PHOTOGRAPHIC FISHEYE LENS DESIGN FOR 35MM FORMAT CAMERAS

by

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DEDICATION

To my parents: Xueping and Zhiming

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ABSTRACT

Fisheye lenses refer to ultra-wide angle lenses that have field of view equal or larger than 180 degrees. Such lenses introduce large amount of barrel distortion to capture at least the entire hemisphere in front of the lens. Fisheye lenses were initially designed for scientific use, such as cloud recording and angle measuring, and were widely used for commercial purposes later. The development of photographic fisheye lenses started in 1960s. However, the lack of detailed references on photographic fisheye lens design makes such design challenging. This thesis provides detailed introduction of photographic fisheye lens design for 35mm format DSLR cameras. A discussion on the history of fisheye lenses is provided to describe the development of fisheye lenses. The tangential and sagittal magnifications are mathematically derived for each fisheye lens projection mapping method to show their differences. The special properties and design issues of photographic fisheye lenses are described in detail. Along with each design issue, some solutions suggested by the author are also provided. The performance of the current diagonal fisheye lenses for 35mm DSLR cameras are evaluated in detail. Then a new diagonal fisheye lens designed by the author is presented and compared with the current diagonal fisheye lenses on the market. Finally, a zoom fisheye lens designed for 35mm DSLR cameras is presented and discussed.

1 INTRODUCTION

A fish's eye underwater can see the entire hemisphere above the water, as such, the field of view of this eye in object space can reach 180 degrees. With appropriate optical design, humans may acquire this fish eye field of view using special lenses. Lenses capable of this extremely large field-of-view (FOV) are referred to as "fisheye lenses."

Since the first fisheye lens, the "Hill Sky Lens" [1] that was introduced in 1924, fisheye lenses have been in development for almost an entire century. The history of a simple fisheye simulation can even go back further to the first fisheye pinhole camera developed by R. W. Wood in 1906 [2]. Many great fisheye lenses were designed in the past century, and the designs have improved, especially on image quality. Due to the pool aberration control, the 'Hill Sky Lens' were only designed to work preferably at F/32, and not to exceed F/22. In addition, this lens was not able to correct for chromatic aberration, so it can only be used for a specific wavelength with a color filter. Today, modern fisheye lenses for commercial cameras can capture very sharp polychromatic image with a maximum aperture of F/2.8. However, despite the great development of fisheye lenses in the past century, there were not many references other than the patents on the topics of fisheye lens design. Even in the standard books for lens design, such as Lens Design (Milton Laikin, 2006) [3] and *Modern Lens Design* (Warren J. Smith, 1992) [4], the topics of fisheye lens were only mentioned in few sentences, while the topics of other lens designs were discussed in detail. Besides that, the corresponding English patents are also hard to acquire. The current photographic lens designs are almost completely dominated by the Japanese companies such as Nikkor (Nikon), Canon, Minolta (Sony), Sigma, Fuji, Pentax and Olympus. Many of their fisheye lens designs were only patented in Japan. A good reference that specifically talks about fisheye lens designs in English is in need. This has become a big motivation for the author to write this thesis report.

1.1 The Use of a Fisheye Lens for Photographic Purpose

Although the initial purpose of fisheye lenses is to record the cloud in the sky, due to their special properties, fisheye lenses are also widely used in many other different areas today, such as sky projection and dome movie projection in the planetariums, surveillance cameras, and military defense. The special fisheye effect with large distortion also drew attentions from the photographers. As a result, fisheye lenses for photographic purposes were introduce since 1960s, and became the most common use of a fisheye lens today.

Based on how the image circle fulfills the camera sensor, photographic fisheye lenses can be categorized by two different types, including the diagonal fisheye lens and the circular fisheye lens.

1.1.1 Diagonal Fisheye Lenses

A diagonal fisheye lens refers to a fisheye lens that has its image circle just fulfills the entire camera sensor. For this kind of fisheye lens, the image height should be equal to the length of the sensor diagonal in order to achieve a 180-degree field of view over the diagonal. For a 35mm 'full frame' DSLR camera, the sensor size is 36 mm by 24 mm. This sensor size is considered standard by most camera manufactures today. Then the diagonal fisheye lens designed for a 35mm format camera has image height equal to 43.2mm to fulfill the entire 35mm sensor [Fig 1.1].



Figure 1.1. Image circle of a diagonal fisheye lens for a 35mm sensor, the shaded parts of the circle are cut off by the sensor.



Figure 1.2. Image circle of a diagonal fisheye lens for an APS-C sensor, the shaded parts of the circle are cut off by the sensor.

Another popular sensor format for DSLR cameras today is the APS-C (Advanced Photo System Type-C) format that was introduced to the industry in 1996. This format was developed by several camera companies (Canon, Fuji, Kodak, Minolta and Nikon) together and is approximately 1.5 times smaller than the 35mm 'full frame' format. Many camera companies such as Nikon and Pentax also designed diagonal fisheye lenses for their APS-C format cameras. For those third party companies, such as Sigma and Tamron, who design fisheye lenses for multiple camera brands, they have to be caution with the image height since the APS-C format sensors are less standard and their diagonal lengths are varied

between different camera brands. Nikon, Sony and Pentax have similar diagonal lengths around 28.4mm for their APS-C sensors, while Canon uses a smaller APS-C sensor with a 26.7mm diagonal length [Fig 1.2]. This 1.7mm short of the diagonal length for the Canon APS-C sensor may not be a big problem for regular lens design. However, for fisheye lenses with such large 180-degree field of view, a 1.7mm short on the sensor diagonal can cause a 10-degree loss of the field of view. Thus, some minor adjustments for the design are needed to make sure that the image height of the lens matches the diagonal length of the APS-C sensor for the specific camera.

For the diagonal fisheye lenses, since the 35mm sensor and the APS-C sensor are both rectangular, the sensor will physically cut off part of the image circle (the shaded part in Fig 1.1 and Fig 1.2). Because these lenses are designed to have 180-degree field of view on the diagonal, the actual horizontal and vertical field of view of the rectangular image are smaller than 180 degrees. The exact horizontal and vertical FOVs are depend on the fisheye lens projection method that is going to be covered in Chapter 3. A sample image taken by a diagonal fisheye lens is provided in Fig 1.3.



Figure 1.3. A sample Image taken by a diagonal fisheye lens

1.1.2 Circular Fisheye Lenses

A circular fisheye lens refers to a fisheye lens that fits its entire image circle on the camera sensor. For this kind of fisheye lens, the image height should be equal or slightly less than the sensor width, so the entire hemisphere in front of the camera can be captured on the picture. Since the image sensor of a DSLR camera is rectangular and larger than the fisheye image circle, it will receive no signal outside that circle. Those areas outside the fisheye image circle will appear to be black on the final image. Fig 1.4 demonstrate a photo taken by a circular fisheye lens.

For a 35mm format DSLR camera, the sensor width is 24mm. Thus, the image height of a circular fisheye lens should be equal or slightly less than 24mm. Recall that the image height of a diagonal fisheye lens for an APS-C camera is around 26.7mm to 28.4mm, which is very close to the image height of a circular fisheye lens for 35mm camera. Therefore, a diagonal fisheye lens for APS-C format camera and a circular fisheye lens for 35mm format camera with same projection method usually have similar specifications.

For an APS-C format DSLR camera, the sensor width is only 14.9 mm to 15.8 mm depends on different camera manufactures. To produce such small image height, a circular fisheye lens for an APS-C format camera needs to have very short focal length, usually shorter than 5mm.

1.1.3 Current Photographic Fisheye Lenses on the Market

Currently, due to the large distortion, fisheye lenses are generally less popular than traditional camera lenses and are rarely used for professional photography. However, the fisheye lenses do attract a specific group of photographers and artists who are looking for



Figure 1.4. A sample Image taken by a circular fisheye lens

this large distortion and field of view on purpose. The popularity of 35mm format photographic fisheye lenses reached its peak in 1980s. In fact, most of the 35mm format diagonal fisheye lenses were designed in 1980s. However, the number of photographic fisheye designs were decreased after 1990s, and most fisheye lenses designed after 1990s are for the newly developed APS-C format cameras. For example, Nikkor released 10 different fisheye lenses for their 35mm SLR cameras from 1962 to 1994. Since that, only one fisheye lens was introduced and it is for their APS-C format cameras [5]. Nevertheless, many new patents of photographic fisheye lens design were filed recently. With the DSLR cameras getting more affordable and popular today, the author believes that photographic fisheye lens designs for DSLR cameras still have a great potential today.

Out of the current fisheye lenses on the market, diagonal fisheye lenses are more common than the circular fisheye lenses, even though the first fisheye lenses that were introduced before 1970s are mostly circular fisheye lenses. The circular fisheye lens designed for APS-C format cameras are even rarer. The extremely short focal length makes the lens design very hard due to the much larger BFD requirement of a DSLR camera. The effect of the BFD requirement to fisheye lens designs will be covered in Chapter 5.

Besides the diagonal fisheye lens and the circular fisheye lens, a zoom fisheye lens combines both kinds into one single lens. In 2010, Canon used an 8-15 mm zoom fisheye lens to replace its diagonal fisheye lens product line. However, zoom fisheye lenses are also very rare. The design of a zoom fisheye lens will be covered in Chapter 7.

1.2 Thesis Content

Chapter 2 provides a detailed discussion on the history of the fisheye lenses, from the first fisheye pinhole camera to modern photographic fisheye lenses. Many designs analyzed in this chapter are important milestones in the fisheye design history. Study these designs helps readers understand the form of structures of the modern fisheye lenses.

Chapter 3 describes the four different fisheye projection methods. The tangential magnification and sagittal magnification for each projection method are mathematically derived to describe what the image look like for each projection method. The distortion of each projection method are also compared and explained.

Chapter 4 discussed the unique properties of a photographic fisheye lens.

Chapter 5 discussed the design issues for a fisheye lens. Some suggestion to solve these issues are also provided. These issues are based on both author's experience during the fisheye lens design, and research on literatures. Some comments on the previous literatures are also presented. In Chapter 6, the author first analyzes the current diagonal fisheye lenses for 35mm DSLR cameras on the market. Then, a diagonal fisheye lens designed by the author is presented. The performances are analyzed and compared to the current diagonal fisheye lens designs.

In Chapter 7, the author discusses about zoom fisheye lenses. The motivation to design a zoom fisheye lens and the current designs are provided in this chapter. A brief discussion on the two-component zoom lens structure is also presented. In the last section of this chapter, author's design of a zoom fisheye lens is introduced along with the performance evaluations.

A conclusion that summarizes this thesis report is given in Chapter 8. In addition, some suggestions for future works are also provided by the author.

In Appendix A1, the macro that is used to generate the cam curve for the zoom fisheye lens is provided.

2 THE HISTORY BEHIND FISHEYE LENSES

Fisheye lenses have a long history that goes back to 1924, when Hill designed the famous "Hill sky lens". The fisheye pinhole cameras that simply simulate the fisheye under water were introduced even earlier. This chapter covers the history from the inspiration of a fisheye, to the modern photographic fisheye lenses.

2.1 The Inspiration of a Fisheye

As well known, a submerged fish points its eye directly at the surface of water sees objects above the water surface compressed into a circular image. The edge of the circular image would be distorted. However, the circular image does contain everything embraced within the complete hemisphere above the water surface. In another word, the fisheye has a field of view of 180 degrees in every direction in air. This can be explained by the Snell's law, which is

$$n\sin\theta = n'\sin\theta' \tag{2.1}$$

where n = refractive index of the incident material

n' = refractive index of the emergent material

 θ = angle of incidence

 θ' = angle of emergence

When light from the water enters the air [Fig 2.1(a)], since the refractive index of water (n = 1.333) is larger than the refractive index of air (n['] = 1), the angle of emergence θ' is always larger than the angle of incidence θ . According to Eq. (2.1), if $\theta' = 90^\circ$, the corresponding angle of incidence is called the critical angle θ_c , and can be calculated by

$$\theta_c = \sin^{-1} \frac{n'}{n} \tag{2.2}$$

Substitute the indexes of refraction to the equation above, then

$$\theta_c = 48.6^{\circ}$$

Based on the fact that rays are reversible, the light enters the water with an angle of incidence of 90 degrees has an angle of emergence equal to the critical angle θ_c [Fig 2.1(b)]. Thus, put a sensor or film under the water can record the entire hemisphere above the water, just like a fisheye. R.W.Wood used this model and created the first fisheye pinhole camera in the world. Detail on Wood's design is covered in the next section.

The explanation above proves that the fisheye can be simulated and can be useful to human being. However, a human eye underwater cannot see the same image as a fisheye. The focus is so poor that almost nothing above the water can be distinguished. The poor image quality of Wood's pinhole camera also proved that other optical elements were needed under the water in order to bring the image to focus. If we consider the fisheye and water to be parts of an entire optical system, then the front surface of the fisheye and the water surface can be treated as a negative thin lens with material of water [Fig 2.2 (a)].



Figure 2.1. Refraction of a ray at critical angle



Figure 2.2. Evolution of the negative meniscus lens in fisheye lenses [6]

Since the first surface of a fisheye usually has a small radius of curvature, the negative lens in Fig 2.2 (a) has very large negative power. If the water is substitute with glass material [Fig 2.2 (b)], then people no longer needs to simulate the fisheye under the water. Thus, an optical system with the field of view of an entire hemisphere can be achieved in air.

However, the plano-concave lens in Fig 2.2 (b) has some flaw. The light come from $\pm 90^{\circ}$ or near $\pm 90^{\circ}$ (Ray A) barely has any energy flux density go through this lens, and will not be detected by the sensor at image plane. This will cause a very low relative illuminance at the edge of the image. To fix this problem, the first surface is replaced by a concave surface, and the curvature of the second surface is increased to maintain the same optical power. The lens becomes a negative meniscus lens [Fig 2.2 (c)]. In this case, the light come from 90° (Ray B) will have enough energy flux density in the image space. This negative meniscus lens then can be used as the first element of a fisheye lens.

