

STUDY OF FISHEYE LENS ATTACHMENTS FOR MOBILE PHONE LENSES

by

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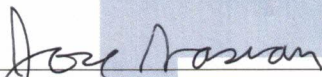
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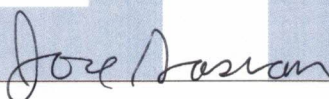


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Final approval and acceptance of this thesis is contingent upon the candidate's submission of the final copies of the thesis to the Graduate College.

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Dedication

*To my Dad and Mom
you are my spiritual support*

Table of Contents

List of Figures	7
Abstract	8
1 Introduction.....	9
1.1 History of Fisheye Lens	9
1.2 Symmetrical Wide-Angle Lens to Fisheye Lens.....	13
1.3 Fisheye Lenses for Smartphones	15
1.3.1 Convertible lens	15
1.3.2 Afocal System.....	16
1.4 Fisheye Lens Adapter	16
1.5 Diagonal Fisheye Adapter	17
1.5.1 Circular Fisheye Adapter	18
1.6 Methods to Test A Camera’s Performance.....	19
2 Structure and Properties of Fisheye Lens.....	20
2.1 Structure of Extra Wide-Angle Lens.....	20
2.1.1 Relationship Between Field of View and Focal Length	20
2.1.2 Short focus wide-angle lenses	22
2.1.3 Reversed Telephoto Structure	23
2.2 Properties of Reversed Telephoto Structure	26
2.2.1 Main Monochromatic Aberration.....	26
2.2.2 Main Chromatic Aberration	27
2.2.3 Distortion, Projection and Mapping Function	28
2.2.4 Relative Illumination in Fisheye lens.....	30
3 Analysis of fisheye lens designs	33
3.1 First order concept.....	33
3.2 Design requirement and specification	34

3.3	Design process	35
3.4	Aberration distribution	40
3.5	Third order aberration contribution for each surface and higher order aberration analysis.....	41
3.6	Relative illumination analysis.....	51
4	Performance test of Fisheye lens adapter in market.....	52
4.1	Slanted edge test	52
4.2	System Setup.....	53
4.3	Solution	53
5	Conclusion	57
5.1	Summary of work.....	57
5.2	Future Direction	58
6	APPENDIX: System Data and Specification	59
6.1	1 st design	59
6.2	2 nd design	59
6.3	3 rd design.....	59
6.4	4 th design.....	60
6.5	5 th design.....	60
	References	61

List of Figures

Figure 1-1 pinhole camera[20].....	9
Figure 1-2 linear projection	10
Figure 1-3 Snell's Window in real world	11
Figure 1-4 Snell's Window	12
Figure 1-5 Hill Sky Lens.....	13
Figure 1-6 J-SH04.....	15
Figure 1-7 diagonal fisheye adapter.....	18
Figure 1-8 circular fisheye adapter	19
Figure 2-1 relationship between focal length and field of view	21
Figure 2-2. First order setting for black box system.....	21
Figure 2-3. Patent 2,031,792.....	22
Figure 2-4. Modern DSLR camera	23
Figure 2-5 (a) telephoto system (b) reversed telephoto system	24
Figure 2-6 The Nikkor 6mm F/5.6 fisheye lens[8]	26
Figure 2-7 Lateral chromatic aberration	28
Figure 2-8 HFOV for different projection	29
Figure 2-9. Relative illumination between fisheye lens and ordinate lens	31
Figure 2-10 spot diagram for incoming beam in (a) ordinate lens and.....	32
Figure 3-1 1 st Design.....	36
Figure 3-2 Footprints from object surface to stop	37
Figure 3-3 Design with only two element.....	39
Figure 3-4 Performance of 1 st design.....	40
Figure 3-5 relative 3 rd order aberration distribution for each surface.....	42
Figure 3-6 OPD after setting conic number.....	43
Figure 3-7 OPD after removing conic number	44
Figure 3-8 2 nd design.....	45
Figure 3-9 RMS of 2 nd design.....	45
Figure 3-10 lateral color and MTF of 2 nd design	46
Figure 3-11 3 rd design	46
Figure 3-12 performance of 3 rd design.....	47
Figure 3-13 4 th design	48
Figure 3-14 performance of 4 th design.....	49
Figure 3-15 5 th design	49
Figure 3-16 performance of 5 th design.....	50
Figure 3-17 relative illumination of previous designs	51
Figure 4-1 setup of slanted edge test.....	53
Figure 4-2. SFR for (a) camera of Iphone 6s and (b)-(f) center field to edge field with fisheye attachment	56

Abstract

Fisheye lenses have a very special lens structure with an extremely wide field of view of about 170 degrees or more. They have some unique specifications, such as 100 percent barrel distortion, one of four different mapping functions, and relative illumination that does not follow the $\cos^4(\theta)$ law. Based on these properties, fisheye lenses are popular in some scientific applications and panoramic photography.

With the development of high-quality smartphone cameras, more and more people use their smartphone instead of dedicated cameras in their daily lives. Convertible smartphone camera kits can expand the photographic capabilities of smartphone cameras, and fisheye lens adapters are one type of kit.

In this thesis, five systems, three having all spherical surfaces, are presented. They all have over 170 degrees field of view with a magnification greater than 0.5 and achieve an RMS wavefront error less than 0.5 waves. An analysis for aberration correction as well as relative illumination is discussed in detail.

In the last chapter, a slanted edge test is utilized to measure and analyze a commercial fisheye adapter's MTF performance.

1 Introduction

1.1 History of Fisheye Lens

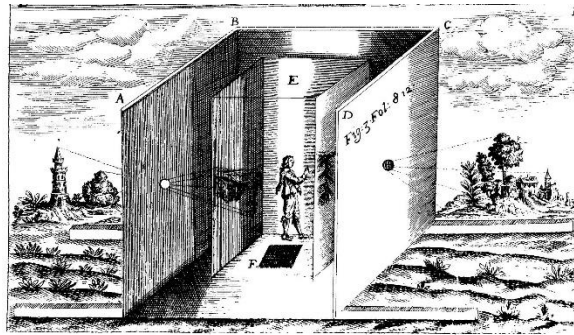


Figure 1-1 pinhole camera[20]

Almost from the beginning of human history, people were trying to record what they saw. With the gradual accumulation of optical knowledge, an instrument was invented—the camera obscura, or pinhole camera. However, the pinhole of this camera must be extremely small if a high resolution is desired, which results in less illumination. To remedy this situation, lenses were introduced to substitute for the pinhole. These camera systems with a lens, it should be noted, did not change the projection relationship between the object and the image.

It is necessary to keep the linear relationship between object and image in most image systems. However, in some special instances, other projections are required. For instance, in meteorology, an extremely large field of view is required to record the whole sky from horizon to horizon. From simple geometric analysis, it is obvious that a 180-degree field of view cannot be linearly projected onto a finite image plane.

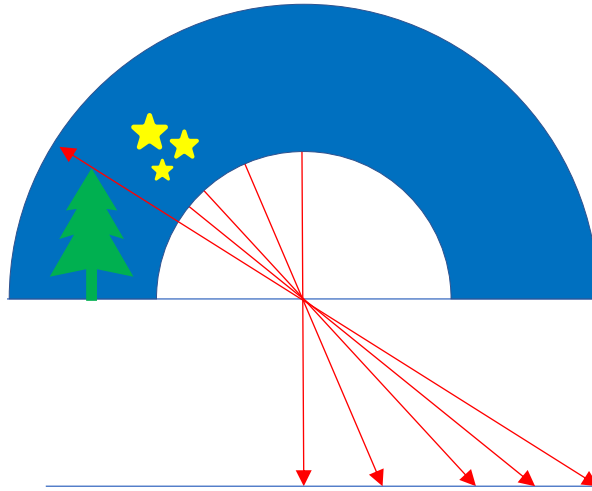


Figure 1-2 linear projection

In figure 1-2, the relationship between object and image in lens system with linear projection is shown. With a finite film, the stars in the sky can be record, but the tree that is located near the horizon cannot be captured. As a result, fisheye lens designers introduced other different projection functions—orthographic projection[Eq. (1.1)], stereographic projection[Eq. (1.2)], equisolid angle projection[Eq. (1.3)] and equidistant projection[Eq. (1.4)]—to map the object reasonably onto the image plane.

$$y = f * \sin(\theta) \quad (1.1)$$

$$y = 2f * \tan\left(\frac{\theta}{2}\right) \quad (1.2)$$

$$y = 2f * \sin\left(\frac{\theta}{2}\right) \quad (1.3)$$

$$y = f * \theta , \quad (1.4)$$

where y is the image height, f is the effective focal length of the system, and θ is the field angle parameter in object space. These projections will be discussed in a later chapter.

Throughout the projection, how to direct a ray from the very edge of field of view onto the film without distortion became an unavoidable issue. The first and the most obvious idea presented was Snell's window (Figure 1-3).



Figure 1-3 Snell's Window in real world

The index of refraction of water is higher than air, so based on Snell's law $n\sin(\theta) = n'\sin(\theta')$, where n and n' are the index of refraction of the two media, and θ and θ' are the incidence and emergence angle, when the light refracts from air to water, its angle in water must be smaller than:

$$\theta' = \arcsin\left(\frac{n}{n'}\right)$$

Thus, when an observer detects a light source that is underneath water, only the light whose incidence angle is smaller than that critical angle is detected. The same phenomenon occurs when the observer is underneath water, e.g. a fish or a diver. When light from the very edge of the field of view just above the air/water interface enters the water, the fish can only observe across the field of view less than the critical angle; however, this encompasses the entire

180-degree field of view above water's surface. The circular field of view that includes everything above water's surfaces is referred to as Snell's window.

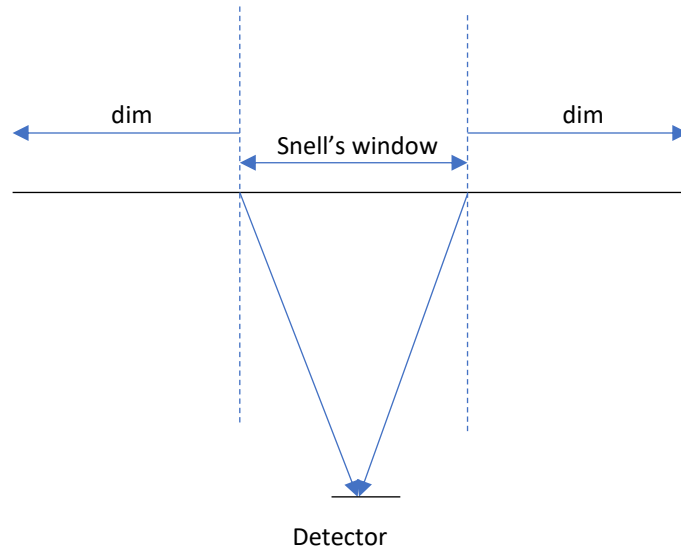


Figure 1-4 Snell's Window

Figure 1-4 shows why Snell's window exists. The blue arrow indicates the ray at the critical angle.

Snell's law inspired people to invent a special camera with a fish eye's view of the world. Originally the term "fisheye" was coined in 1906 by Robert W. Wood in his book[2] that describes a water-filled pinhole camera that is capable of simulating an extreme wide-angle view from beneath water.

In “A LENS FOR WHOLE SKY PHOTOGRAPHS,” published in 1924, the first fisheye lens, the “Hill Sky Lens” (figure 1-5)[1], was introduced. Three different projection methods for fisheye lenses were discussed in this article. This was the first time that a negative meniscus lens was used to be the first lens to produce a near virtual image of a 180-degree field of view within a more limited angle.

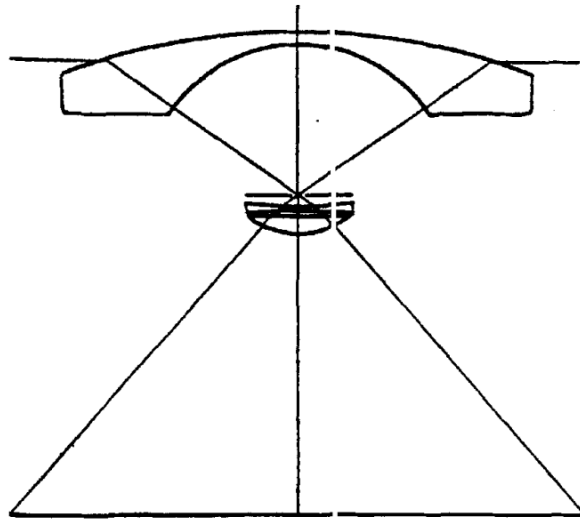


Figure 1-5 Hill Sky Lens

In the beginning, fisheye lenses were utilized solely for cloud imaging. However, because of its unique features, fisheye lenses became not only one of the favorite tools for photographers, but also served as a powerful system for panoramic projection.

1.2 Symmetrical Wide-Angle Lens to Fisheye Lens

An optical system can be categorized by its angular field of view. A standard lens has a field of view under 55 degrees. A wide-angle cellphone lens normally encompasses a field of view from 60 to 70 degrees; This is known as semi-wide-angle (SWA) lens. A regular wide-

angle(WA) lens ranges from over 70 to 90 degrees, while an extreme wide-angle lens has a field of view between 90 and 140 degrees.

There are many methods to improve the field of view of a system. The most typical design involves a symmetrical wide-angle lens whose effective focal length is extremely short. A good symmetry allows these lenses to mostly focus on even aberration correction, because odd aberration such as coma and distortion will be well corrected automatically.

However, when an optical system has a field of view of more than 140 degrees, it has unavoidable distortion and making it difficult to utilize a typical symmetrical design. For a lens that follows rectilinear central (gnomonic) projection,

$$y = f \tan(\theta) \quad (1.5)$$

the image height increases significantly with increase in the field of view. Moreover, its illumination at the edge decreases rapidly because of the $\cos^4(\theta)$ law. So, the value of 140 degrees represents a limit, known as the fisheye limit.

Meanwhile, because of the short focus and short back focal distance, it is difficult to insert other elements after it, e.g. a beam splitter block or penta lenses, which makes the design inflexible. This situation happened in the early 1930s, when a beamsplitter was needed but there was not enough room because of the wide-angle objective lens.

