Turing Patterns (and Their Control) in a Coherent Quantum Fluid

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Turing's original Turing pattern (1952)

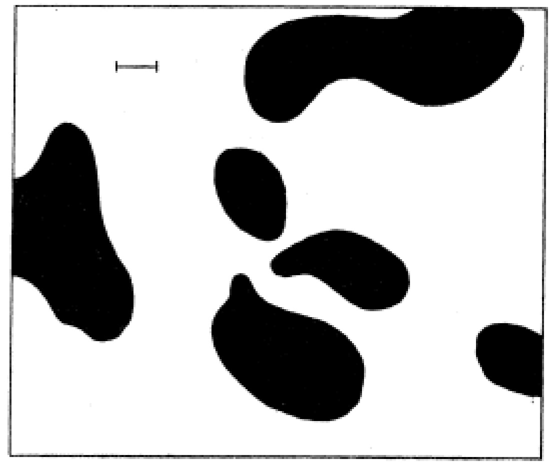


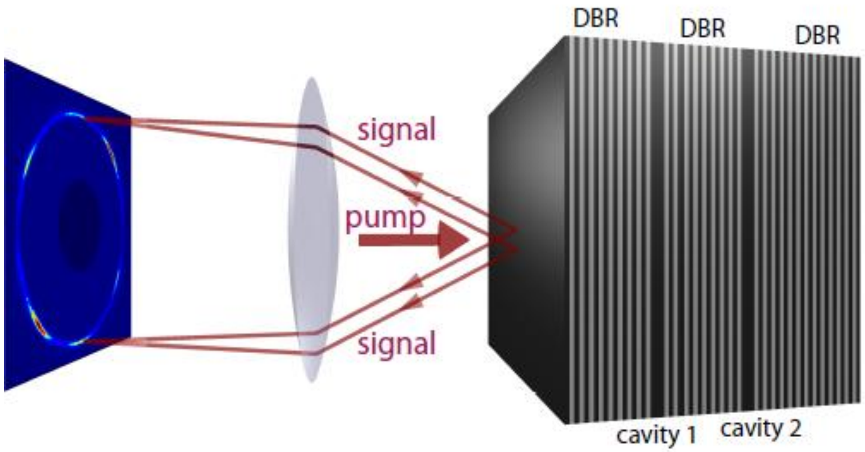
FIGURE 2. An example of a 'dappled' pattern as resulting from a type (*a*) morphogen system.

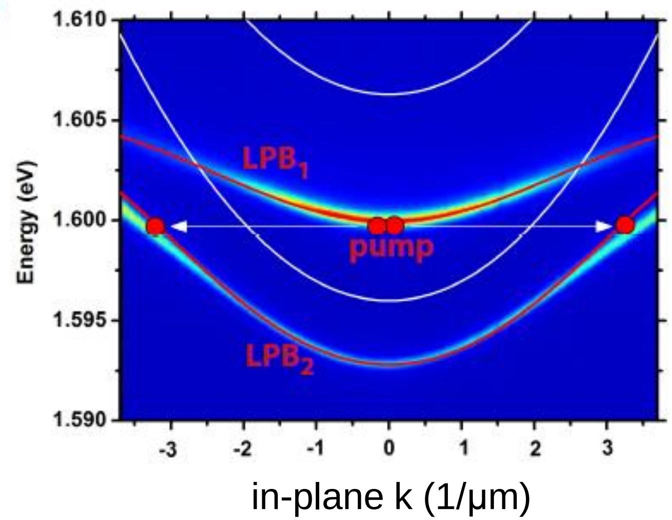
two dimensions. Figure 2 shows such a pattern, obtained in a few hours by a manual computation.

infinite in a finite time. This phenomenon may be called 'catastrophic instability'. In the case of two-dimensional systems catastrophic instability is almost universal, and

Turing, *The Chemical Basis of Morphogenesis*, Phil. Trans. R. Soc. Lond. B 237, 37 (1952)
Thom, *Structural Stability and Morphogenis* (1975)

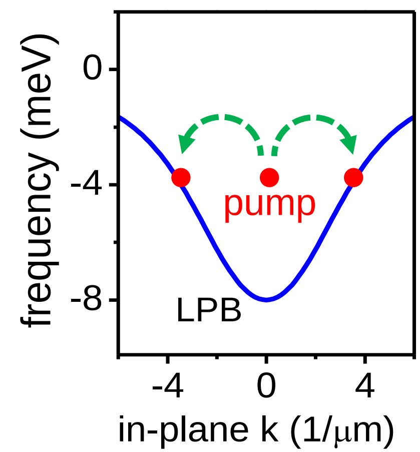
Experiment: double-cavity (only for practical, not fundamental reasons; pump scattering enhanced in double cavity)





