MEASURING REFRACTIVE INDEX INHOMOGENEITY USING DEFLECTOMETRY

by

Evan Mekenney

A Thesis Submitted to the Faculty of the

DEPARTMENT OF OPTICAL SCIENCES

In Partial Fulfillment of the Requirements

For the Degree of

MASTER OF SCIENCES

In the Graduate College

THE UNIVERSITY OF ARIZONA

2022

THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Master's Committee, we certify that we have read the thesis prepared by **Evan William Mekenney**, titled ***Measuring Refractive Index Inhomogeneity Using Deflectometry*** and recommend that it be accepted as fulfilling the thesis requirement for the Master's Degree.

Professor Daewook Kim Date: 8/12/2022

Professor James T. Schwiegerling Date: 8/17/2022

Professor J. Roger P. Angel Date: 8/12/2022

Final approval and acceptance of this thesis is contingent upon the candidate's submission of the final copies of the thesis to the Graduate College.

I hereby certify that I have read this thesis prepared under my direction and recommend that it be accepted as fulfilling the Master's requirement.

Professor Daewook Kim Date: 8/12/2022
Master's Thesis Committee Chair
Wyant College of Optical Sciences

Acknowledgements

I would like to thank my Professor and Committee Chair, Daewook Kim. He presented me with this idea and trusted that I would be able to see it through. Thank you for the guidance and support along the way. Your belief in me helped me understand that I was capable of doing something this new and exciting. Thank you to my other Committee members Roger Angel and Jim Schwiegerling. Thank you for your valuable insight on making this experiment and thesis as great as it could be. I would also like to thank Joel Berkson and Maham Aftab. Joel was extremely helpful when something odd appeared during the experiment and would always help me think through what was happening and how we could fix it. Maham was an amazing help in writing this thesis and making sure I was able to convey the information that I wanted and needed to say. All of you are just as important to this thesis as I am.

Table of Contents

List of Figures

Abstract

Precisely knowing the refractive index of a piece of glass can be a complicated and expensive task and knowing it over a large area gets even more difficult. We are proposing a method that can measure the inhomogeneity of the refractive index of an optic while utilizing a large area and is cost effective. . We are proposing the use of multiple deflectometry measurements that will calculate the difference between two surface maps where the only change is the introduction of small variations to the refractive index under test. This system is based on the geometry of fringe reflection deflectometry to measure the slope of an object rapidly and accurately. By placing a fluid on top of the usual mirror used in a deflectometry system, we can measure the slope of a mirror that is equivalent to this “liquid lens”. Then introduce a piece of glass with a refractive index similar to the fluid and measure again. Subtracting the two images allows for the calculation of the difference in the refractive index thus allowing us to see the inhomogeneity across the glass in question.

1. Introduction

1.1 Motivation

The current methods of measuring the inhomogeneity of refractive index across a piece of glass typically require a relatively expensive interferometer. These measurement systems are also limited to relatively thin windows. This makes it difficult to measure the inhomogeneity of the refractive index when the glass is no longer shaped like a window. The motivation behind this thesis was to create an experiment to measure the inhomogeneity across a glass of known thickness but not limited to a window as well as over a larger field of view. This experiment was initially thought of as a way to test the glass that is to be used in the LFAST astronomical telescope design. This ground-based telescope will utilize hundreds of separate mirrors over a large area for observations. Because there are so many mirrors, it would become increasingly difficult to build a dome for each mirror. The largest downtime for astronomical telescopes comes from having to re-silver the surface of the mirror and this would only increase with the amount of mirrors being used. Without a dome and trying to avoid the need to re-silver, a back-silvered design was proposed. However, the challenge of a back-silvered mirror that is trying to make very precise measurements is the fact that the refractive index of the glass in front of the mirror needs to be very uniform. The goal of this project is to create a mirror that has 50 nm RMS through 50 mm of glass. With the LFAST project in mind, this experiment was created to help design a way to test if this is a possible design for the mirrors.

1.2 Objective

The objective of this thesis is to prove that the homogeneity of the refractive index of a piece glass can be measured accurately without using interferometry, on objects other than windows, and over a large area. This thesis will describe the various experiments that were performed in order to obtain the data as well as the different methods in which the data was processed to achieve the final results.

The objective of this experiment was to measure the refractive index difference between a piece of glass and liquid in order to quantify how the refractive index varies across the piece of glass. This liquid is what is known as an index matching fluid, explained further in Section 1.5. The refractive index of the fluid is very close to the nominal refractive index of the piece of glass but very hard to make exactly the same thus the difference can be measured.

1.3 Current Index Verification

There are traditionally three ways to measure the inhomogeneity of a piece of glass: transmission method, window flipping method, and liquid immersion method [1]. The transmission method uses an interferometer to make four measurements that will allow for the inhomogeneity to be classified. The first measurement is of the empty cavity. Next the window is placed in the cavity and a measurement is taken where the light transmits through the window. The third measurement taken is the reflections from the front surface of the window while the fourth measurement is the reflections from the back surface. These last two measurements are necessary to eliminate any path difference caused by the error in the surface of the window. The window flipping method is similar to the transmission method where four measurements are made. The empty cavity and the transmission are measured but flips the window between steps to measure each surface.

The liquid immersion method is similar to what will be done in this experiment. In interferometry a cavity is created around the window. An index matching fluid, section 1.5, is placed inside the cavity then the window is dipped into the liquid. The transmitted wavefront through the cavity reflects off the flat, and the return wavefront is measured. The contribution from the cavity is removed by taking the difference of the two measurements with and without the window in the cavity. It is this difference of two measurements with and without a piece of glass that is being used in this experiment. The differential measurement method is explained further in section 2.2. While this liquid immersion experiment uses a complicated interferometry system, the method used in this experiment is based on deflectometry. In comparison, the deflectometry system is simpler, less expensive, and can cover a larger field of view.

1.4 Deflectometry

Deflectometry is a surface slope measuring tool. It requires minimal hardware to acquire surface height data with high precision [2, 3]. It directly measures slope data and has a large dynamic range [4, 5]. A deflectometry system must have a screen to display a pattern and a camera to capture images of a specular surface under test, that is being illuminated by the screen. Figure 1 shows what a basic deflectometry measurement setup might look like. The camera can be a commercial camera but one with a low signal-to-noise and fast acquisition time are more beneficial. The screen can also be an off-the-shelf component as long as the pixel pitch is known. The camera is positioned such that it focuses on the surface of the unit under test while being close to the center of curvature. While the camera being at the center of curvature of the optic is ideal, it must be shifted off-axis in order to collect the light from the screen that has been reflected off of the test optic. It becomes easier to visualize the geometry by tracing the rays from the camera to the screen instead of vice-versa. The camera acts as our eye and each pixel will correspond to a point on the mirror and thus the screen. These three

points define the knowledge required to perform the desired measurement. This can be thought of as a mapping between the camera and the screen that was caused by the optic.

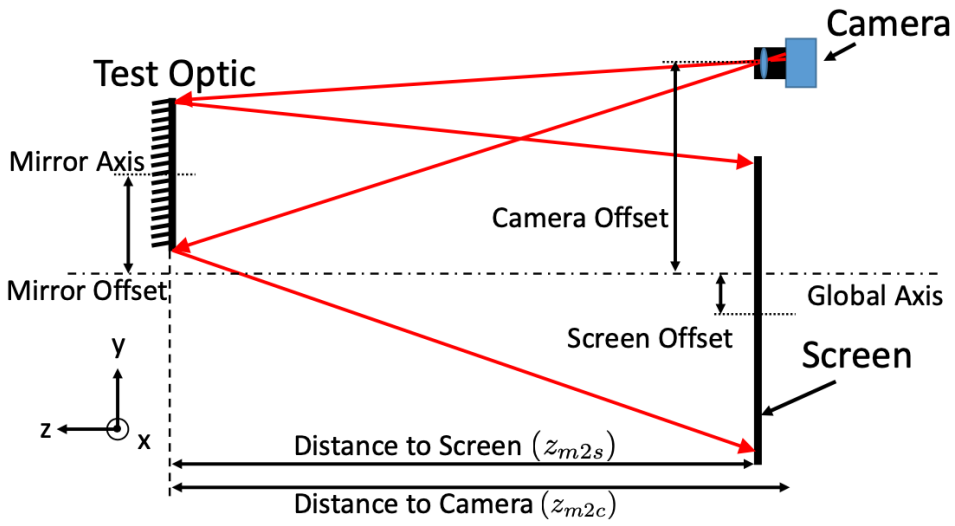


Figure 1 From Instantaneous Phase Shifting Paper, Shows the Layout of a basic Deflectometry System

To create this map, current deflectometry systems display patterns and use different methods to cover the full region of interest such as line-scanning, phase shifting, or even instantaneous phase shifting [6]. In this thesis, the phase shifting approach was used for data collection.

To acquire the slope data, a sinusoidal pattern of fixed frequency is displayed across the screen. The output brightness of each pixel is controlled in order to create the pattern. The pattern is then shifted by a fixed phase until a full 2π phase shift has been produced. This pattern is displayed in both the x and y directions separately from the other. The process and calculations for both directions are identical so only one direction will be explained here. An image is captured of the optic at each phase value. This measurement used four phase shifts where Δφ = π/2. The general expression for the displayed intensity on the screen looks like,

$$I_n(x) = a + bsin(2\pi fx + n\Delta\varphi) \quad (1)$$

where a is the background intensity, b is the amplitude variation, f is the frequency, x is local coordinate on the screen, n is an integer that shows which phase shift has been applied, and

Δφ which is the appropriate phase shift. The optic under test will change the phase of the displayed pattern. The recorded image will be that of a distorted screen pattern in camera space. This intensity can be expressed as,

$$I'_n(x') = a' + b'sin(\phi(x') + n\Delta\varphi), \quad (2)$$

where

$$\phi(x') = 2\pi f[x' + \Delta(x')]. \quad (3)$$

It is now recognized that $x_{distorted} = x' + \Delta(x')$, which is the location on the screen in the coordinate system of the screen, that corresponds to the point x' on the camera, in the coordinate system of the camera. This transformation becomes clearer when a perfect system is imagined. A perfect mirror surface ($2\pi f\Delta(x') = 0$) is observed by a camera that is collocated with the screen that displays the pattern. The measured intensity pattern on the camera is identical to the pattern that is displayed on the screen ($x_{distorted} = x'$). Therefore, when an imperfect mirror is imaged when the screen and camera are not collocated, there needs to be a transformation between the coordinate spaces that is represented by $x' + \Delta(x')$. In Eq. 2 above the phase change due to the mirror under test is represented as $2\pi f\Delta(x')$. Note that the background and modulation values are different from the original display. In both directions, x and y, the system will obtain four separate images of the phase shifted pattern. With four images in a single direction, the wrapped phase of the measured pattern that was distorted from the mirror can be obtained as,

$$\phi_{wrapped}(x') = tan^{-1}(\frac{I'_3 - I'_1}{I'_0 - I'_2}). \quad (4)$$

The next step is to calculate the transformation between the camera and the screen coordinates. This is done with the measured phase information. The phase ϕ is calculated by unwrapping the wrapped phase. This resolves the 2π ambiguity. The local screen coordinate $x_{distorted}$ is calculated from the unwrapped phase as,

$$x_{distorted} = \frac{\phi(x')}{2\pi f}, \quad (5)$$

where f is the fringe frequency given is inverse pixels. This allows for the screen coordinates to be given in pixels. The unit of pixels can be converted to a physical distance using the pixel pitch of the screen. This information defines the mapping between the camera pixels and each screen pixel. From this mapping, the three-dimensional geometry of the experiment, shown in Fig 1, the local slope of the surface under test can be computed using,

$$s_x = \frac{1}{2}\left(\frac{x_m - x_s}{z_{m2s}} + \frac{x_m - x_c}{z_{m2c}}\right), \quad (6)$$

where s_x is the local slope, x_m is local mirror surface coordinate, x_s is the screen pixel coordinate, x_c is the camera pixel coordinate, z_{m2s} is the distance from the mirror to the screen, and $z_{m2c}$is the distance from the mirror to the camera. All of the coordinates in Eq above are with respect to the global coordinate frame. The slope value is calculated pixel by pixel for the entire surface of interest. To generate the surface height map, the slope in both the x and y directions are integrated. In this experiment, Southwell average integration is used [7].

