both the forward and backward ray traces in FIGURE 6.26. The same basic shape can be seen in the residual errors produced using the Zernike representation and the grid phase representation of the measured OPD data, however the grid phase representation shows a much larger peak to valley and rms

error. The final OPD introduced by the form error of the part calculated by comparing the forward propagation wavefront to the backward propagating wavefront at the test part is shown in FIGURE 6.27 (Left) along with the OPD error predicted from the defect design, FIGURE 6.27 (Right). While there is some discrepancy between the two, the overall shape and size of the defect is very similar.

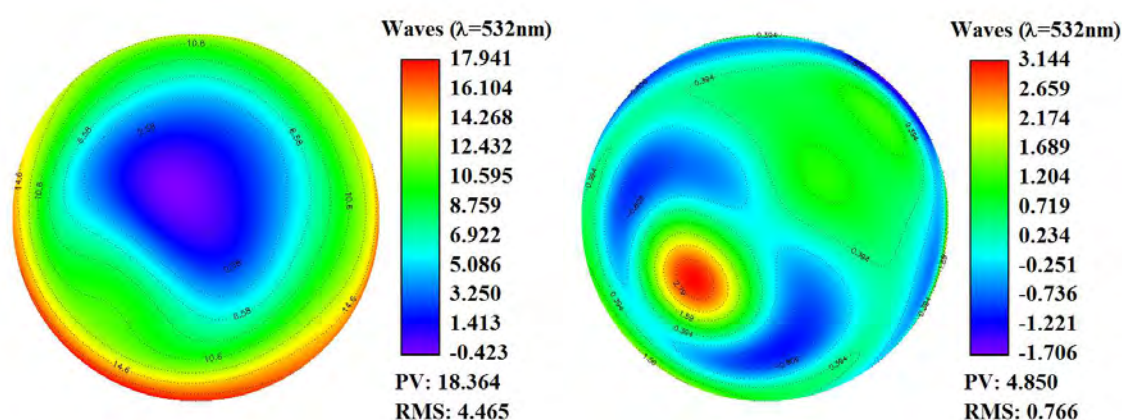


FIGURE 6.24 The residual wavefront error in the model immediately after inserting the measured OPD data into model (Left) and the residual wavefront error after the first few steps of the RO procedure (Right).

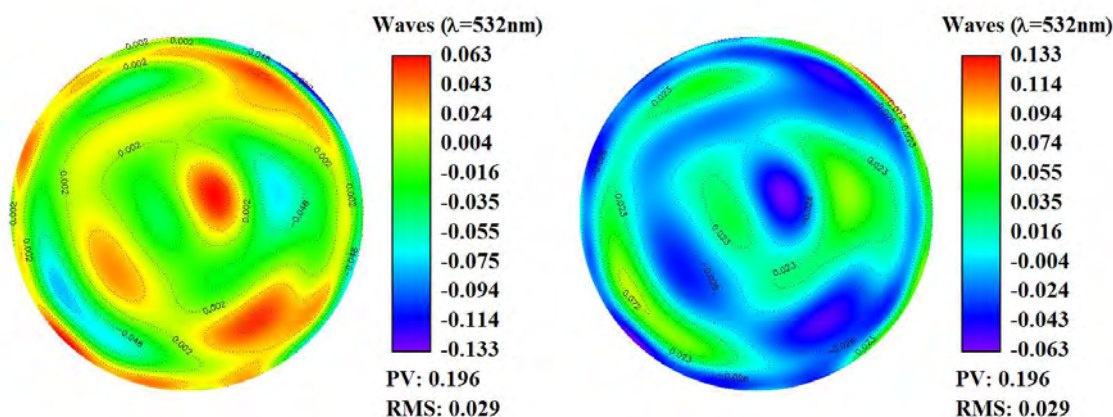


FIGURE 6.25 The final residual wavefront error in the model at the end of the reverse optimization procedure using the Zernike representation of the measured OPD (Left) and the same error present in the reverse ray trace of the model (Right).

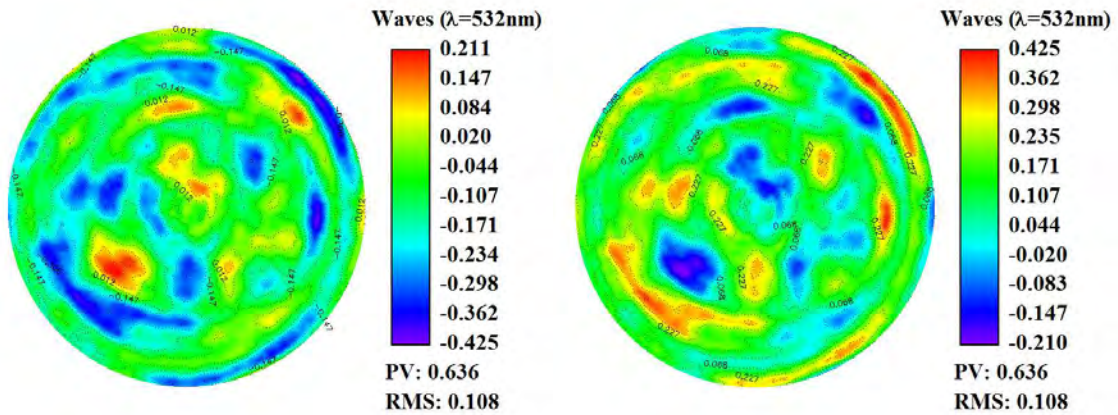


FIGURE 6.26 The final residual wavefront error in the model at the end of the reverse optimization procedure using grid phase surface representation of measured OPD (Left) and the same error present in the reverse ray trace of the model (Right).

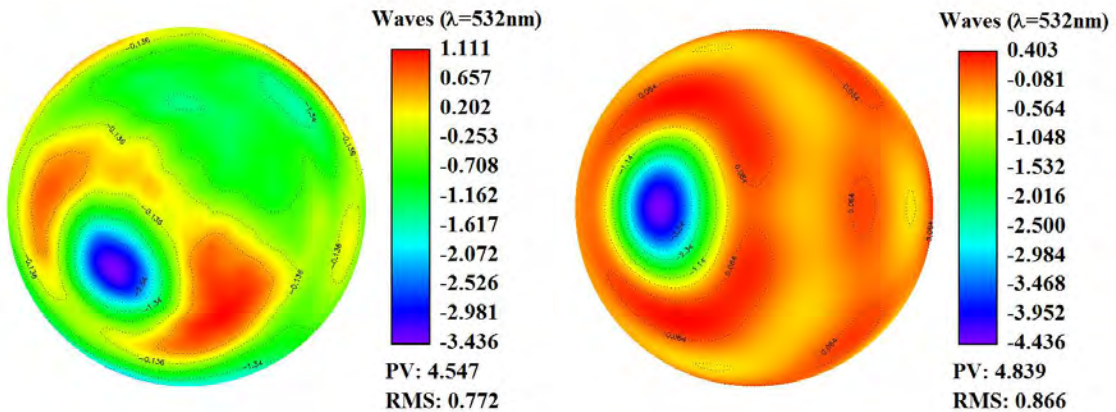


FIGURE 6.27 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The OPD that should be introduced by the designed error (Right).

The entire test was then repeated multiple times, starting at the beginning of the process by first removing the diverger lens and the test surface. The imaging lens and detector were not removed from the system, but the spacing between them was altered, and then remeasured using the magnification target. The rotation about the z-axis of the test was altered for each test. However, because the less cantilevered test part mounting fixture was used for these measurements, the rotation angle could only be coarsely set and

changing it also altered the tip and tilt of the test part. For each measurement the shape of the defect could clearly be seen in final OPD error recovered by the RO process and the measured peak to valley error fell between 4.32λ and 4.74λ . The results of a second test are shown in FIGURE 6.28 (Left) along with difference between the first and second measurement FIGURE 6.28 (Right), after rotating the data to account for the part orientation. The difference between these measurements is artificially high, because of the inability to align the measurements before taking the difference. The process used was to place the final OPD data into two separate mirrored surfaces in a Zemax model. Then Zemax was allowed to change the orientation of the surfaces such that the rms wavefront error produced after reflecting off the surfaces was minimized. The two “aligned” surfaces are shown side by side in FIGURE 6.29, in which apparent misalignment can be seen by visual inspection. A better solution would have been to develop a separate program in the SNI software to compare the surfaces using a technique such as iterative closest point analysis. (Besl & McKay 1992)

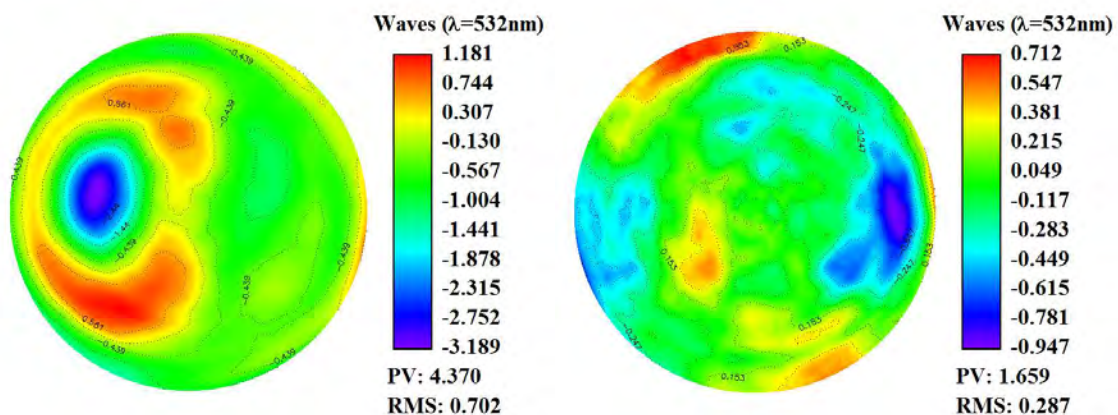


FIGURE 6.28 A second measurement of the OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement is also shown (Right).

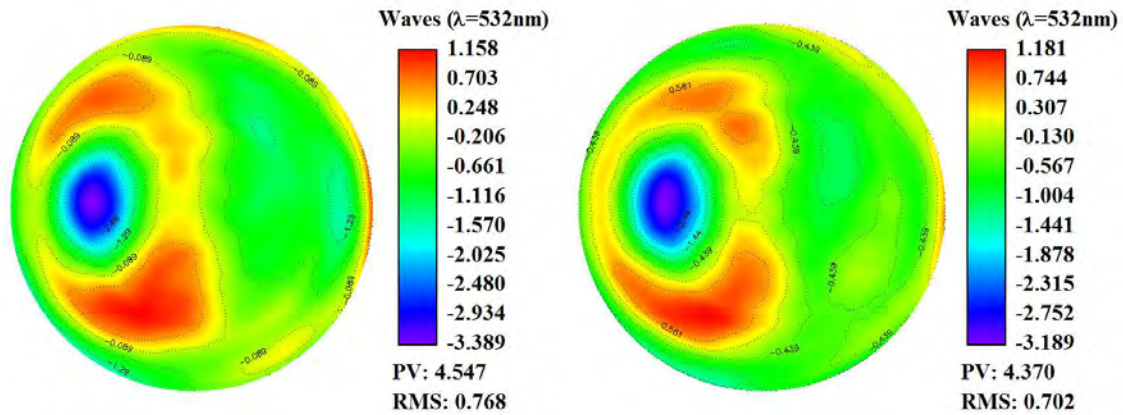


FIGURE 6.29 The “aligned” first (Left) and second (Right) measurements.