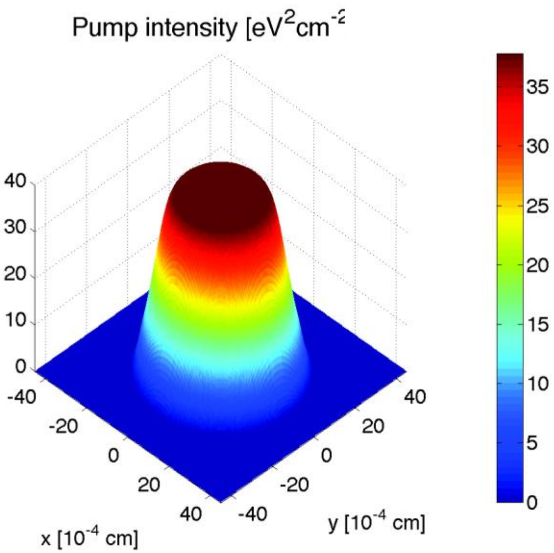
Theory:

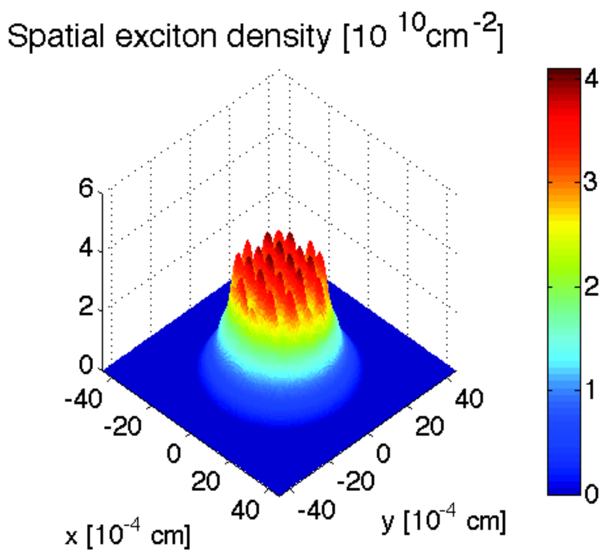
- ❑ Full real-space simulation of double cavity
- ❑ Various simplified models for single cavity



Upper figure from: Ardizzone *et al*., Scientific Reports 3, 3016 (2013)

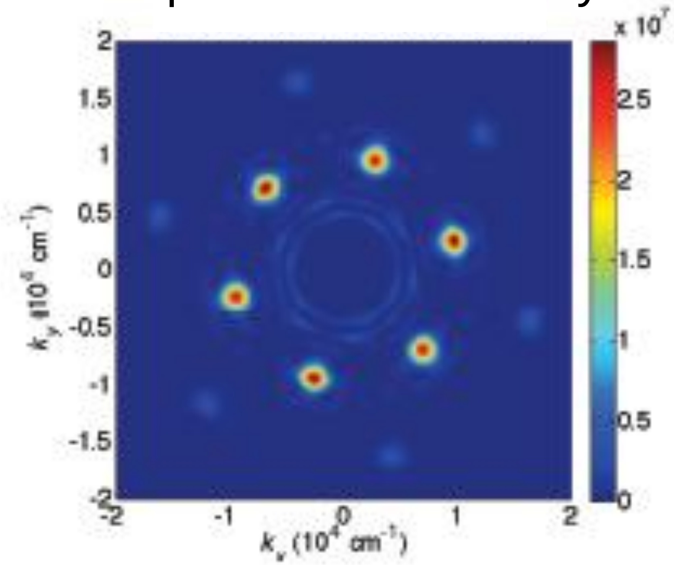
Numerical solution (full 2D real space calculation)