1.5 Index Matching Fluids

Index matching fluids refer to a liquid that has a very similar refractive index to a reference object. This object can be a piece of glass, mineral, or gemstone. These fluids do not always need to be specially made. Common household liquids such as vegetable oil can be used. The fluid used depends on the object that needs to have its index referenced to. The concept of index matching fluid is best explained by a simple experiment. A cup that allows for what's on the other side to be seen is placed on a table. Someone pours a liquid into the cup. The other side is still able to be seen after the liquid is poured. Now a long rod is placed into the cup. The top of the rod is able to be seen above the liquid, but the portion of the rod that is in the liquid appears to not be there. This is because the rod and the liquid have very similar refractive indices. To the observer, the background appears unchanged because the light that

travels through the liquid and the rod take exactly the same path. If the indices were different, the path the light takes would be affected by the rod, thus making it visible to the observer.

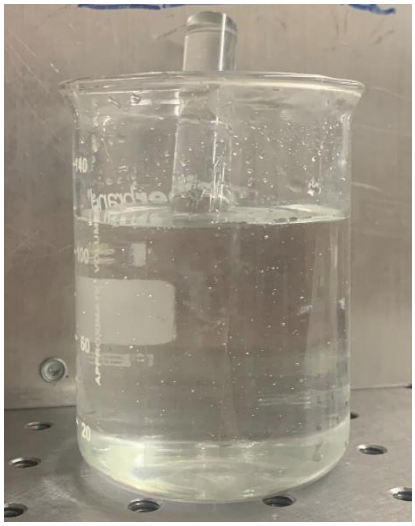


Figure 2 Borofloat Rod Partially Submerged in Index Matched Glycerol

Index matching fluids are not exclusively paired with glass. They are also commonly used in the identification of precious gemstones [8]. For precise information about the gemstones, very specific liquids are used. However, there are plenty of commonly found liquids that have a refractive index in the range of most glasses [9]. This reference was used to find a liquid that had a very similar refractive index to the glass that was picked and can be used find a refractive index matching fluid for other glasses as well.

1.6 Annealing Glass

The annealing of glass is a process used to strengthen a piece of glass. It is done by placing the glass in an oven and bringing it to a very high temperature. This temperature varies depending on the glass that needs to be annealed. Once the glass has been brought up to the correct temperature, the oven will cool at a specific rate to help control the rate at which the glass cools down [10]. This cooling rate directly affects the refractive index of the glass. The slower the cooling rate the more evenly the density of glass becomes. This leads to a more

uniform refractive index. If the rate of cooling is not controlled the glass may cool at a different speed for different parts of the glass. The inside of the glass typically cools at a much slower rate than the outside if the temperature is not properly controlled. While this process is typically used when the tensile strength of the glass is important or to reduce the stress from manufacturing, the capability of altering and possibly controlling the refractive index are what will be used in this experiment. Below is a graph of an annealing process that shows the variation of temperature vs time. In this experiment, an annealing temperature of 565^{o}C is needed for BOROFLOAT. The oven was brought up to 580^{o}C. In the controlled annealing process the oven soaked at the annealing temperature for one hour then cooled at a rate of 3^{o}C per minute. In the fast-annealing process, the oven brought up to 580^{o}C and then door and lid were opened as to allow for a rapid cooling of the glass.

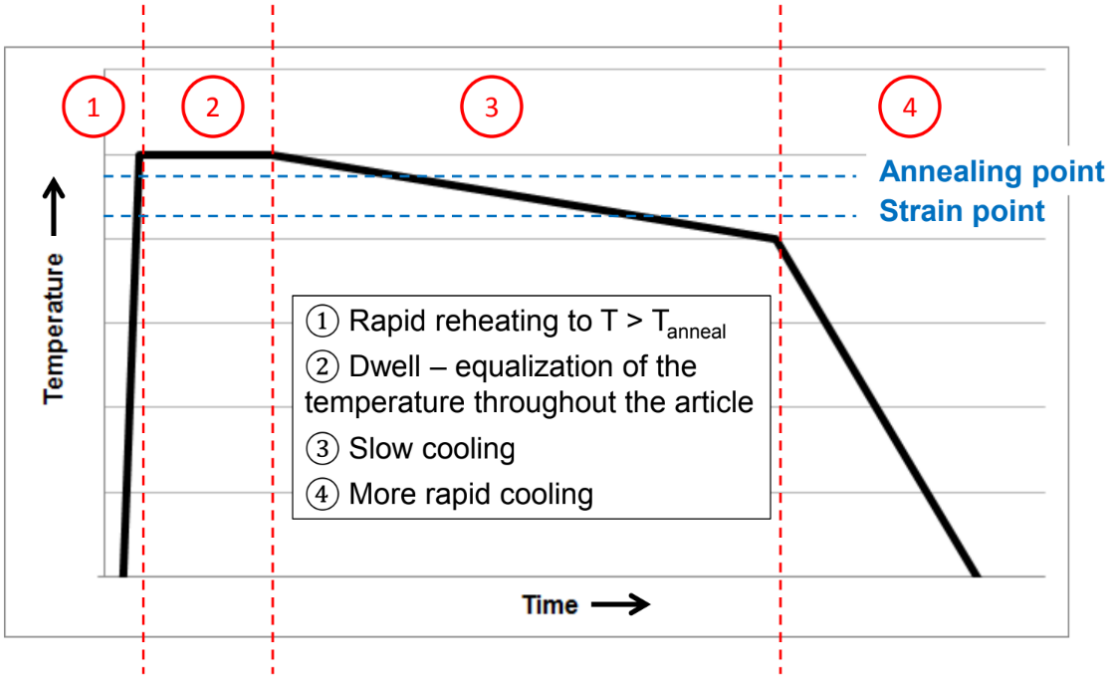


Figure 3 Annealing Method for Glass

2. Experiments

2.1 Set-up

This system contains very few necessary components making it relatively easy and inexpensive to replicate. The four main components of the system are a digital screen, a relatively specular mirror, a camera, and a computer, shown in Figure 4. A thermometer was added to accurately maintain the temperature of the glycerol thus adding a bit more control of the refractive index.

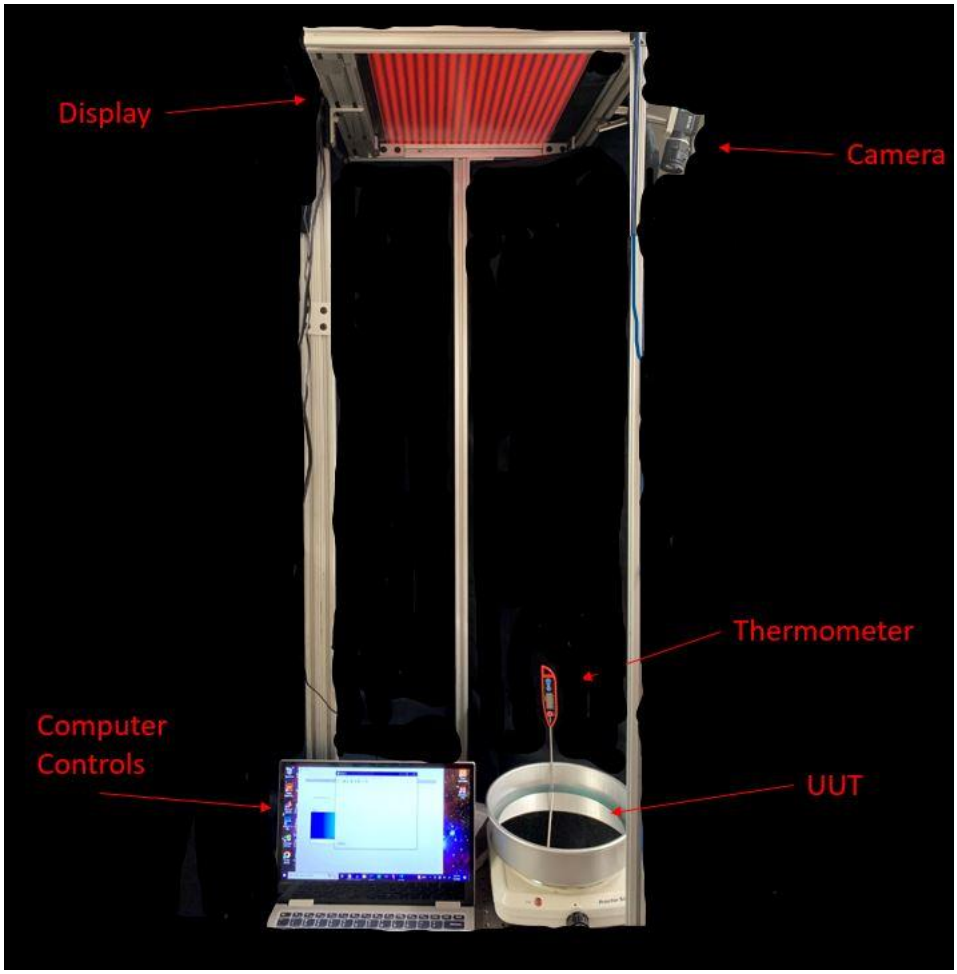


Figure 4 System Component Layout

The digital screen is used to display fringes onto the mirror. These fringes vary the brightness across the mirror that allows for the calculation of slope across the mirror. The fringes on this display also use the phase shifting principle talked about in Section 1.4. Shifting the fringes allows for a specific phase value to be associated with each part of the mirror. Using the method talked about in the earlier section, the phase can be unwrapped and allows the system to match up the points properly. The mirror, commonly used as the unit under test in deflectometry systems, reflects the fringes from the display to the camera. The camera is focused on the surface of the mirror. This is important to distinguish from focusing on the image of the fringes. The purpose of the fringes is to change the brightness of different locations across the mirror. So, the image of the fringes does not benefit the system. Lastly, a computer is used to run the acquisition of images as well as process the data.

The set up for the experiments run in this thesis vary only slightly from the ideal set up for deflectometry systems. Instead of just measuring the slope across the surface of a mirror, the mirror is placed inside of a dam and glycerol was poured on top until it was fully submerged with room to place objects on top of the mirror without protruding the glycerol. The glycerol is shown as $n1$ in Figure 5. It fully covers the surface of the mirror and has a depth large enough to place a piece of glass on top of the mirror such that the piece of glass is also fully submerged. The piece of glass, shown on the right as $n1 + \Delta n$, has a refractive index very close to the refractive index of the liquid. In this experiment a refractive index difference on the scale of 1E-5 was measured. However, the largest possible difference of refractive index was not tested for.

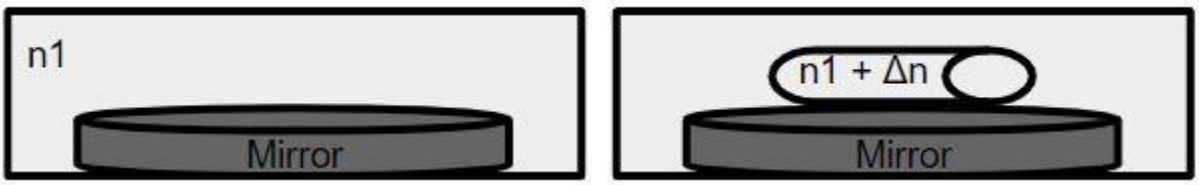


Figure 5 The two set-ups for the UUT in order to perform differential deflectometry

In this experiment, glycerol was chosen as the index matching fluid, $n1$, as its refractive index is very close to the glass that was going to be tested: Borofloat. Placing this fluid on top of the mirror does have a downside as it creates a reflection from the top surface that can be seen in some measurements. This front surface reflection can be avoided by properly positioning the mirror in relation to the camera and screen. These positions need to be clearly known before the measurements are taken and not changed during the experiment. The positions of the screen and the camera are measured relative to the center of the mirror. These values are important to calculating the slope across the mirror so being off by more than a centimeter can drastically change the calculations.