6.3 Measurement of a Toroidal Surface Containing a Designed Defect

A second insert containing this same designed surface error as the previously discussed surface was also manufactured. The base design of this test part was a toroidal surface consisting of a radius of curvature in one axis of 8.15mm and 8.65mm in the crossed axis. The diameter of the surface was also intended to be 8mm, but it was also manufactured with the 1mm transition zone outside the test area. The same approach was followed as was described in the previous section. One difference in testing these surfaces was that only shifts away from the diverger from the nominal part location were used. The nominal testing position for this part is 8.4mm inside the focal point of the diverger lens, which corresponds to the average of the two radii of curvature. The wavefront at the intermediate pupil location is expanding along the axis corresponding to the 8.15mm radius of curvature and collapsing along the axis corresponding to the 8.65mm radius of curvature. Shifts towards the diverger lens will increase the divergence and as a result possibly cause the wavefront along the axis corresponding to the 8.15mm radius of curvature to overfill the imaging lens. Shifts away from the diverger will reduce the

divergence of the light corresponding to the 8.15mm radius of curvature. The rays along the 8.65mm radius of curvature of the test part will converge faster with the shifts away from the diverger lens, but there is no danger of these rays vignetting. In testing this part six part locations corresponding to 0.1mm part shifts away from the diverger lens were used. The resulting interferograms are shown in FIGURE 6.30. The change in the unwrapped measured OPD data between the first part location and the last is shown in FIGURE 6.31. The maximum change in the OPD introduced by the part shifts is approximately 200λ .

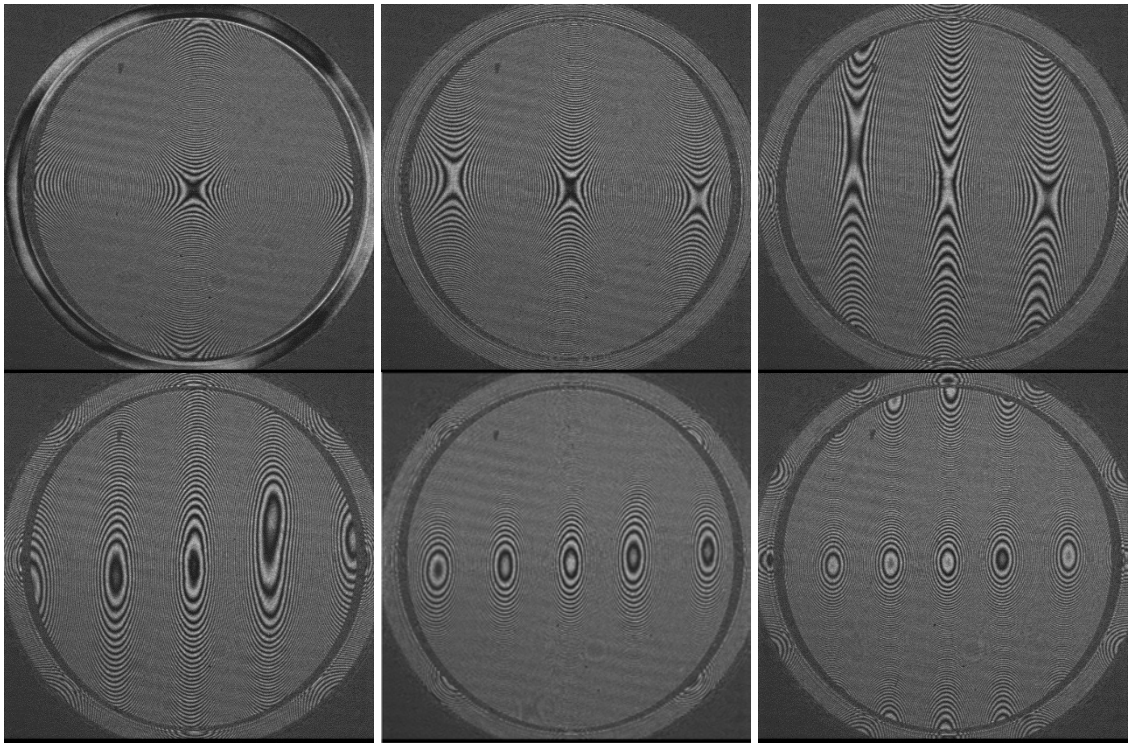


FIGURE 6.30 Interferograms recorded for the convex toroidal test surface recorded in 0.1mm steps progressing away from the diverger from the top left to bottom right.

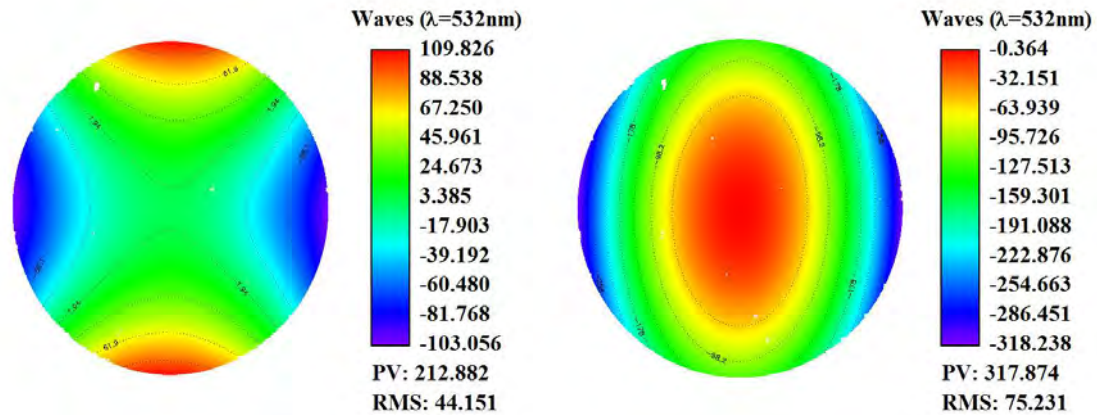


FIGURE 6.31 The unwrapped OPD for the first (Left) and last (Right) interferograms shown in FIGURE 6.30.

The results of two measurements are shown in FIGURE 6.32 (Right) and FIGURE 6.33 (Left). Additionally, the difference between the two measurements after rotating the data, utilizing Zemax, is shown in FIGURE 6.33 (Right). In between these measurements the system was realigned, as discussed in the previous section, starting by removing the test part and diverger and adjusting the system magnification. Also between the two measurements shown, the test surface was rotated by approximately 180° . Again, the basic shape and magnitude of the designed defect was found in the test part for both measurements. The magnitude of the measured error was slightly larger than was found for the convex conic aspheric part, but was still close to the predicted error introduced into the design. Additionally, while both parts were designed to contain the same error there was no guarantee that the same error was introduced during the manufacturing process.

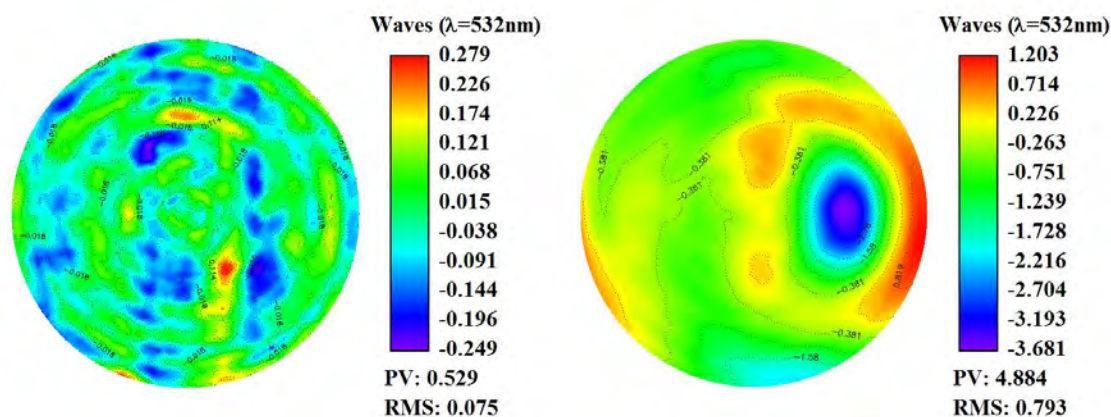


FIGURE 6.32 The final residual wavefront error in the model at the end of the reverse optimization procedure using the grid phase representation of the measured OPD (Left) The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.

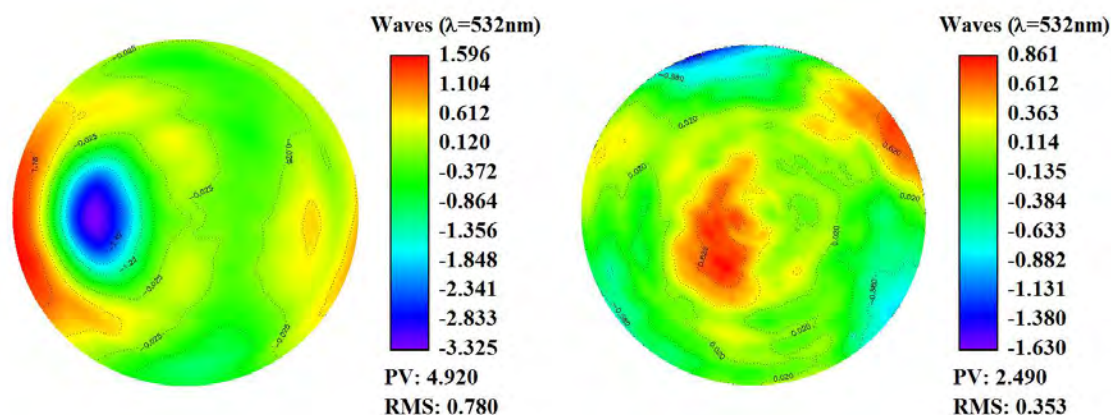


FIGURE 6.33 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure for the second measurement (Left) and the difference between the first and second measurement (Right).

6.4 Measurement of Two Aspheric Contact Lens Tooling Inserts

Measurements were made on two aspheric contact lens tooling inserts. These surfaces were not designed to be used for contact lenses, but were provided by the research sponsor as representative samples. They were made using the same diamond turning process used to make their normal tooling inserts, and we expected to have similar

surface quality. Unlike the previous surfaces these did not have a transition zone outside the 8mm diameter. The first insert was a convex aspheric surface with a radius of curvature equal to 8mm and a 4th order aspheric term equal to $-6.0\text{E-}4$. It was measured utilizing 5 test positions separated by 0.2mm over a range of -0.4mm to +0.4mm from the nominal testing position determined using the simple model, which introduced over 200λ of change into the measured OPD at the detector. The interferograms recorded at these positions is shown in FIGURE 6.34, while the measured OPD for first, third and fifth interferograms are shown in FIGURE 6.35.