hexagonal "lattice" in real space
is 6-spot hexagon in k-space

k-space exciton density



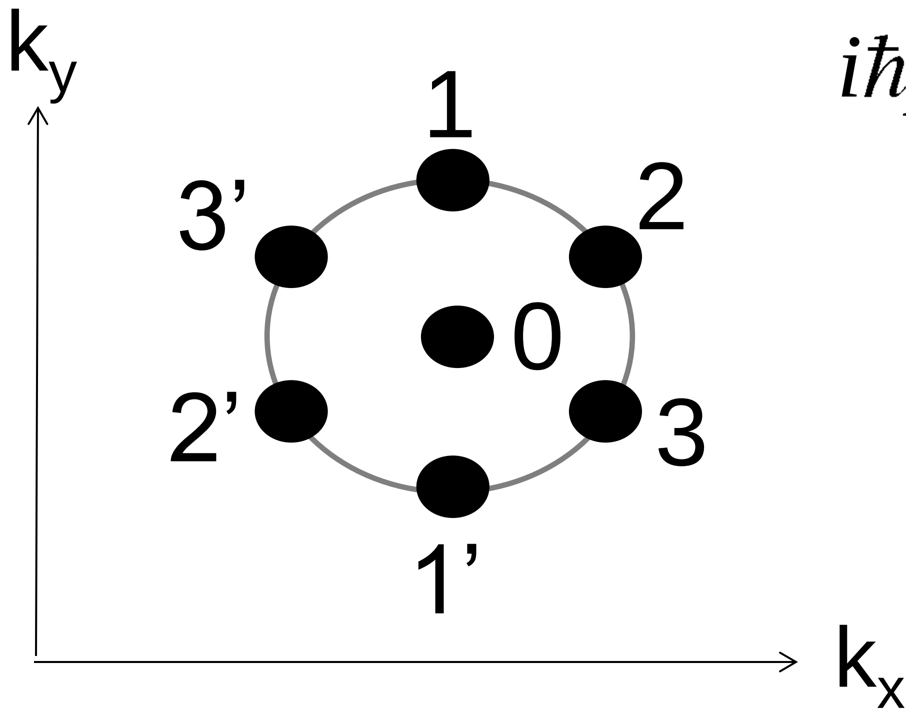
<u>Figures from:</u> Luk,Tse, Kwong, Leung, Lewandowski, Binder, Schumacher, Phys. Rev. B **87**, 205307 (2013)

Origin of patterns: scattering due to four-wave mixing

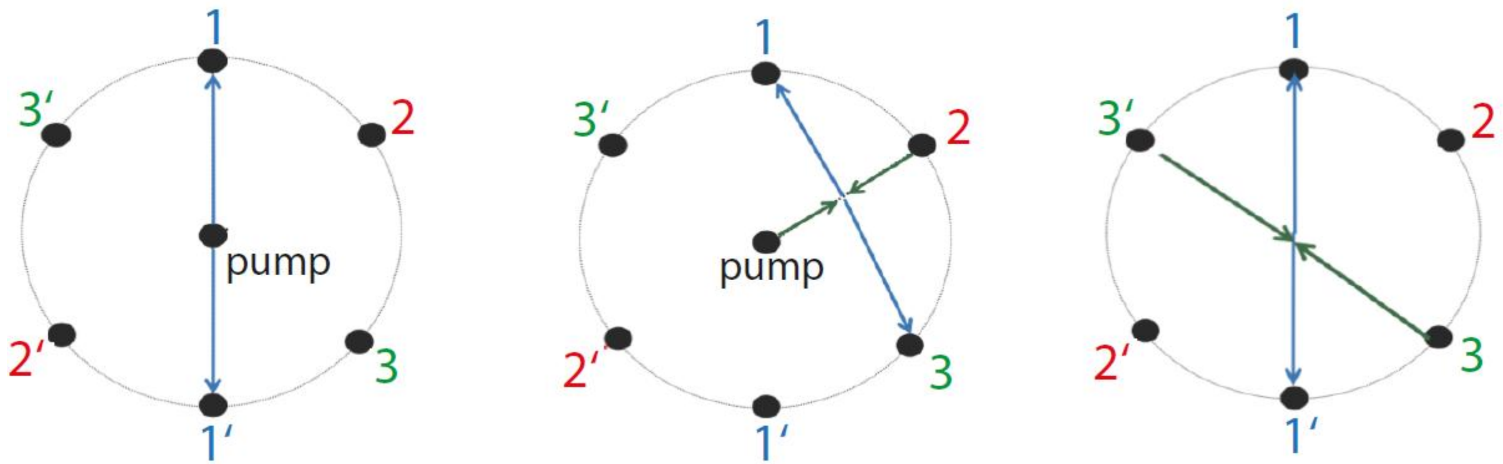
$$i\hbar\dot{p}(\boldsymbol{r}) = \cdots V_{HF}\,|\,p(\boldsymbol{r})\,|^2\;p(\boldsymbol{r})\cdots$$