In order to improve the back focal distance, a design called “reversed telephoto” was introduced. Based on earlier designs for Sky Lenses, designers utilized a negative front group and a positive rear group to shift the principal plane; this design is discussed in a later chapter.

1.3 Fisheye Lenses for Smartphones



Figure 1-6 J-SH04

The first mobile camera phone J-SH04 (figure 1-6) with a 110,000-pixel CMOS was released by Sharp in November 2000, and thereafter camera phones began to flourish. In just 18 years, cellphone cameras have gone from fixed focal length to auto-focus, more than one million pixels, and numerous other features such as a flash and larger field of view. These have become more common and have been assimilated into people's lives over time.

Recently, due to the camera phone's portability as well as its increasingly powerful performance, more and more people are using cellphone cameras to record every detail of their lives. In some cases, cellphone cameras even supersede professional photographic equipment, especially with the smartphone's evolution and the phenomenal growth of social networking. Nowadays, cell phone cameras already have a level of performance that is equal to some digital cameras.

1.3.1 Convertible lens

According to "Some Interesting and Unusual Lenses" by Kingslake, around 1890, "symmetrical lenses were developed in which each half was reasonably well corrected." [6] Then some talented

lens designers tried to design systems consisting of two parts with different focal lengths, so that the system could have three focal lengths: one from each of the two components and one from the combined system.

Historically, this idea was once popular because it provided an economical choice for users as they could buy fewer lenses but have more systems. However, since the development of the zoom lenses with better performance and portability, convertible lenses began to lose their market share.

However, for cellphone cameras, the situation is different. Convertible lenses can be utilized with smartphone equipment. Most cellphones have cameras with a fixed lens, although some cellphones are equipped with optical zoom or more than one camera, which provide a different focal length and field of view. However they still do not have enough ability to achieve extreme requirement, such as an extreme wide-angle field of view or macro photography. In this paper, several fisheye lenses designs for cellphone camera will be discussed.

1.3.2 Afocal System

Because of its short focal length, the objects in cellphone camera lens designs are mostly considered to be located at infinity. Thus, in order to maintain a cellphone lens's performance, any fisheye lens attachment must be reasonably designed as an afocal system, which means an infinite effective focal length. A telescope with its eyepiece represents a typical afocal system. In designing a fisheye adapter, a reversed telescope (not a reversed telephoto) design is required.

1.4 Fisheye Lens Adapter

A regular fisheye lens is a system with rotational symmetry and, a stop located between front and rear groups. However, cellphone sensors have inherent properties, such as a rectangular

shape and specific aperture size. As an attachment, a fisheye adapter should carefully satisfy these original characteristics of the cellphone lens.

Because of the rectangular sensor, there are two different solutions: a diagonal fisheye adapter and a circular fisheye adapter.

1.5 Diagonal Fisheye Adapter

A Diagonal fisheye adapter increases the original field of view as well as the original image height. This means the diameter of the image height coincides with the diagonal length of sensor, and the image fills the entire cellphone sensor. Since the image height of cellphone sensor already fills the sensor, a diagonal fisheye adapter should adjust its output field angle so that it equals the field of view of the cellphone camera. This adapter has the significant advantages that the aberration and dim illumination at the edge of the field are not captured by the sensor.

However, if the field of view of a diagonal fisheye adapter is 180 degrees, its actual field of view recorded by the sensor is less, especially at the shorter side. Thus, if a designer wants to design a fisheye adapter that allows the camera to have a real 180-degrees field of view, the field of view must be larger than 180 degrees.

For example:

For an iPhone 7 sensor that is 1/3", if the mapping function of the optical system follows the equisolid angle projection:

$$y = 2f * \sin\left(\frac{\theta}{2}\right) \quad (1.3)$$

and the horizontal field of view is 180 degrees, then its focal length is 4.98mm, and the maximum diagonal field of view is 233 degrees. However, if it follows stereographic projection:

$$y = 2f * \tan\left(\frac{\theta}{2}\right) \quad (1.2)$$

the diagonal field of view will be only 201 degrees.

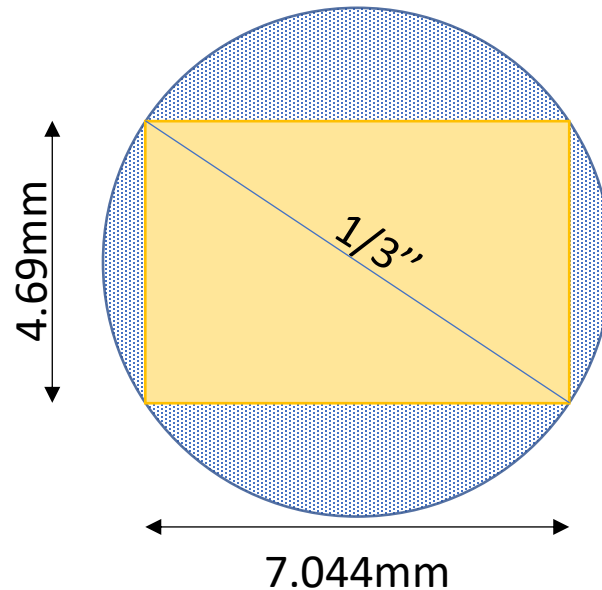


Figure 1-7 diagonal fisheye adapter

1.5.1 Circular Fisheye Adapter

Different from the diagonal fisheye adapter, a circular fisheye adapter reduces its output field of view so that the diameter of the image height coincides with the length of sensor's shorter side. It is popular with some users who are looking for distortion or a real panoramic photograph.

However, these adapters do not only have obvious chromatic aberration and astigmatism at the edge; they also can fill only part of the sensor, so a significant portion of the field of view at the left and right portions is black, especially for smartphones with a significant aspect ratio.

For example, in figure1-7, the 1/3" sensor for an iPhone 7 is presented. The aspect ratio of this sensor is about 1.5. If a circular fisheye adapter is utilized, the real image height for this optical

system should be 4.69mm, and the area of the black portion is 15.76mm², which represents 47% of the entire screen's area.

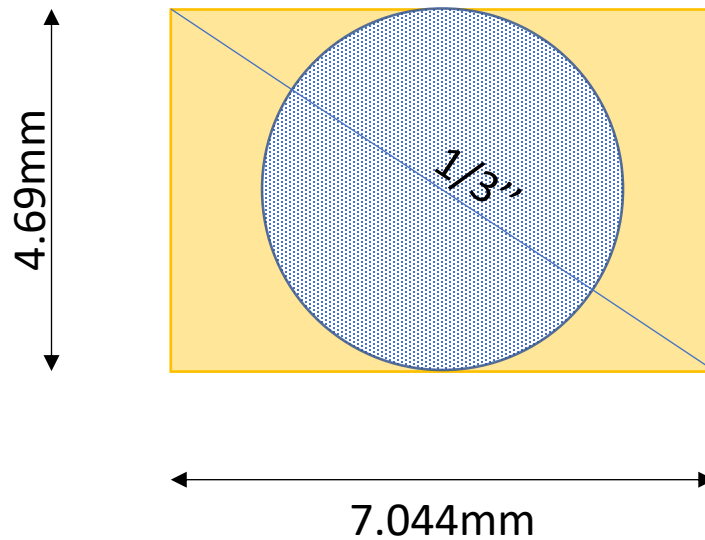


Figure 1-8 circular fisheye adapter

1.6 Methods to Test A Camera's Performance

For photographic systems, the modulation transfer function (MTF) is one of the key performance standards utilized to describe how effectively an optical system can resolve and record the spatial frequency in object. Thus, measuring a system's MTF is one of the essential goals in image testing.

Depending on the system, a variety of testing methodologies for a camera system's MTF are utilizable. For testing microscope, the most popular method is to use a USAF 1951 bar target; for testing a digital camera system, the most common way is an ISO 12233 target. However, since there is a large difference in MTF from lower to higher fields of view as well as significant barrel distortion, it is difficult to test an extreme panoramic system with a regular testing chart. In this paper, the slanted edge test for a fisheye adapter is discussed and utilized.

2 Structure and Properties of Fisheye Lens

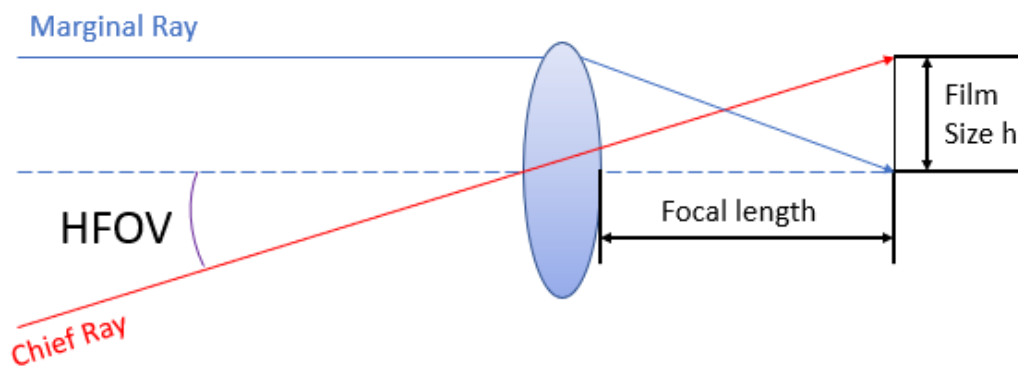
2.1 Structure of Extra Wide-Angle Lens

2.1.1 Relationship Between Field of View and Focal Length

For a conventional lens with fixed focal length, if the detector size is fixed, which means the Field of View in image space is fixed, the angular field of view in object space can only be changed by changing back focal distance. A clear relationship between field of view and focal length for a single thin lens system is shown in figure 2-1 and equation 2.1,

$$h = f * \tan(\theta), \quad 2.1$$

where h is half of detector size, f is effective focal length, θ the angle of the half field of view. This equation indicate that it is necessary to reduce the system's EFL to increase its field of view. From the figure, it is very straight forward to see that with a given detector size, a larger field of view requires use of a shorter focal length.



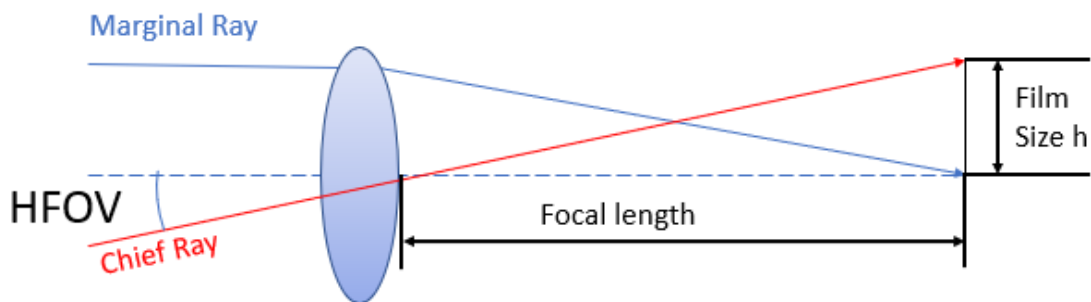


Figure 2-1 relationship between focal length and field of view

In real situations, the system is generally more complex than just a single thin lens. The focal length is obtained by measuring the distance between the rear principal plane and the image plane. However, it is not easy to measure the rear principal plane directly because most of the time, especially for the short focus wide-angle lenses mentioned in the next section, the back focal length is shorter than the effective focal length, which means the principal plane is located inside their mechanical structure (Figure 2-2).

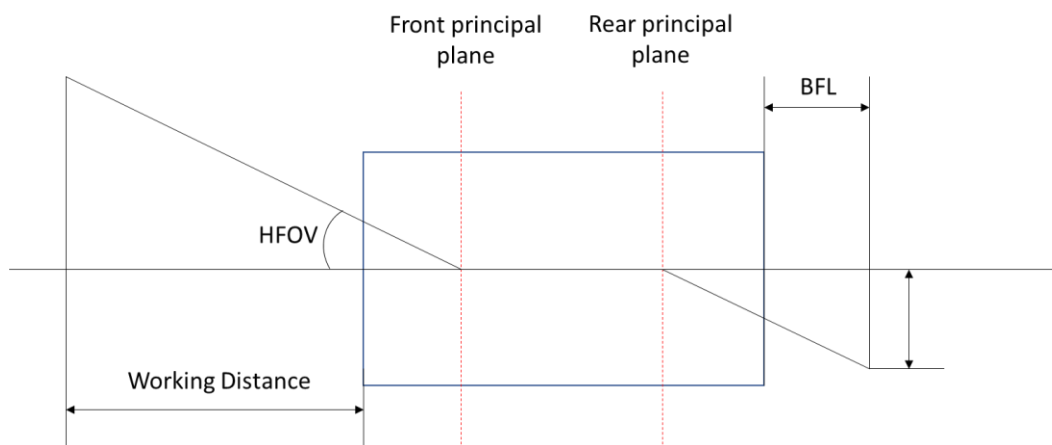


Figure 2-2. First order setting for black box system

So it is not easy to obtain all the parameters of equation 2.1. However, the good news is that the system in real situations still follows the same properties as a paraxial setting. This

means we still can control the field of view by utilizing the same method that is used in a paraxial model: by moving the optical system closer to the image plane to increase the field of view, or by choosing another lens system having a shorter effective focal length. Of course, a sensor with a larger sensor size results in a larger field of view.

Based on this theory, lens designers began to design wide-angle lenses with very short focal lengths beginning in the 19th century. Are those designs the best way for optical designers to obtain a greater field of view? These topics are discussed in the next section.

2.1.2 Short focus wide-angle lenses

As referred to in the previous section, a wide-angle lens is one of the lens systems which has significant small EFL for the format with given size. As a result, compared to a regular lens, a greater field of view can be easily obtained.