For this experiment, two measurements were taken to get the data that allowed for the calculation of refractive index difference. The initial measurement was taken of just the fluid on top of the mirror. The next measurement was taken after a piece of glass was inserted into the fluid. This process was repeated any time a new measurement was taken. More information about this process will be talked about in Section 2.2 differential measurements.

The purpose of this system is to measure the inhomogeneity of refractive index in a piece of glass. This was demonstrated by slightly changing the refractive index of a piece glass through annealing. The glass was measured before and after this process to compare if a change happened and if it is possible to measure that change. Section 2.5 goes into further detail about this process and the results that ensued.

2.2 Differential Measurements

Differential measurements refers to taking two measurements to get the data that allows for the calculation of refractive index difference. The difference map of the refractive index between the fluid and the piece of glass is what allows for the inhomogeneity across the glass to be seen. The initial measurement was taken of just the fluid on top of the mirror. The next

measurement was taken after a piece of glass was inserted into the fluid. By adding the glass into the fluid for the second measurement, the optical path length was changed between the two measurements. If the difference of the two measurements is taken, the optical path difference can be observed. Since the locations of each component stayed the same, the only difference between the two measurements should be the optical path length induced by the difference in refractive index. From this there are two optical path differences that can be related. The first is what is observed by the system. In the measurement, the system sees the entire unit under test; mirror, liquid, and glass; as one mirror system. The calculations assume it is just a mirror and the sag is being measured. Therefore, the OPD is equal to two times the measured sag value.

$$OPD = 2 * D_{sag} \tag{7}$$

This value can be related to the theoretical OPD from the difference in refractive index. This is equal to the difference in refractive index between the liquid and glass multiplied by the thickness of the glass.

$$OPD = 2 * \Delta n * d_{glass} \tag{8}$$

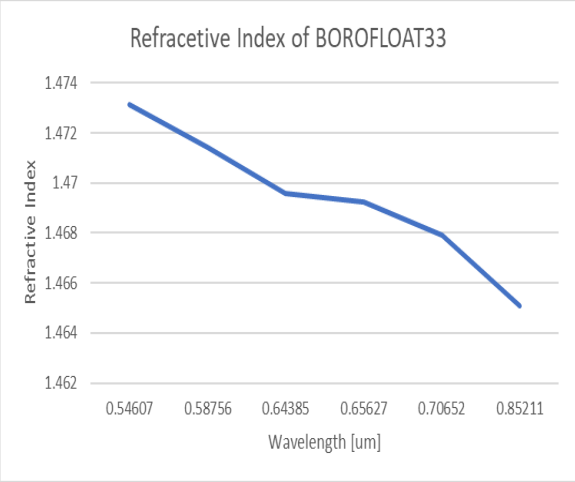
Setting equations 7 and 8 equal to each other, the difference in refractive index can be calculated. This allows for the variation between the refractive index of the liquid and the glycerol to be unknown as long as the thickness of the glass is known.

$$\Delta n = \frac{D_{sag}}{d_{glass}} \tag{9}$$

This equation can be applied to the differential sag map that is calculated. Therefore, dividing the sag value of each pixel by the thickness of glass that it traveled through will yield the difference in refractive index between the liquid and the test glass. Looking at the variations of the difference in refractive index will allow us to understand the inhomogeneity of refractive index across the piece of glass.

2.3 Index Variation Setup

The use of index matching fluids allows for the refractive index of the fluid to be changed. Using an index matching fluid allows for the difference in refractive index between the glass and fluid to be controlled and minimized. Index matching fluids can be customized so the refractive index is precisely controlled. However, there are lots of fluids with known refractive indices that can be matched to the glass that is under investigation. A quick search showed that glycerol was a very good match to the glass that was under investigation in this experiment: borofloat [11, 12]. The figures below show the data from refractiveinfo.net for both BOROFLOAT and glycerol shown on similar scales.



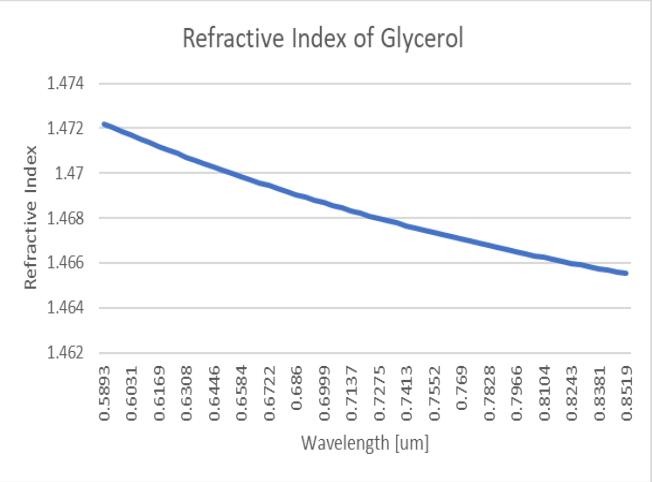


Figure 6a shows the refractive index of Borofloat *Figure 6b shows the refractive index of glycerol*

As seen in the above figures the two have quite similar refractive indices but not exactly the same. The reference refractive index was determined using the values of wavelength around $0.6\mu m$. The values are already quite close to each other at this wavelength and could be made even more similar. There is a way to get the two refractive indices closer to one another. Unlike glass, the refractive index of fluids can be changed more easily by changing the temperature [13]. The change in temperature needed to induce a change in refractive index is much lower for some liquids than it is for glass. As is the case of this experiment where glycerol was chosen as

the refractive index matching fluid. This allowed for the variation of the temperature of the glycerol to be used to give some control over the refractive index. A heating tray was used to keep the temperature of the glycerol stable. This was verified by a thermometer. The UUT, composed of a mirror sitting inside a tray, can be seen sitting on top of the heating tray while the thermometer is sticking out of the tray in Figure 4 System Component Layout.

Another way that glycerol can change the refractive index is through dilution [13]. The more water that is added to the glycerol the lower its refractive index becomes. This one is much harder to control as the weight percentage, wt%, can be different between different bottles of glycerol. Glycerol also collects moisture [14]. This means that if the location of the experiment is even slightly humid, over time the glycerol will absorb some of the moisture in the air and slowly change its refractive index. This can complicate the analysis of data sets that were taken with a large gap in time between them as the system compares the overall index difference between the fluid and glass. If the glycerol changes in the refractive index the overall scale of the refractive index difference will change.

2.4 Measurement Process

This section will describe the process of taking a deflectometry measurement with the software that runs these experiments. Everything is done through MATLAB. An application was created prior to this thesis by Hyukmo Kang and revised by Joel Berkson for another project. The application has five different pages that all play a role in the measurement process. All of the values shown in the snapshots were the ones used in this experiment.

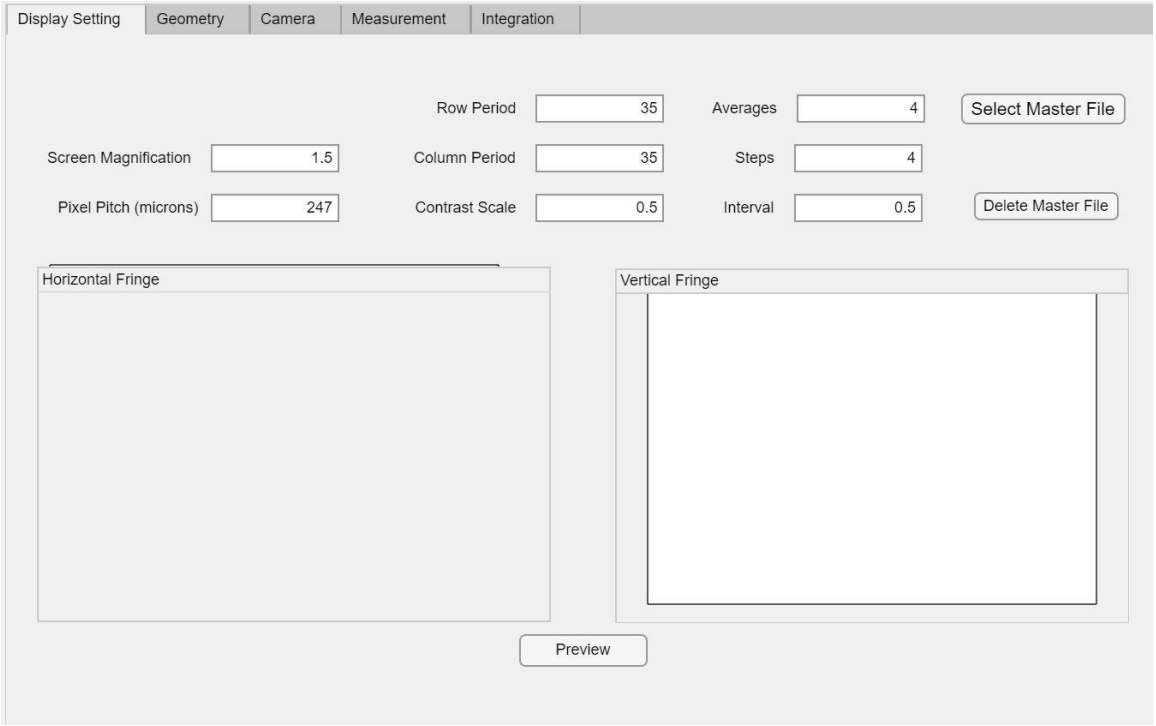


Figure 7 Opening page of GUI

Figure 7 shows the first page of the GUI that shows on start up. This is where the parameters of the screen can be changed. Based on the screen used, the image of the fringes might need to be magnified to fill the entire screen. This is changed with the screen magnification section. Underneath that, the pixel pitch, in microns, of the screen is set. The next column allows the image of the fringes to be changed. The period of the column and row fringes should be small enough such that there are a sufficient number of fringes across the mirror surface but not too small as the screen may not be able to properly display the period. This is done in units of pixels. The contrast can also be changed if seeing fringes is an issue. The final column has three more variables that can be adjusted. The Averages section controls how many images are taken and averaged together. This helps reduce noise but increases the processing time. The Steps section controls how many steps are taken to get across the overall 2π period. This has the same effect as increasing the amount of averages, less noise but more processing time. The interval variable allows for a time delay between displaying the fringes on the screen and the first image capture sequence. It was found that the display takes some time

to occur, so the delay was introduced to make sure the entire fringe pattern is displayed before any images are taken. In the top right corner, there is a button that allows for a master file to be selected. This is used once an image has been taken of the mirror that the user wishes to subtract off. If there has not been an initial measurement, this button can be ignored and used later. Lastly, the preview button at the bottom allows for the vertical and horizontal fringes to be seen before they are used in the measurement.