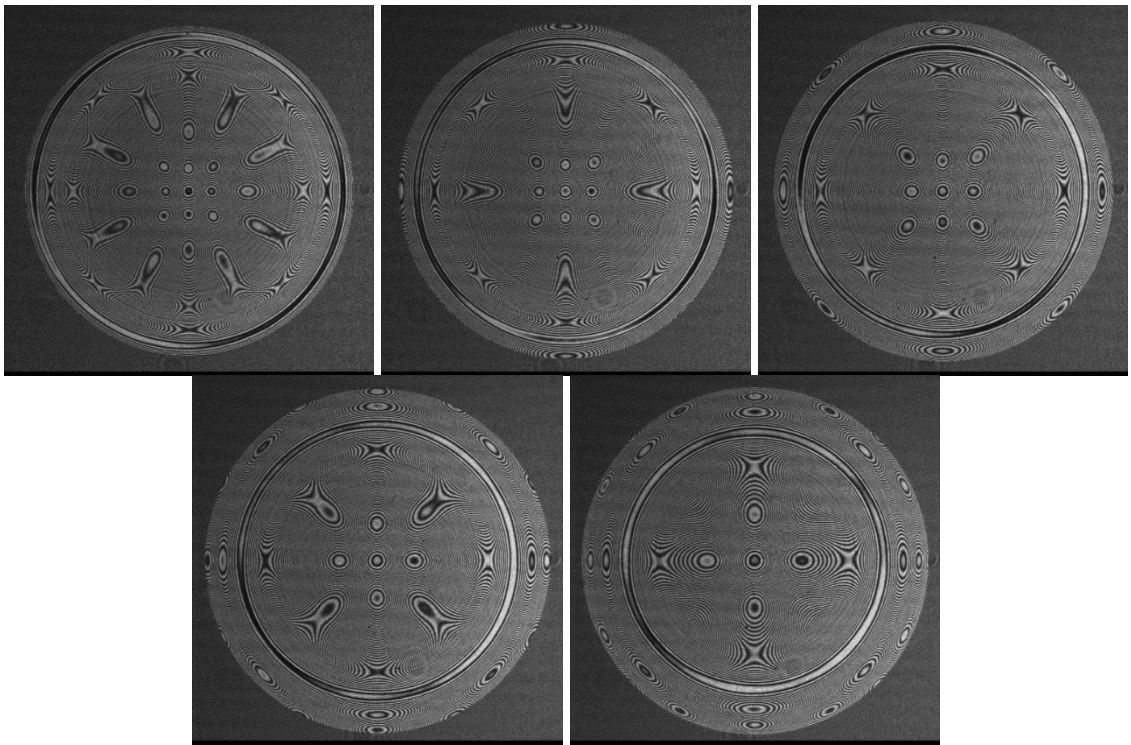


FIGURE 6.34 Interferograms recorded for the aspheric surface with an 8mm radius of curvature and a 4th order aspheric term equal to $-6.0\text{E-}4$. Fringes were recorded in 0.2mm steps progressing away from the diverger from the top left to bottom right.

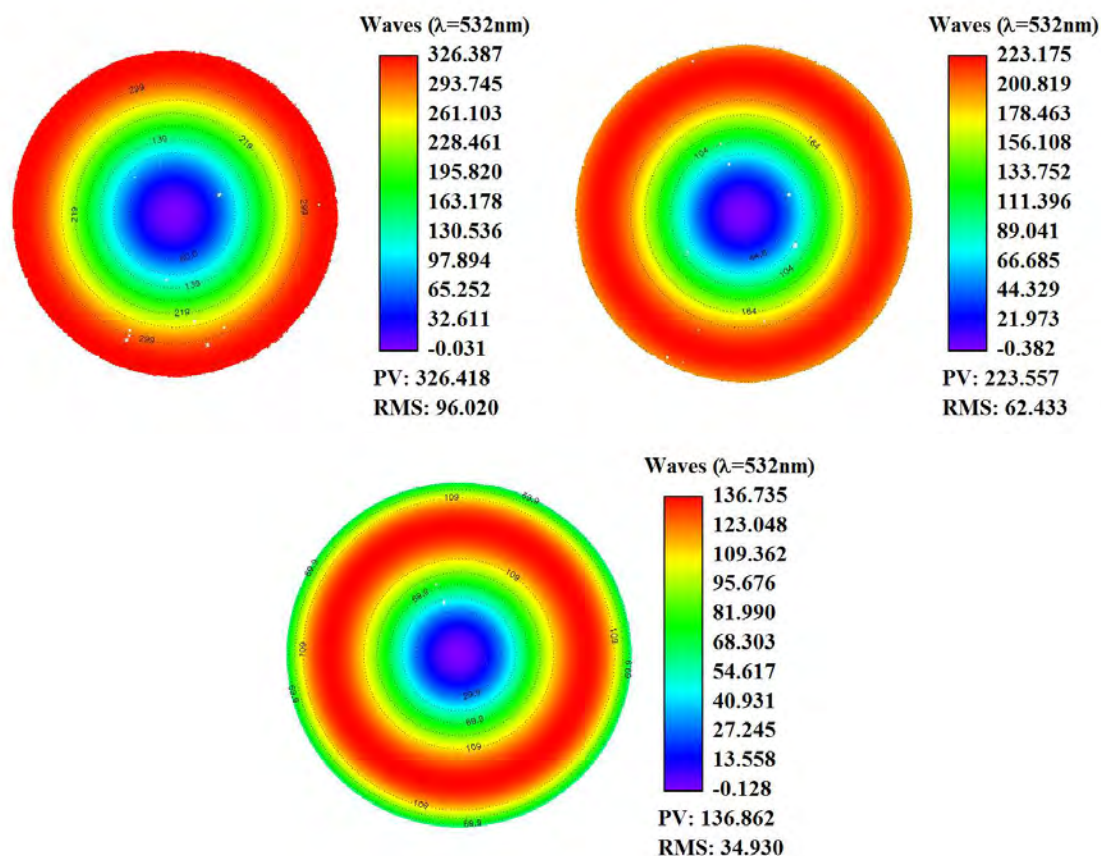


FIGURE 6.35 Unwrapped OPD for the first (Top Left), third (Top Right), and fifth (Bottom) interferograms shown in FIGURE 6.34.

The residual error present in the model, after the completion of the RO procedure, is shown in FIGURE 6.36 (Left). It shows that there was still almost a quarter wave of error that could not be accounted for by the RO model. FIGURE 6.36 (Right), shows OPD error introduced by the test surface recovered using the reverse raytracing model. This measurement was then repeated, FIGURE 6.37 (Left), however unlike the measurements made on the parts with the intentional defect, the test part was not rotated between measurements. The reasons for this will be discussed in Chapter 7. However, this allows the second measurement to be compared to the first measurement by simply taking the difference, FIGURE 6.37 (Right). Here there is fairly good agreement

between the two measurements, with a peak to valley difference of only 0.144waves. The two measurements were then evaluated with the grid phase representation of the measured OPD data incorporated into the model. In this case, the magnitude of both the residual wavefront error and the error introduced by the error in the test surface grew, FIGURE 6.38. However, the same basic shape is present in both solutions. The difference between the two measurements, FIGURE 6.39, using the grid phase representation of the measured OPD is similar in shape and magnitude to the difference calculated using the Zernike representation of the measured OPD.

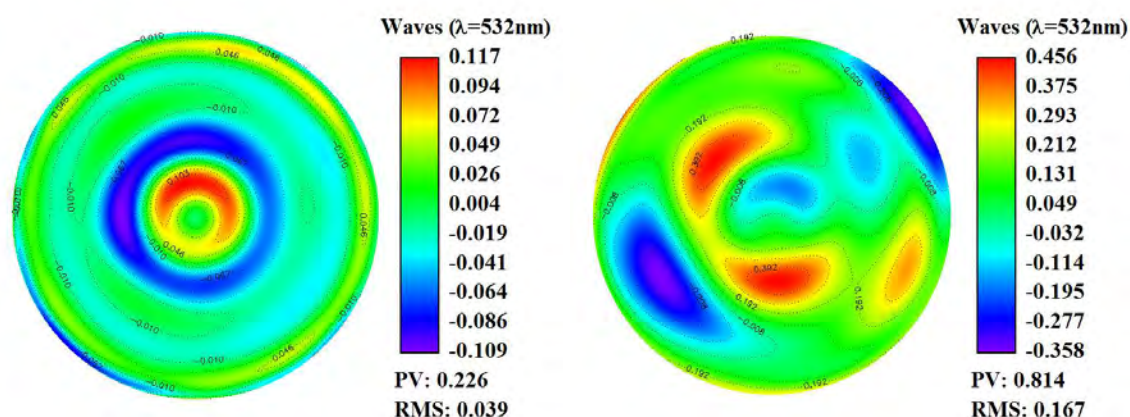


FIGURE 6.36 The residual wavefront error in the model at the end of the reverse optimization procedure using the Zernike representation of the measured OPD (Left). The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.

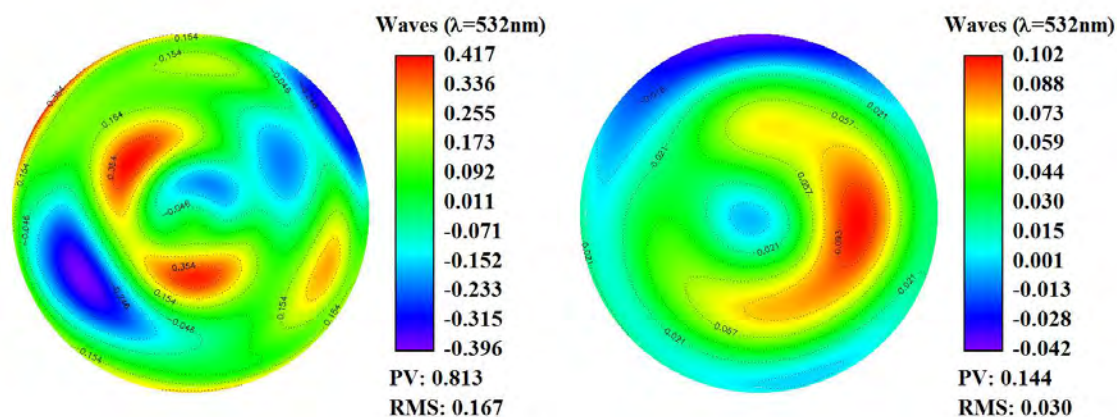


FIGURE 6.37 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).

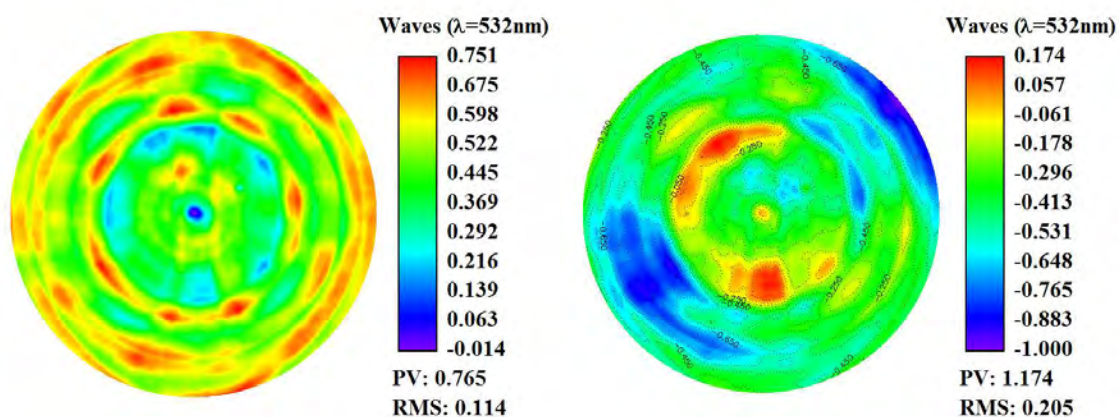


FIGURE 6.38 The final residual wavefront error in the model at the end of the reverse optimization procedure using the grid phase representation of the measured OPD (Left). The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.