With the first element designed, lens designers then use other lens elements to bring the image to focus. R. Hill [1] first used this negative meniscus lens structure and successfully designed a fisheye lens for cloud recording. This lens was then later to be called the 'Hill sky lens' and agreed to be one of the greatest milestone in fisheye lens design history.

The shape of the negative meniscus lens is a great inspiration from the fisheye and it is still being used as the first lens element for every fisheye lens today. The convex front surface of this lens not only ensures that enough light at $\pm 90^{\circ}$ can go through the lens, it also provide the possibility of an optical system with the field of view larger than 180 degrees. As a matter of fact, there are many fisheye lens designs provide field of view larger than 180 degrees, some of them even reach 310 degrees [7]. A few of these hyperfield fisheye lenses can even be found on the market for consumers, such as the Nikkor 6mm F/2.8 camera lens for 35mm SLR cameras patent by Nikkor [8]. This lens has a full FOV of 220 degrees and was introduced to the market in 1972. This lens is still the world's widest fisheye camera lens.

2.2 The Simple Simulation of a Fisheye

The reason why fish can see the entire hemisphere above the water surface can be explained by Snell's law in the last section. Just like any other bionics designs, to create an optical system that can achieve the same large field of view as a fisheye, the first thing come to mind is to simulate the fisheye under the water and try to make an imitation of this model. Based on this, R.W. Wood designed the first fisheye camera in 1906 [Fig 2.3] [2]. He put the photographic plate (image plane) in a water-tight box and filled the box with water. On the top of the box, there was a small pinhole, which made the entire box a water filled pinhole camera. Then he put a piece of glass on the top of the box to seal the pinhole so the water camera could be pointed at the horizontal direction without water leaking out of the pinhole. Wood used his water camera to capture a picture of nine men standing in front of the camera to show how visitors at an aquarium appear to the fishes [Fig 2.4]. He also pointed out that this kind of fisheye camera can be used as a sunshine recorder, which would require no adjustment for latitude or month.



Figure 2.3. The pinhole fisheye 'water camera' designed by R. W. Wood [2]



Figure 2.4. A picture took by Wood's fisheye camera to simulate how visitors at an aquarium appear to the fishes

Although Mr. Wood's water camera was only a simple simulation of the fisheye, it proved that the process of light entering a fisheye could be reproduced by human, and such fisheye camera could be used to benefit people. However, carrying such a 'water box' around was not very convenient. In 1922, W.N.Bond introduced a more practicable design of fisheye camera that contained no water [9]. In his design, he used a single piece of glass with the shape of a hemisphere to replace the water in Wood's design [Fig 2.5]. The aperture of the camera was located at the center of the hemisphere and was very small. Thus, all light went into the camera would approximately go through the center of the hemisphere lens, forming an image that focused at an almost hemisphere image plane. Bond's fisheye camera also had a full FOV of 180 degrees and could be used for cloud recording. A photo of the entire sky captured by this fisheye camera was attached with the design by Bond [Fig 2.6].



Figure 2.5. The fisheye camera designed by W.N.Bond [9]



Figure 2.6. A photo of the entire sky captured by by W.N.Bond's fisheye camera [9]

From the modern lens design point of view, Bond's fisheye camera had a very poor performance with a single hemisphere lens. He did not use any additional lens to control the aberration at all, so the aperture of the system needs to be small enough to produce a fairly low aberrated image. This made his system very slow (about f/50). Even with the small aperture, the field curvature of the final image was still not corrected. The only compensation was to introduce defocus by putting the flat photographic plate in front of the actual curved image plane [Fig 2.5]. Also, both Wood's and Bond's design did not consider the relative illumination of the image. Thus, the image is mostly out of focus and the image quality is not great. With the first surface being flat, the relative illumination would drop rapidly at the edge of the image. Then the useful FOV of their designs were actually smaller than 180 degrees. Despite the poor performance of Bond's fisheye camera, it was still a great innovation in the fisheye lens design history. It demonstrated to people that water is not necessary to be involved in fisheye lens design anymore. With more glass elements for aberration control, high quality fisheye lenses are possible. Besides that, even though his actual design ignored the issue of relative illumination, he did propose in the end of his paper that a plano-convex lens can be placed in front of the aperture to increase the field of view to more than 180 degrees. With the curved surface in the front instead of a plane surface, the relative illumination could be also improved.

2.3 The Hill Sky Lens

The fisheye camera designed by W. N. Bond demonstrated that a reproduction of fisheye view could be achieved without water involved. However, Bond did not consider any aberration correction during his fisheye camera design. In 1924, shortly after Bond published his fisheye camera, R. Hill presented a much improved fisheye lens system in his paper 'A Lens for Whole Sky Photographs' [Fig 2.7] [1]. In his design, he first added a negative meniscus lens in front of the stop, which reduced the maximum ray angle from ± 90 degrees to about ± 60 degrees. The reduced ray angle made the aberrations easier to control with the lenses behind the stop, and prevented the relative illumination from falling off too much at the edge of the image. He also separated the plano-convex lens after the stop into two single lenses to increase the degree of freedom for better aberration control. With this set up, coma, astigmatism and field curvature were well controlled. However, the lack of control of the spherical aberration was still constraining the speed of the lens. The lens was designed to use at F/32, and could be increased to F/22 without serious image quality fall off. The exposure time to take a photo of a well-lighted sky was about 1 second.

The speed of this lens was very slow compare to modern camera lenses, but it was still about 5.4 times faster than Bond's fisheye camera, and was fast enough for cloud recording. However, Hill did not use any method to specifically control chromatic aberration. A color filter had to be used and only monochromatic photos could be captured with this lens.

Not like Wood's and Bond's fisheye camera, Hill's fisheye lens was not only a simple simulation of a fisheye anymore. Hill attempted to improve the image quality by controlling the aberration and maintaining the relative illumination. This lens was later becoming commercially available manufactured by Beck of London [10]. To honor Hill's contribution to modern fisheye lens design, this lens is often referred as the "Hill sky lens" today and is credited as the first prototype of the modern fisheye lens. The negative meniscus shape of the first lens element and the reverse telephoto layout of this design are used in every fisheye lens design ever since.



Figure 2.7. The layout of 'the Hill sky lens' designed by R. Hill [1]

In R. Hill's paper published in 1924, he also mentioned there are three mapping method to project a hemisphere on a plan surface, which are stereographic projection, equidistant projection and orthographic projection. These three projection methods, along with another mapping method called equisolid angle projection, will be further explained in detail in the chapter 3.

2.4 The AEG fisheye lens

The Hill sky lens showed significant improvement in optical performance compare to the previous fisheye cameras. Hill used the negative meniscus lens in the front to reduce the maximum ray angle, then controlled astigmatism and field curvature with an additional negative lens after stop. However, he did not intend to control spherical aberration, coma and chromatic aberration. Even though the coma aberration in his lens turned out to be unexpectedly small, spherical aberration and chromatic aberration were still a problem, which caused the aperture to be very small and only monochromatic photos can be taken. Clearly, there was still a lot of room for improvement in fisheye lens design.

In 1932, the AEG Company (Allgemeine Elektricitats-Gesellschaft) in Berlin filed a patent for its new fisheye lens [Fig 2.8] [11]. It contains 5 lens elements in 4 groups. Two negative meniscus lenses were used in front of the stop instead of one. The additional meniscus lens did not only reduce the maximum ray angle for better aberration control, it also increased the maximum field of view of the lens. According to the patent file, this lens could achieve a 210-degree field of view while maintaining good image quality. Behind the stop, an achromatic doublet was used to correct chromatic and other aberrations. Even though lateral color were still existing in this design, this was the first attempt to correct



Figure 2.8. The layout of the AEG 17mm F/6.3 fisheye lens [11]

chromatic aberration in the fisheye lens design history. Besides chromatic aberration, other aberrations were also controlled by this design, which increase the maximum aperture to F/6.3, which was about 11 times faster than the Hill sky lens.

The AEG fisheye lens is a remarkable design in the fisheye lens history. Some design methods in this lens are continued to use for many modern fisheye lens designs today, such as the double meniscus lens in the front, and the achromatic doublet behind the stop. The fast speed of f/6.3 allowed much shorter exposure time than the previous fisheye camera designs, so the fisheye lenses can be eventually used for normal photography rather than just meteorological photography. Unfortunately, such great fisheye design was rarely mentioned and studied in any references. The longest reference on the AEG fisheye lens is a single sentence comment from R. Kingslake's book 'A History of the Photographic Lens' [10]. In the chapter of 'fish-eye or sky lenses', Kingslake referred the AEG lens as a more elaborate design than the Hill sky lens. The publicity of this lens may be impact due to its major usage by the German military in 1930s and the fact that the designer did not publish any paper talking about the design like Wood, Bond and Hill.



Figure 2.9. The comparison between the AEG and Nikkor fisheye lenses

Many people believe that this design was shared by the German government to their ally Japan, and later became Nikon's first fisheye lens (16mm F/8) in 1938. This Nikkor (a sub-brand of Nikon) lens was used by the former Imperial Japanese Navy for meteorological observation. It was later improved by Nikkor and still listed on their website until today [12]. Although no evidence can be found to prove that the Nikkor fisheye lens was a modification of the AEG fisheye lens. However, these two designs were almost identical by comparing the layout of these two lenses [Fig. 2.9].

2.5 The development and future of photographic fisheye lenses

Since 1960s, with the help of computer optimization, fisheye lenses were well developed. Many great designs and innovations were introduced in the past half century.

For examples, 1n 1964, K. Miyamoto from Nagoya University published a paper talking about a fisheye lens he designed when he was working for Nikkor [Fig. 1.10] [13]. This 8mm F/8 lens was Nikkor's earliest fisheye design for their 35mm SLR cameras.



Figure 2.10. The Nikkor 8mm F/8 fisheye lens [13]

For the first time in the fisheye lens design history, lateral color aberration had been controlled, due to another achromatic doublet in front of the stop. Since then, fisheye lenses were not limited for scientific use and became popular in photographic use.

In 1968, Nikkor released an OP 10mm f/5.6 fisheye lens for their 35mm SLR cameras [14]. This unique lens was the world's first interchangeable orthographic projection (OP) fisheye lens. Compare to most photographic fisheye lenses, which are either equidistance projection or equisolid angle projection, the orthographic projection allows more distortion on the edge of the image. Thus, the center of the object appears larger, and the edge of the object compresses even more on the image plane. In order to achieve such projection relationship, Nikkor had to use an aspherical front element for this lens, which made this lens the first aspherical lens for 35mm format camera.

During the same year, Nikon filed another patent of their fisheye lenses in the U.S [15]. The 6mm f/5.6 lens shared a very similar structure with the OP 10mm f/5.6 lens [Fig 2.11]. However, the field of view was increased to 220 degrees, which made this lens one of the widest fish eye camera lens that ever made. Also, the design changed back



Figure 2.11. The Nikkor 6mm F/5.6 fisheye lens [15]

to equidistance projection, and the aspherical front element was changed back to spherical. In the same patent, Nikkor also proposed another design with a 270-degree field of view. Unfortunately, due to its extreme size and weight, this 270-degree FOV design was never made for mass production. But the 220-degree lens started for mass production in the following year. In 1972, Nikon announced an improved ultra-wide angle fisheye lens [Fig 2.12] [16]. This lens has an increased maximum aperture of f/2.8 while maintaining the 220-degree field of view.



Figure 2.12. The Nikkor 6mm f/2.8 fisheye lens [16]

In the development of photographic fisheye lens, Nikkor seems to be a real pioneer. But their fisheye development slowed down after 1990s. They announced their last full frame fisheye lens in 1994 and only made one other fisheye lenses for their smaller frame cameras during the past 20 years. Meanwhile, other photographic lens companies, such as Canon and Olympus, also saw the potential in the fisheye lens design and had some great designs. Lately, a few zoom fisheye lens appeared on the market, such as the 8-15mm F/4 lens introduced by Canon in late 2010 [Fig 2.13] [17]. These zoom fisheye lenses allow photographers to change the focal length to choose whether they want a fulfilled image or a circular image. However, there were very few zoom fisheye lenses currently on the market. As the pioneer of fisheye lenses, Nikkor does not even have a zoom fisheye lens in their product line. So the zoom fisheye lens still has a big potential now and in the future. A zoom fisheye lens designed by the author is also presented in this thesis report.



Figure 2.13. The Canon 8-15mm F/4 zoom fisheye lens at different zoom setting [17]

3 THE PROJECTION METHODS OF A FISHEYE LENS

For an ideal non-fisheye camera lens, the image height is proportional to the effective focal length and the tangent of half field of view in the object space. For these lenses, the fields of view are the same in both object and image space. The magnification is constant through the entire field. Thus, the final image has no distortion. However, this relationship between image height and the field of view does not hold for fisheye lenses. In this chapter, the standard projection methods for fisheye lenses are described in detail. The tangential and sagittal magnifications for each projection method are also mathematically derived.

3.1 The Projection Methods for Fisheye Lenses

If Y is the half image height from optical axis, f is the effective focal length and θ is the semi field angle in object space, then the projection for typical camera lens with no distortion is expressed by

$$Y = f \tan \theta \tag{3.1}$$

This projection is often called gnomonic projection. And the camera lenses use this projection are called rectilinear lenses. For these camera lenses, the incremental image height corresponds to the incremental object height. And the semi field angle θ ' in image space is equal to the semi field angle θ in object space.

However, this projection will fail if it is used to design a fisheye lens. According to the projection equation, the image height will start to increase rapidly and become unrealistic when θ is larger than 70 degrees. When θ reaches 90 degrees, the half image height Y becomes infinity. Thus, it is impossible to fill the entire hemisphere onto the
camera sensor without introducing large amount of barrel distortion. Equidistant map projection [Eq. (3.2)] is a popular mapping method for fisheye lenses since the Hill sky lens. This projection was mentioned in many literatures about wide angle lenses. Other less popular projection method includes orthographic projection [Eq. (3.3)], stereographic projection [Eq. (3.4)] and equisolid angle projection [Eq. (3.5)].

$$Y = f\theta \tag{3.2}$$

$$Y = f\sin\theta \tag{3.3}$$

$$Y = 2f \tan(\theta/2) \tag{3.4}$$

$$Y = 2f\sin(\theta/2) \tag{3.5}$$

3.2 The Tangential and Sagittal Magnification in the Projection

For a camera lens that has no distortion, the magnification is constant across the field of view. This is not the case in fisheye lens projection. In order to capture the entire hemisphere with a fisheye lens, huge barrel distortion needs to be introduced. That means the image close to the edge needs to be somehow compressed compare to the center of the image. Then the magnification is not constant across the field anymore. The magnification change is not the same in every fisheye lens. In fact, the magnification change of a fisheye lens depends on what projection method this fisheye lens use. Thus, to understand what the final images look like for each projection method, one need to first understand how the magnification changes in each projection method.