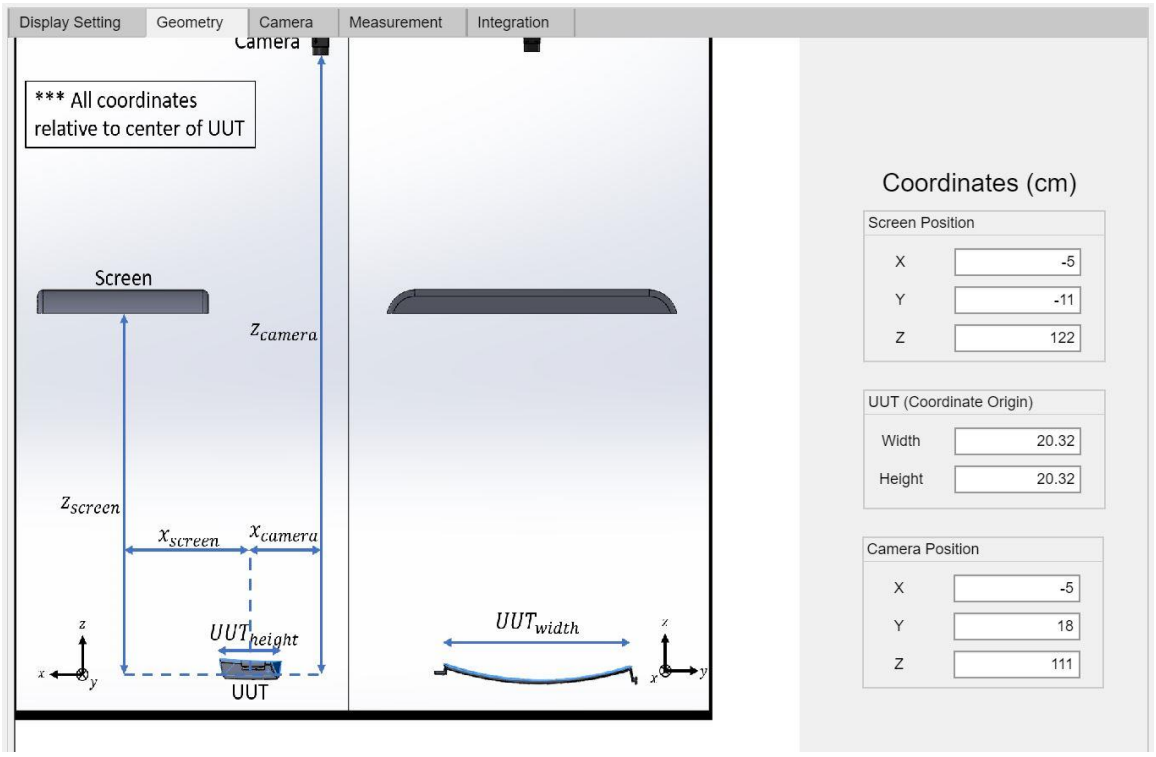


Figure 8 Page 2 of GUI

The Geometry tab is page 2 of the GUI. This is where the geometry of the system is set. As described in Section 1.4, these values are crucial to the deflectometry calculations. The UUT is defined as the coordinate origin and needs the height and width values specified. The screen $x, y,$ and z positions are measured from the center of the UUT. The same goes for the camera positions. Once these values have been measured, in this case with a ruler, it is important not to change them between measurements. Comparing data that was acquired in different locations

can be difficult as measured slopes can vary. However, if the UUT is moved for any reason, it is important to re-measure all locations before any data is taken.

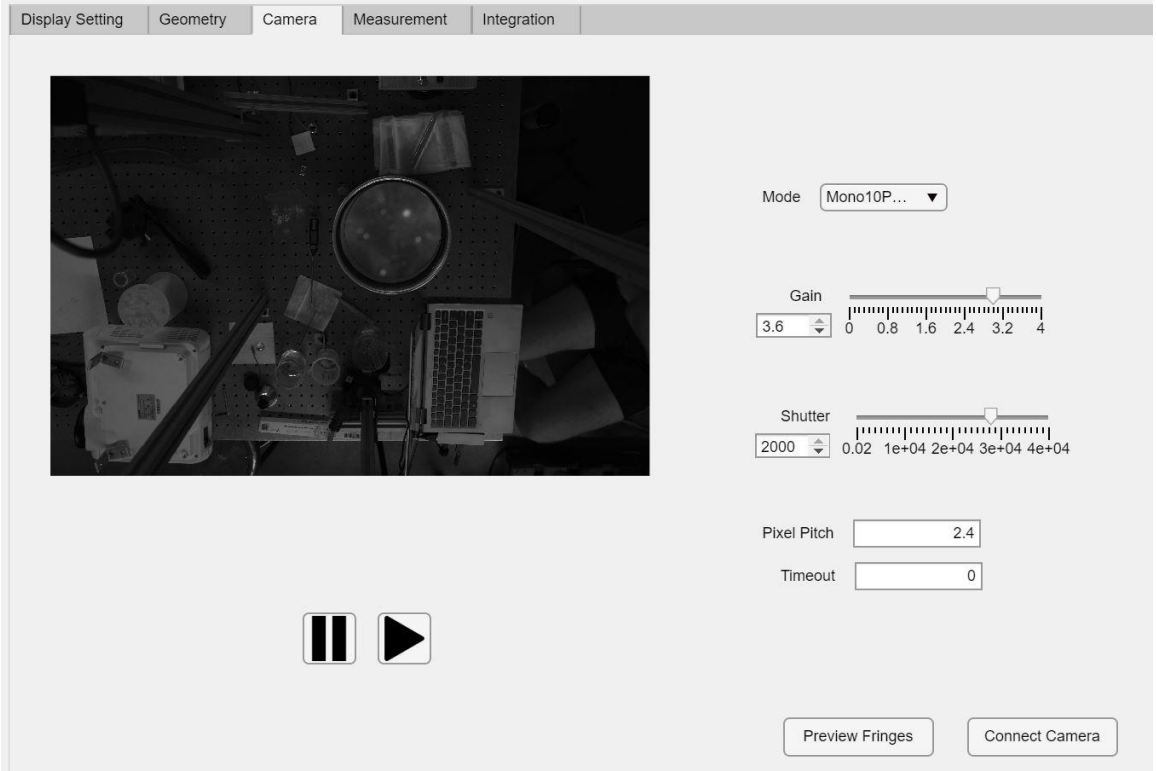


Figure 9 Page 3 of GUI

The Camera tab is the third page of the GUI. This is where the control of the camera is held. The bottom right corner connects the camera to the system. If this button is not clicked there will be no image of what camera sees on the left side of screen. If a measurement is tried to be taken and this button is not clicked, an error message will appear. On this screen, the image on the left is actually a live video of the camera's view. Once a measurement has been taken, this live view pauses. It can be restarted with the play button. Again, there is another button to preview the fringes on this page. Above these buttons are two variables that can be adjusted. The pixel pitch on this page refers to the pixel pitch of the camera in microns. The Timeout variable was not used in this experiment. The next two variables on this page are the shutter speed and the gain of the camera. These values do not need much adjustment unless the images from the camera are not a good quality. Lastly, the mode of the camera can be

adjusted between monochrome or color. This only applies if the camera has the capability of color vs monochrome. For this experiment, monochrome was used for simplicity.

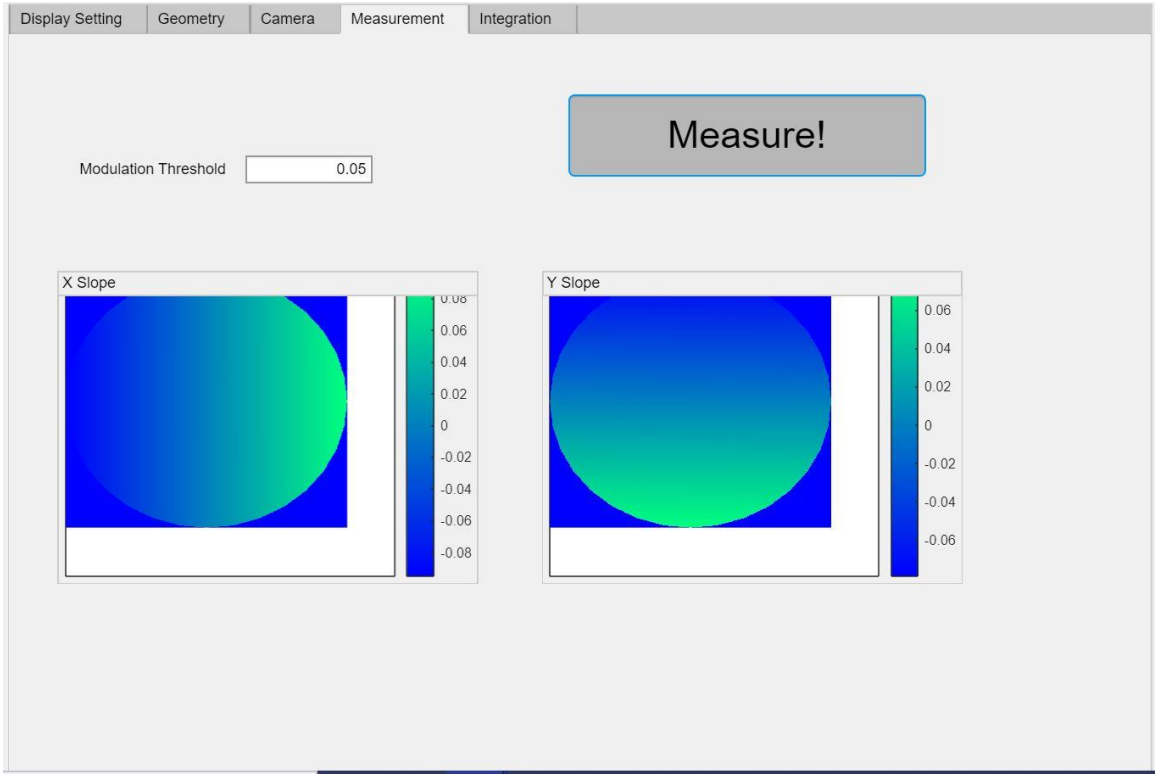


Figure 10 Page 4 of the GUI

Page four of the GUI is the measurement tab. The Modulation Threshold variable sets a minimum change in brightness of the camera pixel to change for recognition. This is primarily used in auto-detection of the UUT. In this experiment, a mask was placed on the UUT manually as the detection would occasionally assume a larger area than just the UUT. The Measure button is how the software starts a measurement. As stated before, if this button is pressed without pressing the Connect Camera button, an error will occur. Once pressed with everything connected, the fringes will display on the screen above UUT. The average image after each step of the period will be displayed on a pop-up window shown below in Figure 11.