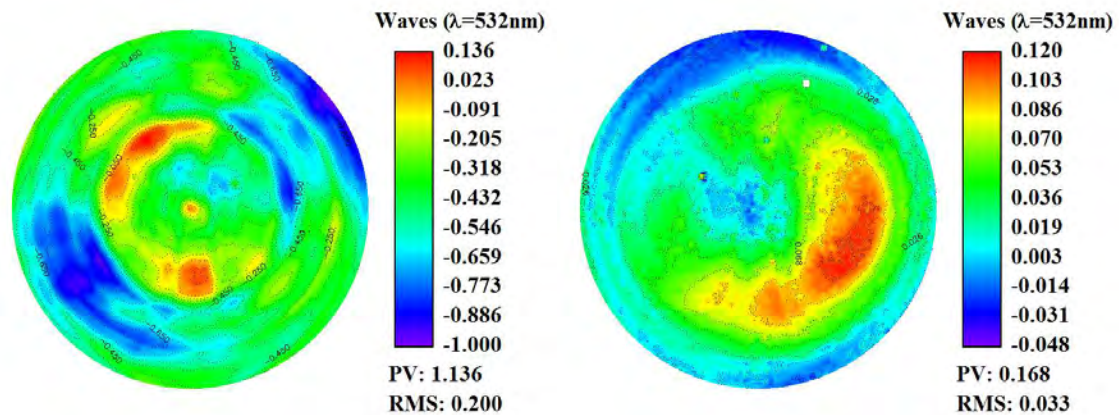


FIGURE 6.39 The OPD introduced by the form error on the test surface from the second measurement calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).

The second insert was a convex prolate ellipsoid surface with a radius of curvature equal to 8mm and a conic constant equal to -0.8. It was measured utilizing 6 test positions separated by 0.1mm over a range of -0.2mm to +0.3mm from the nominal testing position determined using the simple model, which introduced over 200λ of change into the measured OPD at the detector. The interferograms recorded at these positions are shown in FIGURE 6.40, while the measured OPD for first, third and sixth interferograms are shown in FIGURE 6.41.

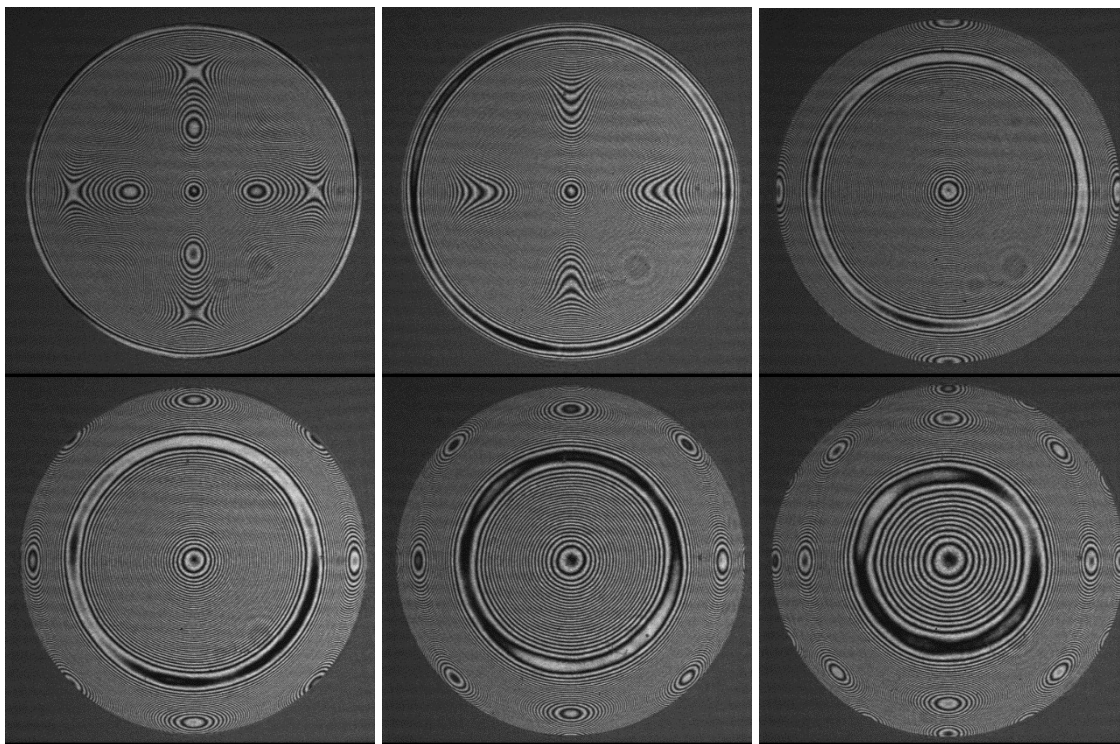


FIGURE 6.40 Interferograms recorded for the aspheric surface with an 8mm radius of curvature and a conic constant equal to -0.8. Fringes were recorded in 0.2mm steps progressing away from the diverger from the top left to bottom right.

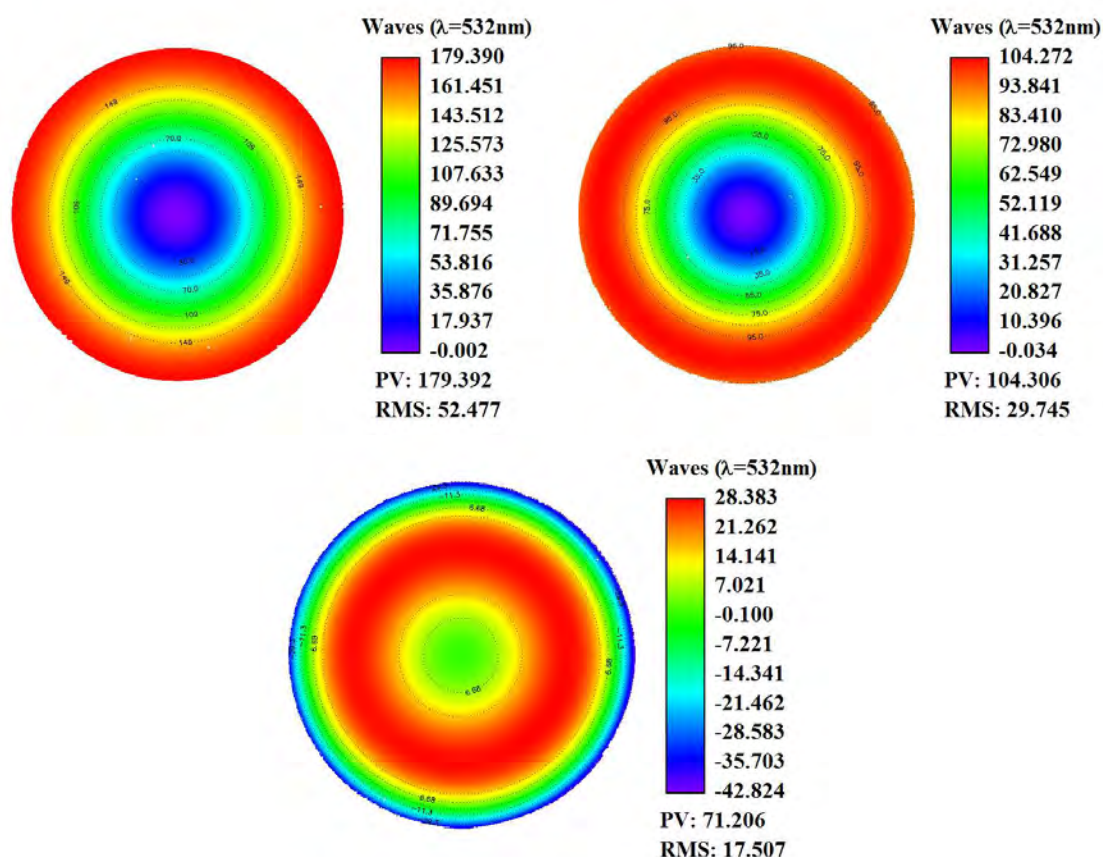


FIGURE 6.41 Unwrapped OPD for the first (Top Left), third (Top Right), and sixth (Bottom) interferograms shown in FIGURE 6.40.

The residual error present in the model, after the completion of the RO procedure, is shown in FIGURE 6.42 (Left). It shows that there was still a sixth wave of error that could not be accounted for by the RO model. However, unlike the previous measurement the residual wavefront error for this part was not circularly symmetric. FIGURE 6.42 (Right), shows OPD error introduced by the test surface recovered using the reverse raytracing model. This measurement was also repeated and, as was the case for the last test part, the test part was not rotated between measurements. However in this case the diverger was removed from the system and realigned before the second data set was

recorded. The second measurement is shown in FIGURE 6.43, along with the difference between the first and second measurements. Here the peak to valley difference between measurements is larger, at almost half a wave, than it was for the previous test part.

Finally the grid phase representation of the measured OPD data is incorporated into the model for both measurements. The residual error, FIGURE 6.44 (Left), is larger when the grid phase representation of the measured OPD is used, at 0.45 waves peak to valley, compared to 0.16 waves peak to valley when the Zernike representation is used.

Additionally, it is difficult to see the same pattern in the residual wavefront error maps shown in FIGURE 6.42 (Left) and FIGURE 6.44 (Left). While the OPD introduced by the form error on the test surface looks very similar for the two measurements using the grid phase representations FIGURE 6.44 (Right) and FIGURE 6.45 (Left), there is still a half wave of peak to valley difference present between them, FIGURE 6.45 (Right).

These results will be discussed in Chapter 7.

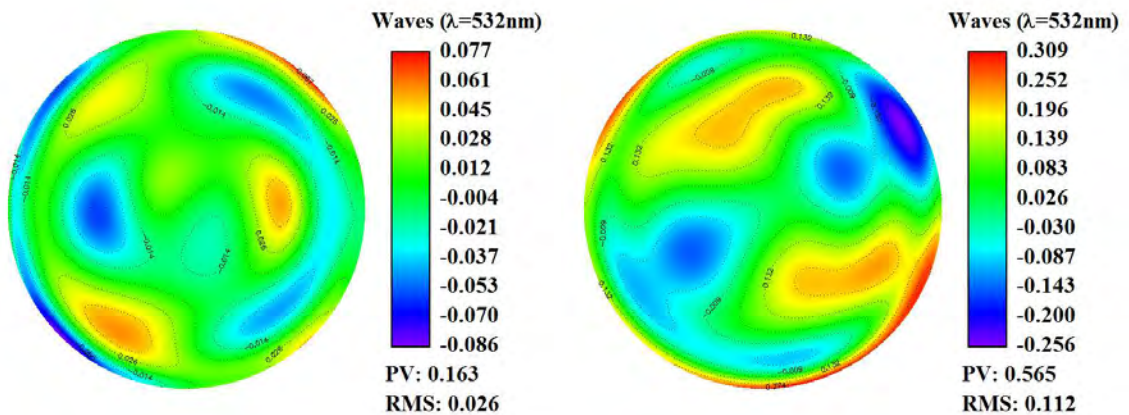


FIGURE 6.42 The final residual wavefront error in the model at the end of the reverse optimization procedure using the Zernike representation of the measured OPD (Left). The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.

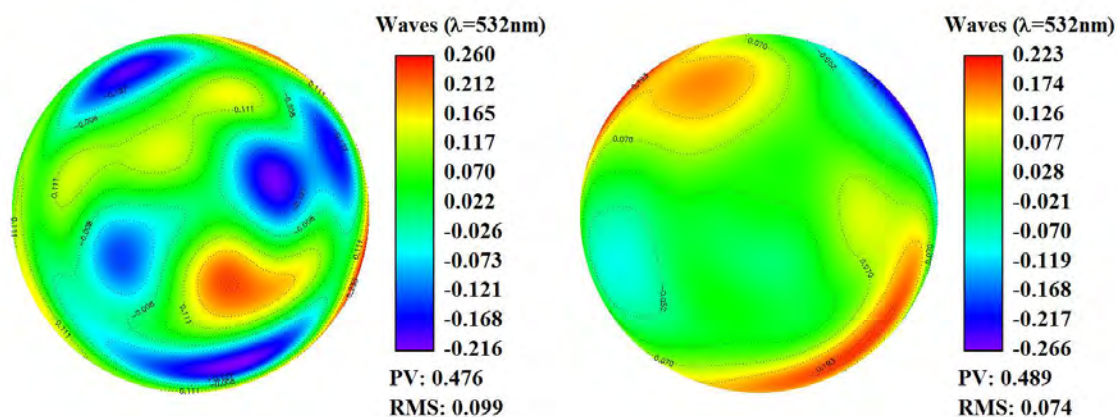


FIGURE 6.43 A second measurement of the OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).