Imagine the hemisphere in Fig 3.1 is the object plane of a 180-degree FOV fisheye lens, and the lens is centered at the origin. In this coordinate system, z axis is the optical



Figure 3.1. The object space of a fisheye lens system

axis. The polar angle θ corresponds to the incident field angle, from -90 degrees to +90 degrees. Angle φ is the azimuthal angle. The radius r is the distance between the object on the hemisphere to the fisheye lens in at the origin. The shaded area ABCD on the surface of the hemisphere represents a very small area dA. Since dA is really small, line AB, AD, BC and CD are approximately straight lines. The corresponding polar and azimuthal angle of this area are small angles d θ and d φ . Based on the small angle approximation, the following relationship holds:

$$AB = DC \approx rd\theta \tag{3.6}$$

$$BC \approx r \sin \theta \, d\varphi \tag{3.7}$$

$$dA \approx AB \cdot BC = r^2 \sin\theta \, d\theta d\phi \tag{3.8}$$



Figure 3.2. The image space of a fisheye lens system

The 3-D hemisphere in the object space is transferred by the fisheye lens system to a 2-D circular image. In this coordinate, the shaded area A'B'C'D' with the size of dA' is the corresponding image of area ABCD in the object space. Y is the semi image height, which is the distance between the origin and the shaded area. Based on the small angle approximation, the following relationship holds

$$B'C' \approx Yd\varphi$$
 (3.9)

$$A'B' = C'D' = dY \tag{3.10}$$

$$dA' \approx A'B' \cdot B'C' = Yd\varphi dY$$
(3.11)

With the coordinates defined in both object and image space, the sagittal magnification M_s is defined by

$$M_s = \frac{A'B'}{AB} = \frac{C'D'}{CD}$$
(3.12)

Combine Eq. 3.12 and previous Eq. 3.6, M_s is then defined by

$$M_s = \frac{dY}{rd\theta} \tag{3.13}$$

For the similar reason, the tangential magnification is defined by

$$M_t = \frac{B'C'}{BC} = \frac{Y}{r\sin\theta}$$
(3.14)

Eq. 3.13 and Eq. 3.14 show that both the sagittal and tangential magnification of the fisheye lens depend on the semi image height Y. Eq. 3.2 to Eq. 3.5 show that different projection methods have different definition of Y. This proves that different projection methods have different sagittal and tangential magnification. The different magnifications cause the image of same object to look different through different projections.

3.3 Equidistant Projection

Recall the equidistant projection equation from Eq. 3.2

$$Y = f\theta$$

Take the derivative of this equation, then

$$dY = f d\theta \tag{3.15}$$

Combine Eq. 3.15 with Eq. 3.13, the sagittal magnification of the equidistant projection becomes

$$M_s = \frac{f}{r} \tag{3.16}$$

Combine Eq. 3.14 with Eq. 3.2, the tangential magnification of the equidistant projection becomes

$$M_t = \frac{f}{r} \left(\frac{\theta}{\sin \theta} \right)$$
(3.17)

Compare Eq. 3.16 and Eq. 3.17, the relationship between tangential magnification and sagittal magnification can be expressed as

$$M_t = M_s \left(\frac{\theta}{\sin \theta}\right) \tag{3.18}$$

And

$$\begin{cases} M_t = M_s, & \theta = 0\\ M_t = 1.57 M_s, & \theta = \frac{\pi}{2}\\ M_t > 1.57 M_s, & \theta > \frac{\pi}{2} \end{cases}$$
(3.19)

Eq. 3.16 shows that the sagittal magnification of this system depends on the focal length and the object distance of the system. It is constant across the field. The tangential magnification, on the other hand, according to Eq. 3.17, is increasing towards the edge of the image. Assume a small circle as the object. If the circle is on axis, its image is also a circle, with the radius of r'. If the same circle is shifting off-axis, its image becomes an ellipse. The length of minor axis is constant and is always equal to r'. The major axis is increasing and reaches 1.57 times of the minor axis at field of 90 degrees. If the fisheye lens has semi field larger than 90 degrees, the major axis of the ellipse will keep increasing while the minor axis remains constant.

Equidistant projection is one of the most common projection method in fisheye lens design. Since the field angle and the image height are proportional, if the image can be located in the image space, its object can be easily located in the object space by simple calculation. This advantage made equidistant fisheye lens popular for scientific use.

3.4 Orthogonal Projection

Recall the orthogonal projection equation from Eq. 3.3

$$Y = f \sin \theta$$

Take the derivative of this equation, then

$$dY = f\cos\theta \,d\theta \tag{3.20}$$

Combine Eq. 3.20 with Eq. 3.13, the sagittal magnification of the orthogonal projection becomes

$$M_s = \frac{f\cos\theta}{r} \tag{3.21}$$

Combine Eq. 3.14 with Eq. 3.3, the tangential magnification of the orthogonal projection becomes

$$M_t = \frac{f}{r} \tag{3.22}$$

Compare Eq. 3.21 and Eq. 3.22, the relationship between tangential magnification and sagittal magnification can be expressed as

$$M_s = M_t \cos \theta \tag{3.23}$$

Different from the equidistant projection, tangential magnification is constant in the orthogonal projection. Sagittal magnification is decreasing towards the edge of the image, and becomes 0 when the semi field reaches 90 degrees. Assume a same circular object as in the last section. When the circle is on axis, the image is also a circle with radius of r'. When the circle is shifting off-axis, the image is becoming an ellipse. The major axis of the ellipse remains the same, while the minor axis of the ellipse keeps decreasing. When the semi field reaches 90 degrees, the sagittal magnification decreases to 0 and the ellipse becomes a line towards tangential direction, with the length of r'.

The orthogonal projection has the highest distortion at the edge of the image among all fisheye projection methods. The information near the edge of the image is nearly unusable due to the sagittal magnification fall off to 0. So these kind of fisheye lens is not good for scientific or metrology use. However, with the edge image being more compressed, the image in the center is less compressed and close to normal undistorted image. The sacrifice of the information at the edge of the image ensures more information in the center area. For such purpose, some fisheye lenses were made using the orthogonal projection, but only for non-scientific use, such as the Nikkor OP 10mm fisheye lens announced in 1968.

3.5 Stereographic Projection

Recall the stereographic projection equation from Eq. 3.4

$$Y = 2f \tan(\theta/2)$$

Take the derivative of this equation, then

$$dY = \frac{f d\theta}{\cos^2(\theta/2)}$$
(3.24)

Combine Eq. 3.24 with Eq. 3.13, the sagittal magnification of the stereographic projection becomes

$$M_s = \frac{f}{r\cos^2(\theta/2)} \tag{3.25}$$

Expand Eq. 3.4 and apply some trigonometric-identities, then

$$Y = 2f \tan(\theta/2)$$
$$= \frac{2f \sin(\theta/2)}{\cos(\theta/2)}$$
$$= \frac{2f \sin(\theta/2) \cos(\theta/2)}{\cos^2(\theta/2)}$$
$$= \frac{f \sin \theta}{\cos^2(\theta/2)}$$
(3.26)

Combine Eq. 3.14 with Eq. 3.26, the tangential magnification of the stereographic projection becomes

$$M_t = \frac{f}{r\cos^2(\theta/2)} \tag{3.27}$$

Compare Eq. 3.27 and Eq. 3.25, the relationship between tangential magnification and sagittal magnification can be expressed as

$$M_t = M_s \tag{3.28}$$

And

$$\begin{cases} M_{t} = M_{s} = f/r, & \theta = 0\\ M_{t} = M_{s} = 2f/r, & \theta = \frac{\pi}{2} \end{cases}$$
(3.29)

For stereographic projection, the image has same tangential and sagittal magnification at any point on the image plane. In other word, the image of a small object has the same shape as the object. Assume the same small circular object as before, the image is always a circle no matter where the object is. When the circle is on axis, the image is a circle with radius of r'. When the circle is shifting off-axis, the magnification in both tangential and sagittal direction is increasing at the same rate. When the same circle is placed at the polar angle of ± 90 degrees in the object space, its image circle has twice the radius of the on axis image circle. This projection has least distortion among the four fisheye projections, and is rarely used in practical.

3.6 Equisolid Angle Projection

Recall the equisolid angle projection equation from Eq. 3.5

$$Y = 2f\sin(\theta/2)$$

Take the derivative of this equation, then

$$dY = f\cos(\theta/2)d\theta \tag{3.30}$$

According to the definition of the small solid angle $d\Omega$ corresponding to the small area dA on the hemisphere in object space, then

$$d\Omega = dA/r^2 \tag{3.31}$$

Recall from Eq. 3.8

 $dA \approx r^2 \sin \theta \, d\theta d\varphi$

Then

$$d\Omega \approx \sin\theta \, d\theta d\varphi \tag{3.32}$$

In the image space, recall the expression for dA' in Eq. 2.11

$$dA' \approx Y d\varphi dY$$

Combine this expression with Eq. 3.5 and Eq. 3.30, then use trigonometric identities, there is

$$dA' = f^2 \cdot 2\sin(\theta/2)\cos(\theta/2) \cdot d\theta d\varphi$$
$$= f^2 \cdot \sin\theta \, d\theta d\varphi \tag{3.33}$$

Compare to Eq. 3.32, then

$$dA' = f^2 d\Omega \tag{3.34}$$

Take the integral of Eq. 3.34, then the relationship between solid angle in object space and image size in image space become

$$A' = f^2 \Omega \tag{3.35}$$

The above Eq. 3.35 tells that when use this projection method, the same area in the image space corresponding the same solid angle in the object space, despite the field angle of the object. That is the reason why this projection method is called "equisolid angle" projection. With this relationship, fisheye lenses that are designed using equisolid angle projection can be used to measure the solid angle in the object space.

For the magnifications for this projection method, combine Eq. 3.30 with Eq. 3.13, the sagittal magnification of the equisolid angle projection becomes

$$M_s = \frac{f}{r}\cos(\theta/2) \tag{3.36}$$

Combine Eq. 3.5 and Eq. 3.14, then use trigonometric identities, the tangential magnification of the equisolid angle becomes

$$M_t = \frac{f}{r\cos(\theta/2)} \tag{3.37}$$

The changing rate of sagittal magnification is the multiplicative inverse of the changing rate of tangential magnification. Assume the same small circular object as before. When the circular object is on axis, its image is also a circle. When the object is shifting off axis, the sagittal magnification is decreasing while the tangential magnification is increasing. Thus, the image becomes an ellipse, with the major axis along the tangential direction. The distortion is larger than the equidistant projection, but smaller than the orthogonal projection at large field. This ensures the center portion of the image becomes larger while still maintaining good details at the edge of the image. Thus, this kind of fisheye projection method become popular recently for photographic fisheye lenses. Most of the photographic fisheye lenses that are currently on the market are designed based on the equisolid angle projection.

3.7 Projection Difference and their Practical Use

Fig 3.3 shows the relationship between the half image height and the half field angle of the four fisheye projection methods and the regular gnomonical projection method.



Figure 3.3. Image height vs. field angle for different fisheye projection system

For the four fisheye projection methods, the lower the image height is at the edge, the larger the barrel distortion is. From this figure, it is clear that the stereographic projection has the least barrel distortion. Orthogonal projection has the largest barrel distortion. Equidistant projection and equisolid angle projection has similar medium amount of barrel distortion compare to other two projection methods.

Practically, fisheye lenses designed based on equidistant and equisolid angle projections are more common than the ones based on stereographic and orthographic projection. However, there are still some photographic fisheye lenses that are successfully design based on stereographic and orthographic projection, such as the Samyang 12mm f/2. fisheye lens (stereographic projection) and the previously mentioned Nikkor 10mm f/5.6 fisheye lens (orthographic projection). There is no easy judgement on which projection method is the best for photographic purpose. They all create different distortion effect and it is really up to the photographer to choose the right kind they need. Less distortion of the

stereographic type fisheye lens makes the final image closer to normal photos. And the more distorted orthographic type fisheye lens focuses more in the center part of the image. One other thing to notice from Fig 3.3 is that for orthographic projection, image height reaches its peak at the 90-degree semi-field and start to decrease after that. For that reason, the maximum FOV of an orthographic type fisheye lens is limited to 180 degrees.



Figure 3.4. Same object under (a) stereographic projection, (b) equidistant projection, (c) equisolid angle projection and (d) orthographic projection

Fig 3.4 shows the comparison between the same object under different fisheye projections. The differences are very noticeable at the edge of each photo. For example, the palm tree appears near the top left edge in the stereographic projection photo is almost vanished in the orthographic projection photo due to the large distortion difference at the edge between these two projections methods.

These four methods are the standard projection methods of a fisheye lens design. However, the manufactures of the fisheye lenses do not always follow these projections exactly, especially when their fisheye lenses are not for scientific and metrology purposes. Depending on how much distortion the manufactures want for their fisheye design, they usually modify their own projection methods based on the standard fisheye projection methods to their desire. For example, the Nikkor 16mm F/2.8 fisheye lens was designed based on equisolid angle projection. However, after a curve fitting applied to the lens projection data [Fig 2.5], it is clear that the projection method used by Nikkor for this lens was modified. The final projection equation after curve fitting is

$$Y = 1.8f \sin(\theta / 1.78)$$



Figure 3.5. The projection curve fitting for the Nikkor 16mm F/2.8D fisheye lens

The small departure from the standard fisheye lens projection is usually allowed for the photographic fisheye lens design. In addition, slightly loosen the constrain on the image projection usually helps with the aberration control during the optimization stage of the fisheye lens design. As long as the projection captures a hemisphere onto a flat plane appropriately, it can be used for photographic fisheye lenses.