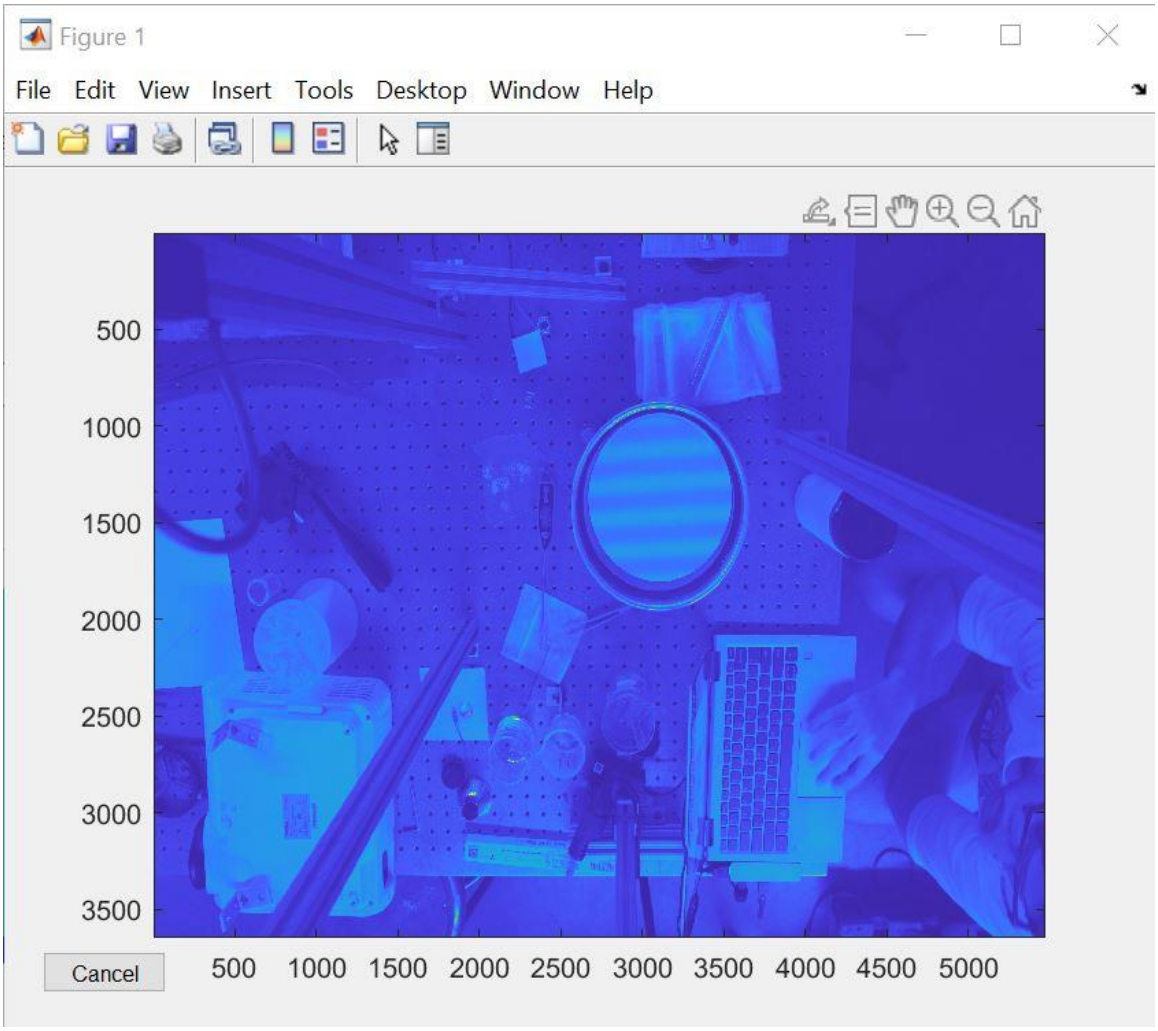


Figure 11 Image displayed during measurement process

This will cycle through twice as many times as the number of steps that were set, once for each direction x and y. After all images are taken this window will automatically close. The software will then calculate the slopes in each direction and display them on the Measurement tab as shown before in Figure 10. After the slopes have been calculated the final page of the GUI can be used.

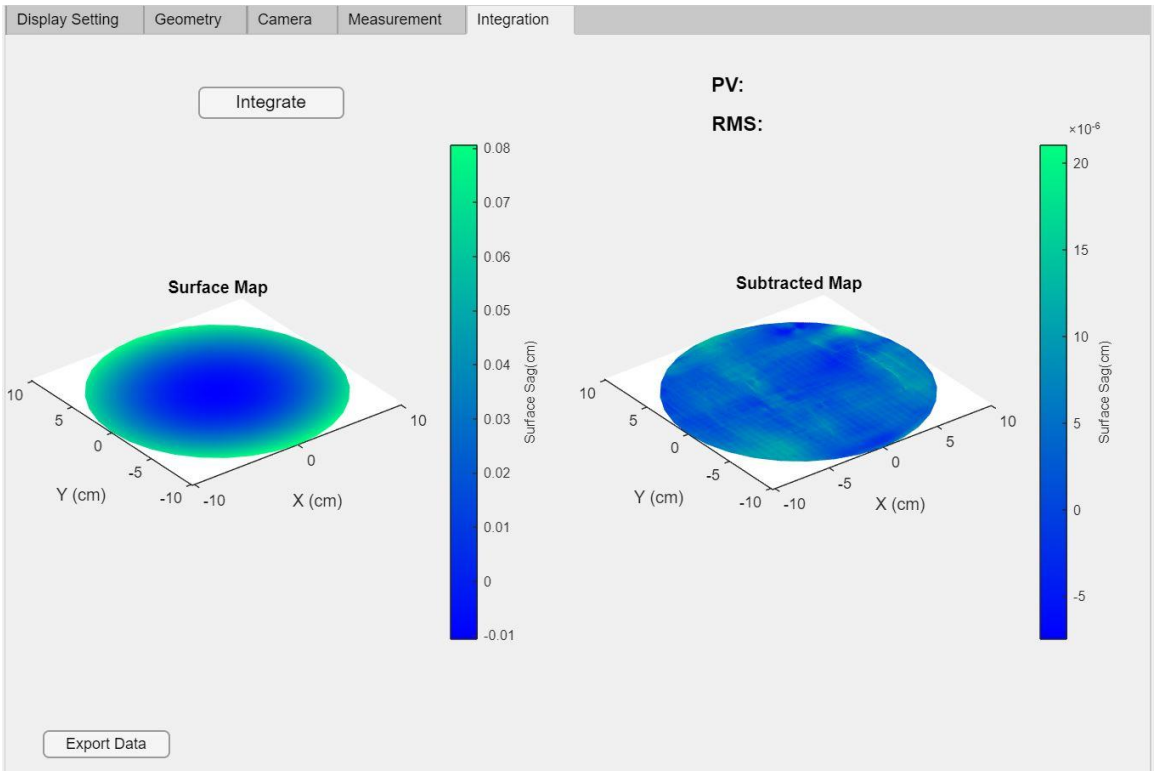


Figure 12 Page 5 of the GUI

The Measurement tab is the fifth and final page of the GUI software. When first navigated to, no maps will be displayed. To generate the Surface Map and the Subtracted Map the Integrate button is pressed. This uses the previously mentioned Southwell Integration technique to integrate both the x and y slopes to create a singular slope map of the surface. The Surface map will appear every time the integrate button is pressed. This will show the measured and calculated sag of the surface as if it were just a reflected surface. In the case for this experiment, it shows the equivalent surface sag of a mirror that has the same power as the liquid lens that has been created in this set-up. If this is the first measurement taken, without a piece of glass to be measured, it is called the master file. The data for the master file can be obtained by hitting the export data button. This allows the user to save this first measurement. The master file is then loaded into the software by using the 'select master file' on page 1 of the GUI. When the time has come to take a second measurement, hitting the integrate button will now subtract the surface sag from the master file from the newly measured surface sag. The

Subtracted Map is created by taking the current slope map and subtracting off the slope map of the Master file that was selected. Lastly, the Export Data button in the bottom left allows for the saving of the data and export from the application. The data must be exported twice. The first time is the initial measurement of the UUT without the piece of glass in question. This will be used as the Master file. The second time the export data button is used is after the next measurement was taken while the Master file was in use and the subtracted map shows nice results. The Export button only saves the Sag Map of each measurement. It will not save and export the subtracted map that is shown in the GUI.. Because of this, the Subtracted Map was created again and difference in refractive index was found using Eq. 9 detailed in Section 2.2. The results are discussed in Section 2.5; however, before this can be done, the repeatability of the system must be defined. This will set the noise floor of the system that lets the user distinguish between good and bad data.

2.5 Repeatability

Before reliable data could be taken with the differential measurement process, the repeatability of the system needed to be tested. This was done by performing the steps in the differential measurement without changing anything between the two measurements. This measurement allowed for the basic noise limit of the experiment to be set.

Below is an image of the mirror before any liquid was placed on top after back-to-back measurements were taken. Here, the fixed pattern noise of the detector can be seen to cause the most amount of noise. This measurement yields an RMS 1.59 nm which is quite good for a deflectometry system.

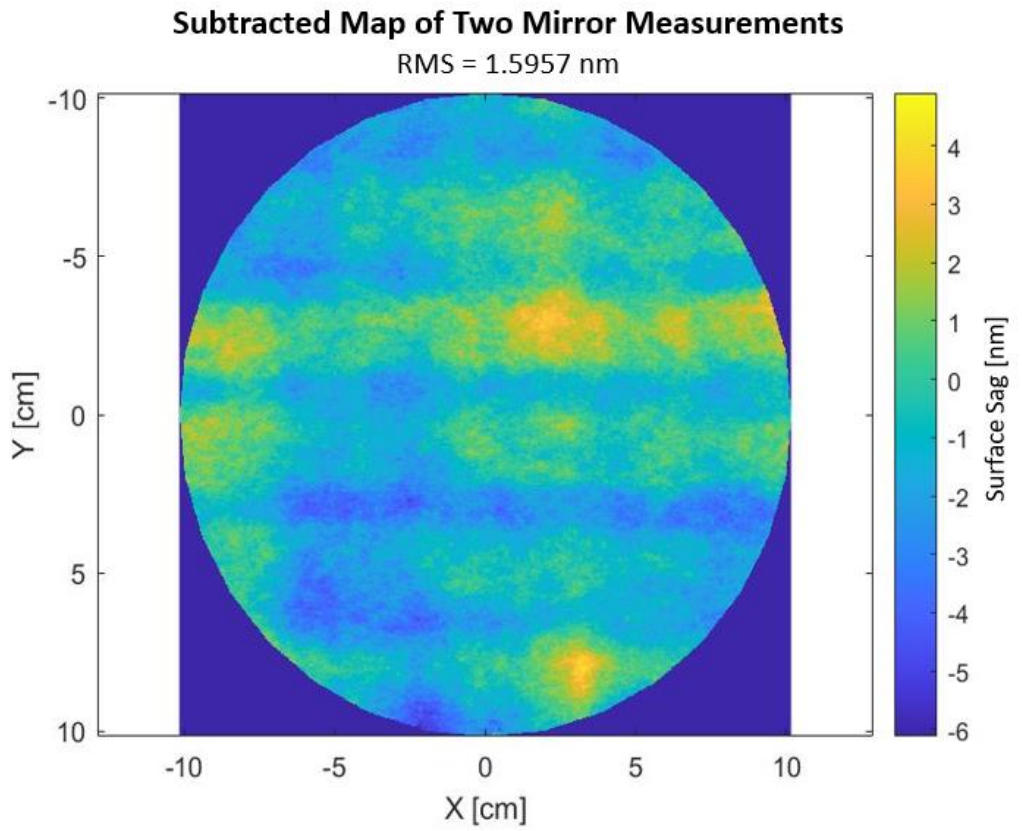


Figure 13 Subtracted Map of Two Independent Mirror Measurements

Another repeatability measurement was done once the liquid was placed on top of the mirror. It had been noted that the glycerol, as a highly viscous liquid, would take a prolonged amount of time to “settle” after it had been disturbed. This can be seen in the following Figure. Here the glycerol was deliberately stirred around, and an image was taken. There are “veins” and large air bubbles in the glycerol that will cause errors in the measurement process. The veins and bubbles can cause parts of the mirror to not be properly illuminated by the fringes which will lead to errors in the phase unwrapping. This leads to massive errors in the surface map.

Figure 14 Disturbed Glycerol

The same repeatability as before with just the mirror needs to be established while the glycerol is on top of the mirror. For this repeatability measurement, the initial measurement was taken well after the glycerol was poured onto the mirror. The glycerol was then stirred around to simulate being disturbed. The secondary measurements were taken at regular intervals of ten minutes to set a reference to the time that should be waiting between measurements where the glycerol had been disturbed. Figure 15 shows the subtracted maps for baseline and at intervals of ten minutes until the RMS approaches the baseline. The intervals start 20 minutes after stirring because the initial measurement and after 10 min, there was too much disturbance for the system to make a proper measurement.