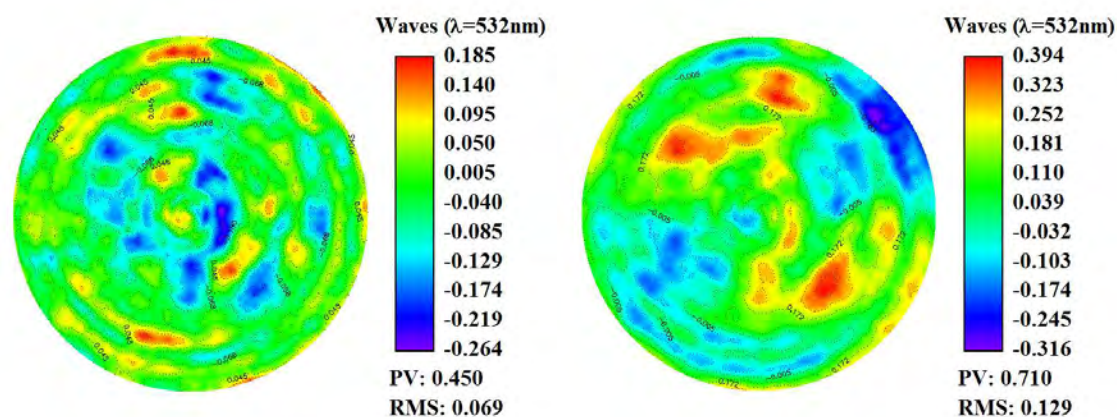


FIGURE 6.44 The final residual wavefront error in the model at the end of the reverse optimization procedure using the grid phase representation of the measured OPD (Left). The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Right). This is twice the surface error.

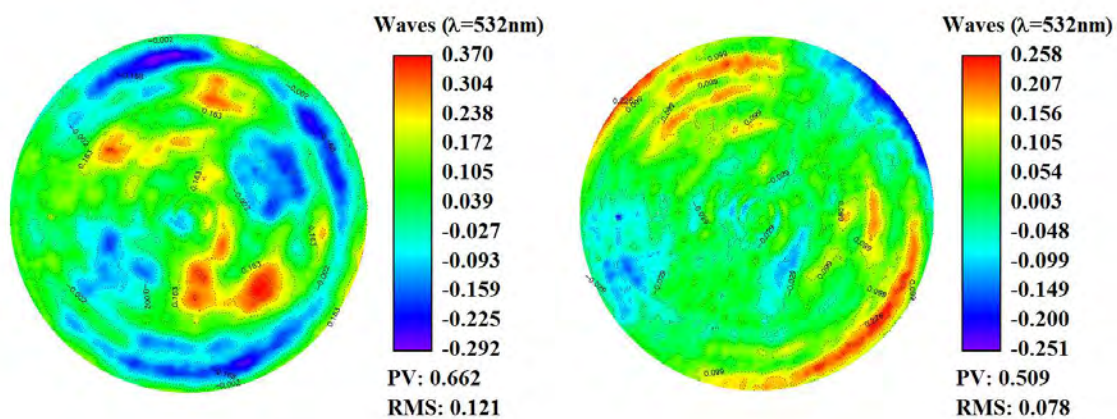


FIGURE 6.45 The OPD introduced by the form error on the test surface calculated using the reverse raytracing procedure (Left). This is twice the surface error. The difference between the first and second measurement (Right).

7 DISCUSSION & FUTURE WORK

This chapter will contain a discussion of the measurements presented in Chapter 6, starting with the cylinder lens measurements. Additionally, a brief review of a few different RO procedures that were investigated will be given. Then the issues that arose when trying to make measurements with the singlet diverger lens will be discussed along with a summary of the system performance made with the doublet diverger lens. Finally some possible improvements that could be made to the system will be discussed.

7.1 Cylinder Lens Testing

In the measurements performed on the cylindrical surfaces, without the diverger lens, the RO procedure was used to recover, not only the form error of the surface, but also the nominal part prescription. For these cylindrical surfaces the prescription is simply the radius of curvature and the diameter of the surface. Without any prior knowledge of the test part prescription or the interferometer, there could be an infinite number of cylinder surfaces with varying radii of curvature and diameters that could produce the recorded OPD on the detector. With prior knowledge of the test part diameter and the imaging lens properties, it would seem that there is enough information to use both the size of the test part and the size of the image of the test part to determine the imaging distances. With this information, the radius of curvature that produces the measured OPD at the detector could be determined. However, determining the imaging distances becomes complicated due to errors induced by the imaging lens, as discussed in Chapter 4.6. In

the presence of pupil aberrations, the size of the image of the test part will depend on the angle at which the ray leaves the edge of the test part. This angle is the field coordinate of the ray as described by the pupil imaging representation. The angle at which the ray reflects off the test part obviously will depend on the radius of curvature of the part. Additionally, Murphy et al (2000a, 2000b) described how the OPD error present in the measured OPD will depend on both the field and pupil coordinate of the test ray. Therefore, because of the interdependencies of the pupil aberrations and the induced OPD errors, separating the location of the test part from its shape becomes increasingly difficult.

Before arriving at the RO process discussed in Chapter 6.1, alternative approaches were attempted. The first method attempted was to rely only on multiple shifted measurements of the part in order to separate the location of the test part from its shape. However, it was discovered that this approach alone was insufficient for solving for the radius of curvature of the lens because the change in the measured OPD at the detector was only on the order of three to six waves out of several hundred waves across the detector. It was observed that only using axial part shifts allowed for multiple solutions to be found by the RO procedure in which the radius of curvature of the surface would vary by over 25mm, corresponding to almost 10λ .

Another method of separating the shape from the position of the part that was explored was to try to tie the measured diameter of the test part to the measured diameter of the

test beam at the detector. If in the model, the rays passing through the edge of the test part, always strike the edge of the test wavefront at the detector, then the magnification of the model should match that of the physical system. However there are several challenges with this approach. First these test parts did not have what could be considered a clean edge or even a constant diameter, which can be seen in the interferograms and unwrapped OPD maps shown in FIGURE 6.13 and FIGURE 6.18. The diameter used to define the edge of the OPD by the SNI software was based either on the user clicking on several points around the edge of the wavefront or the automated masking routine discussed in Chapter 5.2. The absolute accuracy of either of these routines is suspect, as they depend on either the user's ability to select multiple points on the edge of the wavefront or the noise in the image used by the automated edge detection routine. Second, even if these lenses had a constant diameter, a circular beam may not be obtained at the detector due to the difference in the pupil mapping errors of the imaging lens induced by the power difference along the two axes. Third, the normal method Zemax uses to determine semi-diameters of surfaces is inadequate when the surfaces in the system are shifted off axis using coordinate breaks. Discrepancies as large as multiple millimeters have been observed between the Zemax calculated diameter of a surface and the maximum diameter obtained by simply calculating the maximum separation of any two rays out of a large bundle of rays traced to the surface. Multiple attempts were made to develop a custom method of calculating the diameter, such as calculating the displacement of several rays around the edge of the pupil from the chief ray and either using their average distance or maximum distance for the semi-diameter.

Another attempt used the calculated geometric spot size from the Zemax spot diagram. A custom macro was written to calculate the diameter of the aperture by least squares fitting the location of a number of rays traced about the edge of the pupil, similar to the method used by the SNI software. None of these techniques were found to produce reliable results for the surfaces tested.

One compounding factor with using the strategy of binding the diameter of the test part to the diameter of the test wavefront is that the wavefront is only defined up to the edge of the exit pupil at the detector. Attempting to use rays right up to the edge of the pupil will result in rays landing outside the defined measured OPD phase surface at some point during the RO procedure. If a Zernike phase surface is used to encode the measured phase data then the measured diameter of the test wavefront at the detector is used as the normalization radius. Zemax offers the ability for the phase outside the normalization radius to be extrapolated, but this phase will often rapidly depart from the phase data inside the normalization radius. If a grid phase surface is used this problem can become even worse. Since no OPD data is recorded outside the test wavefront diameter these points are often simply set to zero. This can cause a large spike in the phase data, or ringing in the phase data near and often inside the defined diameter. These large changes in the phase near the edge of the pupil can result in problems when trying to use the RO process to null the OPD_z of the RO model.

The method discussed in Chapter 6.1 to measure the cylindrical surfaces used multiple shifted measurements of the same part coupled with measurements of a known test part in order to separate the location of the test part from its shape. The results presented showed that the RO process would find different solutions for the radius of curvature with each measurement. The recovered radius of curvature would vary on the order to a few millimeters, which corresponds to a change in the OPD produced by the surface, of approximately 2λ peak to valley between measurements. This corresponds to a 1λ of change in the surface shape. The RO process was better at finding the OPD error generated by the departure from the cylinder, as this repeated on the order of 0.2λ to 0.5λ for the two surfaces measured. This corresponds to a change in the surface error between measurements of 0.1λ and 0.25λ . This is at least close to the desired accuracy which was originally specified as 0.1λ peak to valley.

7.2 Calibration with a Spherical Standard

The same comparison procedure of using a known part in combination with shifts of the test part was also attempted for the measurements made of the aspheric and toric contact lens tooling inserts. In this case the known part was a grade 3 ball bearing, as specified by the AFBMA ball grade standard (AFBMA Standard). The ball had a radius of curvature of 11.906mm and had surface error of less than 0.01λ as measured with the WYKO 6000 interferometer, FIGURE 7.1.

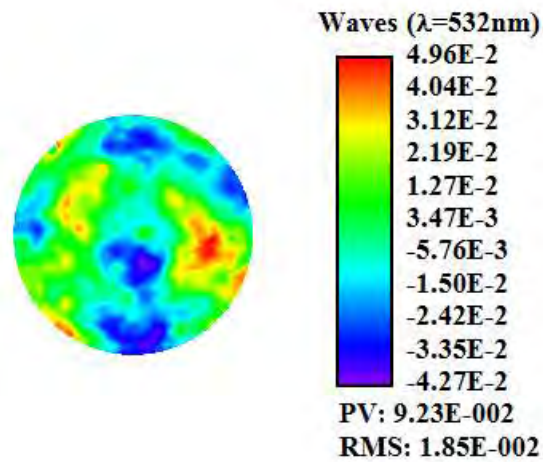


FIGURE 7.1 Surface errors present for a single measurement of the Grade 3 ball bearing as measured with the WYKO 6000 interferometer.

However, this procedure did not work as well as the procedure described in Chapter 6.2, which relied only on shifts of the test part. In some ways, this result is surprising as it seems logical that using a known surface to get the RO model of the system closer to the physical system would be beneficial. However, what was observed is that when the ball bearing measurements and aspheric test part measurements were reverse optimized simultaneously, the RO procedure would allow for more residual wavefront error in the configurations pertaining to the aspheric test part in order to reduce the error from the ball bearing. Using the ball bearing measurements along with the aspheric test part in the model ended up producing about twice the residual wavefront error in the model after the reverse optimization than was observed for the procedure presented in Chapter 6.2. Additionally, the peak to valley OPD error recovered after reverse optimization for the conic aspheric surface with a known defect would vary over a much larger range, from approximately 3λ to 6λ as compared to 4.3λ to 4.7λ for the process previously described. The system performed even worse if the RO process was run in its entirety on the ball

bearing measurements, and the solution found for the position of the interferometer optics was then applied to the model containing the aspheric part.