4 SPECIAL PROPERTIES OF PHOTOGRAPHIC FISHEYE LENSES

As a unique type of photographic lens, fisheye lenses have some special properties compare to other ordinary photographic lenses. These special properties are discussed in this chapter.

4.1 Negative Meniscus Lenses in the Front Group

Since the aperture stop of a fisheye lens is on axis and perpendicular to the axis, any ray with incident angle larger than 90 degrees is impossible to enter the stop directly. Thus, in order to direct the light from an entire hemisphere (or more) into the stop of the fisheye lens, a negative meniscus lens is required for all fisheye lenses. The negative meniscus lens diverges the rays so the field of view after the lens becomes smaller. If this field of view after the meniscus lens is smaller than 180 degrees, then all the rays can enter the aperture stop. Besides that, rays with large incident angle usually cause problem such as missing surface and TIR during the lens optimization. Thus, many fisheye lenses usually have more than one negative element in the front group to further reduce the ray angles. For fisheye lenses with FOV larger than 180 degrees, such as the Nikkor 6mm F/2.8 fisheye lens with 220-degree FOV, even more negative meniscus lenses are needed to reduce the ray angle before the rays reach the stop [Fig 4.1].

4.2 Pupil Shift and Ray Aiming

The aperture stop is a physical surface in an optical system that limits the bundle of light that propagates through the system. In a modern photographic lens, the aperture stop is usually adjustable with an iris diaphragm inside the lens. The entrance pupil, is the image of the stop in the object space. For the on axis ray bundle, the entrance pupil is located at



Figure 4.1. The Nikkor 6mm F/2.8 220-degree fisheye lens with 3 negative meniscus lens in the front

its paraxial location and perpendicular to the optical axis. For the off axis ray bundles, the entrance pupil may start shifting off axis due to the spherical aberration introduced by the lenses in front of the stop. This pupil spherical aberration is also called the pupil shift and is very significant in a fisheye lens. Fig 4.2 shows the pupil shift effect in the Nikkor 8mm F/8 fisheye lens [4]. According to this figure, the entrance pupil is not only shifted in position, but is also tilted in angle. This entrance pupil tilt is necessary for a fisheye lens.



Figure 4.2. The pupil shift effect of the Nikkor 8mm F/8 fisheye lens

If the entrance pupil does not tilt and stays perpendicular to the optical axis at all angles, then the rays from 90 degrees are impossible to enter the entrance pupil. Also, if the lens has a half field of view larger than 90 degrees, the rays from over 90 degrees are entering the entrance pupil from the back. These situations are clearly not practical, so the entrance pupil for the large field angle must be tilted in order for the rays to enter properly. Since the entrance pupil is tilted for off axis rays, it must be tilted at opposite angles for rays from opposite fields (e.g. +90 degree rays and -90 degree rays). This might be difficult to picture for a normal objective lens where the entrance pupils for rays from opposite fields are considered to share the same virtual surface that is orthogonal to the optical axis. But it is the only logical explanation for a fisheye lens system.

Although the shift of entrance pupil does not impact the lens physically since the entrance pupils are only virtual surfaces, it does affect the ray tracing in the lens design CAD software such as Zemax. In the software, the paraxial location and size for the entrance pupil is used to launch rays in the object space [18]. This is acceptable for most lenses with modest FOVs. However, for wide angle lenses with large pupil aberration, using paraxial location and size for the entrance pupil are not accurate anymore. For fisheye lenses with even larger FOV, the chief ray goes through the center of paraxial entrance pupil may miss the stop completely. If the chief ray cannot be traced correctly, then the entire lens design procedure cannot be performed. In these cases, the ray aiming needs to be turned on in the software to accurately trace the rays. With ray aiming, the paraxial pupil entrance pupil position and size is used as the first guess to trace the chief ray at some small field angle. The transverse height at the aperture is then used to determine the new ray height at the entrance pupil. The ray coordinates in object space are then adjusting by the



Figure 4.3. A fisheye lens with ray aiming turned on (top) and off (bottom) in Zemax

software iteratively until the real chief ray is found. Then this procedure is repeated with a small increment in field angle until all rays cross the correct locations on the stop surface.

Ray aiming is extremely important for fisheye lens design. Fig 4.3 shows the difference of a simple fisheye lens [19] before and after the ray aiming is turned on in Zemax. Both files are evaluated at 0 degree, 50 degrees and 75 degree HFOV. Before the ray aiming is turned on, on axis rays can be perfectly trance with paraxial entrance pupil. However, rays from 50 degrees cannot fulfill the stop. And rays from 75 degree missed the stop completely and cannot be traced at all. After the ray aiming is turned on, rays from all three fields can be traced with no problem.

4.3 Inverted Telephoto Structure

Fig. 4.4 shows the structure difference between the telephoto lens and the inverted telephoto lens.



Figure 4.4. The structure of the telephoto lens and the inverted telephoto lens

Both lens structures in Fig 4.4 have same lenses and same distance between two lenses. Thus, the effective focal lengths (from principle plane P' to focal plane F') are same for both lens structures. However, the total axial lengths for these two structures are different due to the different order of the negative lens group and positive lens group. The telephoto structure consists of a front positive group and a rear negative group. Rays from the object converges after the front group. The cone of light is then extended by the rear negative group, resulting the principle plan P' be in front of the front lens group. So the total axial length with such structure is shorter than its effective focal length. For

photographic lenses, especially for the ones with long focal length, the telephoto structure is beneficial for reducing the physical length in order to achieve a more compact design.

The inverted telephoto lens, also called retrofocus lens in some literature, consists of a negative front lens group and a rear positive lens group. Rays from object diverges by the front negative group, then convergence to focus after the rear positive group. Thus, the principle plane P' is behind the rear group, resulting a longer back focal distance (BFD) in relation to its effective focal length (Fig 4.4). For DSLR cameras, certain BFD (usually around 35mm) is required to clear the folding mirror and shutter mechanisms. Thus, the inverted telephoto structure is widely used for wide-angle lenses with short focal length for SLR cameras.

All fisheye lenses are inverted telephoto lenses for two main reasons. First, due to the large FOV nature of the fisheye lenses, their focal lengths are very short. For the fisheye lenses designed for standard 35mm format cameras, their focal lengths are usually 14mm to 16mm depend on what projection method (refer to chapter 3) the manufactures use. For the cameras with slightly smaller APS-C format sensors, the corresponding fisheye lenses usually have the focal length in the range of 8 - 10 mm. Thus, the inverted telephoto structure is required for the fisheye lenses to achiever long BFD in order to clear the space for the mechanisms in the SLR cameras. Secondly, refer to section 4.1, the front group of a fisheye lens contains multiple negative elements in order for the chief ray angle to enter the stop with a small angle. Then the rear group of the fisheye lens needs to be positive to compensate the total power of the lens, which also made the fisheye lens an inverted telephoto structure.

4.4 Depth of Field

Ideally, one image plane in the image space is conjugate with only one object plane in the object space. When a camera is focused on an object plane, only the objects on that plane will appear sharp. And everything on either side of that object plane will be blurred on the image plane. Practically, very small blur on the image is tolerable to human eyes. The limit of a blur on the image plane that is tolerable is also called the circle of confusion, or CoC. This tolerance in the image plane allowed a corresponding tolerance of the object distance in the object plane, which is called the depth of field, or DOF. The images of any object within the depth of field can be considered 'in focus' on the image plane.

There are some different standards of how big the diameter of the tolerable blur (CoC) is, but they all depends on the sensor size. According to Carl Zeiss, the international standard defines the diameter of tolerable blur to be 1/1000th of the camera format diagonal, but 1/1500th of the camera format diagonal is more appropriate for modern 35mm format cameras [20]. Thus, the tolerable blur of a 35mm format sensor with 43mm diagonal is about 0.029mm. With the tolerable blur being settled on the sensor side, the rest of the DOF is purely depend on the camera lens.

Refer to the camera lens in Fig 4.5, C' is the circle of confusion on the nominal image plane, and C is the conjugate circle on the nominal object plane. The distance between object O' and O'' is the DOF of this camera lens. D is the entrance pupil diameter. The diameter of C' and C is proportional by the transverse magnification of the lens system, which is

$$M_T = \frac{f}{r} \tag{4.1}$$



Figure 4.5. The structure of the telephoto lens and the inverted telephoto lens

Then the relationship between C' and C is

$$C = \frac{C'}{M_T} = \frac{C'r}{f}$$
(4.2)

By using similar triangle, the relationship between the back depth of field L_1 and the diameter of C can be expressed by

$$\frac{L_1}{C} = \frac{r - L_1}{D} \tag{4.3}$$

Combine Eq.4.2 and Eq.4.3, the relationship between the back depth of field L_1 and the diameter of circle of confusion C' is

$$\frac{L_1 f}{C' r} = \frac{r - L_1}{D}$$
(4.4)

The definition of the F-number N is

$$N = \frac{f}{D}$$
(4.5)

Combine Eq.4.4 and Eq.4.5, then

$$\frac{L_1 f}{C' r} = \frac{r - L_1}{f/N}$$
(4.6)

Simplify Eq.4.6, the final equation for the back depth of field is

$$L_1 = \frac{NC'r^2}{f^2 + NC'r}$$
(4.7)

For similar reasons, the equation for the front depth of field L_2 is

$$L_2 = \frac{NC'r^2}{f^2 - NC'r}$$
(4.8)

Then the total depth of field can be expressed by

$$L_{TOT} = L_1 + L_2 = \frac{2NC'r^2f^2}{f^4 - N^2C'^2r^2}$$
(4.9)

Since C'² is very small compare to other parameters, the term $N^2C'^2r^2$ can be ignored in Eq. 4.9, so the final approximation of the depth of field is

$$L_{TOT} \approx \frac{2NC'r^2}{f^2}$$
(4.10)

From this approximation, it is clear that with the same object distance and tolerable blur, the depth of field is linear to the F-number, and inverse quadratic with focal length. So the image will have larger depth of field with the increase of F-number and decrease of focal length. Such relationships are showed with real photos took by the author in Fig 4.6 and Fig 4.7. In Fig 4.6, two photos were taken under the same condition with the same



Figure 4.6. Images with the aperture setting of F/5 (left) and F/22 (right) at 70mm focal length setting



Figure 4.7. Images with the aperture setting of F/5.6 at 50mm focal length setting (left) and 140mm focal length setting (right)

focal length setting (70mm) and different F-number setting (F/5 on the left and F/22 on the right). The photo took with large F-number shows significant larger depth of field than the one took with small F-number. In Fig 4.7, two photos were taken under the same condition with the same F-number (F/5.6) and different focal length setting (50mm on the left and 140mm on the right). Note that the photo at 50mm focal setting is cropped to keep the similar image size with the photo at 140mm focal setting. But the object distance was remained the same during the photo shooting. The photo shot at 50mm focal setting shows larger depth of field than the photo shot at 140mm setting.

The derivation of Eq.4.10 is based on the assumption that the light cone in the object space and image space are similar triangles. This assumption is valid in the regular rectilinear image system (f-tan θ projection). For fisheye lenses, since the f-tan θ projection cannot be used anymore, the similar triangle method will also fail in such distorted image systems. However, recall the relationship between different projection method in Fig 3.3, the fisheye projections are the same with f-tan θ projection on axis, and are not deviate much from the f-tan θ projection until the HFOV reaches about 55 degrees. So the light cones in the object space and image space of a fisheye lens are still approximately similar triangles until the field of view reaches 110 degrees. That covers the majority part of the final image. Thus, Eq. 4.10 is still a good approximation to calculate the depth of field even for fisheye lenses.

As discussed in the last section, the focal lengths of fisheye lenses are usually very short. Also, the fastest fisheye lens currently on the market for 35mm format camera is F/2.8. This is relatively slow compare to standard prime lenses (50mm focal length) that usually has maximum aperture of F/1.4, such as the Nikkor 50mm F/1.4G lens and the Canon 50mm F/1.4 USM lens. With such short focal length and relatively small maximum aperture, the fisheye lenses have significantly larger depth of field compare to other prime (fixed focal length) lenses. According to Eq. 4.10, the depth of field of a 15mm F/2.8 fisheye lens is 22 times of the depth of field of a 50mm F/1.4 standard lens.

4.5 Hyperfocal Distance

Recall Eq. 4.8, the front depth of field can be expressed by

$$L_2 = \frac{NC'r^2}{f^2 - NC'r}$$

C'= circle of confusion

r = object distance

f = effective focal length

This front depth of field is extended to infinity if the following condition is met

$$r \ge \frac{f^2}{NC'} \tag{4.11}$$

The hyperfocal distance L_H is then defined as

$$L_H \triangleq \frac{f^2}{NC'} \tag{4.12}$$

When a camera is focused at its hyperfocal distance, the back depth of field, according to Eq.4.7, becomes

$$L_{2} = \frac{NC'L_{H}^{2}}{f^{2} - NC'L_{H}}$$

$$= \frac{f^{2}}{2NC'}$$
(4.13)

The near focus limit $L_{\mbox{\scriptsize NEAR}}$ is

$$L_{NEAR} = L_H - L_2 = \frac{f^2}{2NC'}$$
(4.14)

Compare Eq. 4.14 with Eq.4.12, the near focus limit is half of the hyperfocal distance. Thus, when a camera is focused at the hyperfocal distance, the images for all objects from L_{Near} to infinity are within the tolerable blur limit and are in focus.

According to Eq.4.14, the near focus limit is quadratic with the effective focal length of the objective lens. Fisheye lenses usually benefit from their short focal lengths to have a short near focus limit. For a typical 15mm F/2.8 fisheye lens for 35mm format cameras, assume the circles of confusion to be 0.029mm according to the Carl Zeiss standard, the near focus limit is 1385mm (1.385m). In another word, by focusing this fisheye lens at its hyperfocal distance, any object from 1.385m in front of the camera to infinity is in focus without refocusing the camera. At this condition, the camera achieves a very large depth of field while maintaining a big aperture size with a fisheye lens. To achieve the same near focus limit (1.385m) by using a 50mm standard lens, the F-number needs to reduce F/31. This is very slow and is not even possible for many of the lenses on the market as their lowest apertures are set to F/22. If we slow down a 15mm fisheye lens to F/8, which is still reasonable for normal photographic use, the near focal limit of this fisheye lens becomes 551mm (0.551m). At this condition, the refocusing is barely needed for normal landscape photograph with this fisheye lens.