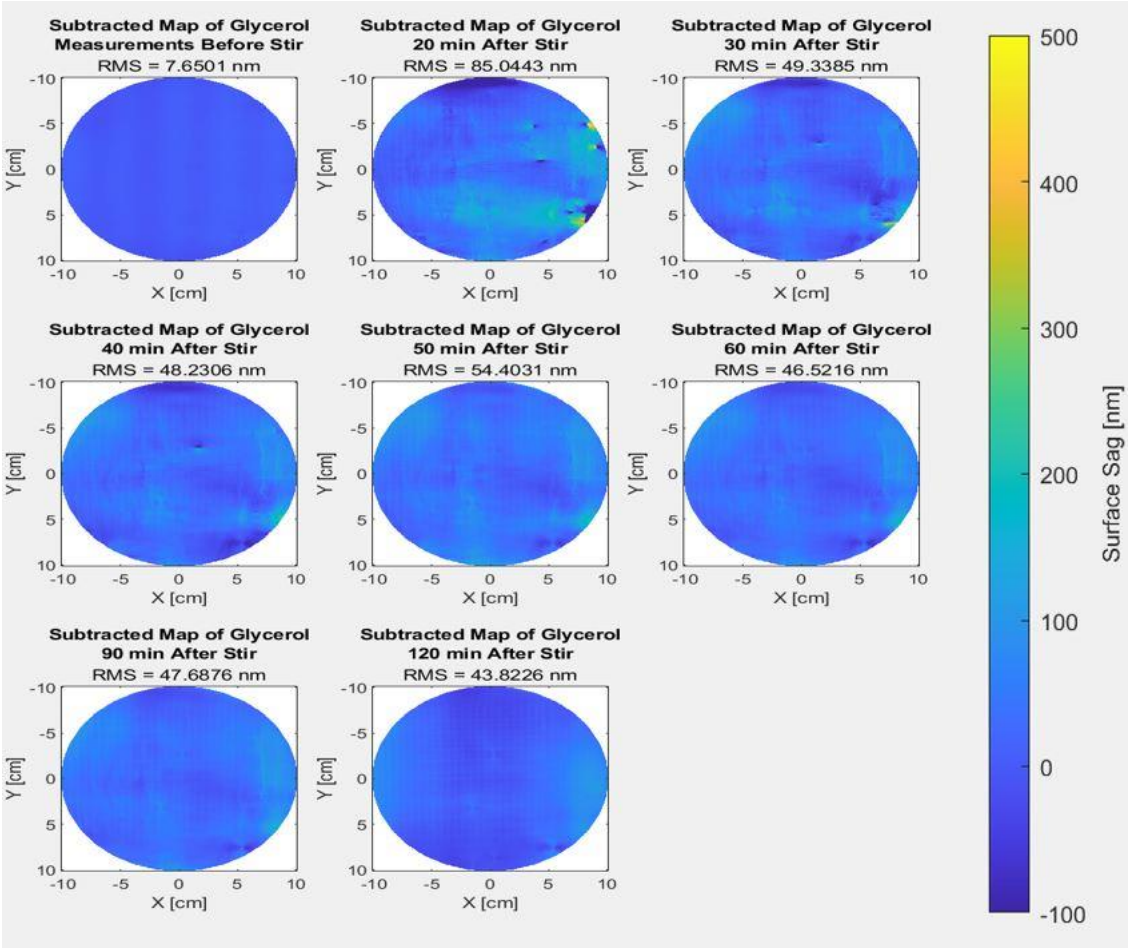


Figure 15 Shows difference map between steady state and at intervals of 10 minutes

The first difference map in the top left is two measurements of glycerol taken roughly ten minutes apart where nothing was done to the glycerol between measurements. This map has an RMS of 7.6 nm. This is quite close to the repeatability of just the mirrors and is a baseline for how well the measurements can be compared with glycerol involved. The glycerol was stirred around to simulate disturbance as glycerol would take a while to become steady after being messed with. Measurements were taken every 10 minutes and the RMS was calculated. After about 20 minutes the glycerol settles back down enough to make a proper measurement and an RMS of 83 nm is calculated. This is a good start as before the measurement was having trouble even being performed. For the next hour, the RMS jumps back and forth between 46 nm RMS and 54 nm RMS as the glycerol is still settling down. After a full two hours, the RMS gets down

to 43 nm. This is not quite as good as the start, but it does provide some valuable information. The RMS stayed fairly constant over the whole second hour of the experiment getting down to 43 nm RMS. This is likely due to some debris being mixed into the glycerol, or even the possibility that over the two hours, the glycerol collected enough moisture to create this amount of change in index. However, in this experiment, the glycerol was disturbed a lot more than what would occur throughout a normal experiment and the time spent on the this section is a lot longer than when a piece of glass is to be measured. This was tested to see how long, if the glycerol was severely messed with, it would take to return to a steady state. Most of the time, the glycerol is only slightly displaced, and 20-30 minutes is enough time to get it back to how it was. As mentioned previously in the motivation, the LFAST project has a goal of 50 nm RMS through 50 mm of glass. This helps set a target for the delta n accuracy across the glass and looks to be achievable with the 43 nm RMS after 2 hours of settling time. Again, in this portion of the experiment, the glycerol was disturbed a lot more than during the measurements where glass is placed inside of the liquid. This gives the experiment even more room in this error budget.

2.6 Effects of Annealing on a Glass Rod

To show that this system does in fact measure the variation in refractive index across a piece of glass, the same piece of glass was broken into three relatively equal sized pieces. All three pieces were examined at the same time for each measurement. The first measurement, nothing had been done to the glass. A surface map was created of the glycerol on top of the mirror without the rods, then a second map was created with all three of the rods in place. The difference between these two measurements is what allows for the calculation of the difference in refractive index. The differential sag map is shown in Figure 16.

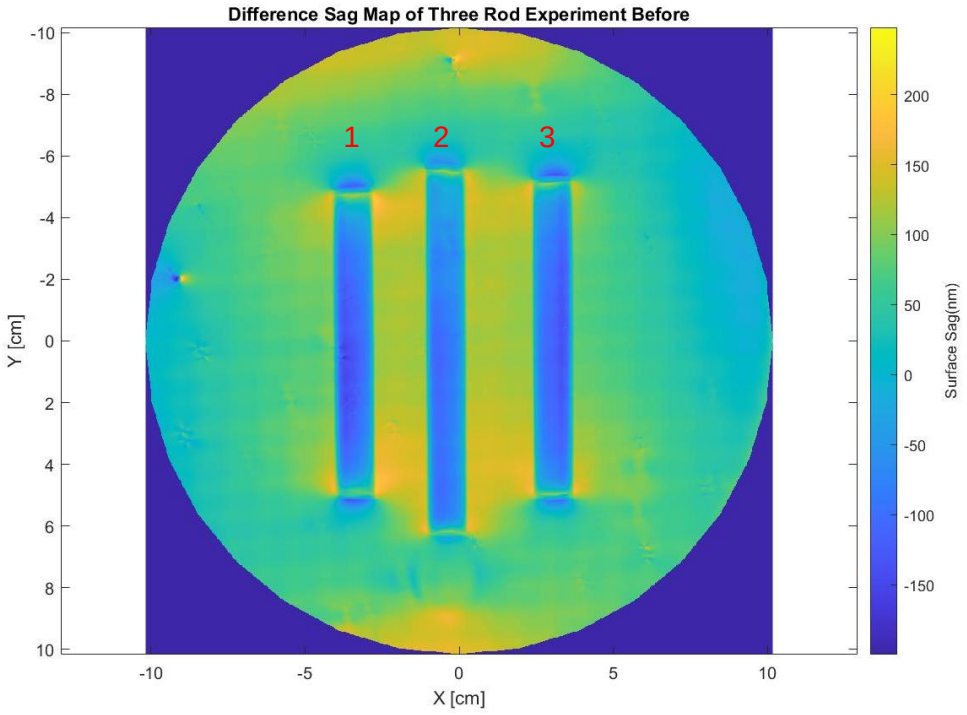


Figure 16 shows the difference in sag of the glycerol with and without the rods

Once this map has been created, the refractive index difference can be calculated using Eq 9. The sag value from this map is used as D_{sag} and d_{glass} is calculated with the chord equation,

$$d_{glass} = 2 * \sqrt{r^2 - d_g^2} \tag{10}$$

where, r is the radius of the rod and d_g is the distance from the center of the rod. It is important to use the same units. In this experiment, d_g was converted from pixels to cm.

Applying equations 9 and 10 properly yields the following maps of the difference in refractive index between the glass and the fluid.

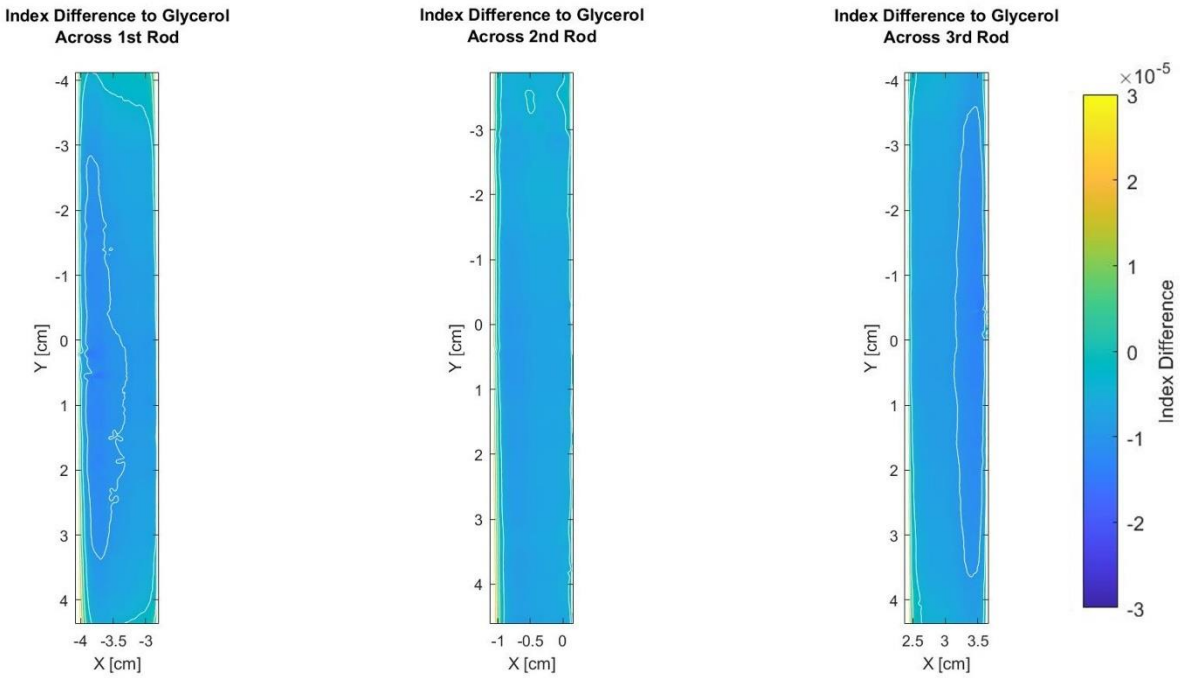


Figure 177 Map of the Refractive Index Difference between each rod and the glycerol

The white lines on each map of the above figures show the contour lines to help see the difference between the values across each rod. It is hard to see any variation when plotted like this, so the cross sections were also looked at to further investigate the results of the experiment. The average index difference was taken at each pixel across the width of the rod for each map. The standard deviation, σ, at each pixel was also calculated and used to show the error. Plus and minus 1σ are depicted by the error bars. All three average cross sections are very similar. This is a good sign as they are expected to be. The three rods were once one long rod so a lot of variation across the length of the rod is not expected.