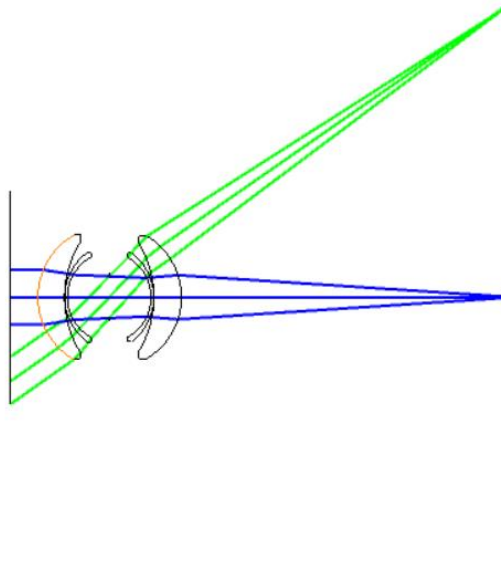


Figure 2-3. Patent 2,031,792

Because of the symmetrical construction, these types of lens are mainly free of curvilinear distortion and also have good illumination based on the Silyusarev effect [20].

A lower distortion with a large field of view of around 120 degrees is useful in some special cases, such as architectural photography, but again, this short focus lens is not able to obtain 180 degrees field of view.

2.1.3 Reversed Telephoto Structure

As previously discussed, conventional symmetric wide-angle optical systems have relatively short effective focal lengths with an even shorter back focal distance. Thus the distance between the vertex of the last surface and the image plane is small. This is a very inconvenient property for a system that needs to insert other optical elements, e.g., beam splitters, penta prism, or mirror shutters, before the image plane for modern photographic systems. For example, here is a modern DSLR camera (figure 2-4).

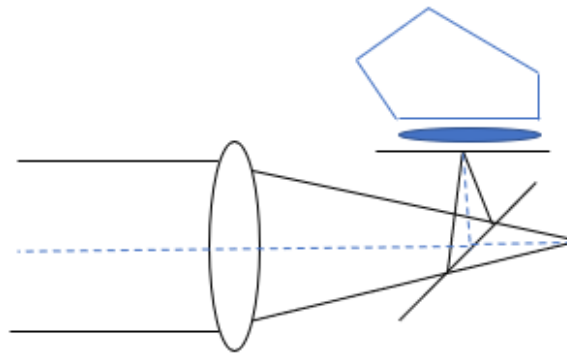


Figure 2-4. Modern DSLR camera

Figure 2-4 indicates that a DSLR camera needs a relatively large back focal distance for positioning a beam splitter.

Furthermore, in some optical experiments, optical designers also need extra optical elements before the image plane. Additionally, based on symmetry, it is impossible to build a conventional wide-angle system with a field of view over 180 degree. In order to obtain greater field of view and larger back focal distance with a reasonable effective focal length, the inversed telephoto, sometimes called a retrofocus system, was introduced. Based on previous work on the Sky lens by Hill referred to in the previous chapter, the combination of a negative front element and a positive rear element moves the principal plane forward to image plane, resulting in a larger back focal distance and a short effective focal length.

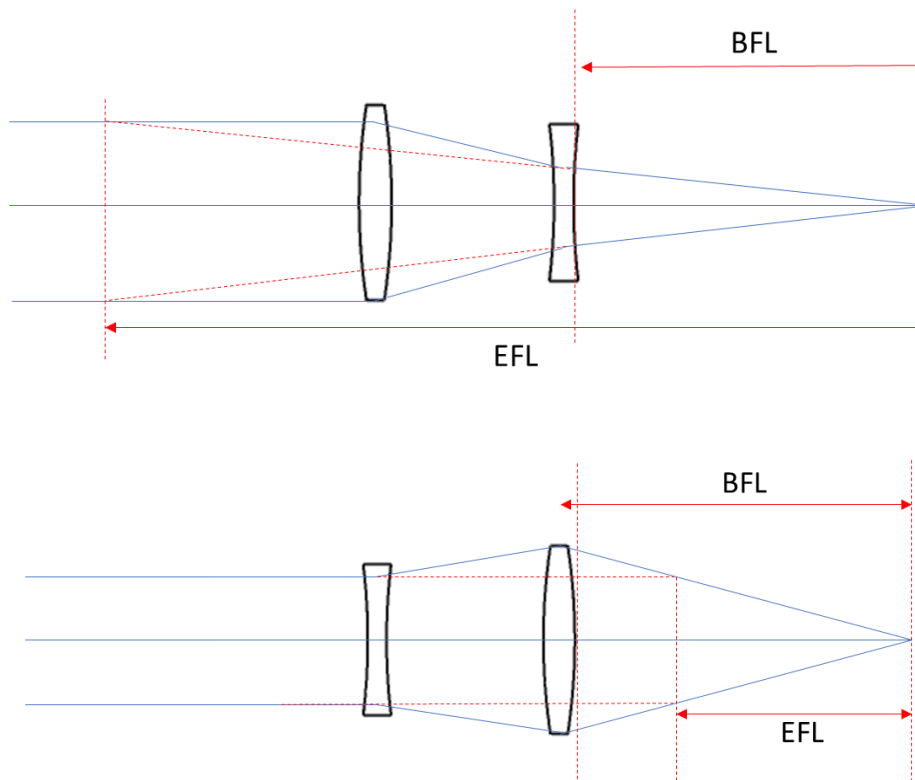


Figure 2-5 (a) telephoto system (b) reversed telephoto system

Figure 2-6 shows the paraxial layout for both a telephoto system and a reversed telephoto system. It illustrates that both of them have one positive and one negative optical element, and

the back focal distance is the same for each design. However, the effective focal length for these systems is different due to the relative position of the positive element and the negative element.

In telephoto structures, the cone of light in the image space is relatively small and the principal plane is more distant from the image plane, resulting in a smaller field of view but more magnification. But in a reversed telephoto structure, because of the reversed position between the negative and positive element, the principal plane is moved closer to the image plane and the cone of light in the image space is relatively larger than in a telephoto structure. The front negative element also helps collect light; this not only increases the field of view but also increases aperture size. Additionally, an asymmetrical structure makes a field of view over 180 degrees possible if the first element of the front negative element is a large meniscus lens; although a convex surface reduces the optical power of the front group, it has the ability to converge the incoming beam with a relatively higher illumination that does not completely comply with the $\cos^4(\theta)$ law. The convex first surface also reduces the angle of incidence. In addition, in contrast to conventional symmetrical wide-angle lenses, the aperture stop of this structure can be one of the degrees of freedom, meaning it can not only be located at the principal plane of the rear element to make the entire system telecentric, but can also be utilized as a powerful tool for aberration correction. Furthermore, the rear positive element helps reduce the field curvature introduced by the front elements. All in all, a reversed telephoto structure, as mentioned before, has a longer back focal distance with a potentially larger field of view and aperture size.

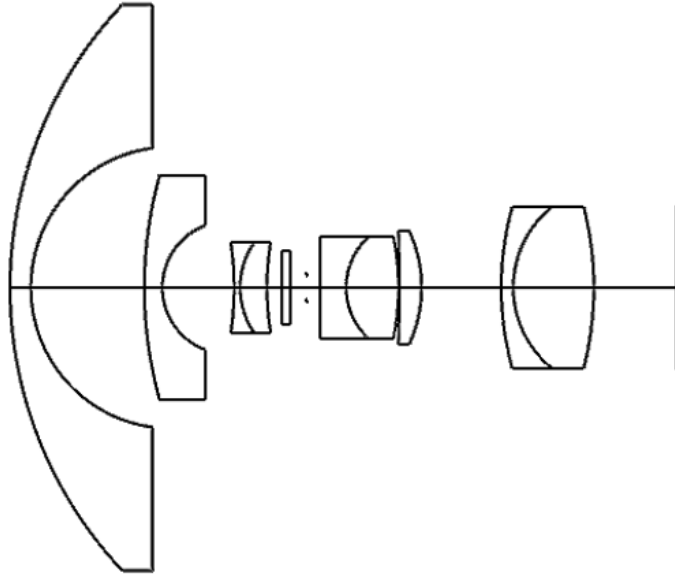


Figure 2-6 The Nikon 6mm F/5.6 fisheye lens[8]

A fisheye lens is one type of reversed telephoto system that has an extremely large field of view, generally over 150 degrees. However, there is no perfect system; this type of structure also has its own problems, which are discussed in coming sections.

2.2 Properties of Reversed Telephoto Structure

2.2.1 Main Monochromatic Aberration

As in all optical systems, aberrations are one of the most essential performance factors that need to be considered in fisheye lens design. A system with good aberration correction has better image quality and higher contrast. Based on centuries of development in lens design, numerous different techniques to correct aberration have been created by lens designers.

In optical design, symmetry is always a very strong tool when designers deal with aberrations because a symmetrical system with unit magnification is free from coma, distortion, and lateral chromatic aberration. On the other hand, if a system has extremely non-symmetric

properties, it typically has significant coma, distortion, and lateral chromatic aberration.

Unfortunately, in order to project a hemispheric field of view onto a finite image sensor, it is unavoidable for extreme wide-angle systems to be nonsymmetrical, resulting in the significant odd aberrations referred to previously. Although coma is an aberration that can be balanced by other degrees of freedom, the noticeable barrel distortion is still difficult to correct. In 1929 H. Lee came up with two ideas to correct the distortion in a reversed telephoto system: either a thin air space or an interface with strong collective ability between the first two elements of the negative group. The thin air space performs like a positive lens.

For a fisheye lens whose field of view is near or over 180 degrees, barrel distortion is unavoidable. However, the mapping functions of fisheye lenses are still controllable and the significant distortion actually helps the system improve the edge illumination, which is discussed in coming chapters.

2.2.2 Main Chromatic Aberration

Chromatic aberration exists because different wavelengths have a different index of refraction for a given material. As previous sections stated, lateral chromatic aberration is one of the essential problems for systems that lack symmetry. Fisheye lenses are no exception. Compared to longitudinal chromatic aberration, also called chromatic change of focus, the lateral chromatic aberration, also referred to as chromatic change of magnification, is significantly more obvious. The reason for this phenomenon is that lateral chromatic aberration exists because the refraction angle of the chief ray for different wavelengths is different; This significantly depends on the field. Unfortunately, fisheye lenses are not only extremely non-symmetric, but also have a large field of view. Thus lateral chromatic aberration is much higher than longitudinal chromatic

aberration. However, lens designers still have a lot of techniques at their disposal to control lateral chromatic aberration.

For chromatic aberration, an achromatic doublet is always a good tool to achieve correction. However, according to ‘Wide Angle Lens System’[3], it is not very effective to put a doublet in front of the stop, though it is useful to insert an **additional** negative doublet in front of the stop. However, for a fisheye lens adapter, all the optical elements are in front of the stop, so the only option for this design is a doublet in front of the stop.

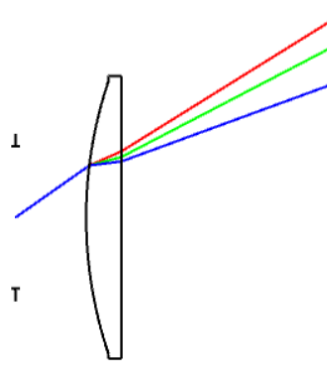


Figure 2-7 Lateral chromatic aberration

2.2.3 Distortion, Projection and Mapping Function

Although the barrel distortion for a fisheye lens is uncontrollable, the mapping function of projection in such a system is controllable, which means fisheye lens systems have different projection methods from ordinary lenses.

An ordinary fixed focus lens follows gnomonic projection, as discussed in chapter 1. The equation for this projection is:

$$y = f * \tan(\theta)$$

Thus based on this equation, when the half field of view is close to 90 degrees, the sensor size is infinity. So without introducing very large barrel distortion, it is impossible for a finite camera sensor to capture the entire image. In order to project the image in a reasonable way, designers came up with four projection methods: orthographic projection [Eq. (1.1)], stereographic projection [Eq. (1.2)], equisolid angle projection [Eq. (1.3)] and equidistant projection [Eq. (1.4)]

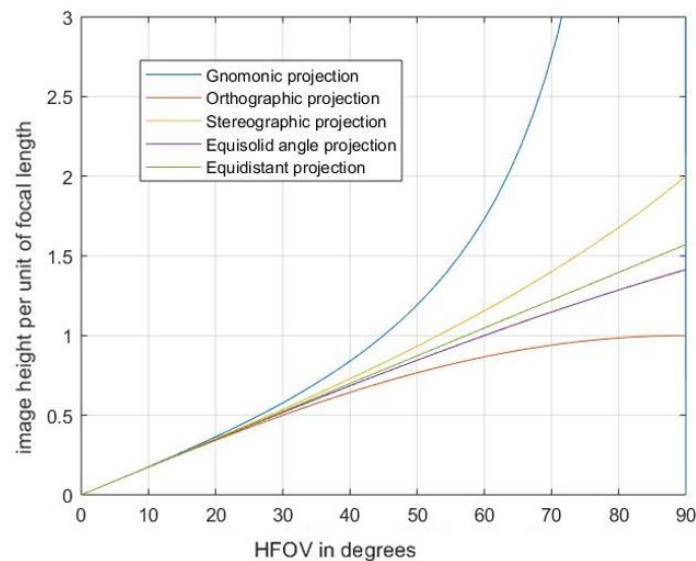


Figure 2-8 HFOV for different projection

Figure 2-8 indicates the relationship between the image height and field of view for different mapping functions. It is obvious that for the four-mapping functions, the barrel distortion is directly related to the difference between it and the gnomonic function. The less this difference is, the less barrel distortion is introduced. Based on this rule, stereographic projection has the least barrel distortion. But that does not mean stereographic projection is the most effective projection method in practice. Which projection method is better really depending on the photographic target. Most of time in practice, equisolid angle projection and equidistance

projection are chosen to be the projection method in real designs. The reason is that their magnification changes smoothly compared to the other two.

2.2.4 Relative Illumination in Fisheye lens

Relative illumination plays a key role in optical system design, especially extreme wide-angle system design. The reason is the relative illumination follows $\cos^4(\theta)$ law without pupil aberration, which causes the illumination with a large field of view to be very small. This means the edge of the image is dim compared to the center portion. In this situation, although a wide field of view is captured, the image is severely compromised.

However, because the mapping functions for fisheye lenses are different from Gnomonic projection and introduce significant barrel distortion, the pupil aberration for fisheye lenses is increased as well, especially coma pupil aberration, which is given by

$$\bar{W}_{131}(\vec{H} * \vec{H})(\vec{H} * \vec{\rho}).$$

However, relative illumination is influenced by both vignetting and pupil aberration. Pupil coma, which changes the cross-sectional size of the beam incoming to the pupil, increases the illumination received by the entrance pupil and is able to play a key role in relative illumination analysis. This is because pupil coma is also mainly relative to system distortion.

$$\bar{W}_{131} = W_{311} + \mathcal{K} * \frac{\Delta(\bar{u}^2)}{2}$$

Thus a fisheye lens with 100% barrel distortion is required to have a higher edge relative illumination of over 50%. This phenomenon is also called the Slyusarev effect.