5 DESIGN ISSUE OF A PHOTOGRAPHIC FISHEYE LENS

The ultimate goal of designing a fisheye lens for photographic purpose is not much different from a normal photographic lens design, which is to optimize for great image quality while keeping the lens practical. This chapter will address some critical design issues for photographic fisheye lenses. Some issues are common for normal photographic lens designs. The other ones are specifically for fisheye lens designs.

5.1 Minimum BFD Requirement

For a DSLR camera, a sufficient BFD is required to clear space for the folding mirror, shutter and other mechanisms. The minimum BFD requirement for a DSLR camera is sometimes confused with the flange focal distance, which is the distance between the lens-mounting flange and image sensor. Practically, for inverted telephoto lenses with short focal lengths, the lens can be further inserted to the camera beyond the flange before hitting the folding mirror [Fig 5.1]. Therefore, the minimum BFD requirement is actually shorter than the flange focal distance. The exact minimum BFD requirements are not given by the camera manufactures. However, by carefully investigating through various patents of fisheye lenses from different camera manufactures, the author is able to conclude that the minimum BFD requirement for a 35mm DSLR camera is no less than 38mm.



Figure 5.1. A lens is able to insert to the camera body beyond the mounting flange

For fisheye lenses, the effective focal length varies by its projection method. The most common fisheye projection method for DSLR cameras today is equisolid angle projection. With the equisolid angle projection, for 35mm format cameras, the effective focal length is around 15 to 16mm for diagonal fisheye lenses, and is 8mm to 9mm for circular fisheye lenses. These focal lengths are much smaller than the minimum BFD requirement for DSLR cameras. During the design, the author experienced a big tradeoff between the image quality and the BFD. Some image quality needs to be sacrificed in order to achieve sufficient BFD. Such tradeoff was also mentioned in a previous paper [7]. For the diagonal fisheye lenses, most designs can still meet the BFD requirement while maintain a good image quality at F/2.8.

However, for the circular fisheye lenses with even shorter focal length, the BFD requirement of a DSLR camera is very hard to meet. Nikkor was a pioneer in fisheye lens design and announced many circular fisheye lenses for 35mm format SLR camera during 1960s and early 1970s. Even they could not find a nice solution to the BFD requirement at that time. Most of their circular fisheye lenses, such as the 7.5mm F/5.6 fisheye lens, were designed without the BFD constrain to maintain good image quality. Those lenses did not have enough BFD to clear space for the folding mirror in the camera. When photographers used those lenses, they needed to hold the folding mirror in the upright position so there was enough space to attach the lens [Fig 5.2]. However, without the mirror, the viewfinder on the SLR camera would not work. So an attachable viewfinder was needed to provide live view [Fig 5.3].



Figure 5.2. A fisheye lens without sufficient BFD, the folding mirror needs to be held at upright position to clear space for the fisheye lens.



Figure 5.3. Nikkor 7.5mm F/5.6 circular fisheye lens mounted on Nikon F SLR camera, the attached view finder in the red box is for live view

In 1995, APS-C format cameras were introduced. Due to the smaller sensor size, the diagonal fisheye lenses for APS-C cameras also have very short focal length and share the same BFD problem with the circular fisheye lenses for 35mm cameras. Today, with modern lens design technology, a few 35mm circular fisheye lenses and APS-C diagonal fisheye lenses were successfully designed with only spherical surfaces to meet the BFD requirement and maintain good image quality at F/2.8. However, applications with less BFD requirement, such as the trending mirrorless cameras, will still make it easier to improve the image quality, and decrease the size and weight of the lens.

5.2 Lateral Color

Just like any other lenses, aberration control is very important for a photographic fisheye lens. Good aberration control does not only ensure good image quality and high contrast, but also increase the maximum aperture, so the lens can be faster. For fisheye lenses, due to their lack of symmetry, good corrections of off-axis aberrations are required. The good news is, for a photographic fisheye lens, the distortion is actually desired and does not require much correction. And axial color aberration is also easy to control because of its very short focal length. However, Lateral color aberration correction becomes a challenge during a fisheye lens design.

Lateral color, also called the chromatic change of magnification, is the chromatic aberration of the chief ray. This is due to the refractive index difference for different wavelengths, which cause the chief ray height difference for each wavelength at the image plane [Fig 5.4]. The lateral color aberration is highly depended on the field of view. Thus, it is very significant for the fisheye lenses with the large 180-degree field of view. Since the chief ray height on the image plane determines the image height, lateral color on the image plane is more noticeable than axial color. Therefore, lateral color aberration is the worst offender for a fisheye lens and requires good correction to maintain great image quality.

To correct the lateral color in the fisheye lens, at least one achromatic doublet is needed behind the aperture stop. Based on Milton Laikin's paper of 'Wide angle lens systems' [21] and the author's own design experience, putting the achromatic doublet in front of the stop is less efficient for lateral color control. However, Laikin did suggest in his paper that an additional negative achromatic doublet could be used in the front to help



Figure 5.4. Chromatic aberration of the chief ray, different wavelength has different chief ray height on the image plane. C (656nm), d (587nm) and F (486nm) are standard wavelengths to represent the visible spectrum.

with the lateral color. Nevertheless, it is proved in many fisheye lens patents that the achromatic doublet in the front group is not required.

In author's experience, it is very challenging to start the design with achromatic doublet and optimize the chromatic aberration with other aberrations. A useful technique called 'the buried surface' can be used to help with the optimization. To apply this technique, one should first design the lens for a desired single wavelength with only singlets. For at least one of the singlet behind the stop, one should keep in mind to use the material that another material with similar index and different Abbe number can be found, such as SF-1 ($n_d = 1.717$, $V_d = 29.62$) and LAK-10 ($n_d = 1.720$, $V_d = 50.62$). After the monochromatic design is finished, insert a surface in this singlet to separate the singlet into a doublet with these two materials. Then vary the curvature of the surface and optimize the lens again. Since the doublet has the similar index, the optimization at this stage will not affected much of the monochromatic aberration that has already been corrected. Thus, all of the optimization power can be used for the chromatic aberration. After an initial polychromatic design is finished, one can start to substitute the glass in the doublet for better performance.

Some other methods that help with the lateral color include using the extra-low dispersion glass (ED glass). With ED glass, only one achromatic doublet is needed to achieve very good lateral color correction in author's design. However, such glass cost more than traditional crown glass, so it should be placed near the stop to reduce its size to help with the cost. Besides that, shifting the stop position can also be helpful with the lateral color. However, sometimes the stop shifting is conflict with another issue of the image space chief ray angle that is going to be addressed in the next sections

5.3 Relative Illumination

For all camera lenses, chief ray angle in image space directly affects the edge relative illumination of the final image. For the ideal camera lens with no distortion, the relative illumination follows the $\cos^4 \theta$ law that is state below:

$$E = E_0 \cdot \cos^4 \theta \tag{5.1}$$

where E = Off-axis illumination

 E_0 = On-axis illumination

 θ = Chief ray angle in image space

Since normal photographic lenses usually have very small amount of distortion, their relative illumination before vignetting obey the $\cos^4 \theta$ law well. The edge relative illumination is usually required to be over 50% so it can be corrected digitally by the camera [22]. Thus, by the $\cos^4 \theta$ law, the image space chief ray angle needs to be smaller than 33 degrees in order to maintain at least 50% edge relative illumination. However, if the lens designer wants to control the off axis aberration by using vignetting, which is very common aberration control method for photographic lenses, he needs to further reduce the chief ray angle to increase the edge illumination in order to compensate the illumination reduce cost by vignetting.

For fisheye lens, due to the large barrel distortion, the light flux is distributed over increasingly smaller areas towards the edge of field of view. In this way, the edge illumination can be improved. This illumination improvement towards the large field of view can be also explained by the entrance pupil coma aberration. Because of the large distortion, the entrance pupil is suffering from pupil coma aberration. The entrance pupil for the off-axis ray bundle is deformed and it size is increased compare to the on-axis paraxial entrance pupil [Fig 5.5]. This allows the entrance pupil to accept more off-axis light to help increase the off-axis illumination. This is known as the Slyusarev effect [23]. With the Slyusarev effect, an over 100% edge relative illumination is possible. Fig 5.6 shows a monochromatic fisheye lens designed by the author with 104% edge relative illumination (compare to the center).



Figure 5.5. Coma aberration of the entrance pupil. Black solid lines indicate the paraxial entrance pupil with grid. The red dashed lines indicate the off-axis entrance pupil with corresponding grid.



Figure 5.6. A monochromatic fisheye lens design with 104% edge relative illumination (compare to center illumination)

With the large distortion and pupil coma, the edge relative illumination is usually not a big problem for fisheye lens design. Extra edge relative illumination allows lens designers to use vignetting to further control off axis aberrations, which is very beneficial for fisheye lenses.

5.4 Maximum Allowed Chief Ray Angle

For digital cameras today with CMOS sensor, there is a limit on the maximum allowed chief ray angle in the image space due to the structure of the current CMOS sensor. Fig 5.7 demonstrates a pixel level structure of a classic CMOS sensor with Bayer Filter. The microlens on top of each pixel is used to concentrate the incoming light onto the photodiode. When the incident ray angle is too large, some of the rays are missing the photodiode at the bottom, causing the actual sensor response to be smaller than expected. The loss of photons is on top of the reduced edge illumination caused by the lens, and further reduce the edge relative illumination. In addition, some of the light may even go through the barrier between two pixels, causing color crosstalk [Fig 5.7].


Figure 5.7. The pixel level structure of a normal CMOS sensor. The ray bundle in the center pixel is the on-axis ray bundle. The right pixel shows the issue when the acceptance angle of the ray bundle is too large.

The maximum allowed incident angle was increased during the past 10 years due to the development of CMOS sensors. Currently, 30 degrees is usually the maximum allowed chief ray angle in the image space. This was increased from 20-degree constraint in 2006 [24]. Meanwhile, scientists are working on new CMOS designs to further expend the sensor acceptance angle. For example, CMOSIS developed a new 35mm CMOS sensor for Leica to use on their new digital cameras [25]. The sensor increased the power of the microlens and reduced the distance between the photodiode and the color filter to improve the angular response of the sensor.

One of the biggest tradeoff to design a lens with small image space chief ray angle is the compactness of the lens. In order to reduce the image space chief ray angle, it usually requires shifting the physical stop towards the front focal point of the rear lens group. When the stop is at the front focal point of the real lens group, the system is telecentric in image space, and the image space chief ray angle reaches its minimum (0 degree, chief ray parallel to optical axis). Such stop shifting may increase the total axial length of the lens system. Also, the further the stop is from the rear lens group, the bigger the last lens element is. If the system is telecentric in image space, then the last lens element needs to be at least the same size of the image sensor, that increase the total weight of the lens system. Besides the compactness, shifting the stop may also conflict with the correction of off-axis aberrations, such as coma, astigmatism and lateral color, since these aberrations are depended on stop position. These tradeoffs should be considered during the design.

5.5 Other Issues

During the optimization of the fisheye lens design, the front elements sometimes collide with each other. Fig 5.8 shows an example of such situation. Fig 5.8 (a) shows the entire layout of a monochromatic design similar to the system in Fig 5.6. This design is fully optimized, except the first three lens elements are partially overlapped. This design is not practical to manufacture due to the overlapping. Because of the big curvatures of the meniscus lenses, controlling the center thickness between two elements does not help much with the collision issue. The overlapping usually looks like the situation in Fig 5.8 (b). In this case, constrain the edge thickness between two elements to be positive usually solve the collision issue. However, sometimes lens designer design the system with diameter margin for mounting purpose. If the diameter of the second element in Fig 5.8 (b) is increased with some margin, then the lens edges of the first two lens elements are not colliding with each other. However, there is still overlapping between the first two lens elements [Fig 5.8 (b)]. In this case, constraining the edge thickness to be positive does not help with the overlapping. One good approach in Zemax is to constrain the real ray radial coordinate of the outermost ray (H = 1, ρ = -1) by using the optimization operand 'REAR'.



Figure 5.8. Lens elements collision during a fisheye lens design.

By constraining the radial distance on the next surface to be smaller than on the previous surface, the operand forces the outermost ray to be closer to the optical axis on the next lens element, so the elements can be physically separated [5.8 (d)].



Figure 5.9. A meniscus lens with a hyper-hemisphere rear surface. Ray A and Ray B are two rays from maximum field

Another issue during the optimization is that the rear surface of the first meniscus lens become hyper-hemisphere in order to provide enough negative power of the first lens element [Fig 5.9]. Even though the hyper-hemisphere surfaces are possible to manufacture, it is bulky and unnecessary and should be avoid. One way to avoid this type of surface during the optimization is to trace two close rays from maximum field, such as Ray A (H = 1, ρ = -0.9) and Ray B (H = 1, ρ = -1) in Fig 5.9. Then constrain the radial distance of the outer ray (Ray B) to be larger than radial distance of inner Ray (Ray A) on the potential hyper-hemisphere surface. This method was used in author's fisheye lens design. Another effective method to avoid the hyper-hemisphere surface is to constrain the sag of that surface to be smaller than its radius of curvature.

For a practical design of a commercial camera lens, the size and weight of the lens elements should be concerned. The early fisheye lens designs are bulky due to the large meniscus lenses in the front. An extreme example is the 6mm Nikon 220-degree fisheye lens [16]. The diameter of its front element is over 200mm, and the total weight is over 5kg. Due to the size of the first element in a fisheye lens, Laikin suggest in his paper to use classic crown glass, such as BK-7, as the material for the front element [21]. The cost of BK-7 is low for such large element, and its relatively low dispersion helps with the lateral color control of the system. However, based on the current patents and author's own design experience, increase the index of the first element is extremely effective on reducing its diameter. Higher index allows higher power of the first element, while still maintaining appropriate curvatures for the control of spherical aberration. Higher index of the first element also helps to avoid the hyper-hemisphere shape of the rear surface that was mentioned before. However, higher index glasses usually have larger dispersion, which makes the control of lateral color harder. Besides the index of first element. To reduce the total weight, the designer can sacrifice some image space telecentricity to help reduce the diameters of the rear lens group. In addition, the manufactures usually cut the unnecessary edge part of the meniscus lenses to further reduce the weight [Fig 5.10].