The exact reason for the degradation in performance is unclear; however there are several likely suspects. First, using the ball bearing measurements in the model requires an additional set of unique variables being introduced into the model that are not required when the RO procedure is performed on the aspheric surface alone. These variables represent the position of the ball bearing relative to the diverger lens. When aligning the test part to the interferometer it is difficult to position the test part such that its vertex is perfectly aligned to the optical axis of the diverger lens. While this can be aided by observation of the interferogram, with several hundred fringes across the detector it can be difficult to find the optimal lateral position and tilt of the test part. Taking measurements and looking at the OPD at the detector can help, but the manual positioners used to adjust the test part location make small adjustments difficult. Additionally, the motion of the stage on which the test part is mounted is not perfectly aligned to the optical axis of the interferometer, which means as the test part is shifted axially there is some small lateral motion of the test part. The reason all these details are important is that when the ball bearing and test part are RO together it is likely that their vertices are not aligned to each other or the interferometer. This is further complicated by the fact that the test part is the aperture stop of the system and for an unknown reason the Zemax optimization procedure tends to favor not laterally shifting the aperture stop. Rather, the RO procedure tends to shift the diverger to match the test part location rather

than vice versa. This ultimately means that the solution found using the RO model for the ball bearing may not match the ideal solution for the test part, leading to the increase in the residual error. A method or stages which allowed for the two surfaces to be better aligned to the interferometer and to each other may help minimize this effect.

A ball bearing may not be a very good part to use as a calibration optic for the non-null interferometer. One problem with the ball bearing used is that it was significantly larger in diameter than the test beam. As the ball bearing is shifted axially more or less of the surface is used and therefore it is not the true aperture stop of the system. However in the model, the test surface is set to be the aperture stop of the system. While the RO process described in Chapter 6.2 does not try to relate the size of the test part to the measured diameter of the test wavefront at the detector for all configurations, it does try to keep the measured OPD at the detector filled for one configuration. However, without a defined aperture on the part it is difficult to determine this relationship. Therefore, the RO process needs to stop down the aperture such that all measured OPD at the detector is under filled for all configurations. This may give the RO model too much freedom to adjust the imaging distance in order to help match the measured OPD without the normal constraint of keeping both the test part and measured test wavefront diameter filled. A test part with a defined aperture, which will act as the aperture stop of the interferometer, would help address this issue.

Another possible reason for the increase in the residual error and the change in measured peak to valley OPD error of the conic aspheric surface is that the solution found by the RO procedure, is that the ball bearing deviates too much from the aspheric part under test. Because of the difference in their surface shapes, rays from each surface will encounter different OPD errors and pupil aberrations on their respective paths through the interferometer. The RO process targets the modeled OPD to match the measured OPD for each configuration. It is likely that the final model produced by the RO process does not match the physical interferometer but rather is simply a solution which minimized the OPD difference. As such, the RO process may use some properties of the model interchangeably, such as the power of the diverger lens and the distance between the diverger lens and the test part. The RO process may make tradeoffs that have a small impact on the measurement of the ball bearing or that have larger impact on the aspheric surface measurement or vice versa.

Using a surface as a calibration test surface that produces a test wavefront more similar to the test part to be tested may produce better results. In Chapter 6.2 and 6.3 measurements are shown for test parts with designed in surface defects. The original plan of these measurements was to also test parts with the same prescription which didn't have the surface defects. The measurements of the test parts without the defects would then have been used to calibrate the measurements of the parts with the defects. Unfortunately, a mix up in the manufacturing of the parts resulted in the test parts with and without the errors being generated with the opposite sag profiles. This means the parts with the

defect were generated as convex surfaces while the parts without the error were generated as concave parts, making the direct comparison impossible. This could still be an interesting measurement and comparison to make by a future system.

7.3 Singlet Aspheric Diverger Lens

In Chapter 5.4.4, the defect present in the aspheric surface of the singlet diverger lens was presented. This defect took the shape of a $1\mu\text{m}$ divot in the central 10mm of the surface. In order to use this lens in the non-null interferometer, this defect would have to be accounted for in the RO model. However, as shown in FIGURE 5.66 the Zernike representations of this surface do not account for over 0.7λ of the peak to valley surface error. Therefore, a grid sag surface was required in order to reproduce the surface defect in the model. A similar problem is encountered when trying to model the measured OPD at the detector for measurements made with the singlet diverger. Several waves of OPD error are introduced into the measured OPD by the defect in the diverger lens. Due to the shape and magnitude of this defect, a Zernike phase surface will not contain the high frequency content necessary to create an accurate representation of the measured OPD. This leads to the OPD error resulting from the diverger defect not being included in the measured OPD of the model. When the reverse optimization procedure is run the RO model will not have a way to cancel the error introduced by the surface defect. For an example of this, a measurement was performed using the grade 3 ball bearing previously discussed. When the ball bearing is positioned so that its center of curvature is located at the focus of the diverger lens a null test should be produced. The OPD recorded at this

position in shown in FIGURE 7.2 (Left), in which the defect on the surface of the aspheric surface of the diverger lens is clearly visible. Furthermore, the inability of this measured OPD to be represented by 37 Fringe Zernike terms is shown in the FIGURE 7.2 (Right) which represents the difference between the measured OPD and the Zernike fit. Multiple measurements of the ball bearing were then made as it was shifted over a range of 0.4mm in 0.1mm steps.

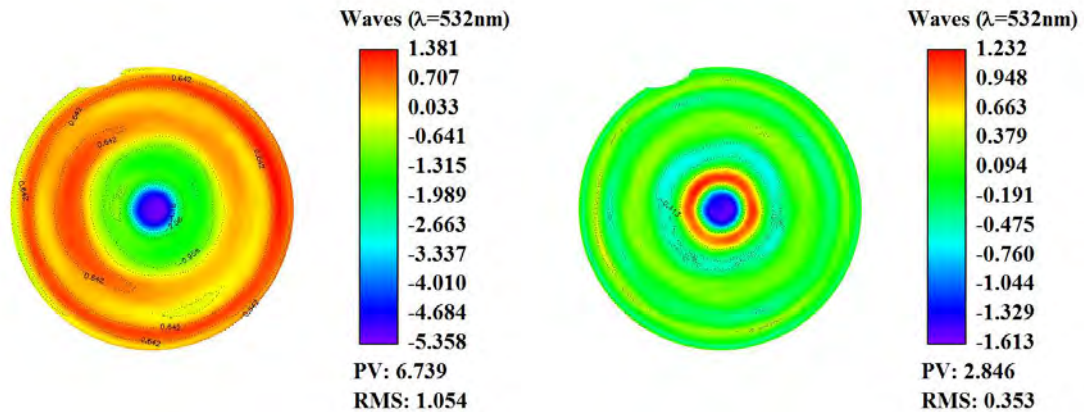


FIGURE 7.2 The measured OPD with the ball bearing located at the “null” position made using the singlet diverger lens (Left) and the difference between the measured OPD and the Zernike Fit of the OPD (Right)

In this measurement it was assumed that the ball bearing was perfect, so that any residual error left over after the RO procedure is a failure of the RO process to produce a model of the interferometer which matches the measured OPD at the detector. While the ball bearing is not actually a perfect surface, this assumption is valid because the residual errors present in the system are much greater than the surface errors of the ball. The result of this measurement is shown in FIGURE 7.3 (Left), in which the measured OPD was introduced into the model as a Zernike fringe phase surface. In this case, the RO process was unable to find a solution that nulled the residual wavefront to better than 1.8λ of the

peak to valley. However, even after the representation of the measured OPD was switched to a grid phase surface the residual error after the RO process was still larger than 1.5λ of the peak to valley, FIGURE 7.3 (Right).

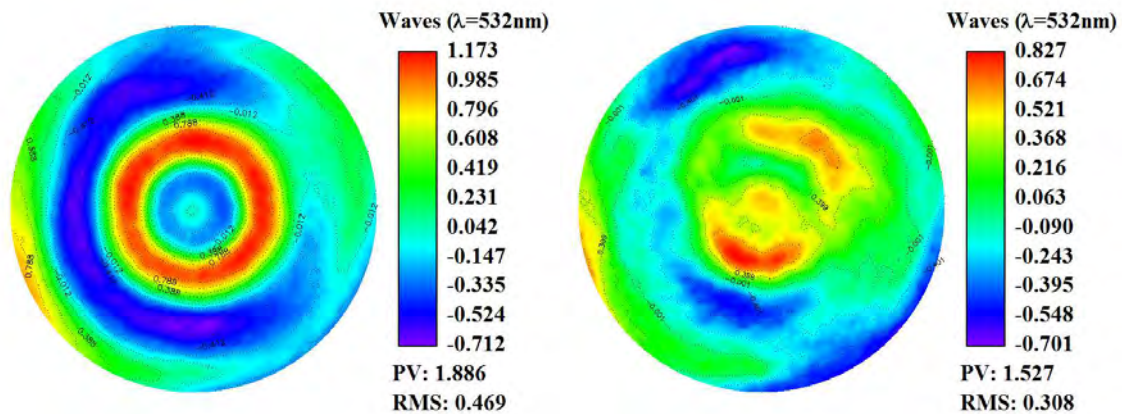


FIGURE 7.3 The residual OPD error left in the model after the RO procedure for the grade 3 ball bearing measured with the singlet diverger in which the measured OPD at the detector is modeled as a Zernike phase surface (Left) and as a grid phase surface (Right).

If the singlet diverger was used to measure an aspheric surface the results were even worse. When an attempt was made to measure the aspheric conic insert tested in the second half of Chapter 6.4, with the singlet diverger, the surface defect on the diverger lens would print through onto the test part if the measured OPD was represented in the model as a Zernike surface. This can be seen in FIGURE 7.4, here the calculated OPD introduced by the error in the test surface is plotted (Left) along with the same error minus the power (Right). These plots should represent twice the surface error of the test part, however it is clear from these images that the error in the aspheric diverger lens can clearly be seen in the measurement of the test part, by comparing FIGURE 7.4 to FIGURE 5.64 or FIGURE 7.2. This surface error in the diverger can be suppressed by replacing the Zernike representation of the measured OPD with a grid phase

representation. However, unlike the measurements discussed in Chapter 6, the RO procedure must be run with the grid phase surface in place. The result of this is shown in, FIGURE 7.5, here the peak to valley OPD error introduced by the form error in the test part is still over twice what it was when measured with the doublet diverger lens, however the central artifact have been removed. Additionally, in performing the reverse optimization and the reverse raytracing required to generate FIGURE 7.5, not only did the raytracing run incredibly slow, but the Zemax raytracing program crashed no fewer than four times. The exact reason for the crashes was never pin-pointed but it probably had to do with use of the combination of the grid sag surface on the diverger lens, and the multiple grid phase surfaces used to represent the measured OPD at the detector. The abrupt change in the aspheric surface at the edge of the defect also probably hindered the ability of the program to aim rays onto the test part that passed close to the edge of the defect, which likely contributed to the slow ray tracing and instability of the program. The bottom line is that the use of this singlet diverger lens with the RO process as described was both inaccurate and unstable.