Figure 2-9 shows the relative illumination for a fisheye lens and an ordinary singlet. Obviously, the ordinary lens follows the $\cos^4(\theta)$ law, while the fisheye lens does not.

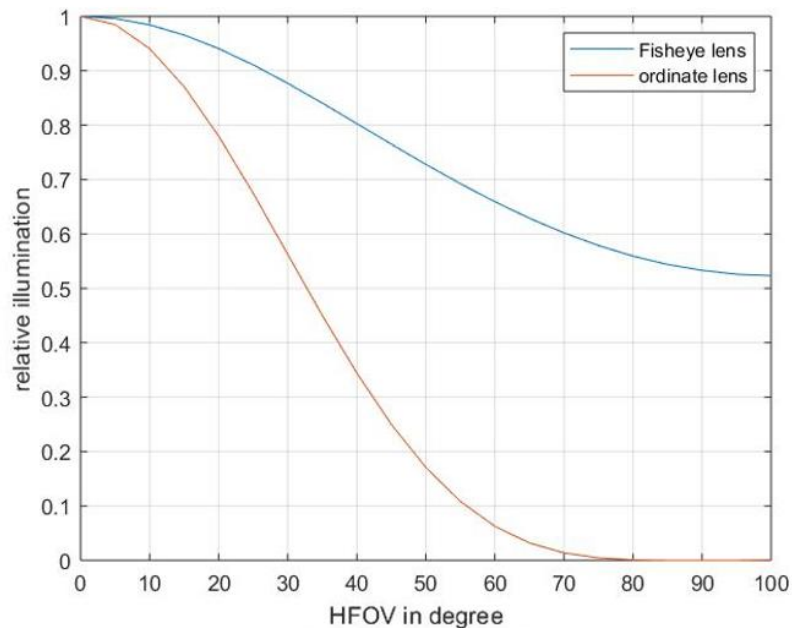


Figure 2-9. Relative illumination between fisheye lens and ordinate lens

The other method to explain the reason that fisheye lenses have a better relative illumination is checking the spot diagram before the first surface of the system. Figure 2-10(a) indicates that for an ordinary lens spot diagram, different fields have the same spherical shape, which means their incoming beam has the same starting illumination. However, because of the $\cos^4(\theta)$ law, the illumination at high fields is reduced significantly at the image plane.

Figure 2-10(b) indicates that when the field of view increases, the spot diagram becomes enlarged, which is because pupil coma is introduced, and the amount of incoming beam is improved, meaning starting illumination is increasing with the field. Thus, its final relative illumination is higher than shown by $\cos^4(\theta)$.

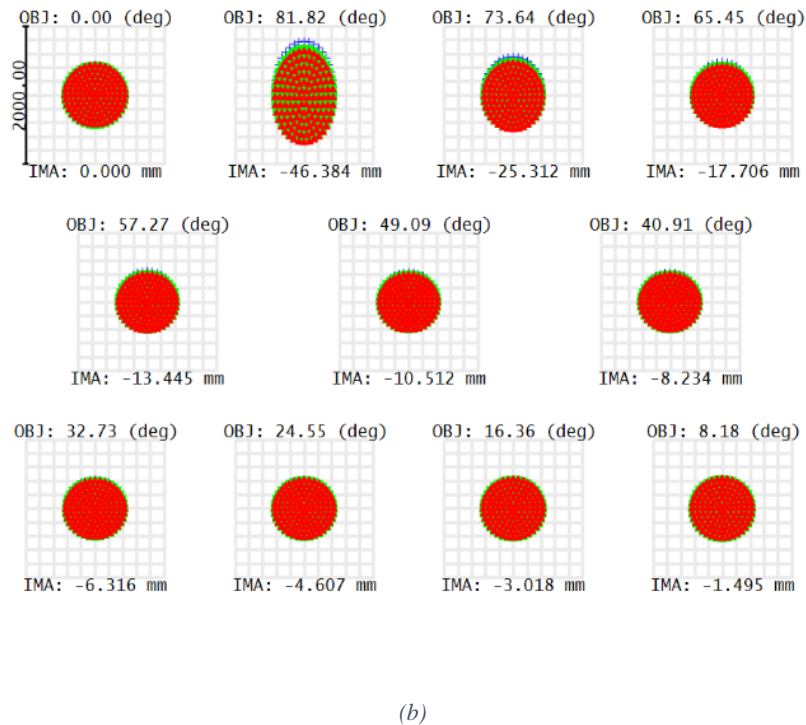
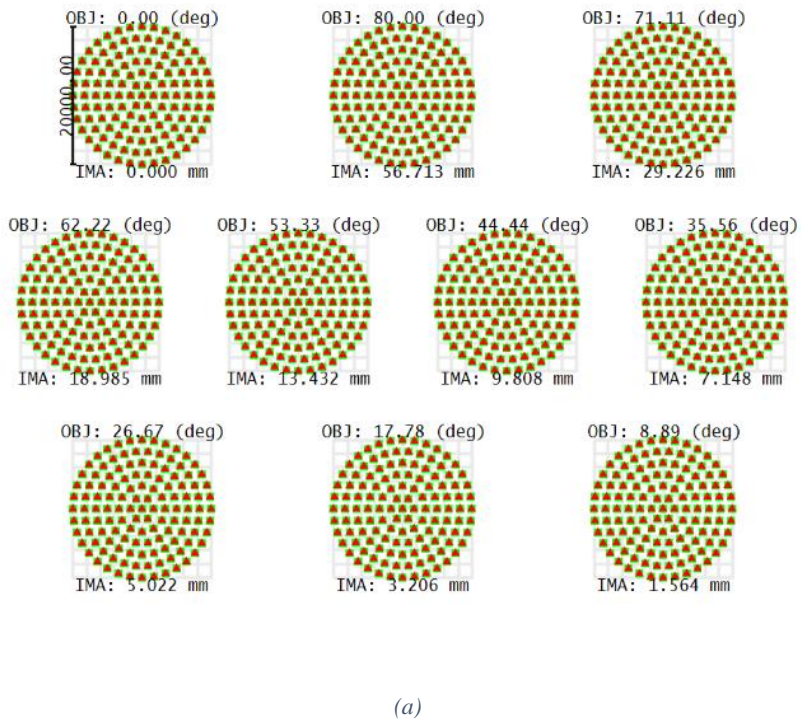


Figure 2-10 spot diagram for incoming beam in (a) ordinate lens and
(b) fisheye lens

3 Analysis of fisheye lens designs

3.1 First order concept

Fisheye lenses are retrofocus systems, which consist of a front negative lens group and a rear positive lens group. This combination helps shift the principle plane toward the image surface. Thus the back focal length of this system is significantly larger than the focal length.

According to this, the design of a fisheye lens starts from a negative lens with a positive front surface which helps converge the incoming ray into the axis. And the second part of this system is a rear positive group. In general, this group is the focusing part of fisheye lens, but in convertible system, the focusing part is the photographic image system after the stop. As a convertible attachment, a suitable positive group is needed to make the whole system afocal. Additionally, the power distribution of the front negative group needs to be large due to the magnification of FOV for image and object space, which causes conspicuous field curvature and astigmatism. Those large aberration of the front group are balanced by the rear group, which has a power contribution with reverse sign.

In this chapter, different structures of convertible fisheye lenses for smartphone shall be discussed, including their aberration contribution and design thinking.

3.2 Design requirement and specification

As a smart phone attachment, the basic requirement for this design is the location and size of the exit pupil in this system should match the entrance pupil of the smartphone camera, which typically has a diameter of 1.6mm at the first surface of the smartphone camera lens. The entrance field of view should be at least 150 degree to reach the fisheye limit; the exit field of view depends on the type of the attachment. For a diagonal fisheye adapter, the exit field of view should match the diagonal entrance field of view of the smartphone camera, which is between 60-66 degrees. And of course, as an afocal system, the effective focal length of the attachment should be close to infinity.

The lens material can be either glass or plastic, and in this chapter both are discussed. Glass designs always have lower absorption and a larger range of index of refraction and Abbe number, which is good for correcting chromatic aberration and maintain first order specifications. Compared to plastic material, glass also has better environmental stability such as lower thermal sensitivity and resistance to scratching. However, in fabrication, plastic lenses are easier to have aspheric surface with lower cost since injection molding fabrication is available for plastic material. Furthermore, plastic lenses also have relatively lighter total weight.

The specifications of the fisheye attachment are summarized in the Table 1.

<u>Specification</u>	<u>Requirement</u>
Focal Length (mm)	Afocal
Design f/#	2
f/# Range	1.8-2.3
Is f/# Image or Object Space? Infinite or Working?	Image, infinite
Field of View (horizontal)	over 160 degrees
Field of View Range	NA
Primary Wavelength	587.6nm
Design Waveband	Visible
Telecentricity - Image Space	Not telecentric
Image Space MTF	70lp/mm >30%
Relative Illumination	50% at maximum field
Lens material	Glass & plastic
Element count	3 or 4
Min / Max Lens Length	20mm(Not including BFL)
Min / Max Lens Diameter	20mm

Table 1

3.3 Design process

As discussed before, a fisheye lens should have at least one negative group and one positive group to reach the retro focus structure, and the first element must be a meniscus lens with a positive surface facing the object to capture the light from large field of view and improve

the relative illumination as well. For a 4 elements design, the degrees of freedom left are the shape factors of the 2 middle lenses and the aspheric surfaces of each element.

The design shown in Figure 3-1 briefly introduces a second element with double negative surface to provide large negative power so that the angle of refraction at each surface in the negative group is smaller, which helps control aberrations and tolerances. The thin air gap between the second and third element helps control higher order aberration that have higher pupil dependencies, such as spherical aberration and coma.

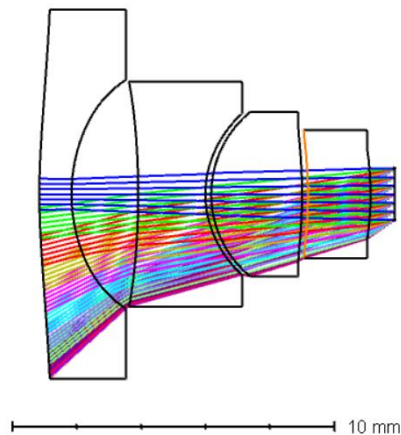
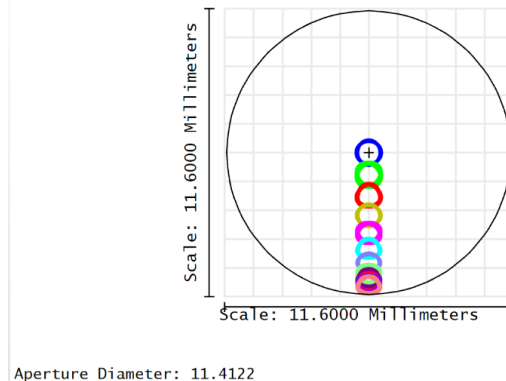
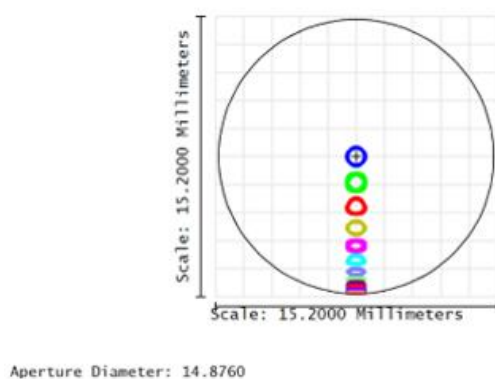


Figure 3-1 1st Design

The dependencies of aberrations can be checked by footprint plots for each surface which can be obtained in the Zemax lens design program .