Figure 5.10. Manufactures usually cut the unnecessary edge of the negative meniscus lens (indicate in red shadow) to reduce total weight.

6 DIAGONAL FISHEYE LENS DESIGN FOR 35MM DSLR CAMERAS

The basic theories and design difficulties of a photographic fisheye lens were covered in the previous chapters. In this chapter, diagonal fisheye lenses for 35mm format DSLR cameras that are currently on the market are evaluated. A new diagonal fisheye lens design by the author is then presented. The lens aberrations are evaluated independently. The other performance assessments, including lateral color, distortion, relative illumination and contrast (MTF) are analyzed and compared to the current designs.

6.1 Diagonal Fisheye Lenses and Their Current Designs on the Market

Recall from Chapter 1, a diagonal fisheye lens covers the entire 35mm sensor and has a 180-degree field of view on its diagonal [Fig 1.1]. This type of fisheye lens is more common than a circular fisheye lens. Currently on the US market, there are five diagonal fisheye lenses for 35mm DSLR cameras. These lenses are listed in Table 6.1. Note that one of these lenses, the Canon 15mm F/2.8 fisheye lens, was just discontinued by its manufacture. However, since this lens is still available from many lens vendors, the author still kept it in this list. The following information can be acquired from the table:

- The newest design from Samyang (highlighted in red), used stereographic projection method. This is an unusual design since most of the photographic fisheye lenses today, including the other four fisheye lenses in this table, are using the equisolid angle projection.
- In order to achieve the stereographic projection, the Samyang fisheye lens used two aspherical surfaces. The other four designs used all spherical surfaces.

- Due to the special stereographic projection, recall from chapter 3, the Samyang fisheye lens has less distortion than the other ones. In order to achieve the same image height with the equisolid angle fisheye lenses, the focal length of the Samyang fisheye lens is significantly shorter at 12mm. The other four lenses all have similar focal lengths in the range of 15mm to 16mm.
- All of these designs have same maximum aperture at F/2.8.
- The equisolid angle designs were all kept simple with less than 10 lens elements.
- The corresponding patent for these designs were provided in the table. The author was not able to acquire the patent file for the Samyang fisheye lens. It is highly possible that this lens was only patented in Korea.

Manufacture	Canon [26]	Sigma [27]	Nikkor [28]	Sony [29]	Samyang	
Focal Length	15mm	15mm	16mm 16mm		12mm	
Maximum Aperture	F/2.8	F/2.8	F/2.8	F/2.8	F/2.8	
Projection Method	Equisolid Angle	Equisolid Angle	Equisolid Angle	Equisolid Angle	Stereographic	
Element Number	8 Elements 7 Groups	7 Elements 6 Groups	8 Elements 5 Groups	10 Elements 7 Groups	12 Elements 8 Groups	
Aspherical Elements	None	None	None	None	2 ASP element	
Patent Number	JP63- 017,421	JP02- 248,910	US 5,434,713	US 3,589,798	Unknown	
Patent Date	01/25/1988	10/04/1990	07/18/1995	06/29/1971	N/A	

Table 6.1. Current diagonal fisheye lenses for 35mm format DSLR cameras

6.2 Evaluation of the Current Designs

This section provides evaluation and comparison of the current diagonal fisheye designs for 35mm DSLR cameras. Some explanations of this evaluation are described in words below:

- Only the equisolid angle fisheye lenses listed in Table 6.1 were evaluated and compared. The Samyang stereographic fisheye lens would not be a fair comparison due to its different projection method and the use of aspherical surfaces.
- The lenses were reconstructed based on the patent files. The actual product on the market may not have exactly same lens prescription with the patent file. For each patent file, the embodiment that has the closest lens structure with the actual product was evaluated.
- The information of vignetting and the exact location of physical stop were missing in some of the patent files. The author has made every effort to carefully reconstruct the lens intended by the inventor.
- The Canon fisheye lens and Sony fisheye lens were evaluated in the SPIE proceeding paper 'Fish-eye lens designs and their relative performance' [30]. These lenses were reevaluated in this thesis report and checked with this paper for best accuracy.
- Lateral color, relative illumination and distortion were directly evaluated from the reconstructed lenses. Due to the typographical errors (intentional or not) in the patents, the OPD plots and MTF plots from the reconstructed lenses were not accurate enough to be used since the wavefront and contrast are very sensitive.

- The MTF provided by the lens manufactures were also presented in this section. These plots should only be used as rough references due to the testing method and testing condition differences between manufactures.
- All lenses were evaluated at their maximum aperture (F/2.8).
- All lenses were evaluated in visible spectrum, using the wavelength of F (486nm), d (588nm), and C (656nm). The lateral color was evaluated between F and C. The distortion was evaluated for d (588mm) line.



6.2.1 Lens Structure

Figure 6.1. Lens structures of the (a) Canon, (b) Nikkor, (c) Sigma, and (d) Sony fisheye lenses.

The Lens structures of the four evaluated fisheye lenses are presented in Fig 6.1. Other than the similar first meniscus lens, not many regular patterns can be found between these designs. The Sony fisheye lens, which was originally patented by Minolta in early 1970s, has the most lens elements. Compare this lens with other three ones designed around 1990s, it is clear that the lens designers were trying to reduce the number of lens elements for the new designs. In addition, the new designs all have less telecentricity in image space, so the rear group of the new designs are smaller than the old Sony (Minolta) design. This helps reduce the total weight of the fisheye lens.

6.2.2 Relative Illumination

The relative illumination plot of the four evaluated fisheye lenses are presented in Fig 6.2. And the exact amounts of edge relative illumination are presented in Table 6.2. As mentioned before, relative illumination is affected by both the image space chief ray angle and vignetting. It is not required to have very high edge relative illumination for digital cameras, since the dark corners that are caused by low edge relative illumination can be fixed digitally. The rule of thumb that also mentioned before is to have at least 50% of the relative illumination. From this evaluation, all of these lenses meet this criterion. The Sony (Minolta) fisheye lens has the best image space telecentricity [Fig 6.1], thus, its relative illumination drops slowly until the vignetting started to dominate at large field angle. The Sigma lens has the worst relative illumination performance that barely meets the 50% criterion over the edge.



Figure 6.2. Relative illumination of the (a) Canon, (b) Nikkor, (c) Sigma, and (d) Sony fisheye lenses.

Manufacture	Canon	Nikkor	Sigma	Sony
Edge RI	59.6%	56.2%	50.6%	68.0%

Table 6.2. Edge relative illumination of the evaluated lenses

6.2.3 Lateral Color Aberration

The lateral color aberration of the four evaluated fisheye lenses are presented in Fig 6.3. For clear comparison, the lateral colors are plotted at the same maximum scale of 100µm.



Figure 6.3. Lateral color of the (a) Canon, (b) Nikkor, (c) Sigma, (d) and Sony fisheye lenses, the maximum scale is 100µm.

According to Fig 6.3, the Nikkor fisheye lens and the Sony fisheye lens have better lateral color control. From the lens structure in Fig 6.1, these two lenses both have multiple achromatic doublet and at least one achromatic doublet in front of the stop. This kind of structure gave the system more achromatize power to correct the lateral color aberration. Out of these four lenses, the Sigma fisheye lens has the worst lateral color aberration, which exceeds 60µm near the edge. This is about 12 pixels on a modern 35mm DSLR camera with 5µm sensor width. With this amount of lateral color, color fringes are highly noticeable around the corner of the image.

6.2.4 Distortion

At 180-degree field of view, the image height is infinity with the traditional f-tan θ projection method. Thus, all the fisheye lenses have -100% distortion at the edge of the image circle. In this situation, the traditional distortion criterion simply lost its meaning.

A common way to assess the distortion for fisheye lenses is to use the equidistant $(f-\theta)$ projection as the reference. This distortion is also called the f- θ distortion. In the paper 'Fisheye lens designs and their relative performance' [30], the f- θ distortion criterion is used to assess the distortion of the photographic fisheye lenses. However, the author of this thesis report decided not to use such criterion to evaluate the current photographic fisheye lenses. As mentioned before, all fisheye lenses under evaluation in this project are equisolid angle fisheye lenses. Since the equidistant projection was not intended to be used in the first place, it should not be used as the reference.

Instead of the equidistant projection, the standard equisolid angle projection is used to evaluate the distortion in this chapter. Recall the standard equisolid angle projection from Chapter 3:

$$Y = 2f \sin(\theta/2)$$

For diagonal fisheye lens, the image height is equal to the diagonal of a 35mm sensor. Thus, if a diagonal fisheye lens was made using the standard equisolid angle projection, the focal length should be 15.3 mm. This focal length is then plugged in the above equisolid angle projection equation to provide a reference to evaluate the equisolid angle distortion in this project. The raw image height data are presented in Fig 6.4, and the distortions in percentage are presented in Fig 6.5.



Figure 6.4. Half image height vs. half field of view of each fisheye lens under evaluation and the standard equisolid angle fisheye lens



Figure 6.5. Equisolid angle distortion vs. half field of view of each fisheye lens under evaluation.

According to Fig 6.4 and Fig 6.5, all the fisheye lenses under evaluation follow the equisolid angle well, with very small departures. As mentioned in Chapter 3, for a

photographic fisheye lens that is not designed for metrology, the small amount of departure from the standard equisolid angle projection is not a problem at all.

6.2.5 MTF Evaluation

The modulation transfer function, also known as MTF, is the evaluation of the contrast performance of an optical system at different spatial frequency. The traditional MTF plot is usually contrast (percentage) vs. spatial frequency, and is evaluated at several fields. However, for photographic lenses industry, contrast vs. image height is served as the standard MTF plot. For such MTF plots, contrast is evaluated at both 10lp/mm and 30lp/mm as an industrial standard. The MTF plots for the lenses under evaluation are presented in Fig 6.6.

As mentioned before, the MTF plots provided in this section are directly from the manufactures' websites as a rough reference. Since different manufactures plot the MTF slightly differently, a guide to read the MTF plots is provided below:

- For all MTF plots, the independent variable on X axis is the half image height (distance from the image center). Due to the symmetry of the lens, the other half of the image height should have the same MTF. The dependent variable on Y axis is the contrast in percentage from 0% to 100%.
- For all MTF plots, the solid lines represent the sagittal contrast, and the dashed lines represent the tangential contrast.



Figure 6.6. MTF plots for the (a) Canon [31], (b) Nikkor [32], (c) Sigma [33], and (d) Sony [34] fisheye lenses.

- For Canon MTF plot, the contrast is measured at both maximum aperture and F/8. To compare with other lenses, the contrast measured at F/2.8 in black lines should be used. The bold lines represent 10lp/mm frequency, the thin lines represent 30lp/mm frequency.
- For Nikkor MTF plot, the red lines represent 10lp/mm frequency, the blue lines represent 30lp/mm frequency.

- For Sigma MTF plot, the red lines represent 10lp/mm frequency, the green lines represent 30lp/mm frequency.
- For Sony MTF plot, the contrast is measured at both maximum aperture and F/8. To compare with other lenses, the contrast measured at F/2.8 in green lines should be used. The bold lines represent 10lp/mm frequency, the thin lines represent 30 lp/mm frequency.
- The corresponding MTF curves at the maximum aperture (F/2.8) are also labeled in red on the plots.

The ideal lens would have a contrast performance that is limited by diffraction. However, this is usually not achievable in real design. The contrast is highly affected by the aberration in the system. Also, due to the diffraction limit, higher frequency image usually has lower contrast. There is currently no specific standard on the contrast performance of a photographic lens. A rule of thumb of a camera lens for 35mm sensor with good contrast performance is to have at least 50% contrast for the 30lp/mm frequency [35]. For this evaluation, all of the lenses have good contrast performance in the center. The all have over 90 % contrast for the 10lp/mm frequency and at least 70% contrast for the 30lp/mm in the center. However, the contrast falls off quickly near the edge of the image circle. This is due to the large off-axis aberration at the extreme field of view of the fisheye lenses. Other than the contrast, the difference between the tangential curve and sagittal curve for the same frequency also shows the astigmatism aberration in the lens design. MTF plots in Fig 6.6 show that Canon, Sigma and Sony (Minolta) preferred the sagittal contrast over the tangential contrast and use the astigmatism to sacrifice the tangential MTF curve for better sagittal MTF curve.

6.3 Design Goals of a New Diagonal Fisheye Lens for 35mm DSLR Camera

According to Table 6.1, besides the new Samyang fisheye lens, most diagonal fisheye lenses for 35mm DSLR cameras were designed more than two decades ago. Although these old designs still provide good performance today, however, with the aid of modern optical CAD software, there is a great potential for a new design with better performance.

According to the evaluation in the last section, all lenses under evaluation have good relative illumination and distortion performance. Lateral color is the most offender to the performance of the current fisheye design. Besides the lateral color, the contrast performance of the current designs should also have room for improvement. Even though the evaluated lenses all have very good contrast in the center, the MTF curve falls off too quickly. A more consistent contrast performance is needed for most part of the image. These are two areas that needed to be improved in the new design.

For current designs that were evaluated in the last section, the Nikkor lens has the least lateral color aberration of 23.3µm. The design goal is to further reduce this number by 50%. For the MTF curve, the goal is to keep the contrast level of 30 lp/mm consistently over 70% for 50% of the field, and over 50% for 70% of the field at maximum aperture. Another MTF criterion mentioned by Richard Juergens from Raytheon is to have 20% contrast at 30lp/mm for over 90% of the field at maximum aperture [35]. Besides that, the author believes that the tangential contrast is as important as sagittal contrast, so the dispersion of sagittal MTF curve and tangential MTF curve should be minimum in the new

design. In addition, relative illumination, maximum aperture and distortion in the new design should be kept at the same level with the current design.