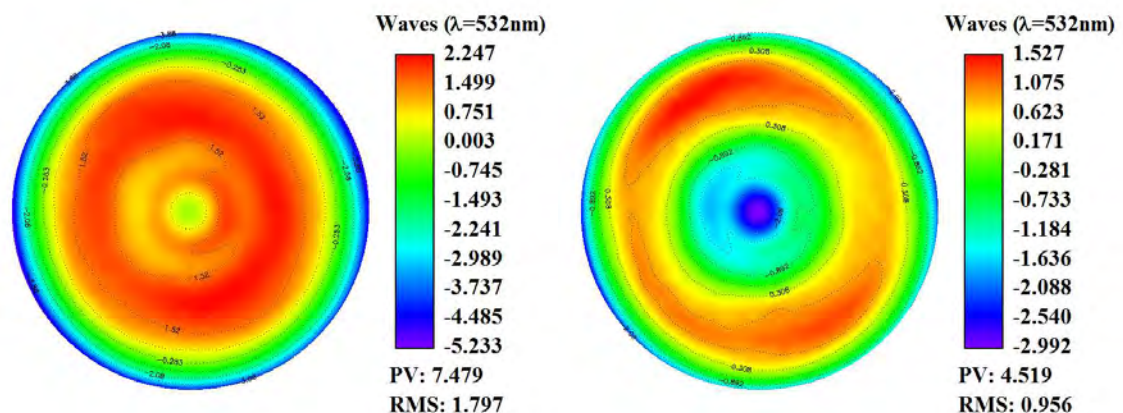


FIGURE 7.4 The OPD error incorrectly attributed to surface errors on the conic aspheric test part by the RO procedure (Left) and the same error minus the Zernike power term

(Right) These errors are actually generated by the surface of the diverger lens and the method used to model the measured OPD at the detector.

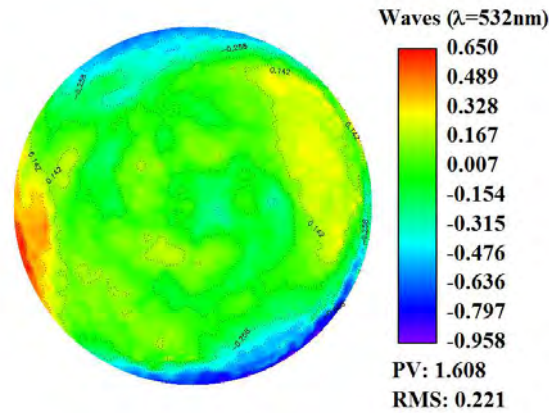


FIGURE 7.5 The OPD error attributed to surface errors on the conic aspheric test part tested using the singlet diverger lens and a grid phase representation of the measured OPD.

7.4 Doublet Diverger Lens

As was the case for the singlet diverger, the doublet diverger lens surfaces, especially the aspheric surface, also contain high frequency error. Although in the doublet, the magnitude of the error is smaller and it is not concentrated in the center of the aspheric surface. In the results presented in Chapter 6.4 for the aspheric test parts without the introduced defect, the OPD error that was calculated by the reverse raytracing procedure as being generated by errors on the test surface, also show high frequency errors on the order of 0.6λ to 1.2λ peak to valley. These results are shown in FIGURE 6.38 and FIGURE 6.44. These errors would correspond to surface errors on the order of 0.3λ to 0.6λ peak to valley. While the exact method used to generate the test parts was unknown, it is assumed that they were diamond turned. However, the magnitude and shape of these surface errors appear to be out of step with what would be expected from single point

diamond turned parts. If the part was turned on a lathe, the test part would be spun radially at some high angular velocity. The diamond cutting tool would then be simultaneously translated across the semi-diameter of the test part while plunging into the surface in order to trace out the profile of the part. In this cutting scheme it would be expected that the surface errors would be radially symmetric errors due the diamond not tracing out the correct part shaper. Non-rotationally symmetric errors could be caused by things like chatter of the diamond against the work piece, but these would be expected to be even higher frequency than what was observed. This leads to the conclusion that either another process was used to make these parts, or the errors attributed to the surface of the test part are actually generated by another source in the interferometer. In order to investigate this issue, the ball bearing that was previously discussed in this chapter was measured using the doublet diverger lens. The results of a measurement made of the grade 3 ball bearing are shown in FIGURE 7.6 (Left). For comparison the measurement results for the conic aspheric test part, shown in FIGURE 6.44, are plotted again in FIGURE 7.6 (Right). While not identical, from visual inspection it is clear that there striking similarities between these two measurements, both in the magnitude and the shape of some of the defects. Since these measurements are from two different surfaces it is likely that the error that was measured as coming from the surface of the test part is actually being introduced from another component, or a combination of components, in the interferometer that is not included in the RO model. The most likely culprit is the high frequency errors on the aspheric surface of the doublet, which can be seen in FIGURE 5.67 (Right). The surface errors on the aspheric surface of the diverger, shown

FIGURE 5.67, seem to be consistent with errors that would be generated from a sub aperture polishing technique.

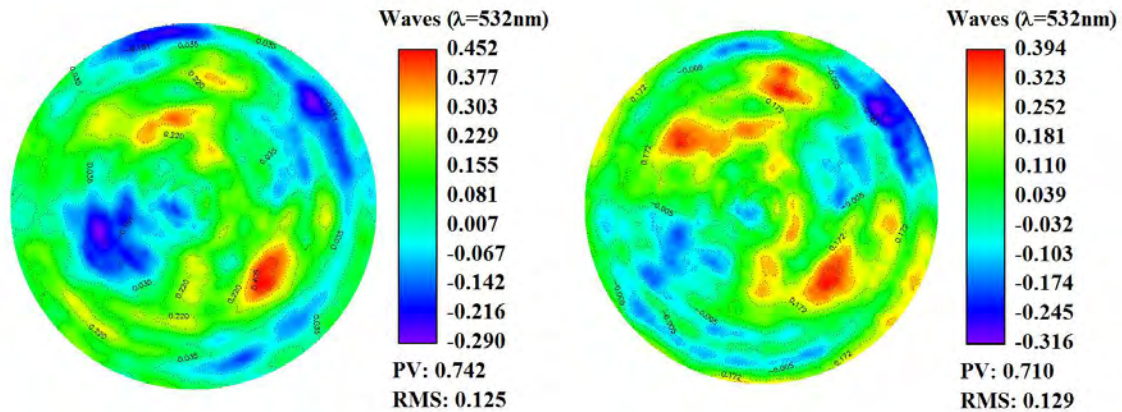


FIGURE 7.6 The OPD introduced by the form error on the ball bearing surface calculated using the reverse raytracing procedure (Left) and the OPD introduced by the form error on the conic aspheric test part calculated using the reverse raytracing procedure (Right).

Unsuccessful attempts were made to account for this error in the RO model. First, the measurement data from the Zygo Verifire Asphere of the aspheric surface of the diverger was incorporated into the RO model as a grid sag surface. However, there were problems with this approach. First, while the orientation of the diverger lens surface during the Zygo Verifire Asphere measurement was marked and attempts were made to preserve the orientation in the non-null interferometer, the exact rotation angle of the data relative to the rotation angle of the mounted diverger was unknown. Attempts were made to allow the RO process to rotate and shift this surface in the model in order to minimize the residual wavefront error and properly align the data to the model. However, in order to accurately duplicate the error using a grid phase surface a grid of several hundred points across each dimension was required. This slowed the raytracing down to an unacceptable level, similar to what was witnessed with the singlet diverger lens. Either because of the

slowdown, or because the merit function would get caught in local minima, the high frequency content of the aspheric surface could never be registered to the data such that the residual wavefront error was significantly decreased. Attempts were also made to measure the high frequency error in place, by making null measurements of the ball bearing.

A similar process for calibrating a transmission sphere by ball averaging, as described by (Parks et al, 1998) and (Griesmann et al, 2005) was attempted in which it was assumed that all the error was the result of surface errors on the aspheric diverger lens surface. While this approach would suppress some of the error visible in FIGURE 7.6, it would often introduce other residual wavefront errors that were either on the same order of magnitude, or only slightly smaller. This was probably due to the fact that although the aspheric surface was the primary contributor of these high frequency errors, it was not the only contributor. Therefore, combining all the residual errors onto this single surface did not bring the RO model any closer to matching the physical system.

This means that for the measurements presented in Chapter 6.4, the errors that the RO and reverse raytracing procedure attributed to the surface of the test part are likely a combination of errors on the test part surface, and errors generated by surface errors of the interferometer optics and other discrepancies between the model and physical system. While the exact split is not known it appears that at least for the conic aspheric surface the errors shown are primarily the result of the interferometer and not the test surface.

The other measurement presented in Chapter 6.4 is for a 4th order aspheric surface test part, shown in FIGURE 6.38 (Right). Here the OPD errors attributed to the surface are about 1.7 times larger peak to valley than those shown for the conic surface or the ball bearing. Additionally, the residual wavefront error at the end of the RO process appears to be made up of a greater component of circularly symmetric errors, as one would expect to see in a part turned on a lathe, FIGURE 6.38 (Left). Therefore for this part, while the recovered OPD errors attributed to the test surface is still a combination of test part errors and errors introduced by the interferometer, there is likely a higher percentage of surface errors present in this measurement than were present in the measurement of the conic aspheric test surface. Finally, for the measurements presented in Chapters 6.2 and 6.3, where the test parts were designed to have surface defects on the order of 2.5λ , the errors in the test part surface appear to dominate over the errors introduced by the interferometer. The results in Chapter 6.2, for the conic aspheric test part with a known defect, are replotted below in FIGURE 7.7. While the defect is clearly dominating the measurement FIGURE 7.7 (Right), the residual error at the end of the RO process shows similar high frequency error in both shape and magnitude as was shown in FIGURE 7.6. However looking at the lower left quadrant of FIGURE 7.7 (Left), it also appears that the defect in the test surface has introduced its own contribution onto the residual error. This demonstrates that the residual error plots are a combination of contributions from the unaccounted for errors in the interferometer, such as the high frequency error on the aspheric surface of the diverger, and surface errors on the test part that were not fit by the

Zernike polynomials used to alter the test surface during the RO process.

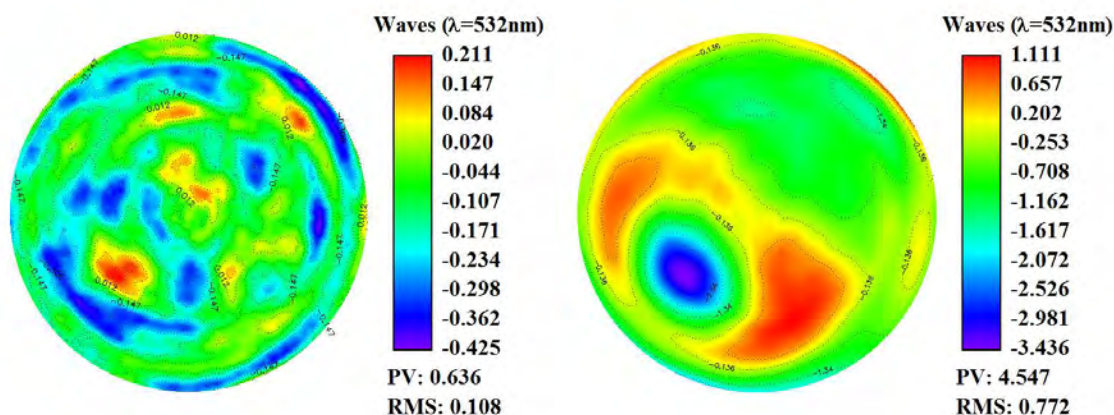


FIGURE 7.7 The final residual wavefront error in the model at the end of the reverse optimization procedure using grid phase surface representation of measured OPD (Left) and the OPD introduced by the form error on the conic asphere test surface calculated using the reverse raytracing procedure (Right)

If a series of test parts were available with different magnitudes of surface errors, it would be interesting to see where the transition occurs from measurements that are dominated by errors in the interferometer to measurements that are dominated by errors in the test part. From the measurements presented here, the good estimate of when this transition would occur is when the surface errors are on the order of 0.75λ to 1λ peak to valley.