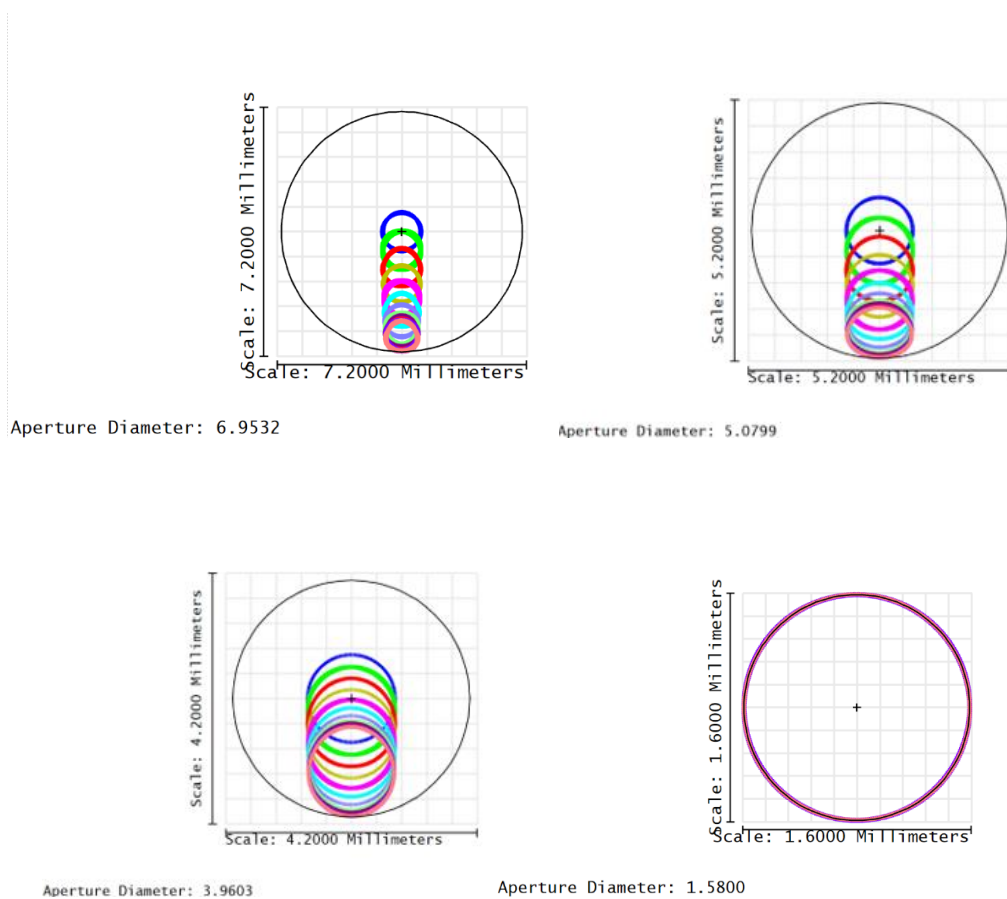
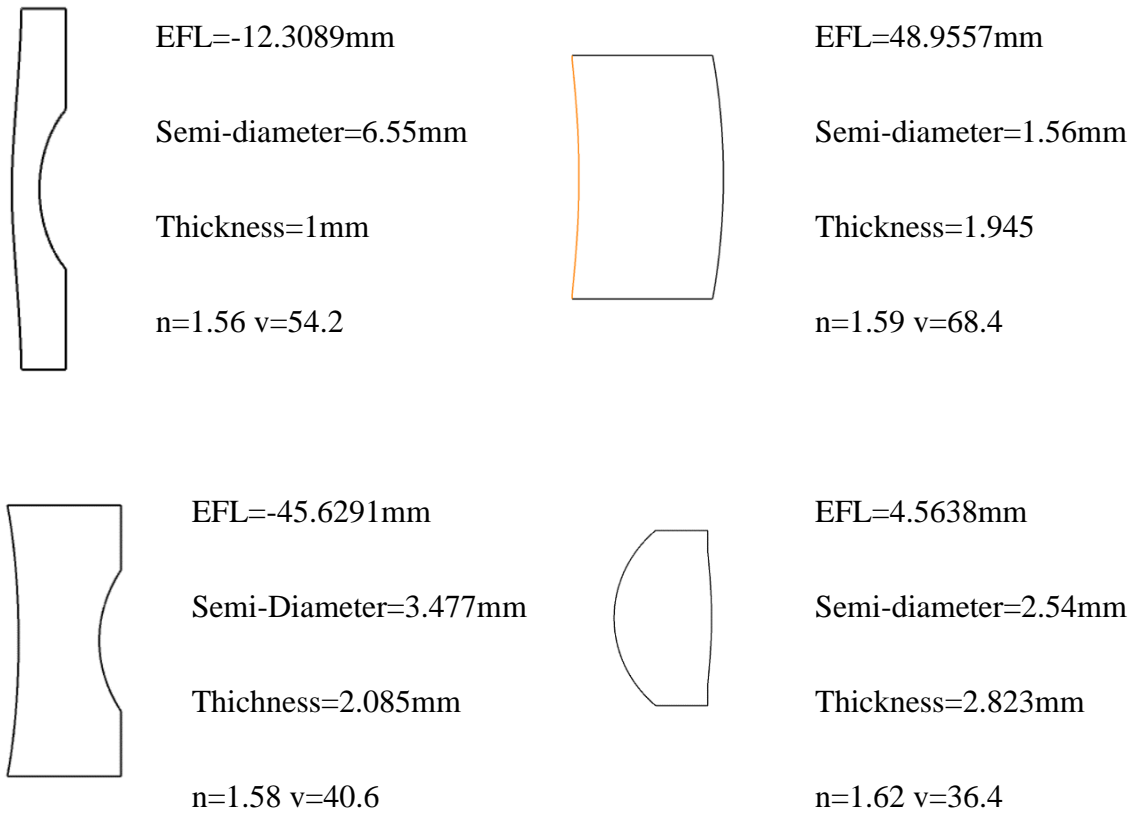


Figure 3-2 Footprints from object surface to stop

Figures 3-2 is footprint plots from object surface to stop. According to them, it is obvious that if the element is near the stop, its contribution for aberration is mainly for on-axis aberration or the aberration mostly depends on pupil factor, for example Coma and Spherical Aberration. Otherwise, it contributes more off-axis aberration, especially those significantly influenced by field factor, such as astigmatism, field curvature, lateral chromatic aberration and distortion. However, distortion is not able to be corrected in fisheye lens.

In order to control both the number of elements and aberration performance, the design is required to have enough degrees of freedom and must satisfy first order specification at the same time. Thus the essential concept of the design approach is to employ larger negative power in the second lens. This allows better aberration correction by subsequence in the overall design.

However, this concept will intensify the imbalance of fisheye lens and introduce more field of curvature.



Lens data above shows that the optical power of the 2nd and 3rd lenses are almost the same value with reversed sign, which means their main contribution is aberration correction. As figure 3-3 shows, when they are removed from this design, the system can still be built with only slight defocus (about 0.1mm axial shift), but the aberrations are significantly worse.

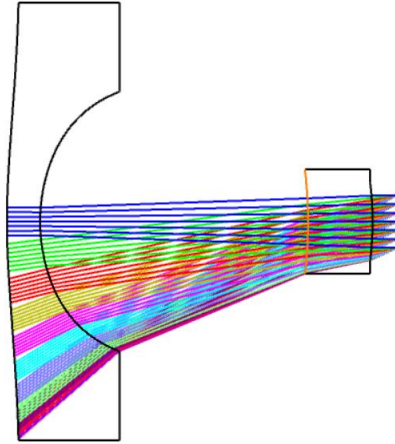


Figure 3-3 Design with only two element

Additionally, compared with the 1st and 4th lens, the middle part -2nd and 3rd elements- consist of flint lenses that help balance chromatic aberration, especially lateral chromatic aberration based on the large field of view.

3.4 Aberration distribution

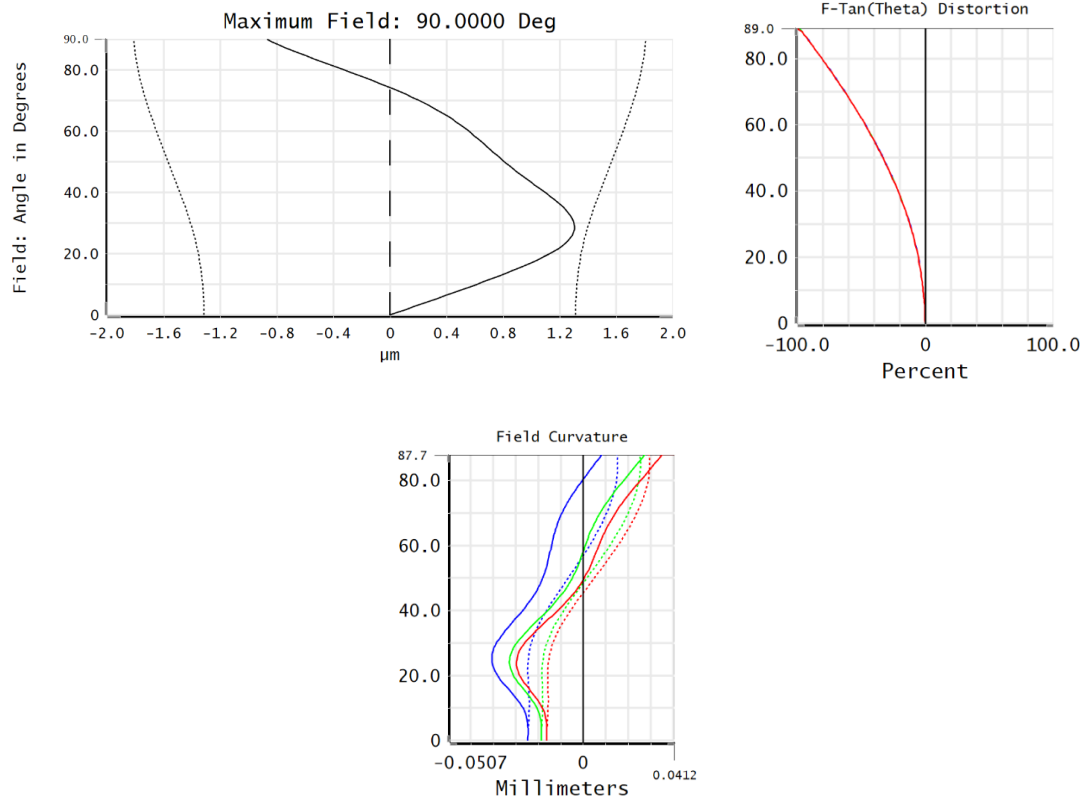


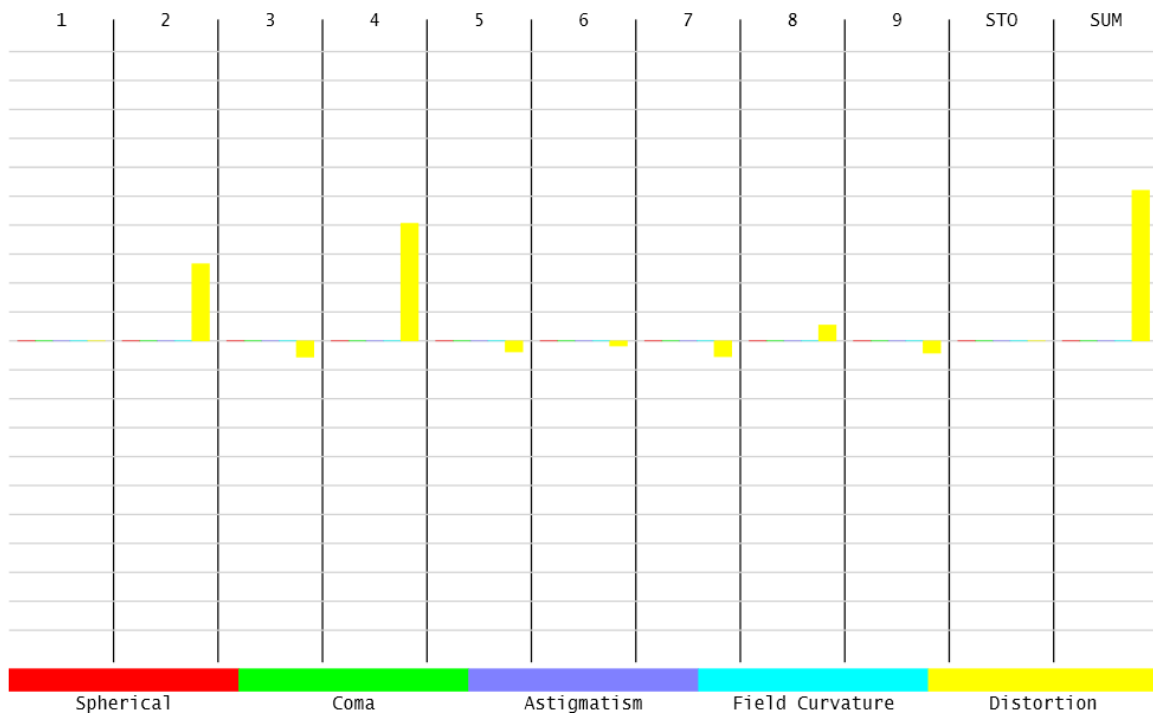
Figure 3-4 Performance of 1st design

Figures 3-4 shows the balancing of lateral chromatic aberration, field of curvature and distortion of this system. According to the extreme asymmetry, distortion of fisheye lens is expected to be uncorrectable, thus the correction of this aberration is ignored.

The lateral chromatic aberration is corrected by flint elements of both the front negative and rear positive group, which also helps the designer correct axial chromatic aberration, although it is small compare to lateral chromatic aberration. In this process, higher order terms from aspheric surfaces are introduced to balance this aberration.

Field curvature is also one of the main aberrations of fisheye lens. In this design, defocus is introduced to balance the field curvature, the irregular curve also shows the existence of higher order terms.

3.5 Third order aberration contribution for each surface and higher order aberration analysis



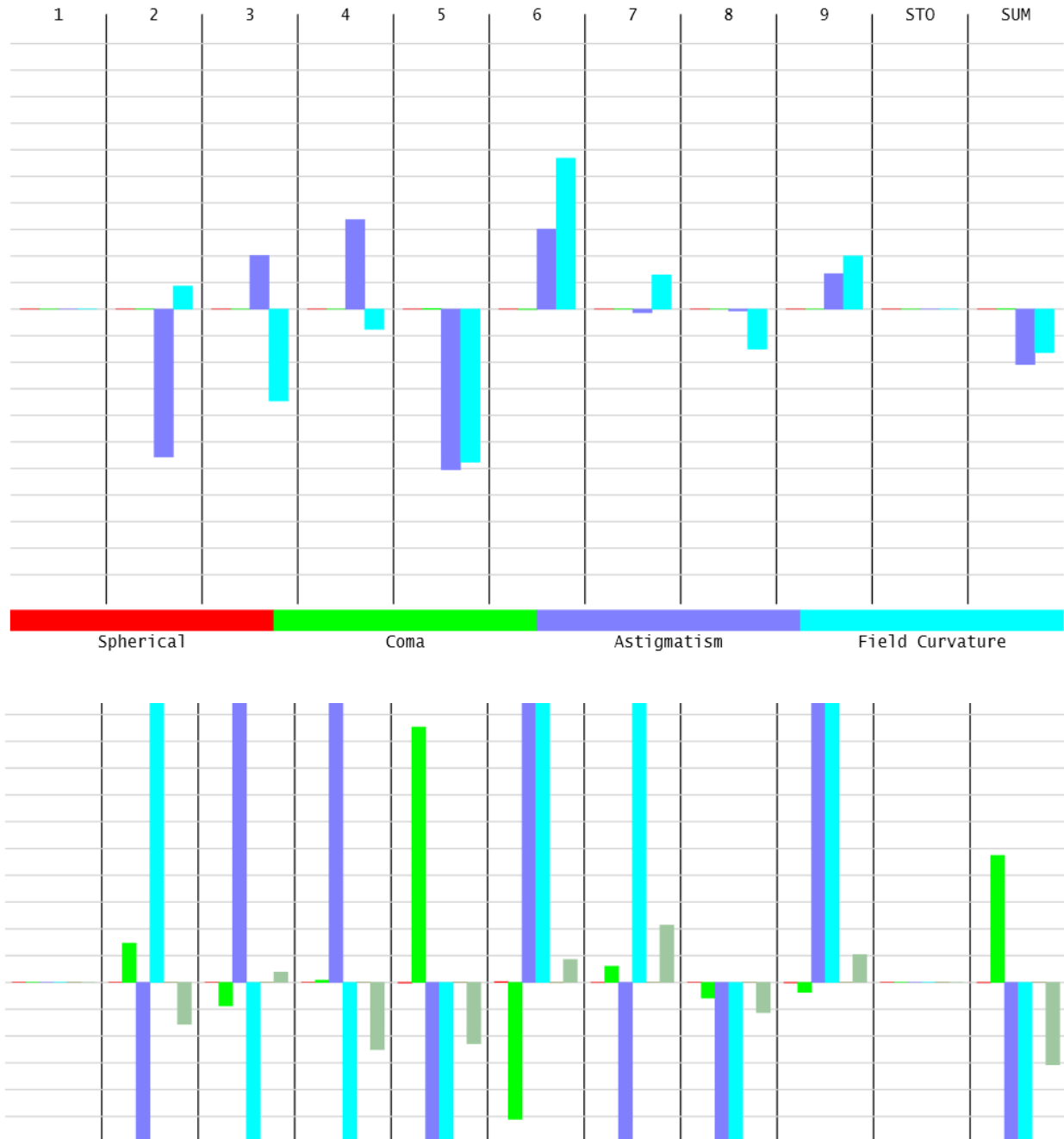


Figure 3-5 relative 3rd order aberration distribution for each surface

Figures 3-5 show the third order aberration contributions of this fisheye lens. The distortion distribution is larger than any other aberrations in this system and mostly contributed by the front negative group.