While improving the lens performance, the author does not wish to break the simplicity of the current designs. Therefore, the number of lens elements should be constrained, and only the spherical surfaces are used in the new design. The summary of the design goals is provided below in table 6.3.

Fisheye Lens Type	Diagonal fisheye lens			
Image Height	41.2 mm (35mm sensor diagonal)			
Focal Length	15mm – 16mm (varied by distortion)			
Maximum Equisolid Angle Distortion	$\pm 5\%$			
Field of View	180 degrees			
Maximum Aperture	F/2.8			
Maximum Lateral Color	11.7µm			
MTF	At least 70% for 50% of the image height, 50% for 70% of the image height, and 20% for 90% of the image height at maximum aperture for 30lp/mm.			
Image Space Chief Ray Angle	Less than 30 degrees			
BFD	More than 38mm			
Maximum Number of Elements	8			
Aspherical Elements	0			

Table 6.3. Design goal summary of a diagonal fisheye lens for 35mm DSLR camera

6.4 Design Structure and Evaluation

F/# = 2.84									
f = 15.99 mm									
	HFOV = 90 Degrees								
Back Focal Distance = 38.09 mm									
Surface No.	r d N _d V _d Clear Aper								
1	76.461	3.000	1.69680	55.4	69.5				
2	20.615	16.324	-	-	39.4				
3	-144.269	2.500	1.69680	55.4	38.0				
4	19.579	10.993	-	29.7					
5	168.743	4.000	1.51112	60.4	30.0				
6	-118.106 0.000 -			-	30.0				
7	26.797 4.000 1.51680		1.51680	64.2	30.0				
8	35.702 5.000 1.64769		33.8	30.0					
9	-198.347 15.645 -		-	-	30.0				
10 (Stop)	-	3.575	-	-	14.2				
11	83.789	4.715	1.49700	81.6	17.6				
12	-13.597	1.500	1.80518	25.4	17.8				
13	-30.051	0.000	-	-	19.8				
14	123.727	4.000	1.51680	64.2	22.0				
15	-52.823	-	-	-	22.0				

The lens prescription and system layout are presented in Table 6.4 and Fig 6.7.

Table 6.4. Lens prescription of the new diagonal fisheye lens design



Figure 6.7. System layout of the new diagonal fisheye lens design

This lens has eight total elements in six lens groups. The back focal distance is over 38 mm so there is enough room for the folding mirror. The image space chief ray angle is controlled at 29 degrees to fulfill the acceptance angle requirement of the CMOS sensor. The front element diameter is 69.5mm, this is larger than many of the camera lenses, but is quite normal for a high-speed prime lens. The total length from the first surface to the last surface of this lens is only 75.25mm, so the lens has an overall compact structure.

The total aberration is presented in three methods: OPD fan plot [Fig 6.8], RMS wavefront vs. field [Fig 6.9], and RMS spot size vs. field [Fig 6.10]. The field curvature, astigmatism and f- θ distortion are presented in Fig 6.11. The lens projection and the equisolid angle distortion are also calculated and presented in Fig 6.12 and Fig 6.13.



Figure 6.8. OPD fan plot of the new diagonal fisheye lens design



Figure 6.9. RMS wavefront vs. field of the new diagonal fisheye lens design



Figure 6.10. RMS spot radius vs. field of the new diagonal fisheye lens design



Figure 6.11. Field curvature, astigmatism and f-θ distortion of the new diagonal fisheye lens design



Figure 6.12. Half image height vs. half field of view of the of the new diagonal fisheye lens design (thesis design) compare to the standard equisolid angle fisheye lens



Figure 6.13. Equisolid angle distortion vs. half field of view of the of the new diagonal fisheye lens design

The aberration is well controlled in this design. The peak to valley OPD is controlled under 2 waves for most of the fields up to 60 degrees. The OPD performance is falling off when the HFOV is over 70 degrees, but the peak to valley OPD can still be maintained around 3 waves by carefully introducing some vignetting. The RMS wavefront error is under 0.5 waves for most of the field, and under 1.4 waves for the entire field. The RMS spot radius is under 10µm for most of the field, and under 25µm for the entire field.

According to Fig 6.11, the field curvature is maintained at a good level. The offset of the field curvature on axis is due to the defocus that introduced to the lens for aberration balancing. From the field curvature plot, astigmatism is also presented by the deviations of the tangential and sagittal lines. The astigmatism is well controlled and is only 0.3 mm at the edge of the field. In this figure, Zemax also provide the f- θ distortion of the lens. However, as mentioned before, this distortion is not very useful since the lens is not designed as an equidistant f- θ fisheye lens in the first place. However, the image projection plot and equisolid angle distortion plot from Fig 6.12 and 6.13 shows that this lens has very low amount of distortion from the standard equisolid angle projection. Note that the final half image height of this design is slightly larger than the reference image height in Fig 6.12. This is because a 0.2 mm margin is added to the half image height of the design in case of the small difference between the sensor diagonal lengths from different manufacture.

Next, the relative illumination and its comparison with current lenses are presented in Fig 6.14 and Table 6.5. The lateral color and its comparison with current lenses are presented in Fig 6.15 and Fig 6.16. The MTF is presented in Fig 6.17.



Figure 6.14. Relative illumination of the new diagonal fisheye lens design

Lens	Canon	Nikon	Sigma	Sony	Thesis Design
Edge RI	0.60	0.56	0.51	0.61	0.68

Table 6.5. Edge	Relative	illumination	comparison
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Figure 6.15. Lateral color of the new diagonal fisheye lens design, maximum scale is 100µm



Figure 6.16. Lateral color comparison



Figure 6.17. MTF plot of the new diagonal fisheye lens design

The relative illumination of this lens is 68% at the edge. This is well beyond the 50% requirement, and is also better than all of the current designs. The lateral color is controlled at an excellent level of 10.3µm at the edge, which is only about 2 pixels on a modern 35mm CMOS sensor. And the lateral color is within the airy radius for half of the field. The excellence of the lateral color control can be easily seen from the comparison of the lateral color between the thesis design and the current designs in Fig 6.16. This lens also has a very good MTF performance. At 30lp/mm, the contrast is consistently over 70%, mostly close to 80%, for 60% of the fields. Even though the contrast falls off quickly after that, it is still maintained at 50% for nearly 80% of the fields. Since this is a diagonal fisheye lens and the image only reaches its maximum image height on the diagonal, only the corners of the picture have low contrast, which is acceptable. Also, due to the excellent

control of astigmatism, the contrast is consistent for both sagittal and tangential direction. Current lens MTF plots from Fig 6.6 can be used as rough comparison.

6.5 Design Goal Achievement and Conclusion

The final design is compared to the design goal in Table 6.6.

	Design Goal	Final Design	
Fisheye Lens Type	Diagonal fisheye lens	Diagonal fisheye lens	
Image Height	41.2 mm (35 mm sensor diagonal)	41.6 mm (0.4 mm margin)	
Focal Length	15 mm – 16 mm (varied by distortion)	15.99 mm	
Maximum Equisolid Angle Distortion	± 5%	4.49%	
Field of View	180 degrees	180 degrees	
Maximum Aperture	F/2.8	F/2.84	
Maximum Lateral Color	11.7 μm	10.3 μm	
MTF	At least 70% for 50% of the image height, 50% for 70% of the image height, and 20% for 90% of the image height at maximum aperture for 30 lp/mm.	70% for 63% of the image height, 50% for 79% of the image height, and 20% for 100% of the image height at maximum aperture for 30 lp/mm.	
Image Space Chief Ray Angle	Less than 30 degrees	28.9 degrees	
BFD	More than 38 mm	38.09 mm	
Maximum Number of Elements	8	8	
Aspherical Elements	0	0	

Table	66	Comparise	n hetweer	final	decign	and	initial	design	nool
I able	0.0.	Comparise	III Detween	i iiiiai	uesign	anu	muai	uesign	guai

According to Table 6.6, the design goals are well achieved. In conclusion, a new design of a equisolid angle diagonal fisheye lens for 35mm DSLR cameras is presented in this chapter. The aberrations in this design is well controlled. The design is able to achieve better relative illumination, much better lateral color performance, and more consistent MTF performance. At the meantime, other criteria of this design, such as distortion and lens simplicity, are able to be kept at the same level with the current designs on the market.

7 ZOOM FISHEYE LENS DESIGN FOR 35MM DSLR CAMERAS

In this chapter, current zoom fisheye lens designs and the two-component zoom structure are briefly discussed. A zoom fisheye lens designed for 35mm format DSLR camera by the author is presented along with the performance evaluations. In the end, the real cam curve calculated by the Zemax macro is also provided.

7.1 Motivation and Current Designs of Zoom Fisheye Lenses

As mentioned in the previous chapters, there are two kind of fisheye lenses for 35mm format DSLR cameras, the diagonal fisheye lens and the circular fisheye lens. The unique characteristics of each kind usually leave photographers with a tough choice. A zoom fisheye lens has enough focal length coverage to provide both circular and diagonal fisheye image is definitely a solution to that problem.

For such design, the zoom fisheye lens should form an image circle with a diameter equal or slightly small than the width of a 35mm sensor to provide a circular fisheye image at its shortest focal length [Fig 4.1 (b)]. In addition, the lens should form an image circle with a diameter equal or slightly larger than the diagonal of a 35mm sensor to provide a diagonal fisheye image that fulfills the entire sensor at its longest focal length [Fig 4.1 (d)]. Between the shortest focal length and the longest focal length, such lens can also zoom to a point where the image circle diameter is equal to the diagonal of an APS-C sensor, so the lens can be also used on an APS-C format camera as a diagonal fisheye lens [Fig 4.1 (c)]. The focal length to cover an APS-C sensor is varied by the distortion and the size of the APS-C sensor, but it should be close to the shortest focal length since the diagonal of an APS-C sensor is close to the width of a 35mm sensor.



Figure 7.1. (a) Size comparison between a 35mm sensor and an APS-C sensor, (b) zoom fisheye lens at its shortest focal length, (c) zoom fisheye lens providing diagonal fisheye image for APS-C sensor, (d) zoom fisheye lens at its longest focal length.

There are two zoom fisheye lenses currently on the market, the first one is the Tokina 10-17mm fisheye lens. However, this lens was designed for APS-C cameras, and does not provide a full circular image at its shortest focal length when using on a 35mm format camera. The other zoom fisheye lens is the Canon 8-15mm fisheye lens, which was also mentioned in chapter 2 [Fig 2.13] [17]. This lens provides both circular fisheye image and diagonal fisheye image for Canon's 35mm DSLR cameras. This lens has a good reputation among the photographer community for its excellent image quality. However, there is still room for improvement for this design. The lens has a relative small maximum aperture of F/4.1 throughout the entire zoom range, and the field of view is 175 degrees instead of 180 degrees at both wide end and telephoto end. In addition, the lens has 14 lens elements with one aspherical surface, the complicity of the design may increase the

manufacture and mounting cost. Thus, the motivation of this thesis design is to achieve larger maximum aperture, 180-degree field of view with less complex structure, while still maintaining good image quality.

7.2 Zoom Lens Structure

Zoom lens has a long history that is over 100 years. The early zoom lens designs contain two moving groups, the variator group and the compensator group. The variator group usually moves linearly to vary the focal length, and the compensator usually moves none linearly to maintain a fixed image position. Modern zoom lenses sometimes use more than 2 moving groups to achieve better aberration control. However, more moving groups means more complex optical and mechanical design, which usually increase the cost. Thus, two component zoom structure is still widely used for the modern zoom lenses. For a zoom fisheye lens, the purpose is to change the image circle size so that it can serve as both circular fisheye lens and diagonal fisheye lens. For such purpose, the lens does not have to have a large zoom ratio (usually less than 2). Thus, the two component zoom structure is sufficient enough to provide good aberration control while keeping the structure simple.

For the two component zoom lens, the early designs used positive front group and negative back group. In this telephoto structure, the lens groups move toward the image plane quickly when zooming towards the short focal length. For SLR cameras, sufficient BFD (around 40mm) is needed for the folding mirror, which limit the movement of the lens groups in this structure. As such, the zoom ratio is limited for zoom lenses with telephoto structure. Later, the two component zoom lenses were modified to have the negative group in the front and positive group in the back to form an inverted telephoto structure that provides larger BFD and has less limitation on the zoom ratio. The difference between the telephoto and inverted telephoto zoom structure is presented in Fig 7.2. For this plot, the focal lengths of the components are arbitrarily set at 30mm and -40mm. Both groups are assumed to be thin lenses, so the group movement can be simply plotted according to the following equations:

$$\varphi = \varphi_1 + \varphi_2 - \varphi_1 \varphi_2 t \tag{7.1}$$

$$BFD = f - \frac{\varphi_1}{\varphi}t \tag{7.2}$$

where $\varphi = \text{total power of the zoom lens}$

 φ_1 = power of the front lens φ_2 = power of the rear lens t = separation between two lenses f = focal length of the zoom lens BFD = back focal distance

The plot shows the movements of components in a telephoto and inverted telephoto structure before the rear group reaches the BFD limitation (40mm from the image plane). Since the movements of the components are linked with a mechanical cam in the actual zoom lens, this kind of plot is often referred as the cam curve, and should be analyzed for every zoom lens design.



Figure 7.2. Movements of components in a telephoto and an inverted telephoto zoom

According to the plots in Fig 7.2, inverted telephoto structure is more useful for zoom lens with small minimum focal length. Currently, many wide-angle zoom lenses have reached a short-focus limit of 24mm. A few of them even reached 16mm. For fisheye lenses, the short-focus limit is even smaller. Thus, an inverted telephoto structure should be used for a zoom fisheye lens design.