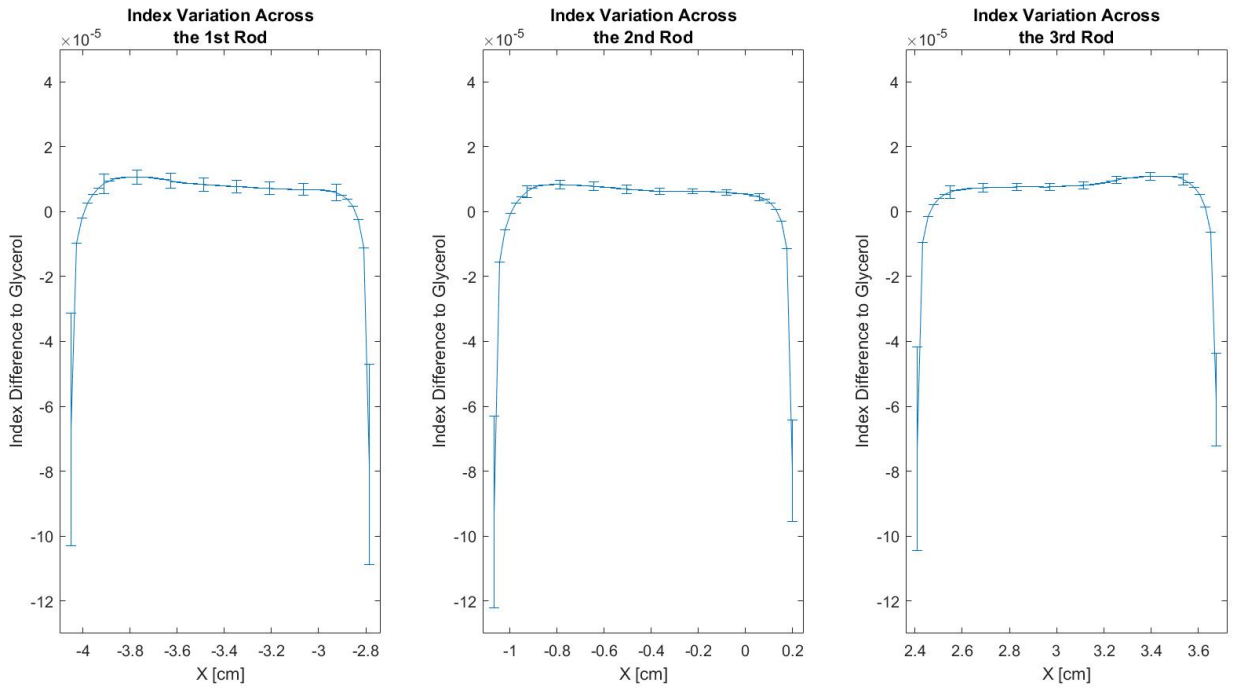


Figure 18 Average Cross Section of the Delta N Map of Figure 17

The refractive index is quite stable near the center of the rod; however, as it gets closer to the edge of the rod the refractive index begins to change. Not only does the refractive index increase at the edge of the rods, the error associated with those measurements also increases. This is due in part to the fact that it is pixel limited in this area. At the edge of the rod, it is going through a very small amount of glass. If the calculation is not exact or the rod is not exactly lined up with the pixels on the camera, it is possible that some of the pixels have more glycerol than rod. This could lead to a very different calculation as it is not taking into account that some pixels are seeing rod than others along the column. Another source of error that arises at the edge of the rod is due to the slope discontinuity between the edge of the rod and the pure glycerol. If the refractive index of the two are too different, the system will not see it as a smooth transition. Since the calculations use the Southwell integration method, the algorithm has trouble if the slopes have too large of a difference between them. These maps will be used as a reference as the next step is to change the refractive index of the glass and see if the change can be measured.

Now that a reference was created, two of the three rods were annealed individually. Of the three pieces, the first rod from Figure 16 was not placed in the oven at all. This ensured that a control piece of glass could be related back to the initial experiment. The second rod was placed in the oven was brought to a temperature of 580 degrees Celsius, just above the annealing temperature for borosilicate glass [10]. Once this temperature was reached, the door and lid to the oven were propped open as to let the glass cool off very quickly. From before, the cooling rate of annealed glass is correlated to the refractive index of the glass. The oven was left alone overnight so it would reach a solid equilibrium temperature for the next heating. Afterwards, the third rod was placed into the oven and brought to the same annealing temperature. This time, the rate at which the oven cooled off was controlled around ~3 degrees Celsius per minute. This ensured that the rod would cool down slowly and more uniformly. A new measurement without the rods was taken in case the refractive index of the glycerol had changed between the two experiments. All three rods were then placed back into the respective spot in the glycerol and another measurement was taken. The differential sag map of the rods after annealing process is in Figure 19.

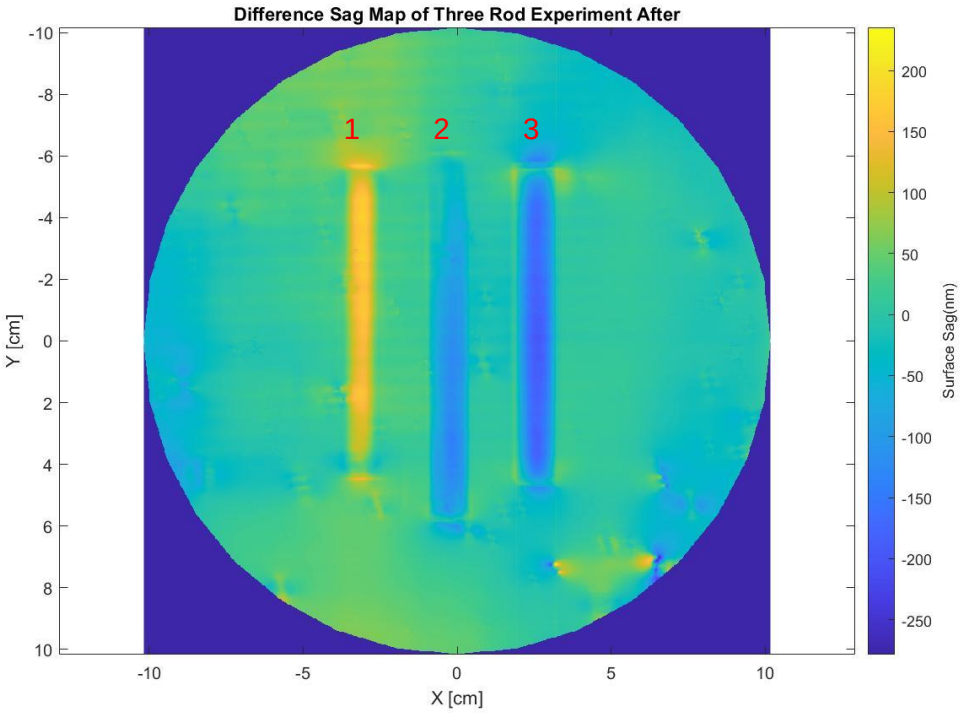


Figure 199 the total sag difference map for the three rods after some were annealed

It can be seen just in the sag difference map that the three rods are no longer the same. To fully understand how the refractive index has changed, the same calculations from the first experiment have to be performed. Applying equations 9 and 10 again to each of the rods from this map, the index difference maps can be seen for each rod. For comparison the sag maps for before and after annealing are shown next to each other in Figure 20.

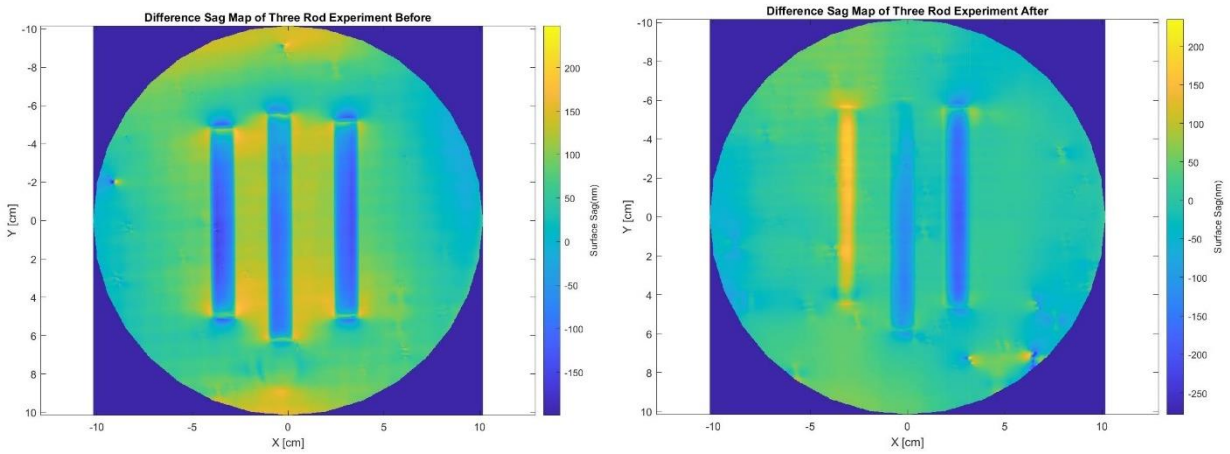


Figure 2020 Sag Map Comparison of Before and After Annealing Process

Here we can see that all of the rods before the annealing process have a very similar sag to one another. After annealing, all of the rods have a different amount of measured sag. This is what was expected to happen. It is then possible to determine how the refractive index changed between all three rods after the annealing process. The same steps were taken to determine this difference and it detailed below. Each rod was examined separately, and maps of the refractive index difference are shown here in Figure 21.

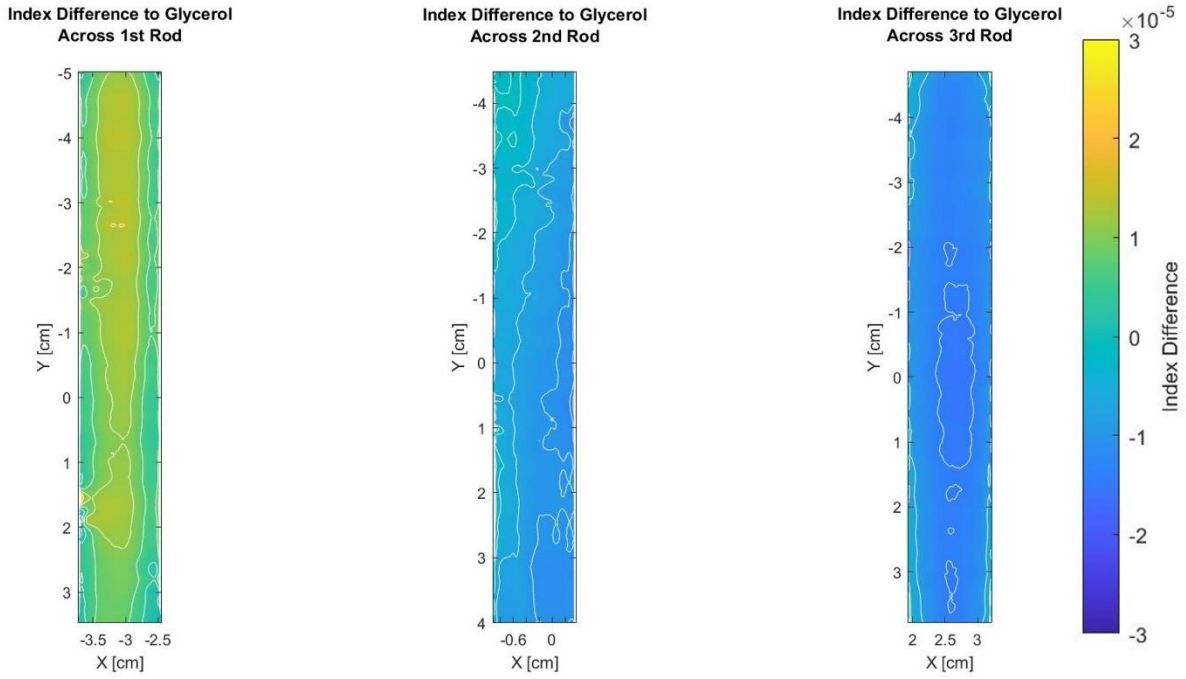


Figure 211 Map of the Refractive Index Difference between each rod and the glycerol

Again, it is initially hard to see if the annealing process had any effect on the refractive index of the two rods that were exposed. Performing the same calculations to show the average refractive index difference across the width of each rod and the error associated makes these maps easier to understand.