As discussed previously, the astigmatism and coma are the main monochromatic aberration because of the uneven power distribution and the location of the front elements. Field curvature is mainly balanced by the participation of rear positive group (surface 6-9). For the third order, astigmatism is corrected by power distribution as well as the fourth order terms, especially at the 5th surface (rear surface of second element). However, it is easily seen that in order to balance astigmatism at surface, a lot of coma is involved, but balanced by the fourth order term of 6th surface.

In order to analyze higher order terms, I tried to set each aspheric term to zero and see the change of the performance, which indicates the contribution of this variable. Here is an example:

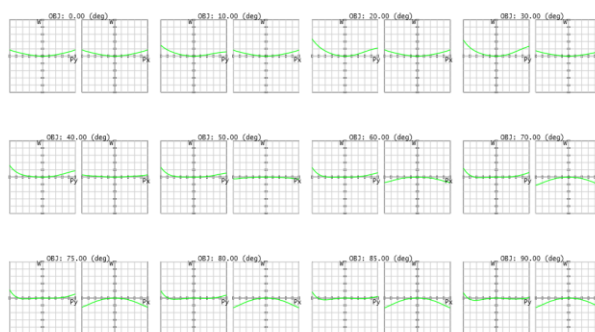


Figure 3-6 OPD after setting conic number

Figure 3-6 shows the final OPD, after setting the conic number of the last refractive surface to zero, the OPD fan becomes the result shown in Figure 3-7.

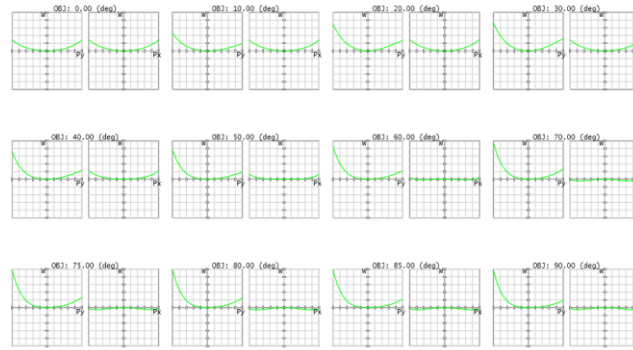


Figure 3-7 OPD after removing conic number

It can be clearly seen that it is used to balance linear coma. Moreover, a similar result can be obtained when the 4th order term of this surface is set to be zero. Theoretically speaking, the last surface is the most effective surface to correct spherical aberration, however in this situation, because of the limitation of degrees of freedom, huge astigmatism and relatively small spherical aberration, the last surface helps more at correction of coma, especially linear coma, which has less field dependency than astigmatism.

From this way of analysis, it is possible to list the contribution of each surface.

The second similar system is a system with plastic material LF5HT for first second and last element, F2 is chosen as the material for third element for correcting chromatic aberration.

The special property of this system is a very large first element with diameter 28mm is introduced to increase the magnification and allow more light to come in. The second lens in this system is still a meniscus lens with negative surface facing into the object, another advantage of this design is air lens between the first two element introduces inverse astigmatism in high field, which improves the sagittal ray's performance but sacrifices the tangential ray's performance so that improves the lowest MTF of this system.

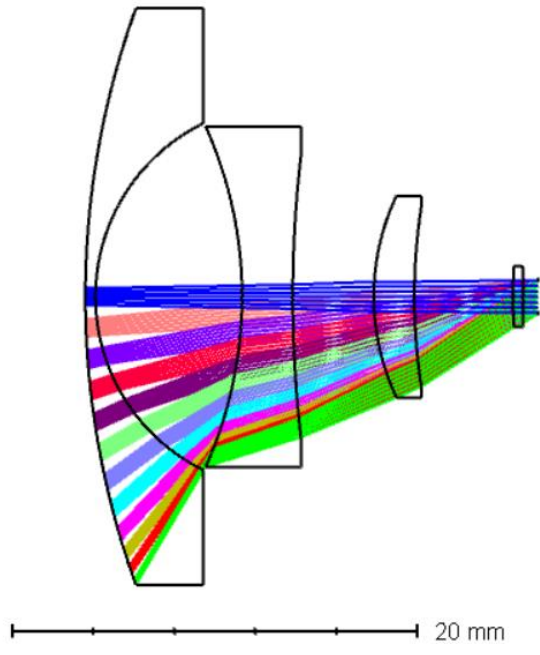


Figure 3-8 2nd design

Although the field of view is extremely big, the RMS wavefront error for this system is still under 0.5 waves for all fields.

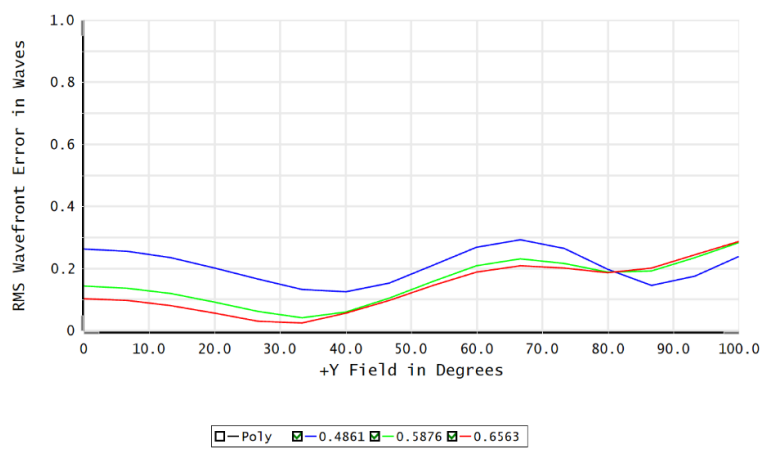


Figure 3-9 RMS of 2nd design

The main problem of this design is also caused by the first surface of the second element with significant large negative power which introduces exacerbates the lateral chromatic aberration at high field.

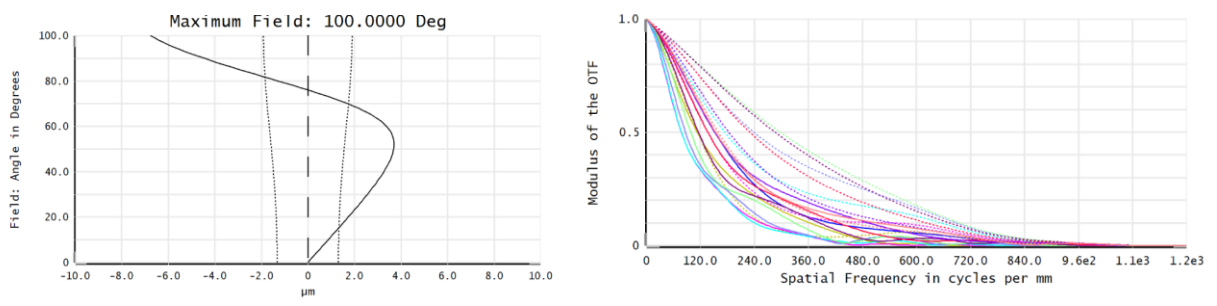


Figure 3-10 lateral color and MTF of 2nd design

In glass design, there are more material options, in order to reduce the lateral chromatic aberration at very high field, a glass with large abbe number should be utilized at first and second element, and the third element shall be a material with relatively small abbe number.

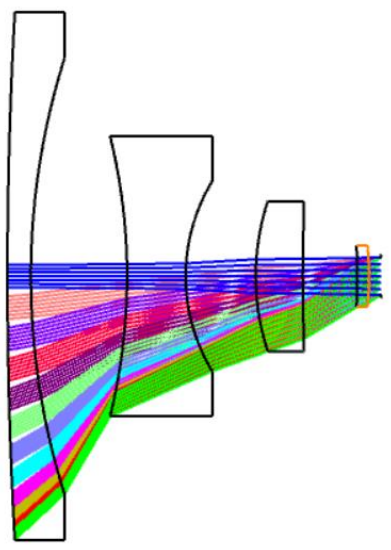


Figure 3-11 3rd design

The performance of the design with similar structure using N-PSK58 in first element, SF5 at third element is shown in the figure.

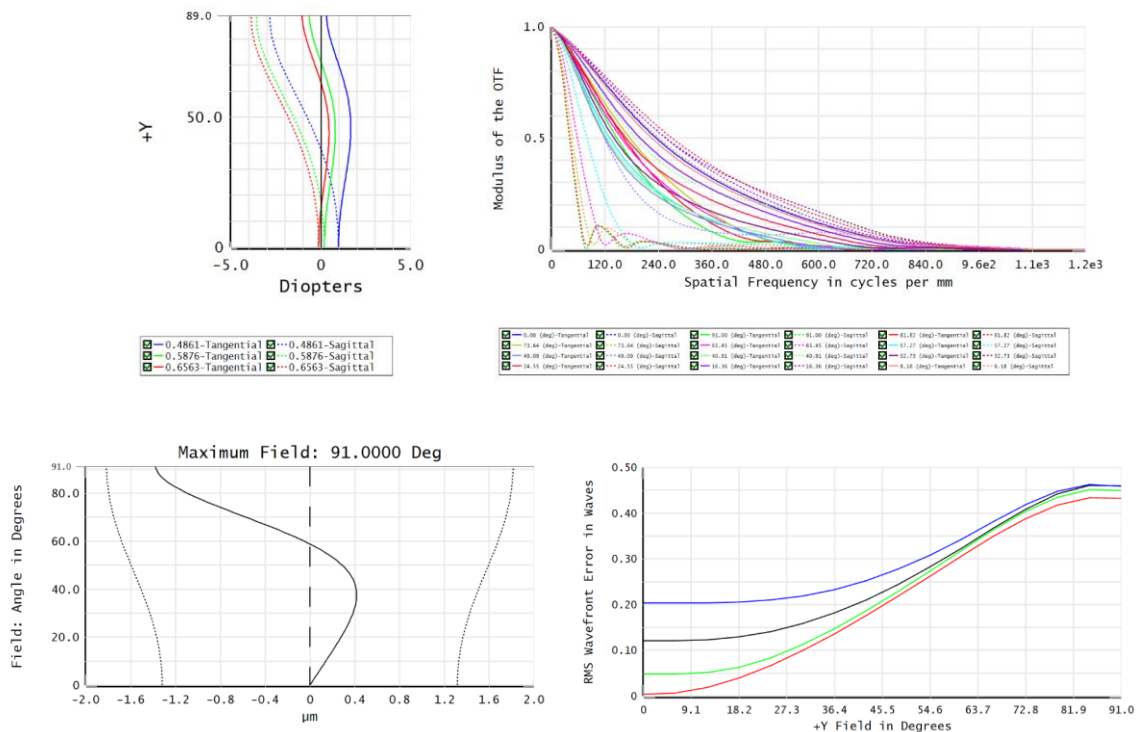


Figure 3-12 performance of 3rd design

By reasonably chose the material of the elements, lateral chromatic aberration is well controlled to be under 1.6 μm at maximum field. RMS wavefront error is still under 0.5 waves in maximum field. A F/1.8 aperture provides enough speed for smartphone camera. However, since the air lens between first two elements has less power than previous design, higher order astigmatism increase significantly and reduces the MTF curve of high field sagittal rays.

The third design uses positive front surface at second element.