The design procedure of a zoom fisheye lens is similar to the design of a fixed focal fisheye lens. Some common limitations on the camera end, such as the image space chief ray angle and the minimum BFD, are also applied to the zoom lens design. For a twocomponent zoom structure, the physical aperture is generally located in one of the group and moves with it [10]. In this design, the diaphragm is located in the rear group. Since the stop is moving relative to the image plane, consequently, the relative aperture (F/#) changes during the zoom. Largest maximum aperture during the zoom usually achieves at the shortest focal setting (wide end), and the smallest maximum aperture usually achieves at the longest focal setting (telephoto end). To have equal or larger aperture than the Canon zoom fisheye lens with constant maximum aperture at F/4.1 through the entire zoom range, the aperture of the thesis design should achieve at least F/4.1 at its telephoto end, and larger relative aperture at its wide end. In addition, the automatic exposure control in most of the modern DSLR cameras can maintain the aperture consistently at F/4.1 or smaller during the focus if the user desire. This lens is designed as an equisolid angle fisheye lens. Some equisolid angle distortions are allowed since the departure from standard equisolid angle projection is usually not a problem for photographic fisheye lenses. The short-focus limit and the long-focus limit are varied by the distortion to provide a circular fisheye image near the short-focus limit, and a diagonal fisheye image near the long-focus limit. The lens is optimized at both short-focus limit and long focus limit, along with an intermediate focal point. However, since the main purpose of this lens is to capture either a circular fisheve image or a diagonal fisheye image, the longest focal point and shortest focal point have more weight than the intermedia focal point during the optimization. The specific design
F/#		3.2		3.6		4.1		
Focal length		9.1		12.:	12.5		16.2	
FOV		180		180	180		180	
BFD		38.000		45.60	45.602		53.762	
d7		26.604		12.580		4.072		
d17		38.000		45.602		53.762		
Surface No.	r		d	Nd	Vd		Clear Aperture	
1	108.248		2.238	1.67790	55.2		68.2	
2*	15.908		22.694				35.5	
3	-41.131		2.000	1.61800	63.4		30.0	
4	23.403		5.818	1.78472	25.7		30.0	
5	137.397		5.921				30.0	
6	-20.462		2.685	1.51680	64.2		25.9	
7	-25.4	480	variable				26.7	
8	8 29.2		2.000	1.50847	61.2		19.0	
9	631.432		12.375				19.0	
10 (Stop)	Stop) Infinity		0.179				12.6	
11	25.937		5.232	1.49782	67.0		18.0	
12	-16.698		1.000	1.88300	40.8		18.0	
13	25.856		7.861	1.60342	38.0		18.0	
14	-28.408		0.000				18.0	
15	149.053		1.305	1.80518	25.4		20.6	
16	37.205		3.805	1.61800	63.4		20.6	
17	-39.784		variable				20.6	

parameters and lens data are provided in Table 7.1. And the design layouts at both wide end and telephoto end are presented in Fig 7.3.

Table 7.1.	Zoom	fisheve	lens	design	parameter
					r

*Aspheric data (2 nd surface)							
K = -2.45355E-01	A4 = -1.74172E-06	A6 = 6.94079E-09					
A8 = -6.54051E-11	A10 = 2.04548E-13	A12 = -3.48448E-16					

Table 7.1 (cont.). Zoom fisheye lens design parameter



Figure 7.3. Lens structure at (a) wide end, and (b) telephoto end

This design contains only 10 elements in 6 lens groups. Cemented surfaces are used for easy mounting and better tolerance purposes. The maximum diameter of the lens is 68mm, and the maximum total length from the first surface to the last surface is 101.7mm. The size is compact compare to other wide angle zoom lenses. The rear surface of the first element is an aspherical surface for better aberration control.

The lens performances are evaluated under the same criteria with the fixed focal fisheye design presented in Chapter 6. The evaluations are performed at both wide end and telephoto end. The total aberration is presented in three methods: OPD fan plot [Fig 7.4], RMS wavefront vs. field [Fig 7.5], and RMS spot size vs. field [Fig 7.6]. The field curvature and astigmatism are presented in Fig 7.7. The lens projection and the equisolid angle distortion are also calculated and presented in Fig 7.8 and Fig 7.9.





Figure 7.4. OPD at (a) wide end, and (b) telephoto end, both plots have maximum scale of ± 2 waves.



Figure 7.5. RMS wavefront error vs. field at (a) wide end, and (b) telephoto end, both plots have maximum scale of 2 waves.



Figure 7.6. RMS spot radius vs. field at (a) wide end, and (b) telephoto end, both plots have maximum scale of 50 µm.



Figure 7.7. Field curvature and astigmatism at (a) wide end, and (b) telephoto end



Figure 7.8. Half image height vs. half field angle of the thesis design compare to the standard equisolid angle fisheye lens at (a) wide end, and (b) telephoto end



Figure 7.9. Equisolid angle distortion vs. half field of view of the thesis design at (a) wide end, and (b) telephoto end

In this zoom fisheye lens design, aberrations are well controlled. The peak to valley OPD is controlled under 2 waves for the entire field of view at both wide end and telephoto end. The RMS wavefront error is under 0.5 waves for the entire field of view at wide end, and under 0.8 waves for the entire field at telephoto end. The RMS spot radius is under 15µm for the entire field at wide end, and is under 25µm for the entire field at telephoto end. The aberrations are controlled better at the wide end due to the smaller image height.

According to Fig 7.7, the field curvature is maintained at a good level, especially at the wide end. The offset of the field curvature on axis are due to the defocus that introduced to the lens for aberration balancing. From the field curvature plot, astigmatism at both ends are also presented by the deviations of the tangential and sagittal lines. The astigmatism is almost not existed at the edge of the field, and remains good correction for the entire field at wide end. At telephoto end, some astigmatism is showing when the field angle reaches 40 degrees. However, it is smaller than 0.2mm for the entire field, and is well corrected at the edge.

The maximum equisolid angle distortion at both ends are around 6%. According to the image projection plot in Fig 7.8, this design has less barrel distortion in the center part of the image, and larger barrel distortion around the edge. This small amount of the departure from the standard equisolid angle fisheye lens should not be a problem for photographic purposes.

Next, the relative illumination plots are presented in Fig 7.10. The lateral color plots are presented in Fig 7.11. The MTF plots are presented in Fig 7.12.



Figure 7.10. Relative illumination at (a) wide end, and (b) telephoto end



Figure 7.11. Lateral color at (a) wide end, and (b) telephoto end, both plots have maximum scale of 50 µm.



Figure 7.12. MTF plot at (a) wide end, and (b) telephoto end

The design has excellent relative illumination at both wide and telephoto end. The edge relative illuminations are well beyond the 50% requirement. The maximum lateral color is controlled at an excellent level of 3.2µm at the wide end, which is less than the size of a pixel on a modern 35mm CMOS sensor. At the telephoto end, the maximum lateral color is controlled within 12µm. The contrast performance of this design is also excellent. At wide end, the contrast of the 30lp/mm frequency is over 70% for most of image, and is over 0.6 for the entire image circle. At the telephoto end, the contrast reduces significantly towards the edge of the image. However, the lens can still maintain a 50% contrast at 30lp/mm frequency for over 90% of the image. At the telephoto end, the lens provides a diagonal fisheye image that only reaches its maximum image height on its diagonal. Thus, a low contrast at extreme image height is not a big problem at the telephoto end.

Other than the optical performance, for a zoom fisheye lens, the mechanical cam curve is just as important. A smooth cam curve is required so that the lens is possible to zoom mechanically. Since the front and rear group of this lens are not thin lenses anymore, Eq. 7.1 and Eq. 7.2 cannot be used to calculate the actual cam curve. To accurately plot the actual cam curve, the author wrote a macro in Zemax to gradually change the separation of two groups in small increments, and track the actual movement of both groups. The resulted cam curve is presented in Fig 7.13. In this plot, the distances are measured from the rear surface of each zoom group to the paraxial image plane. Note that defocus is usually used to balance the aberration to achieve the best image quality, so the paraxial image plane may not be the real image plane. However, such defocus should be achieved by shifting the entire lens when focusing the lens during photo shooting. Thus, it should not be controlled by either of the zoom groups and is not reflected on the cam curve.



Figure 7.13. Cam curve of the zoom fisheye lens

According to Fig 7.13, the real cam curve of the zoom fisheye lens is very smooth. The movement of each group is very small during the zoom, so is the total axial length of the lens itself.

In conclusion, a zoom fisheye lens with simple structure is presented in this chapter. The use of less lens elements and cemented surfaces reduces the manufacture cost. The maximum aperture at wide end is increased from F/4.1 to F/3.2 compared to the current Canon zoom fisheye lens. A consistent F/4.1 aperture can be achieved by the automatic exposure control provided by the modern DSLR cameras if user desires. This design is able to provide full 180-degree FOV at both zoom ends to provide both circular and diagonal fisheye images, while maintain excellent image quality.

8 CONCLUSION AND FUTURE PLANS

8.1 Conclusion

In this thesis report, author described the photographic fisheye lens design in detail from the history, the fisheye projection method, the fisheye lens properties and the design difficulties of the fisheye lenses. A diagonal fisheye lens design and a zoom fisheye lens design for 35mm DSLR cameras are presented.

Fisheye imaging has a long history that goes back to more than 100 years ago. In 1906, R. W. Wood designed the world's first fisheye imaging device. A water tank pinhole camera is used to simply simulate the fisheye under water. In 1922, W. N. Bond used a hemispherical lens to replace the water in Wood's pinhole camera. Two years later, R. Hill introduced the world's first fisheye lens, which was later referred as the 'Hill sky lens'. The lens was capable of capture the entire hemisphere in front of itself, and was used as a cloud recording device. The negative meniscus shape of the first element from the Hill sky lens became a standard on every single fisheye lens until today. After the long development of fisheye lenses, they are no longer limited for scientific use. Today, fisheye lenses are widely used in many different areas, photographic fisheye lenses were also well developed since 1960s.

To capture the entire hemisphere onto the image plane, large barrel distortions are required. The traditional gnomonical projection is not useful anymore. Thus, special projection methods are needed for fisheye lenses. The original Hill sky lens and most of the early fisheye lenses were strictly following the equidistant projection. In this projection, same angle in the object space corresponding to same distance on the image plane. This projection is great for metrology purposes and is practical for angle measurements. However, such property of the equidistant projection are not necessary for photographic fisheye lenses. Today, modern photographic fisheye lens often uses equisolid angle projection for easy optimization. A very few photographic fisheye lenses were also made using stereographic projection and orthographic projection. These fisheye lenses are very rare and often requires using an aspherical surface in the front lens element to achieve the stereographic or orthographic projection.

Fisheye lenses have some unique properties. A negative meniscus front element is required to collect the light from the entire hemisphere and aim the rays at the stop. Due to the large pupil aberration, during the design of a fisheye lens, ray aiming is needed in order to find the real entrance pupil position for ray tracing. A photographic fisheye lens usually has very short focal length, which requires an inverted telephoto structure to extend its BFD in order to clear enough space for the folding mirror inside a SLR camera. In addition, fisheye lenses have very large depth of field. Refocusing is rarely needed when using a photographic fisheye lens.

Designing a photographic fisheye lens is challenging. Minimum BFD requirement of a SLR camera is hard to achieve due to its very short focal length. Lateral color is also hard to control because of the extremely large field angle. Some shape constrains on the meniscus lenses are needed to make sure the lens structure is practical. The designer also needs to carefully balance between the image space chief ray angle and the size of the lens elements in the rear lens group. Fortunately, due to the pupil coma of the fisheye lenses, relative illumination is rarely a big issue. Over 100% relative illumination at the edge is possible for a fisheye lens. In this thesis report, the author presents a diagonal fisheye lens design for 35mm DSLR cameras. The design has excellent optical performance, while keeping the lens structure simple and easy to manufacture. The diagonal fisheye lens designed in this project has overall better performance than all the equisolid diagonal fisheye lenses that are currently on the market. To capture a circular fisheye image, instead of designing an individual circular fisheye lens, a zoom fisheye lens was also designed in this project. The zoom fisheye lens can be served as a circular fisheye lens at its wide end, and as a diagonal fisheye lens at its telephoto end. This kind of zoom fisheye lens is very rare on the market today. The zoom lens designed in this project has a large maximum aperture varied from F/3.2 to F/4.1 throughout the zoom range. The aberrations are well under control and the contrast performance is excellent. On the mechanical side, the lens structure is simple with only 10 elements, and the cemented surfaces are used to reducing the mounting difficulty. The design also has a smooth cam curve that makes the design of the mechanical cam for this zoom fisheye lens possible.

Fisheye lens design is challenging, and the lack of references makes the design even more difficult. Currently, there are very few references that specifically talked about fisheye lens design in detail. Most references only present some lens prescription with a brief discussion in a few sentences, so are the patents. The author wishes to provide as much information about photographic fisheye lenses as possible, so this thesis report can serve as a good reference on the topic of fisheye lens design.

8.2 Suggestions for future work

The maximum aperture for a photographic fisheye lens is limited at F/2.8 for over three decades. This is due to the tradeoff between good aberration control and the required minimum BFD for a SLR camera. A few designs with larger aperture were patented. However, none of them is on the market yet due to their high complicity level of lens structure for aberration control. Nevertheless, mirrorless cameras are becoming more popular recently. Sony even issued several 35mm 'full frame' mirrorless cameras to replace their current DSLR cameras as their new professional camera product line. With much smaller BFD constrain from a mirrorless camera, fisheye lens with higher maximum aperture is possible without affecting the simplicity of the current designs. The research on fisheye lens design for mirrorless cameras then has a great potential.

Besides the use of fisheye lenses in the visible spectrum, recent research showed that a fisheye lens can also be used in the IR spectrum for the purpose of surveillance camera, border patrol, fire alarm and military defense. However, design a fisheye lens for the IR spectrum is even more challenging than design for the visible spectrum. Broad range of the IR spectrum requires more chromatic aberration control. Longer wavelengths require smaller F/# (usually less than 1) to maintain good resolution. Glass choices are also limited for the IR spectrum. Further research is needed for a good fisheye lens design for IR spectrum.

APPENDIX A – ZEMAX MACRO

This macro generates the cam curve of the zoom fisheye lens design presented in chapter 7. The positions of two zoom group are recorded at 270 different zoom setting to accurately plot this cam curve.

for i = 0,27,0.1

THIC (7) = i

update

T = THIC (18)

GETSYSTEMDATA 1

PRINT "Distance Between Lens Groups = ", i, " BFD = ", T, " Focal Length =", VEC1(7)

next

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