HF yields four-wave mixing

<u>Fourier transform (k-space):</u>



$$i\hbar\dot{p}_{\boldsymbol{k}} = \cdots V_{HF}\,p^*_{\boldsymbol{k}'}\,p_{\boldsymbol{k}''}\,p_{\boldsymbol{k}'''}\cdots$$



linear terms
basic instability,
no directional preference
(ring)

strong cross saturation

one two-spot pattern
survives

quadratic terms
favor hexagon,
no directional preference
(ring)

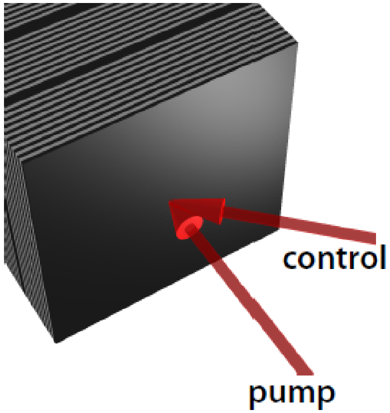
strong cross saturation

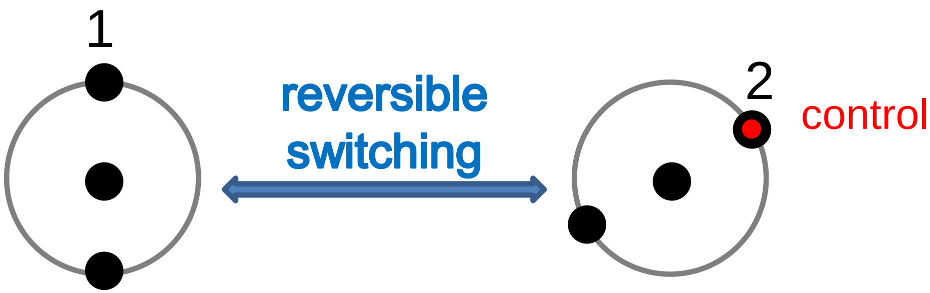
one hexagon
survives

cubic term,
incl. self-saturation and
cross-saturation
("winner takes all")

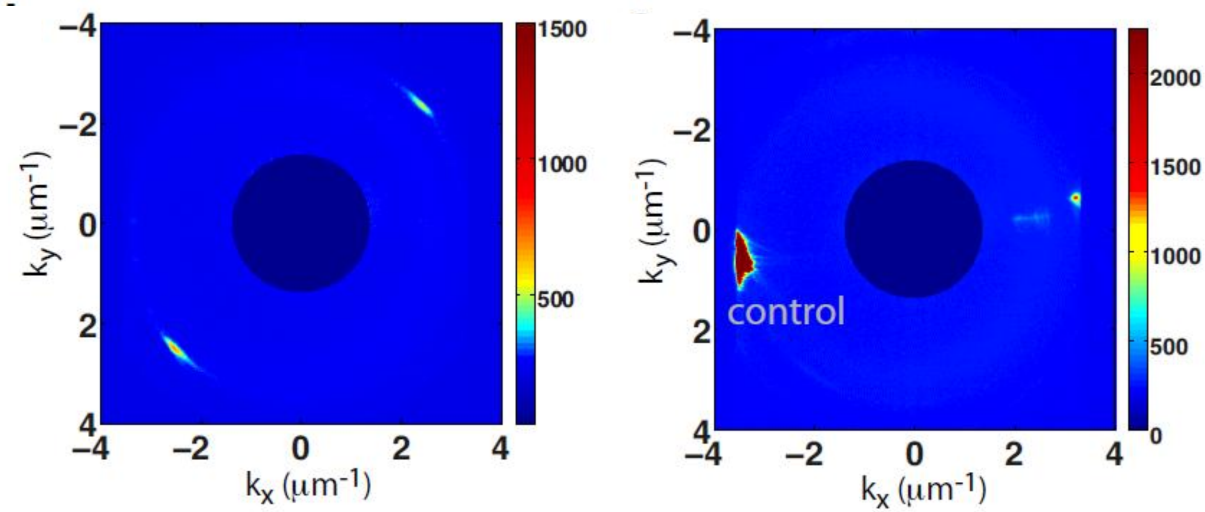
Luk,Tse, Kwong, Leung, Lewandowski, Binder, Schumacher, Phys. Rev. B **87**, 205307 (2013)

Reversible directional switching





Experiment (pump light blocked for clarity):



<u>Figures from</u>: Ardizzone *et al.*, Scientific Reports 3, 3016 (2013)
<u>Also compare</u>: Dawes, Illing, Clark, Gauthier, Science 308, 672 (2005)
Schumacher, Kwong, Binder, Smirl, Phys. Stat. Sol. (RRL) **3**, 10 (2009)