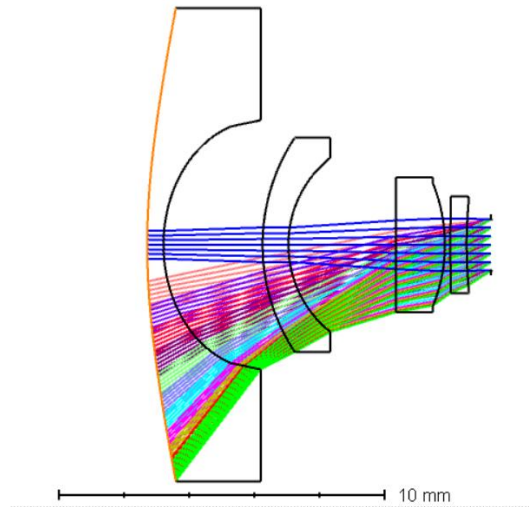
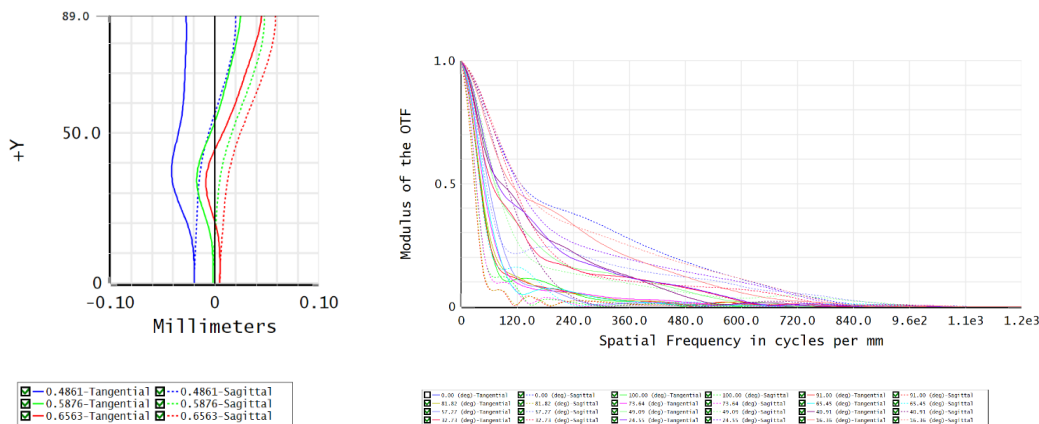


Figure 3-13 4th design

The advantage of this design is a positive or non-negative power front surface at second element can ease the angle of incidence and angle of refraction between first two element so that the tolerance can be well controlled. Another advantage is it introduces more pupil coma so both the maximum field of view and the relative illumination becomes larger. However, by using two positive surfaces to converge the incoming beam, less degrees of freedom are used to balance aberration and provide enough negative power for first group, thus in order to get better performance, aspheric surface needs to be introduced.



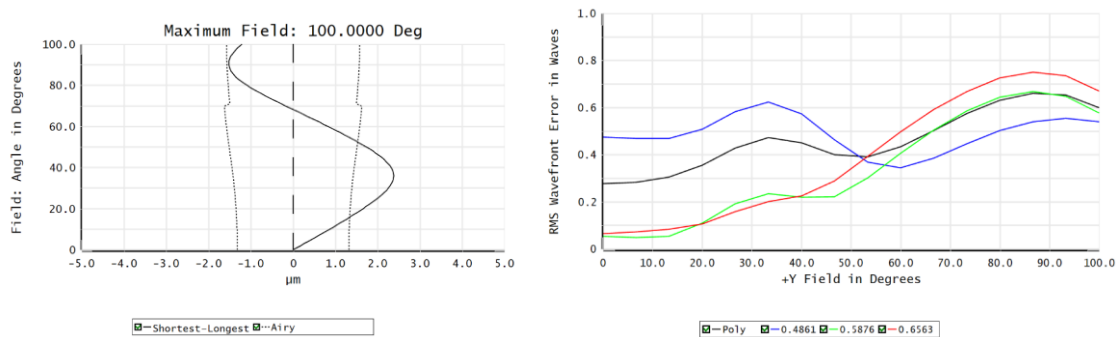


Figure 3-14 performance of 4th design

Figure 3-14 illustrates that even with aspheric surfaces in this design, the performance is still barely fit the requirement.

However, with glass material, even with all spherical surface, a good performance with high edge relative illumination is obtained.

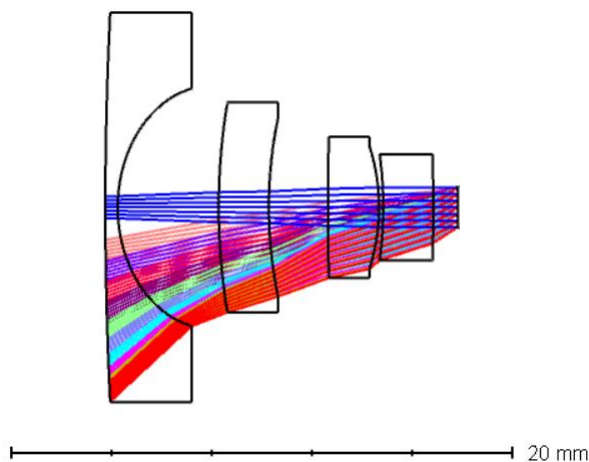


Figure 3-15 5th design

Figure 3-15 shows the design with material PSK50, BK7, N-SF5 and N-KZFS8, an interesting correction method for lateral chromatic aberration during the design process is the radius of curvature of the first surface have a bigger contribution than expect. An element with flatten first surface has less chromatic aberration than the element with curved surface. Based on previous section, a flatten first surface significantly reduces the relative illumination, it is a

tradeoff. However, with a meniscus second element, relative illumination is improved. The other surprising correction method is the thickness of last element. Most time thickness has relatively small contribution on aberration correction, but in the situation that the degrees of freedom are limited, thickness is the only weapon for aberration correction. In Addition, based on previous section, properties of elements far away from pupil has more contribution on high field off-axis aberration, however in this design, thickness at last element has much more contribution on lateral color correction.

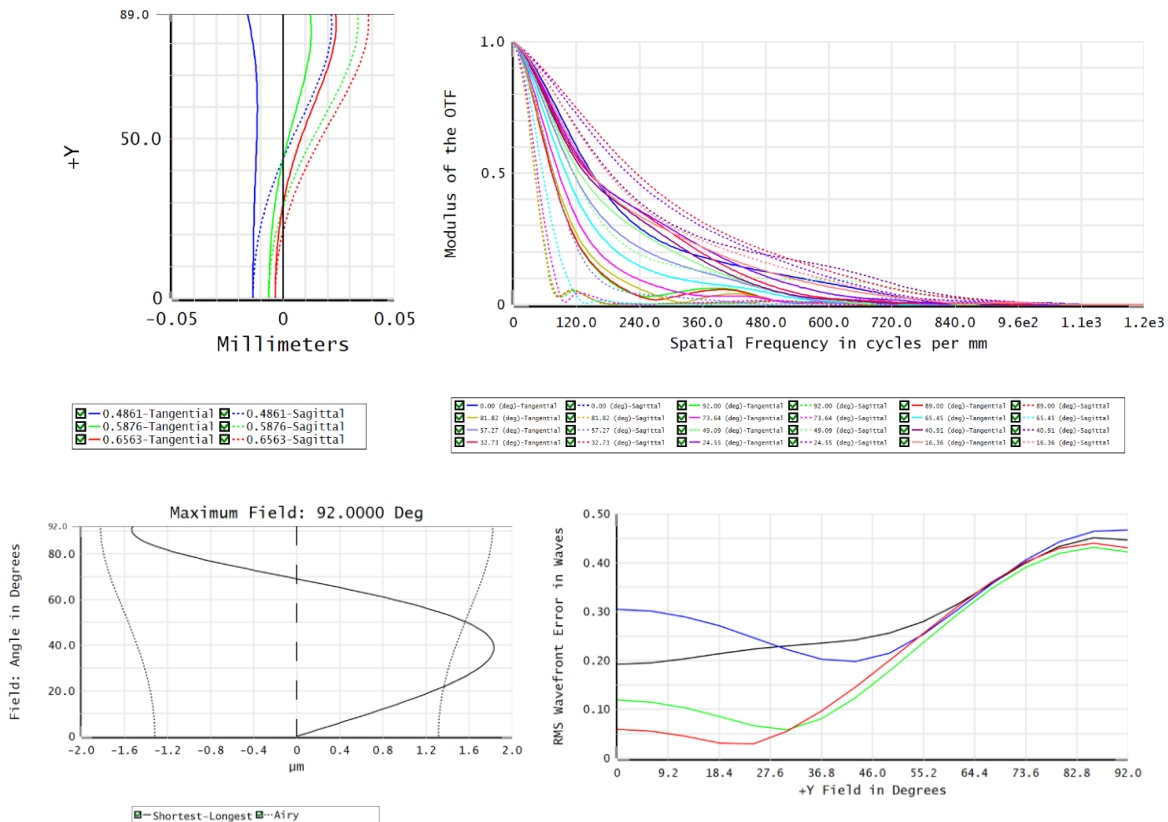


Figure 3-16 performance of 5th design

Figure 3-16 indicates that most aberrations are controlled in this design, however the lateral color at 40 degrees of field is little larger than airy disk. The lateral color around half field is always the problem for all the fisheye adapter design that have only spherical surfaces.

3.6 Relative illumination analysis

As discussed in previous chapter, the relative illumination of fisheye lens does not follow the $\cos^4(\theta)$ law, the big pupil coma aberration enlarges the entrance pupil size in object space so that the light from edge field has larger cross-section at their entrance pupil.

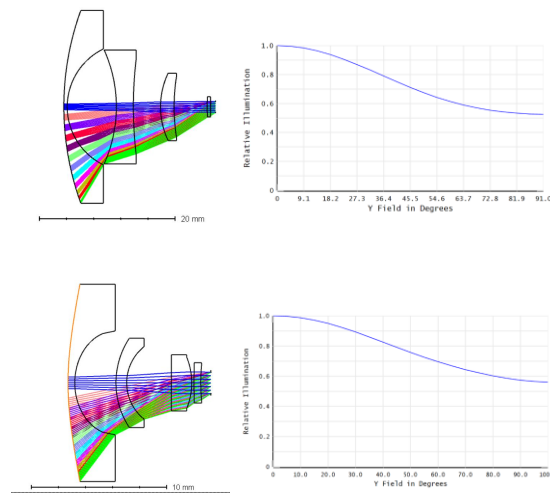


Figure 3-17 relative illumination of previous designs

The plots above show the relative illumination curves for the designs in previous section. Designs with positive or less negative power at the front surface of second element has better relative illumination at maximum field of view. For the 4th design, even the maximum field of view is upon 200 degrees, the edge relative illumination is over 55%. The reason is positive front surface has better ability to converge the incoming beam, although the later chromatic aberration and higher order astigmatism is difficult to control with this surface.

4 Performance test of Fisheye lens adapter in market

The performance of fisheye lens adapter for smartphone camera was presented and characterized in previous chapters. In this chapter, the MTF performance of the entire system consisting of a market available fisheye adapter and smartphone camera was measured utilizing slanted edge tests. Because of the large field of view of the fisheye lens, MTF varies significantly at different fields of view so that it should be sampled at different angle.

4.1 Slanted edge test

The method called the slanted edge test is used to measure the spatial frequency response of the entire system. This is one of the most efficiency and important methods to test the image quality in practice and has been recognized by IEEE and ISO.

In the slanted edge test, a picture of a sharp edge is taken and an Edge Spread Function (ESF) is obtained. Then the Line Spread Function (LSF) is accessible by taking the derivative of ESF. Finally, the local MTF can be obtained by taking the one-dimensional Fourier Transform of the ESF.[9]

4.2 System Setup

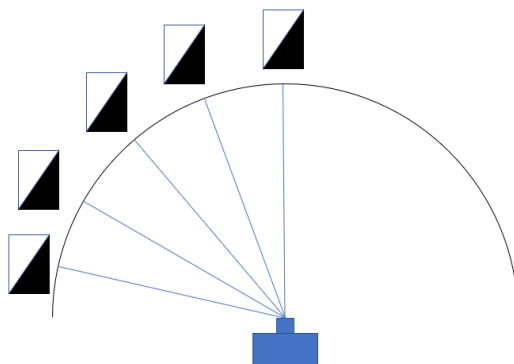
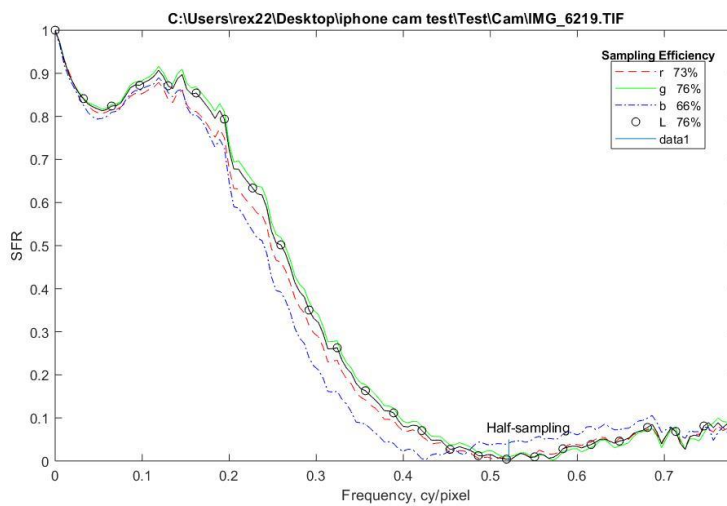


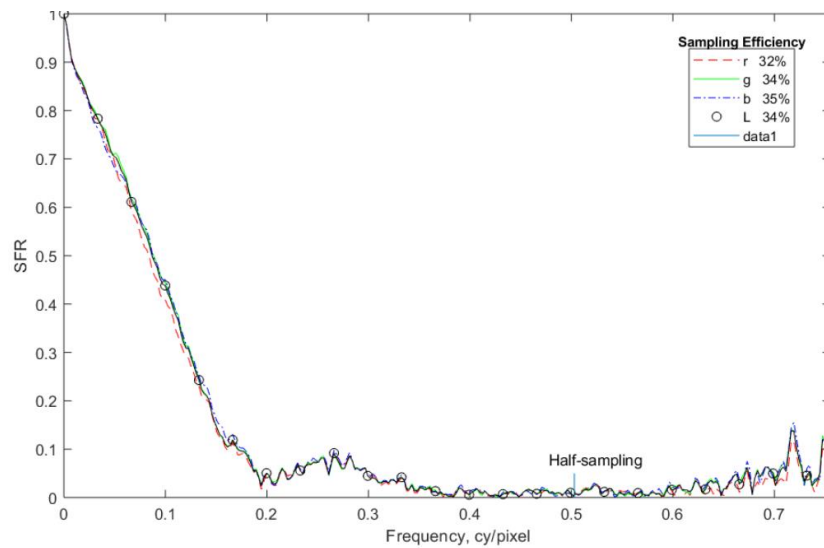
Figure 4-1 setup of slanted edge test

Figure 4-1 illustrates the setup of the slanted edge test for fisheye lens adapter. Since the significant difference between different field of view, several edge tests are required to describe the total performance.