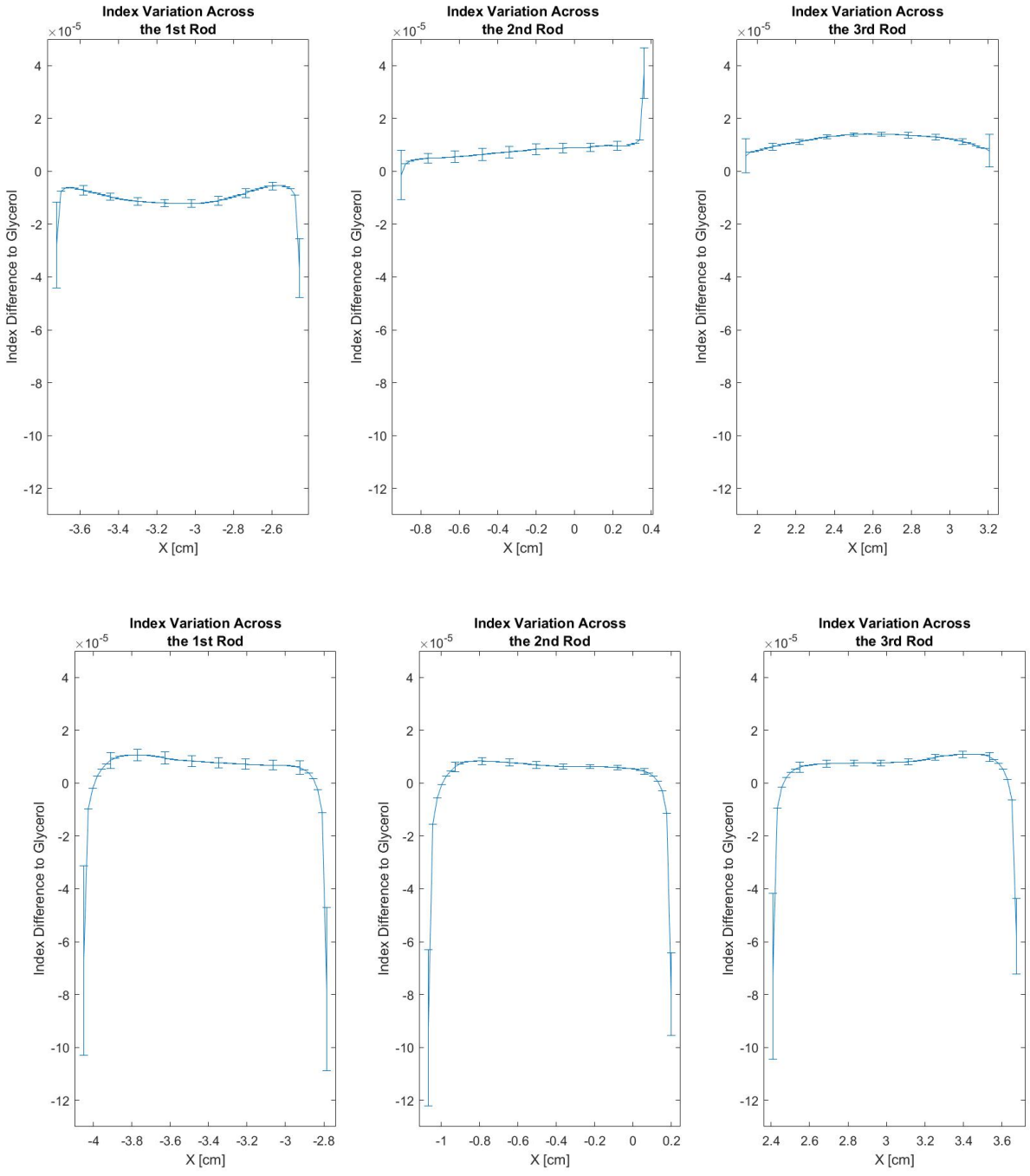


Figure 222 Average Cross Section of the Delta N Map of Figure 21 versus the average cross section of Figure 17

For comparison, the original average cross sections are shown underneath the new measurements. Unlike the first experiment, all three of these cross sections are different from one another. While the changes may only be slight, it is important to note that they were still detected. The left rod has an almost identical shape as before. This is a good sign as nothing had happened that would change the refractive index across the glass rod. The peak-to-valley of the left rod is much lower this time. This can be attributed to the fact that the refractive index of the glycerol most likely changed between the two experiments. In the second experiment, after annealing, the rods seem to be better index matched than before. We see less variation at the edge of the rod. There is still more than near the center due to the pixel limitation, but the error due to the slope discontinuity has gotten lower. While the refractive index is quite constant throughout the center of the rod, it still has a variation towards the edge of the rod. The middle rod that was annealed quickly shows a large change from the initial map. The edges of the rod now have a different refractive index from each other on the same rod. This is likely attributed to how it was cooled quickly in the oven. The oven door was opened, and the top was propped open. From convection, it is possible that even the left and right sides were cooled at different paces as well as inside and outside of the rod. In the future, it would be smarter to place the rod such that an end face is closest to the door that way this might not happen again. The left rod, that had its annealing time controlled, has the best refractive index difference seen yet. The edges of the rod still have a slight change to them; however, the error associated with edges of the rod is much lower than before which is to be expected with the slower cooling time. The error associated with the pixel limitation is still there, but the slope discontinuity error has decreased.

3. Conclusions

3.1 Summary

This thesis has outlined the process of a new technique that will allow for the measurement of inhomogeneity of refractive index across a piece of glass of known shape. A bowl with a mirror at the bottom that is filled with an index matching fluid is used as the unit under test in a deflectometry measurement. Two measurements are taken where the only difference is the addition of a piece of glass into the liquid on top of the mirror. The goal is to calculate the refractive index variations across the piece of glass. The optical path length is the same everywhere in the image except where the glass was inserted. Since the index of the liquid and the glass are very similar, the only optical path difference comes from the difference in the refractive index. This knowledge of the optical path difference allows for the difference in refractive index between the glass and liquid to be calculated from the sag that was measured. This calculation does not require either of the refractive index values to be known precisely but performs better when the two refractive indices are quite similar. The inhomogeneity can then be determined as the difference in refractive index between liquid and glass changes across the sample region. The theory and experimental process were confirmed when a refractive index variation was measured across three identical pieces of glass. Two of the pieces underwent a different annealing processes to change the refractive index across the glass while the third was held constant as a control. Measuring the three glass pieces again and seeing a change consistent with how each piece was annealed verified the theory.

3.2 Usefulness of Proposed System

This system allows for accurate measurements of the difference in refractive index down to the fifth decimal place. Current interferometric systems are limited to small regions of a window of glass. This new system can measure much larger areas of glass, for example, three 2 square inch areas examined after an 8-inch diameter circle was measured. However, there is no reason to assume a larger area cannot be measured across. This system can also measure across a piece of glass with a known amount of curvature such that the distance through the glass can be known for each point. It is important to note that the system works at its best when it is measuring smooth changes in index. This is demonstrated by the low degree of error found when measuring inside the rod. It begins to struggle at the boundaries of the rod. This is a principle of deflectometry where boundaries are harder to measure. In this case, if the refractive index difference between the glycerol and the rod was too high, a slope discontinuity would appear which would cause an error in the Southwell integration. This system will work at its best if a large, in area, piece of glass is to be measure such that a sufficient portion of the glass can be measured without the interference of the boundary. This will get rid of the issues from boundary discontinuities and provide a much more accurate measurement.

3.3 Future Work

There were a lot of environmental factors that could be controlled better to improve the accuracy of tests run over time. When using an index matching fluid, the dilution percentage is a key factor in the overall refractive index. For the case of this experiment, glycerol was used because its refractive index is very close to the refractive index of BOROFLOAT33. This system works at its best when the general refractive index of both the liquid and the glass are known and quite close to each. Looking at the difference maps allows for the homogeneity of the refractive index of the glass to be seen. As previously mentioned, glycerol collects the moisture

in the air. This gradually lowers the refractive index over time. Because of this lowering, the magnitude of the refractive index difference calculated becomes larger. The inhomogeneity across the glass does not change but having better control over the dilution would allow for a more reliable comparison between different measurements. This issue could be solved by performing the experiment in a sealed environment with low humidity or by using a different index matching fluid. The temperature of the liquid is another large factor in determining the refractive index. It was found that the temperature of the liquid could vary as much as two degrees Celsius over the course of an experiment and almost five degrees Celsius between experiments run on different days. Adding a temperature control to the system would allow for more accurate control of the liquid refractive index.

Another aspect of the experiment that needs to be looked into is the largest possible refractive index that can be measured the yields reliable results. This experiment relies on the assumption that the only difference between the two measurements is the refractive index of the optic placed in the liquid. So the fringes are very slightly deformed by the index difference but still measure the same area on the mirror. If the refractive index difference between the optic and the liquid, refraction will occur at the surface of the optic. This will cause the fringes to measure a different location on the mirror. Once the two maps are subtracted from one another, they will no longer be comparing the same part of the mirror. They will be showing the difference in sag between the two locations, no longer just the refractive index. Some work in determining the largest possible index difference would be a great step to show the ranges of refractive index difference that can be measured using this technique.

Continued testing with other shapes of glass would provide more information in how well this experiment works. So far, only a rod of constant radius was measured reliably. There is no reason to believe that the experiment will no longer work, but it is always a good idea to verify. It is important that the distance through the optic be known for all points, as it is used in equation 9. As long as the thickness of the optic can be known for all points, any shape should be able to

be measured. A flat microscope slide was previously measured previously and there did not seem to be any trouble with the shape, but the refractive index was too large to obtain reasonable results.

4. References

[1] C. Ai and J. C. Wyant, "Measurement of the inhomogeneity of a window," *Optical Engineering 30(9),* 1991.

[2] M. Knauer, J. Kaminski and G. Hausler, "Phase Measuring Deflectometry: a New Approach to Measure Specular Free-Form Surfaces," *SPIE 5457,* pp. 366-376, 2004.

[3] D. Kim, T. Su, P. Su, C. Oh, L. Graves and J. Burge, "Accurate and Rapid IR Metrology for the Manufacture of Freeform Optics," *SPIE,* 06 July 2015.

[4] P. Su, R. Parks, L. Wang, R. Angel and J. Burge, "Software Configurable Optical Test System: a Computerized Reverse Hartmann Test," *Applied Optics,* vol. 49, no. 23, pp. 4494-4412, 2010.

[5] J. Choi, D. Ryu, S. Kim, L. Graves, P. Su, R. Huang and D. Kim, "Integrated Ray Testing (IRT) Simulation of SCOTS Surfae Measurement of GMT Fast Steering Mirror Prototype," *Optical Manufacturing and Testing XI,* 2015.

[6] I. Trumper, H. Choi and D. W. Kim, "Instantaneous Phase Shifting Deflectometry," *Optical Express 24,* pp. 27993-28007, 2016.

[7] W. Southwell, "Wave-front Estimation From Wave-front Slope Measurements," *J. Opt. Soc. Am.,* vol. 70, pp. 998-1006, 1980.

[8] R. Berenguel, "Gemstones and Refraction Liquids," International Gem Society, [Online]. Available: gemsociety.org/articale/gemstones-refraction-liquids/. [Accessed 22 June 2022].

[9] R. Berenguel, "Refractive Index List of Common Household Liquids," International Gem SOciety, [Online]. Available: gemsociety.org/article/refractive-index-list-of-common-household-liquids/. [Accessed 22 June 2022].

[10] M. Hubert, *IMI-NFG Course on Processing in Glass Lecture 9: Annealing and Tempering,* Eindhoven: International Materials Institute for New Functionality in Glass, pp. 20-31, 2015.

[11] J. Rheims, J. Koser and T. Wriedt, "Refractive-Index Measurements in the Near-IR using Abbe Refractometer," *Meas. Sci. Technol.,* no. 8, pp. 601-605, 1997.

[12] *Datasheet "BOROFLOAT® 33 glass - Optical Properties",* SCHOTT North America Inc, 2014.

[13] L. Hoyt, "New Table of the Refractive Index of Pure Glycerol at 20C," *Ind. Eng. Chem.,* no. 26, pp. 329-332, 01 March, 1934.

[14] M. Wernke, "Glycerol," in *Encyclopedia of Toxicology (Third Edition)*, Tallahassee, Elsevier Inc., pp. 754-756, 2014.