7.5 Performance Summary

For the cylinder surfaces tested, without the use of the diverger lens, the RO procedure used would recovered a consistent radius of curvature to within 5mm of the average measurement. The average radius of curvature measured was also within 10mm of the radius of curvature measured using a profiler. These ranges correspond to a change in the

surface shape from the average prescription of 1λ to 2λ peak to valley. However the OPD error generated from the departure of the surface from a crossed cylinder shape, repeated on the order of 0.2λ to 0.5λ for the negative and positive respectively. This corresponds to a change in the surface error of 0.1λ and 0.25λ .

For the test parts with the manufactured error, the RO process was clearly able to identify the defect on the surface of the part. For the conic aspheric test surface, the magnitude of the OPD introduced by the defect was within 0.25λ to 0.5λ of what was predicted by the design of the defect. While for the measures made of the toric surface the magnitude of the OPD introduced by the defect was within 0.1λ of the predicted value. However, comparisons of the shape of the OPD error either between measurements or to the nominal design proved difficult. The difference plots showed peak to valley values ranging from 1.7λ to 2.5λ , however these high values appeared to be dominated by the fact that the data sets were not properly registered to each other before difference was taken.

Finally, for the two test parts presented without the introduced surface defect, the results were inconclusive. It appears that the surface errors were below the resolution of the interferometer and RO process. While, the RO process would produce constant results between measurements, the results appeared to be dominated by errors introduced by the interferometers which were unaccounted for in the model.

7.6 Improvements

The goal of this system was to test relatively fast surfaces. As a result, the design of the diverger lens became complicated and, in order to minimize the number of elements, required the use of an aspheric surface. It is believed that much of the residual error found in the measurements is the result of surface errors in the diverger lens. Even when characterized, these errors are difficult to include in the interferometer model and degrade the RO performance. The goal of testing fast surfaces was perhaps too aggressive for this attempt to test aspheric surfaces in reflection in a non-null configuration. A slower test requirement would have potentially reduced some of the requirements on the diverger design and allowed for an all spherical, lower tolerance diverger.

There are many steps that could be taken in order to improve the performance of a next generation non-null interferometer. One area is clearly reducing the surface errors on the interferometer optics that cannot easily be incorporated into the model. While low frequency errors can be represented with Zernike surfaces, high frequency errors are difficult to model. The best practice may be to simply ensure that all surfaces are manufactured to a high enough level of quality that any remaining surface errors are insignificant for the reverse optimization and reverse raytracing processes. This leads into the issue of the need for a deeper understanding of the tolerances of the system so that errors in manufacturing of the individual optics, or the alignment of the multi-element optics do not contribute significant error into the system. This could reduce the number of properties that have to be included as variables in the RO process. In Chapter

5, some rudimentary tolerance analysis was performed on the individual interferometer components in order to determine which properties can be ignored during RO process and which would have to be included as variables. However, what should really be done is a total system analysis in which the interdependencies between the all the optical elements and their misalignments and manufacturing errors are investigated. Using a process like principal component analysis (Jain 2015) may be useful in order help understand the correlations between all the possible variables in the RO model. This may also offer insight into which properties of the model can be recovered using reverse optimization and which cannot, or which properties of the system will need to be well known in order for the RO procedure to be able to accurately recover the properties that are unknown.

Another change which may, or may not, help improve the performance of the system but would definitely help in the understanding the ray tracing that is performed using the RO model would be to transition away from Zemax, or any commercial ray tracing program, and into a custom ray tracing program. Ideally the program would be written specifically for the reverse optimization and reverse raytracing of a non-null interferometer. Too many times over the course of this research, the Zemax ray tracing engine acted as a black box in which the processes it was doing behind the scenes were unknown and out of the control of the user. These issues include: the program would stall or run slowly when high density grid sag and grid phase surfaces are utilized, FIGURE 7.8, and the OPD_z calculation would sometimes have a step change in it FIGURE 7.9 (Left), which

may or may not disappear on subsequent ray traces. These raytrace issues also included completely unexplainable results like the result seen in FIGURE 7.9 (Right)

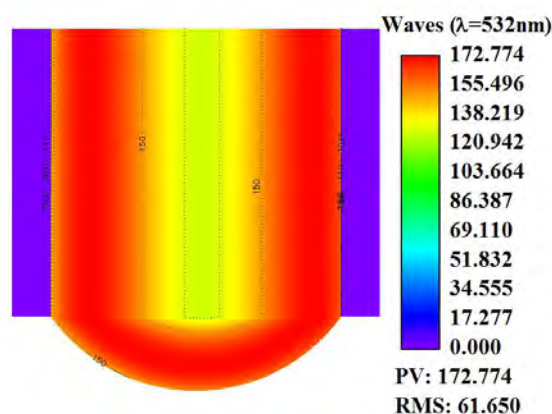


FIGURE 7.8 Example of a Zemax ray trace that stalled out for no apparent reason.

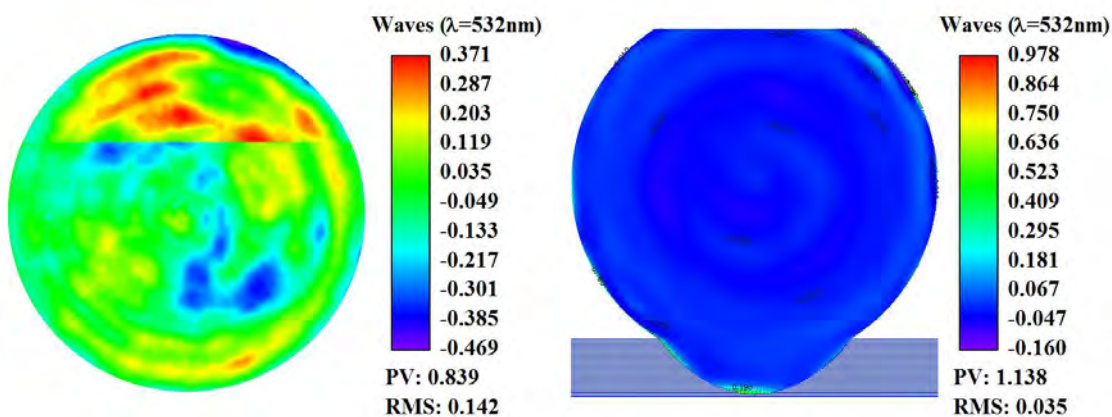


FIGURE 7.9 A random step change in the OPD calculated by Zemax (Left), a ray trace in which the results are unexplainable (Right).

Additionally, too many tricks or work arounds had to be developed in order to get Zemax to perform the required calculations or ray traces. Issues like the fact that Zemax only keeps track of the OPL of rays that are traced by pupil coordinates, and then only the OPL relative to the chief ray are recorded. Or the fact that in order to trace rays both forward and backwards through the system each optical component had to be inserted

into the model at least twice. While it would be a considerable amount of effort to build a raytracing program from scratch that can perform all the necessary tasks needed for reverse optimization and reverse raytracing a non-null model, it would undoubtedly offer a greater degree of flexibility.

A better method of aligning the part to the desired initial testing position may be beneficial. In the system as built, the distance from the cat's eye position was used to set the test part at the initial testing position. Then the interferogram generated at this position was visually compared the interferogram predicted by the model of the interferometer. Next a phase shifted measurement can be captured and the magnitude and shape of the unwrapped OPD can be compared to the values predicted by the model. However, this process relies on the vertex of the test part being aligned to the focus of the diverger lens. It is also slow because phase shifted measurements would have to be recorded and unwrapped. A more direct method, which provides instantaneous feedback, for determining when the test part is in the proper location would be beneficial. One method would be to compare the predicted interferogram, such as the one shown in FIGURE 5.24 (Left) directly to the live interferogram being recorded by the sensor, FIGURE 5.24(Right) and to observe the moiré beat pattern that is generated. This is a technique that has been developed by Mahr-ESDI (Tucson, AZ). They have produced a commercially available sub-Nyquist interferometer which uses the predicted interferogram generated by raytracing as a computer generated hologram. (Engineering Synthesis Design, Inc. 2014)

Another area of improvement would be to improve the positioning of the test part and the part shifts used to generate multiple measurements for the RO process. In the system as constructed there is an uncertainty, on the order of a few microns, for the length of the introduced axial part shifts. In the beginning of the RO procedure it is assumed that the lengths of these shifts are known exactly. Therefore the size of the part shifts used was large so that the uncertainty in the motion as a percentage of the shift length was small. In order to account for the uncertainty in the part motion, at the end of the RO procedure the length of the each shift was allowed to vary by a few microns. The use of long axial shifts also means that the change in the OPD at the detector was large, generally on the order of hundreds of waves. In some regards, having drastically different measured OPDs at the detector is a benefit to the RO process, since the RO process needs to find the single surface, and its starting position, that will produce the measured OPD at the detector for each part shift.

However there are also disadvantages to these large changes in the OPD. One is that it will limit the range of testable parts. This is because not only does a single location for the test part need to be found that produces an interferogram which has fringe frequencies resolvable by the sparse array sensor, but multiple part locations must be found that meet this criteria. For an aspheric surface that is just on the edge of being testable, multiple test part locations may not exist. Another problem with introducing large changes to the OPD is that this will induced larger changes to the induced OPD error between test part locations. Additionally, axial shifts will move the test part away from the plane

conjugate to the detector. The test part will move out of focus as the length of the test part shifts grow which make determining the edge of the test wavefront at the detector more challenging, and can introduce diffraction effects into the edge of the measured OPD.

A better solution may be to use smaller part shifts, with a more accurate measurement of the exact length of the shift. Using motorized stages equipped with high resolution linear encoders could allow not only for more accurate knowledge of the part shifts, but for the test part to be aligned to the interferometer more accurately. With the manual stages it is difficult not to overshoot or undershoot the desired position of the part and it is almost impossible to return the part to a given position after it has been moved. These problems could be overcome using computer controlled stages.

It may also be beneficial to introduce other perturbations than simply axial part shifts. Rotating the part about the Z-axis could be a good perturbation to add to the system, as suggested by (Lowman 1995). For rotationally symmetric test parts the surface errors on the part would rotate as the test part is rotated, while the interferometer errors would remain fixed. For non-rotationally symmetric parts the induced aberrations of the interferometer may in fact rotate with the test part. Rotational perturbations would not cause the test part to move out of focus, because the imaging distance would be kept constant. However they would likely be insensitive to rotationally symmetric surface errors on the test part. Ideally the part would be rotated over large angles to maximize

the change to the system, but also the angle at which the part is rotated should be well known, so that it doesn't have to be included as a variable in the RO process. Using an air bearing spindle with a high resolution encoder could solve this problem.

7.7 Conclusion

This dissertation has described the design and implementation of a non-null Sub-Nyquist interferometer built to test aspheric contact lens tooling inserts. The system has been found to be capable of measuring parts with aspheric departure of over 60λ from the best fit sphere, which with introduced part shifts, generated over 300λ of OPD at the detector. The OPD introduced by the parts was measured to an accuracy of at least 0.76λ peak to valley and 0.12λ rms. A reverse optimization procedure was developed and used to characterize both the interferometer as well as the part location. Multiple axial shifted part locations were used to provide sufficient information to solve the multi-variable problem. Reverse ray tracing of the reverse optimization model was utilized to calculate the surface errors of the test part from the measured OPD at the detector. The system performance appears to have been degraded by surface errors in the diverger lens which could not fully be incorporated into the reverse optimization procedure. Limitations in the use of commercial ray trace software for this process were also found. In future work, it is recommended that custom software be developed. While there is a tradeoff in the modeling complexity seen when using a larger number of optical elements in the interferometer, it is imperative that only high-quality, well-characterized and simply-modeled optical elements be used in the system. Additionally, since the reverse

optimization process uses multiple measurements of the test part at shifted locations in order to separate the position of the test part from its surface errors, higher precision and motion controlled mechanics would be of value.

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