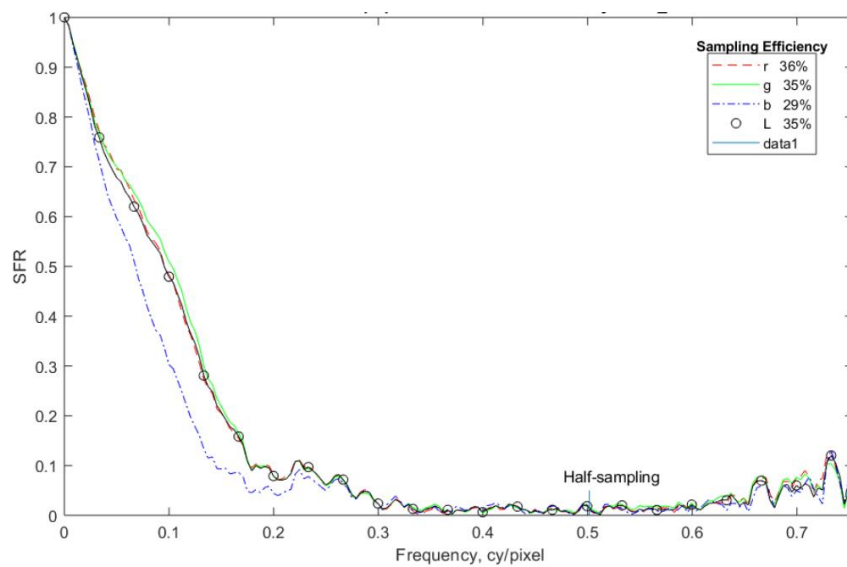
4.3 Solution



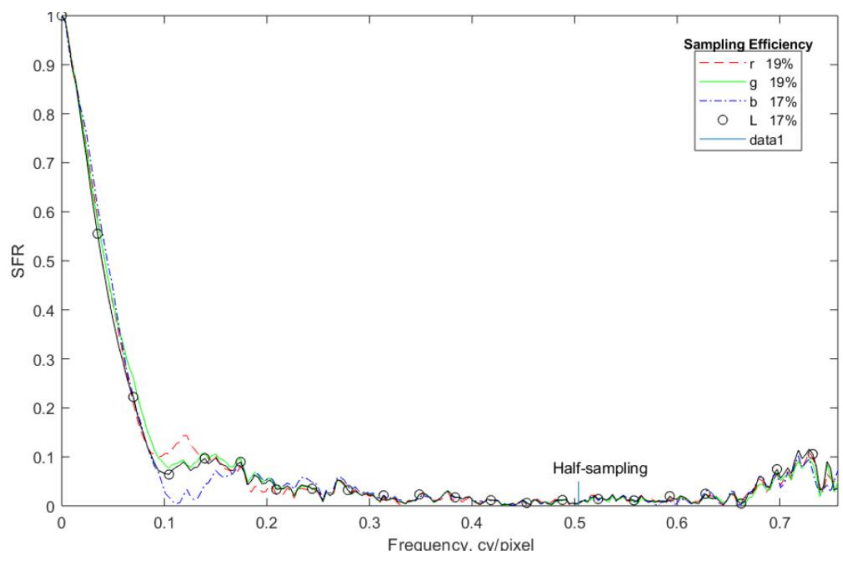
(a)



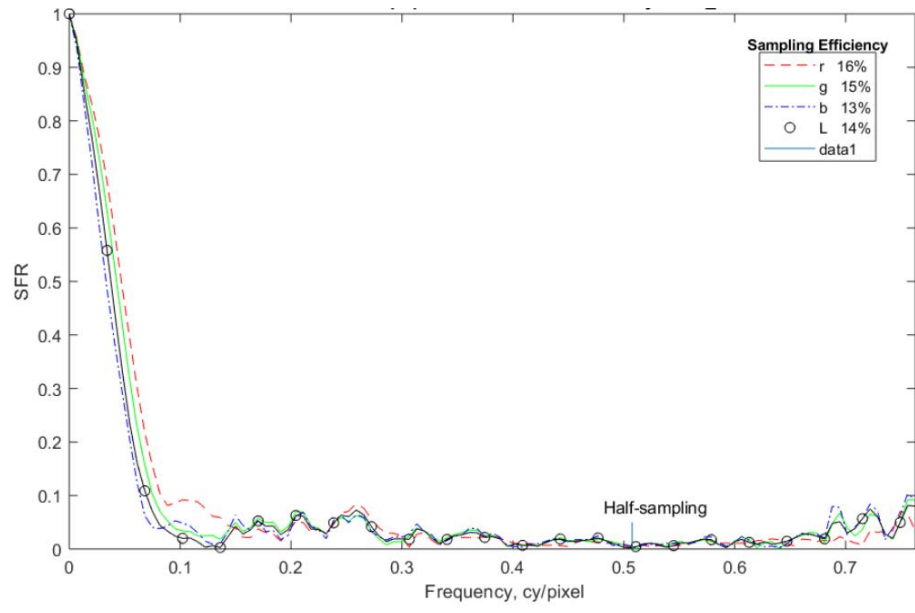
(b)



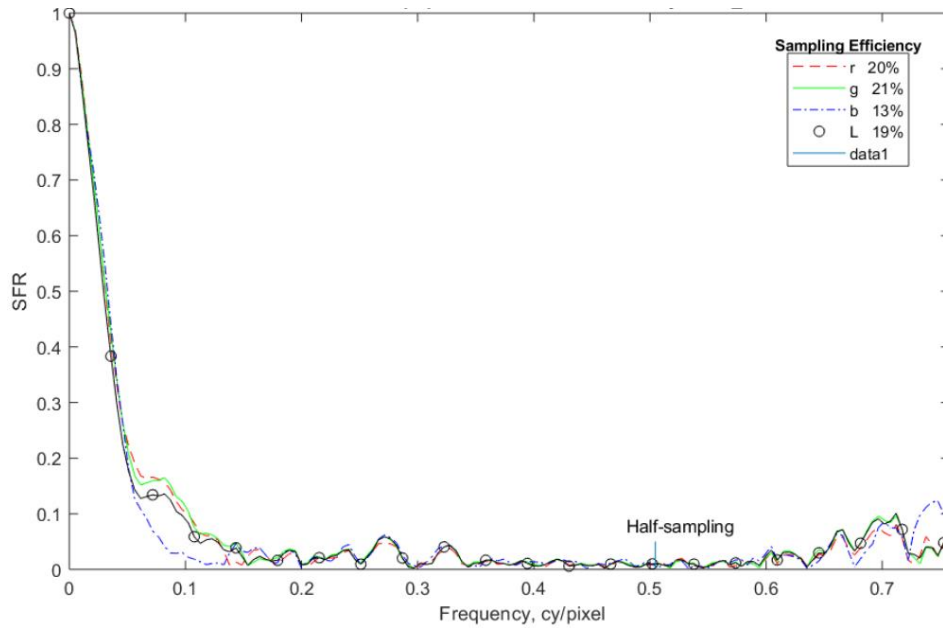
(c)



(d)



(e)



(f)

Figure 4-2. SFR for (a) camera of iPhone 6s and (b)-(f) center field to edge field with fisheye attachment

The pixel size of the camera utilized in this test is $1.22\mu\text{m}$, thus the cut off frequency of this adapter at zero field is 163.93 cycles/mm, and 81.967 cycles/mm at the edge of the field. Compared to the MTF of the designs in the previous chapter, this commercial fisheye lens adapter has a poorer resolution performance. However, this could be the result of misalignment of the adapter lens during the installation process.

5 Conclusion

5.1 Summary of work

In this thesis, the historical development, the relationship between focal length and field of view, aberration properties, and projection methods for both fisheye lenses and fisheye adapters are discussed. Ideas for lens systems with an extremely wide field of view started with Snell's Window and pinhole cameras. During the process of designing systems with a wide field of view, the earlier concept involved a short-focus symmetric lens; however, based on its design limitations, it is not possible to get a field of view as large as 180 degrees with the required BFL. Thus a structure called retro-focus was invented to increase both the field of view and BFL; this is still very widely used today.

Because smartphone cameras have become more and more popular in daily photographic activities, fisheye lens adapters are increasingly in demand for panoramic photography. This thesis presents five different designs for a four-element fisheye lens adapter for smartphone cameras. The primary distinguishing characteristic is the shape of the second lens. The first design type introduces a second lens with a strong negative first surface, which effectively balances higher order astigmatism but has a smaller relative illumination at the maximum field of view. The second design type utilizes a relatively flat first surface on the second lens and a large first element, resulting in greater illumination but with more higher order astigmatism. Two versions are presented, one utilizing plastic and the other featuring glass. The third design type utilizes a meniscus second element with a positive surface facing the incoming beam, which significantly improves the field of view to 200 degrees and increases the relative illumination up to a value of 55%; better materials or more degrees of freedom from aspheric surfaces are

required to balance the aberration caused by this structure. Two versions, one plastic and one glass are compared. When compared with plastic designs, glass designs always have better performance since a wider variety of materials are available during the design process. However, plastic materials cost less and are more practical in fabrication that involves aspheric surfaces.

5.2 Future Direction

Tolerances for fisheye adapters are not discussed in detail in this thesis. However, since it is a system that is attached by customers, tolerances are very important for fisheye lens adapters, especially for any decentering of the stop location. In addition, based on material and element number limitations, it is challenging to design a system with very good performance over the full field. There are always some tradeoffs among properties, such as relative illumination and lateral color, or magnification and total length. Further research is needed to refine and enhance fisheye adapter designs.

6 APPENDIX: System Data and Specification

6.1 1st design

Surface Type	Radius	Thickness	Material	Conic	Par 2(unused)	Par 3(unused)	Par 4(unused)
C Standard ▾	Infinity	Infinity		0.00000			
Even Asphere ▾	20.14400	1.00037	KZFSN2	-116.698...	-5.46160E-05	-1.25145E-07	0.00000
Even Asphere ▾	5.04781	2.07141		0.45729	-1.17133E-03	1.32490E-04	-1.57670E-07
Even Asphere ▾	-23.53766	2.08522	LF5HT	7.25752	6.57529E-05	-2.13483E-05	4.96451E-07
Even Asphere ▾	3.10614	0.18180		-0.06386	-2.14471E-03	-2.15818E-05	4.60190E-06
Even Asphere ▾	3.27708	2.82256	F2	-0.02016	6.77282E-04	-7.11506E-05	-4.98836E-06
Even Asphere ▾	-14.48843	0.18249		-10.78186	5.77094E-04	-3.12481E-05	4.30490E-05
Even Asphere ▾	-12.02947	1.94469	N-PSK57	-13.43307	1.09845E-04	3.41888E-04	-2.79913E-06
Even Asphere ▾	-9.02325	0.75000		-9.58614	1.92543E-03	-1.69853E-03	2.13973E-04

Magnification	0.607
Field of view(degrees)	180
Total Length(mm)	11.03854
Maximum Diameter(mm)	11.4

6.2 2nd design

Surface Type	Comment	Radius	Thickness	Material
OB Standard ▾		Infinity	Infinity	
Even Asphere ▾		41.397... V	0.50000	F52R
Even Asphere ▾		9.46891 V	7.23257	
Even Asphere ▾		-20.28... V	2.50000	F52R
Even Asphere ▾		59.380... V	3.99998	
Even Asphere ▾		11.536... V	2.00000	OKP-A2
Even Asphere ▾		28.512... V	4.87052	
Even Asphere ▾		36.559... V	0.50000	PMMA
Even Asphere ▾		-142.2... V	0.75000	

Magnification	0.520
Field of view(degrees)	180
Total Length(mm)	22.35308
Maximum Diameter(mm)	15.2

6.3 3rd design

Surface Type	Comment	Radius	Thickness	Material
OB Standard ▾		Infinity	Infinity	
Even Asphere ▾		41.397... V	0.50000	N-PSK58
Even Asphere ▾		6.97741 V	4.47886	
Even Asphere ▾		-11.74... V	2.30180	N-PSK58
Even Asphere ▾		-50.87... V	1.24253	
Even Asphere ▾		10.738... V	2.00003	SF5
Even Asphere ▾		25.990... V	2.55681	
Even Asphere ▾		74.580... V	0.50000	BK7
Even Asphere ▾		-22.03... V	0.50000	

Magnification 0.5568
 Field of view(degrees) 185
 Total Length(mm) 14.08
 Maximum Diameter(mm) 17.21736

6.4 4th design

Surface Type	Radius	Thickness	Material	Conic	6th Order Term	8th Order Term	10th Order Term
OB Standard ▾	Infinity	Infinity		0.00000			
Even Asphere ▾	19.81233	0.50000	F52R	-16.98420	-1.19428E-07	6.42062E-10	3.58886E-11
Even Asphere ▾	4.93235	3.06396		0.57058	-3.02136E-05	2.21553E-09	6.55132E-08
Even Asphere ▾	6.03239	0.62275	F52R	0.45758	1.34702E-06	-7.05798E-08	1.58207E-08
Even Asphere ▾	3.39711	3.57120		-0.08394	-7.27240E-06	1.06128E-06	5.70375E-08
Even Asphere ▾	-89.31336	2.10163	OKP-A2	595.19747	-4.51158E-05	-5.29364E-06	5.87934E-07
Even Asphere ▾	-5.14194	0.06495		-0.21855	4.24761E-05	-7.53440E-06	-1.75726E-05
Even Asphere ▾	601.01660	0.43603	OKP-A2	1.92497E+05	-4.22025E-04	-1.58462E-04	-2.00261E-05
Even Asphere ▾	15.37860	0.79632		-11.40220	-9.78696E-05	-1.47719E-04	-2.84013E-04

F/# 1.771232
 Field of view(degrees) 200
 Total Length(mm) 11.15684
 Maximum Diameter(mm) 14.42304

6.5 5th design

Surface Type	Comment	Radius	Thickness	Material
OBJECT Standard ▾		Infinity	Infinity	
Standard ▾		154.78859	0.49552	PSK50
Standard ▾		6.16035	4.03044	
Standard ▾		25.14889	2.00001	BK7
Standard ▾		17.28319	2.35763	
Standard ▾		97.01416	2.03951	N-SF5
Standard ▾		-8.00978	0.17010	
Standard ▾		-15.54720	1.99994	N-KZFS8
Standard ▾		-53.24033	0.97131	

F/# 1.837281
 Field of view(degrees) 184
 Total Length(mm) 14.06
 Maximum Diameter(mm) 15.5

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